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The nature and applications of an irradiation ionization front in cataclysmic variables

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Abstract. In order to explain the photometric oscillations of several minutes observed in CVs, A.R.King proposed a model based on the nature of IF on the irradiated surface of the secondary in 1989. Having carefully studied King's theory, we propose a revised version of King's theory in this paper and get some results quite different from King's. The upper limit of oscillation period is found to be much larger than $160P_{hr}$ and it can vary in a wide scope in real cases in stead of being strictly at $160P_{hr}$. And our further calculations rule out the application of this mechanism to any disk-feed system including dwarf novae in outbursts.

Key words: accretion, accretion disks – novae, cataclysmic variables – stars: atmospheres

1. Introduction

Oscillations and flickerings are common features in the photometry observations of cataclysmic variables (CVs, e.g. Patterson 1981). A lot of theories have been proposed to explain various of oscillations and there are many possible origins for them, such as the disk, boundary layer, the compact object or the secondary etc.

A.R.King(1989) had discussed the motions of the ionization front (IF), which is formed by irradiation, in the extended atmosphere of the secondary , and proposed a model to explain the oscillations of several minutes in CVs. King's model is based on the nature of the IF which controls the mass overflow rate just as a piston. The physical picture of King's theory is the following: Because the flow between the IF and L_1 point is sonic, the IF must be D-critical, which means the density ahead of it should be ρ_D (Shu 1992). In the compression phase, IF moves inwards the interior of the secondary with a isothermal shock ahead to compress the density between the IF and shock to ρ_D. That inwards motion reduces the $\dot{M}_{ov}$ greatly. When the star density ahead of the shock reaches ρ_D, the IF then moves out with a rarefied wave and $\dot{M}_{ov}$ is enlarged by $1 + V_{out}/c_2$, until the IF reaches L_1 and the new circle begins. King's result implies a strict relation between the oscillation period and the orbital period: $P_{osc} \sim H_*/c_* = 160P_{hr}$.

The physical picture of King's model is quite clear. But King's derivation of his equation A10 which describes the motion of the shock is not correct: The integration $\int e^{z^2} dz$ can not lead to an erf function. In fact, in equation A10, he mistook the sign for the power index, which resulted in an incorrect convergent integration. Because of the incorrect equation A10, King did not emphasize the difference between the motions of the IF and the shock, which is proved in our following analysis to be very important. Furthermore, King did not consider what would happen if the irradiation from the primary is not strong enough to push the IF to a large depth below L_1. In Sect.2, we propose a revised version of King's model and get some results quite different from King's in Sect.4. In Sect.3, we investigate to what extent the modulation on $\dot{M}_{ov}$ can affect the photometric property of a disk-feed system including dwarf nova in outburst.

2. Motion of the ionization front

2.1. Model

The atmosphere near L_1 is divided into 3 regions as shown in Fig. 1. ρ_D and v_D are the D-critical density and velocity. The detailed discussion about the IF was given by Shu (1992).

The following conditions are satisfied:

1. The IF (ionization front) is ionization driven rather than pressure driven. As we will discuss in Sect.2.2, even if $n_i > \dot{M}_{ov}/Q\mu_e m_H$ (the condition given by King is satisfied), the ionization driven IF can not reach a very large depth because the ionizing photons from the hot component have a maximum penetration depth into the secondary's extended atmosphere, z_{max}. For simplicity and clearness, the calculations in Sect.2.1.1 is based on the assumption that $z_{max} > z_{0s}$ (z_{0s} is defined below). If the irradiation is not so strong, the IF would reverse to move outwards at z_{max} and the calculations are similar. The time needed for the IF to reach z_{max} in the compression phase can be easily gotten from the equations we will give below and as an example,

