

# A comparison of synthetic and observed spectra for G-K dwarfs using artificial neural networks<sup>★</sup>

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**Abstract.** A library of synthetic spectra, based on Kurucz model atmospheres, has been used to compare the spectroscopic observations in the wavelength range 4850–5384 Å by using statistical and supervised artificial neural network methods. The effective temperatures assigned by these methods for G-K dwarfs are compared with those given in Gray and Corbally (1994) and found to be closely matching their calibration curve. This result provides a promising new technique for determination of fundamental stellar atmospheric parameters on the basis of comparison between the model generated synthetic spectra and observed stellar spectra.

**Key words:** stars: general; atmospheres; fundamental parameters – methods: statistical

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## 1. Introduction

A programme to determine observational properties of stars from their spectra using the statistical ( $\chi^2$  minimisation) and supervised artificial neural network (ANN) methods has been initiated by our group with the aim of understanding stellar populations in an objective manner. This programme originated from the project of determining the stellar spectral classification of optical library, where Gulati et al. (1994) (hereafter referred to as G94a) have classified Jacoby et al. (1984) library in terms of Silva and Cornell (1992) library. The project has become possible through the availability of digitised optical spectra in the data archives. Later, these methods have been applied to a homogeneous set of ultraviolet spectra of the IUE low dispersion catalogue (Gulati et al. 1994 (hereafter referred to as G94b) and Gulati et al., 1996 (hereafter referred to as G96)). The performance of these methods has shown that with limited set of data, the classification can be replicated to an accuracy of 1–2 subclasses when compared to the respective catalogue classes.

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<sup>★</sup> Based on observations using Vainu Bappu Telescope, VBO, Kavalur, India.

To understand the physical phenomenon in stars, it is essential to tie-up the observed spectra to synthetic spectra based on realistic stellar atmospheric models. With the aim of determining physical properties in an objective manner, we have extended these methods to G-K dwarfs spectra. This approach is similar to the one adopted by Gray and Corbally (1994) (hereafter referred to as GC94), for the calibration of MK classes using synthetic spectra. However these approaches differ on two aspects: firstly the GC94 makes use of classical approach to compare library of synthetic spectra in terms of MK standards, whereas we use computer based methods to assign temperature classes from synthetic spectra to the observed spectra; secondly the GC94 works in the blue end of the optical spectra, while we use the yellow part of the optical spectra, where one finds more spectral features which are diagnostic of the physical properties for cool stars.

Although the automated classification methods described above find their efficient utilisation in determining the properties of bulk data sets; as a first ever attempt we have applied these methods to a small number of dwarf stars so as to compare the calibration and the distribution of semi-empirical properties with GC94 calibration scale.

The paper is divided into the following sections:

In Sect. 2, the observational data of programme stars with the library of synthetic spectra are described. The application of the methods and results are discussed in Sect. 3. Section 4 concludes the paper and lists out various future possibilities of application of these methods in this field.

## 2. The input data

### 2.1. Spectroscopic observations

The observations were carried out in two phases totaling of four nights in 1994–95, from Vainu Bappu Observatory, Kavalur, India with the 2.3 meter Vainu Bappu Telescope. A standard Boller and Chivens spectrograph attached to the Cassegrain focus was used with a 1200 grooves/mm grating blazed at 5000 Å in first order. With a detector consisting of a liquid nitrogen cooled blue enhanced EEV P8603 CCD housed in a Oxford In-

struments MN1815 INV dewar, the effective spectral resolution obtained was 2.4 Å in the observed wavelength range of 4850-5500 Å. Stars up to 6th magnitude required integration times of about 5-6 minutes to build up good signal to noise ratios ( $S/N \sim 100$  or better). The routine steps during the observations consisted of taking bias frames, dome flats, standard star spectra and the lamp spectra (FeAr lamp). Lamp spectra were taken after every two object spectra, so as to take into account the spectrograph flexure. The effects due to the flexures were estimated to be within 0.1 Å.

## 2.2. Data selection and reduction

We had selected stars from the Cayrel et al. (1992) catalogue as this catalogue contains a compilation of various stellar atmospheric parameters viz.  $T_{\text{eff}}$ ,  $\log g$ ,  $[Fe/H]$  etc. The present analysis was restricted to the G and K type dwarfs of nearly solar chemical composition and the list of the 10 observed programme stars is given in Table 1. The basic spectral type for these stars was taken from the Cayrel et al. (1992) catalogue. IRAF version V2.10.4 DOSLIT package was used to reduce the raw data which were in the form of FITS files. All the four individual nights were treated separately for the purpose of data reductions. However, for checking the consistency of the data, two stars were observed repeatedly on different nights and the flux calibrated spectra were within 2-5 % uncertainty level. The zero and flat field frames were obtained by using median combined images of the respective nights. Wavelength calibration was accurate to within 0.1 Å and the flux calibrations of the standard stars were found reliable between the nights. All the spectra, after the DOSLIT procedure, were spline fitted to bring them on a common wavelength bin and range of 1.2 Å and 4850-5384 Å respectively. Then each spectra was normalised to unity at the individual peaks. Thus the final observed spectra for the 10 programme stars used for the present analysis were having 446 flux values in the range of 4850-5384 Å with 1.2 Å sampling and 2.4 Å spectral resolution.

## 2.3. Synthetic spectra

Gulati (1991) (G91) had built a library of synthetic stellar spectra based on Kurucz models, spanning an effective temperature range ( $T_{\text{eff}}$ ) from 4000-6000°K (in steps of 250°K), surface gravity ( $\log g$ ) from 1.5 to 4.5 (in steps of 0.5 dex) and metallicity  $[M/H]$  of -0.5, 0.0 and 0.5 dex. Even though the details of the library are given in G91, here we outline the salient features useful for readers of this paper. The library makes use of line blanketed stellar atmospheres models of Kurucz (1992) which are the same generation of models as used in GC94. These models are proved to be realistic because they reproduce most of observed photometric and spectrophotometric properties of large number of stars and are widely used for interpretation of stellar observations. Using these models G91 had computed the synthetic spectra at the computational resolution of 250,000 and all the spectra are normalised to continuum flux at each wavelength point. The quality of synthetic spectra depends upon the relia-

**Table 1.** A list of programme stars

HD	Sp. Type	$T_{\text{eff}}^1$	$T_{\text{eff}}^2$	$T_{\text{eff}}^3$	$T_{\text{eff}}^4$
32147	K3V	4755	4860	4700	4800
32923	G4V	5727	5610	6000	6000
37394	K1V	5196	5160	5400	5400
38392	K2V	4990	5010	4900	5000
52711	G4V	5860	5610	6000	6000
75732	G8V	4460	5430	