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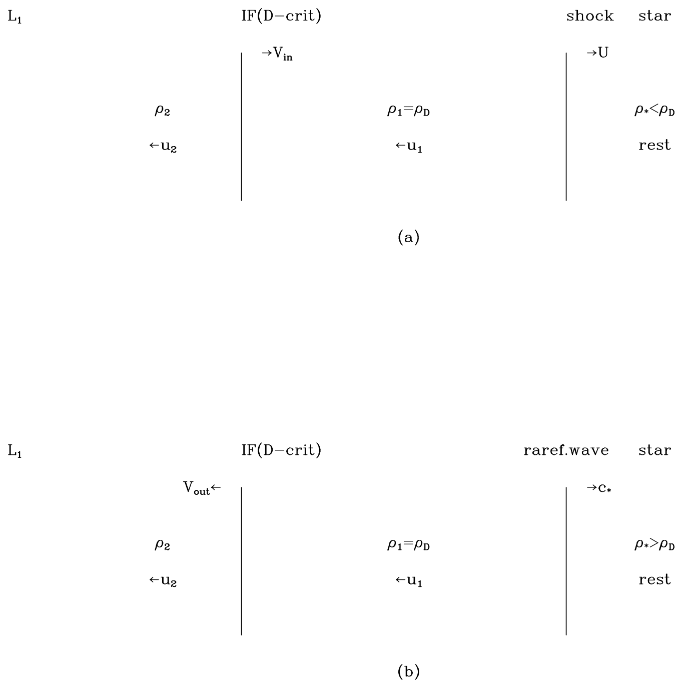

Fig. 1a and b. This figure is adapted from King (1989), which shows the symbols used in this paper. **a** is for the first period of the compression phase, while **b** is for the expansion phase. All velocities are measured in the rest frame of the star.

can be directly read from the X-axis of Fig. 3 by specifying Y value as z_{max}.

2. The mass density behind IF which is ρ_2, almost remains constant (=ρ_{L1}, except for a short period at the beginning of the expansion in some cases, as will be mentioned in Sect.2.1.2). The reason is that in region 2, $T_2 \sim 100T_*$ which means the scaleheight $H_2 \sim 10H_*$ (Hameury et al. 1986) and the maxium depth the IF can penetrate is much less than $10H_*$, which will be proved below.

2.1.1. The compression phase

Under the above conditions, the compression phase can be further divided into two periods.

(a). At first, the D-critical IF moves into the secondary with a isothermal shock in front of it and the mass overflow rate is reduced. The IF controls the shock in order to keep the density in the region between the IF and the shock as ρ_D. From the equation describing the motion of the shock

$$U^2 = \frac{\rho_D}{\rho_*} c_*^2, \tag{1}$$

and the matter distribution ahead of the shock, which is near L_1,

$$\rho_* = \rho_{L1} e^{(z/H_*)^2}, \tag{2}$$

we get the time when the shock reaches depth z

$$t(z) = \int_0^z \frac{dz}{U} = \left(\frac{\rho_{L1}}{\rho_D}\right)^{1/2} \cdot \frac{H_*}{c_*} \cdot f_1\left(\frac{z}{H_*}\right) \tag{3}$$

where

$$f_1(x) = \int_0^x e^{\frac{1}{2}x^2} dx. \tag{4}$$

The result here is different from King's(1989). In this period, the velocity of IF is

$$V_{in} = v_D + U - \frac{c_*^2}{U}, \tag{5}$$

which is shown by King(1989). The motion of IF can be solved as

$$t(z) = \int_0^z \frac{dz}{V_{in}} \tag{6}$$

This period will end when the D-critical condition is satisfied ahead of the shock at

$$t_0 = \left(\frac{\rho_{L1}}{\rho_D}\right)^{\frac{1}{2}} \cdot f_1\left(\left(\ln\frac{\rho_D}{\rho_{L1}}\right)^{\frac{1}{2}}\right) \cdot \frac{H_*}{c_*},$$

$$z_{0s} = H_* \cdot \left(\ln\frac{\rho_D}{\rho_{L1}}\right)^{\frac{1}{2}} \tag{7}$$

At that time, by solving Eq.(6), the position of the IF can be gotten as z_{0IF}. Under the assumption of typical parameters: $T_2 = 100T_*$ $c_2 = 10c_*$ $v_D = 0.0501c_*$ and $\rho_D = 199.6\rho_{L1}$, the motions of shock and IF are plotted in Fig. 2.

(b). In the second period, the IF moves from z_{0IF} to z_{0s} with a velocity v_D. In fact, the shock vanished at the end of period (a) leaving the region between z_{0s} and z_{0IF} a uniform density ρ_D. Because the IF dominates the density ahead of it, the density of the region between the IF and z_{0s} will remain constant ρ_D before the IF reaches z_{0s}. And there is no shock leading the matter into motion now, so in the whole period (b), the matter ahead of the IF can be considered as being rest with a density ρ_D. The time spent in this period is

$$t_1 = \frac{z_{0s} - z_{0IF}}{v_D} \tag{8}$$

After IF reaches z_{0s}, it can not move in any further to make the jump condition from D-critical to strong D-type which would violate the sonic-flow condition in the region between L_1 and the IF. If the irradiation is strong enough, the IF reverses at z_{0s}. Or, if it is not so strong, the IF reverses at z_{max} as discussed above.

2.1.2. The expansion phase and the total time scale

King discussed the expansion phase on condition of $z_{max} > z_{0s}$. If $z_{max} < z_{0s}$, the IF stops before the density ahead of its corresponding shock reaches ρ_D. When it stops, the matter between the IF and L_1 is still flowing out with no matter entering this region through the IF. Therefore the density in a layer just behind the IF is rarefied quickly and the condition (2) is not

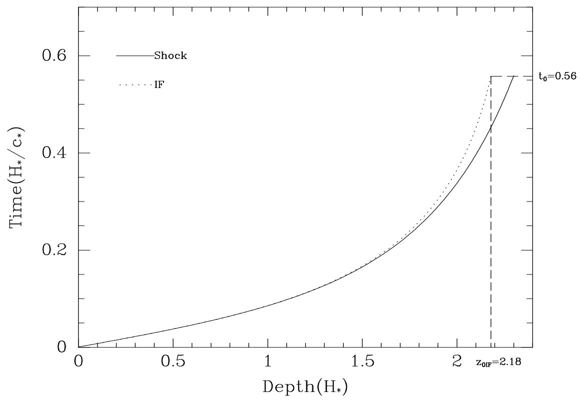

Fig. 2. The motions of the shock and the ionization front in the first period of the compression phase under the condition of $T_2 = 100T_*$ are shown in this figure. That period stops when the shock reaches the depth $(\ln(\rho_D/\rho_{L1}))^{1/2}H_* = 2.3H_*$. The X axis is the depth measured in units of H_* and the Y axis is the time in H_*/c_*.

satisfied now. When ρ_2 is so rarefied that the current D-critical condition is satisfied:

$$\frac{\rho_{L1}e^{(z_{max}/H_*)^2}}{\rho_2} = \frac{c_2}{v_D}, \tag{9}$$

the IF moves outwards by generating a rarefied wave just as what happens in the case of $z_{max} > z_{0s}$, which had been discussed by King (1989).

According to King, we get the velocity of the IF in the expansion phase

$$V_{out} = 3c_* \left[1 - \left(\frac{\rho_D}{\rho_*}\right)^{\frac{1}{3}}\right] - v_D, \tag{10}$$

where

$$\rho_* = \rho_D \cdot e^{(z^2 - z_{0s}^2)/H_*^2}. \tag{11}$$

Please note here that the above equation is also different from King's Eq.(A18). Because the velocity in expansion phase changes smoothly from 0 to $3c_*$, the case here is much simpler than that in the compression phase. We define an average velocity to treat the motion of the IF in expansion phase

$$V_{out} = \beta c_*. \tag{12}$$

Subsituting $z = z_{0s} + c_* t$ and $t \sim z_{0s}/V_{out}$ to Eq.(11), we get

$$\beta c_* = 3c_* \left(1 - e^{\frac{1}{3}(-2z_{0s}^2/\beta - z_{0s}^2/\beta^2)/H_*^2}\right) - v_D. \tag{13}$$

If $z_{0s} = 2.3H_*$, we get $\beta = 2.45$. Then the time spent in expansion phase is

$$t_{out} = \frac{\min(z_{0s}, z_{max})}{2.45c_*}. \tag{14}$$

When the IF reaches L_1, its control on density vanishes, IF and a new cycle begins (King 1989).

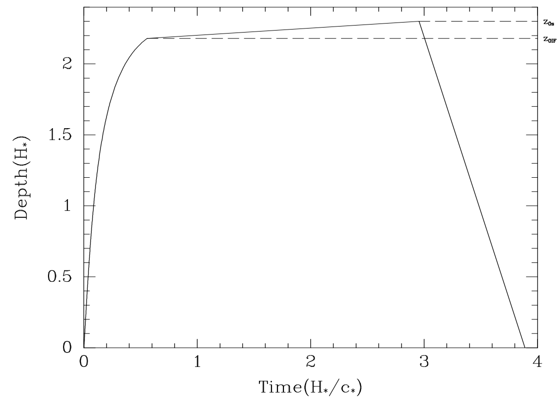

Fig. 3. The motion of the IF in a circle, when $z_{max} > z_{0s}$.

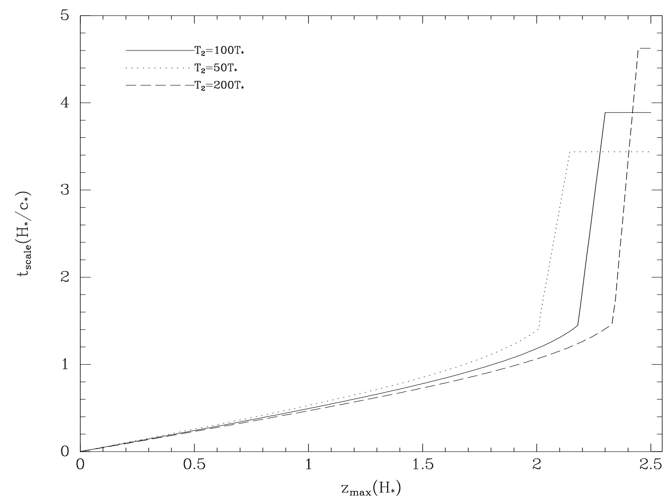

Fig. 4. This figure shows the oscillation time scale as a function of the maximum penetration depth under different temperature parameters. If z_{max} lies between z_{0s} and z_{0IF}, t_{scale} is very sensitive to z_{max}. If $z_{max} > z_{0s}$, such time scale does not vary with z_{max}.

Because in our discussion the time scales both in compression and expansion depend on z_{max} ,if $z_{max} < z_{0s}$, it is apparent that the total time scale t_{scale}, which is the sum of the time spent in a circle and can be mentioned as the oscillation time scale, is sensitive to z_{max}. But z_{max} is decided by the physical conditions in the system. The main difficulty in determining the oscillation time scale t_{scale} is the estimation of z_{max}. We discuss the cases for $z_{max} > z_{0s}$ $z_{0IF} < z_{max} < z_{0s}$ and $z_{max} < z_{0IF}$, and show the results in Fig. 3 and Fig. 4.

Fig. 3 shows the motion of the IF and it shows a long period is been spent in compression phase(b) from z_{0IF} to z_{0s}. Fig. 4 shows the relation of the maxium penestration depth and the total time scale. The sharp jump means that if z_{max} lies between z_{0IF} and z_{0s}, the total time scale will be extremly sensitive to z_{max}. It is reasonable because the IF moves very slowly in that region as shown in Fig. 3 and Eq.(8).

The variation of mass transfer rate in a cycle for a $z_{max} < z_{0s}$ system is complex. In the beginning of the expansion phase,

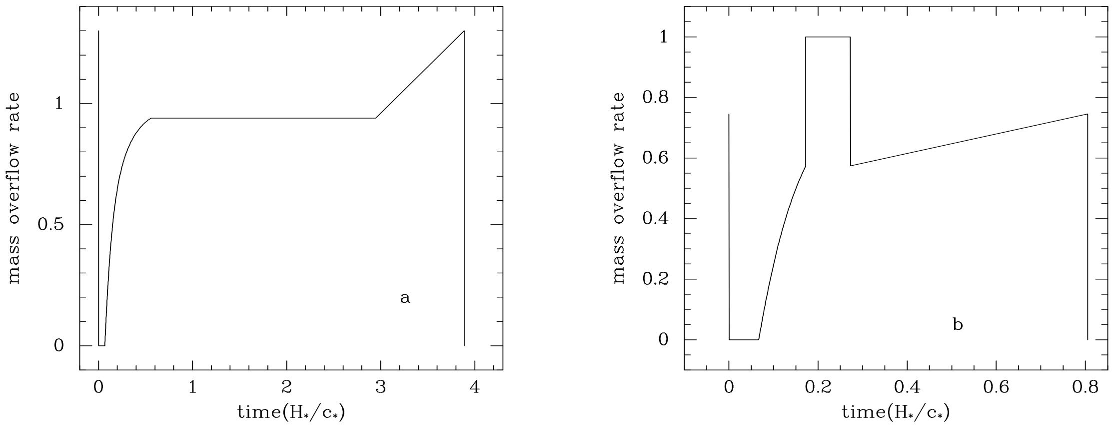

Fig. 5a and b. Approximate form of the overflow rate $\dot{M}_{ov}$ as a function of time. **a** is for the case of $z_{max} > z_{0s}$. **b** is for $z_{max} = 0.66z_{0s}$.

the rarefied layer (ρ_2) is expanding to L_1. Before it reaches L_1, the overflow matter has a density ρ_{L1} and after it reaches L_1 the density is reduced to ρ_2. According to Eq.(16) below, $\dot{M}_{ov}$ is determined by the overflow matter density. Therefore $\dot{M}_{ov}$ is high for a short time about $H_*/c_2 = 0.1H_*/c_*$, and then is reduced to $\exp((z_{max}^2 - z_{0s}^2)/H_*^2) = \rho_2/\rho_{L1}$. The above process is illustrated in Fig. 5b.

2.2. Estimation of z_{max}

If the King's condition is satisfied $n_i > \dot{M}_{ov}/Q\mu_e m_H$, the maximum penetration depth is determined by the recombinations along the path of the irradiation flux. The basis of the following calculations is that the photosphere of the secondary can not reach the Roche lobe (Ritter 1988). We define P_{hr} as the binary period given by hours, T_2 as the temperature of the ionized gas, M_1 and R_1 as the mass and radius of the compact object, R_L as the radius of the Roche lobe, η as the efficiency defined by King and $\dot{M}$ as the mean mass transfer rate.

The stream's cross-sectional area is

$$Q = 1.9 \times 10^{17} \left(\frac{T}{10^4}\right) P_{hr}^2 \quad (cm^2). \qquad (15)$$

Then from the mass transfer rate

$$\dot{M} = Qc_2\rho_{L1} \quad (g/s), \qquad (16)$$

we can deduce the number density

$$N = \rho_{L1}/m_H \qquad (17)$$

$$= 1.57 \times 10^7 \times \dot{M} P_{hr}^{-2} T^{-\frac{3}{2}} \quad (cm^{-3}) \qquad (18)$$

The ionizing photon density near L_1 can be derived as following:

$$n_i = \eta \cdot \frac{L_{acc}}{4\pi R_L^2 < h\nu >} \qquad (19)$$

$$= 3.32 \times 10^3 \times \eta \frac{M_1 \cdot \dot{M}}{R_1 \cdot R_L^2} \quad (cm^{-3}) \qquad (20)$$

Then the maximum penetration depth can be gotten from the Stromgren-type relation

$$z_{max} = 192.42 \times \eta \cdot M_1 P_{hr}^4 T^3 R_1^{-1} R_L^{-2} \dot{M}^{-1} \quad (cm) \qquad (21)$$

where the recombination efficiency α is adapted as $7 \times 10^{-14} cm^3/s$ (Tucker 1975).

For a number of typical parameters, we get $z_{max} \leq z_{0IF}$. For example, if we chose $P_{hr} = 3$, $T_2 = 3 * 10^5 K$, $M_1 = 10^{33} g$, $R_1 = 10^9 cm$, $R_L = 10^{10} cm$, $\eta = 1$ and $\dot{M} = 10^{16} g/s$, we can get $z_{max} = 1.43H_*$ and the time scale is $0.63H_*/c_*$. In a specific system, the change of the mass transfer rate can cause the oscillation period vary with time.

For two typical z_{max}, the $\dot{M}_{ov}$ is calculated through one cycle, and the results are given in Fig. 5. We note that in Fig. 5, $\dot{M}_{ov} = 0$ at the beginning of a cycle, which is caused by $V_{in} > c_2$ at that time.

3. Discussions in a disk-feed system

In the mechanism discussed above, the oscillation of luminosity is caused by the IF modulating the mass overflow rate like a piston. If the overflow mass is directly accreted to the compact object, as what happens in AM Her stars, the change of $\dot{M}$ has an instant effect on the luminouncy variation. In this section, we discuss the effects of such mechanism on a viscous disk.

3.1. Outbursts of dwarf novae

If the outburst of a dwarf nova is caused by disk instability, the oscillation mechanism discussed in this paper can not be applied to such outburst. In a disk outburst system, $\dot{M}_{ov} \ll \dot{M}_{acc}$, the luminosity is determined by $\dot{M}_{acc}$. Then changes of $\dot{M}_{ov}$ which is much smaller than $\dot{M}_{acc}$ can have little effect on the total luminosity.

3.2. Other disk-feed systems

In a disk-feed (or a ring plus one or two accretion columns at magnetic poles) system with $\dot{M}_{ov} \sim \dot{M}_{acc}$, if the disk shielding

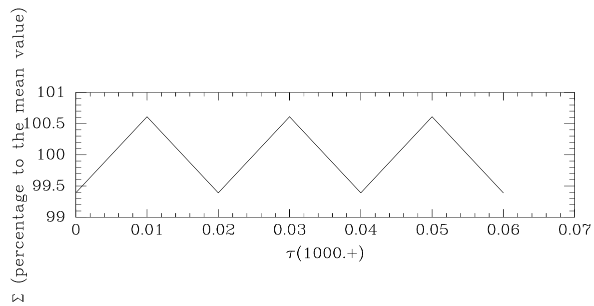

Fig. 6. The surface density fluctuation at $0.9R_{circ}$ introduced by the modulation of $\dot{M}_{ov}$ in period of about $\tau = 0.02$ when a stable oscillation have been established. X axis is $\tau - 1000.$ in which τ is the integration time we used and Y axis is the time dependent surface density in percentage to the mean value.

is not significant, the effect of the $\dot{M}_{ov}$ modulation described above on the system luminosity needs some detailed discussions. We adapt a simplified mass input function: In the first half of circle $\dot{M}_{ov} = 1$ and in the second half $\dot{M}_{ov} = 0$; The time-scale of one circle is several minutes.

The basic disk equation is

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left\{R^{1/2}\frac{\partial}{\partial R}[\nu \Sigma R^{1/2}]\right\}. \tag{22}$$

Under the α disk assumption, the viscous vary as $\nu = \nu_0 r^n$(Mineshige & Wood 1989). For a simple solution, we adapt $n = 0$ and the solution is

$$\Sigma(x,\tau) = C\int_0^\tau \frac{\dot{M}(\tau)x^{-1/4}}{\tau} e^{\left(-\frac{x^2+1}{\tau}\right)} I_{\frac{1}{4}}\left(\frac{2x}{\tau}\right) d\tau, \tag{23}$$

where

$$x = R/R_{circ}, \quad \tau = 12\nu t R_{circ}^{-2} \quad C = 1/(12\nu_0\pi). \tag{24}$$

R_{circ} is defined as circular radius at which the mass initially carrying angular momentum forms a Keplarian orbit. Here the mass input function is considered and the starting time is taken as $\tau = 0$. For reasonable parameters $\nu_0 = \alpha c_s H$, $\tau = 1$ correspond to $t \geq 2 \times 10^4 s$. We take the $\dot{M}_{ov}$ variation period as $\tau_p = 0.02$ and the mass input function as

$$\dot{M}(\tau) = 1. \qquad k\tau_p < \tau < k\tau_p + \tau_p/2 \tag{25}$$

$$\dot{M}(\tau) = 0. \qquad k\tau_p + \tau_p/2 < \tau < (k+1)\tau_p, \tag{26}$$

where k is an integral. Fig. 6 shows the relative density fluctation at $x = 0.9$. Such fluctuation will decrease with the deviation of x from unity.

It is clear that such small density change causes little change on the disk's luminosity. The mass overflow rate modulated by IF will not have any photometrically observable effect in a disk-feed system. The disk masks such rapid variation of $\dot{M}_{ov}$ effectively.

4. Conclusions

In our work above, we revise King's model and test the observational effects that mechanism introduces. We get three main conclusions:

1. From the correct equation we derived for the motion of the IF, we find that the difference between the motions of the IF and the shock is important. Therefore we calculate the time the IF spend on moving from z_{0IF} to z_{0s} and show it is quite long. That result gives an upper limit of oscillation time scale which is much longer than that given by King.
2. After introducing the parameter z_{max}, the oscillation time scale in real cases can range in a wide scope, which is quite different from King's result of being strictly at $160P_{hr}$. That enables us to explain more kinds of oscillation in different time scales using this model.
3. This model can not be applied to any disk-feed system including dwarf nova in outburst, because the viscous disk masks any short period modulations on $\dot{M}_{ov}$.

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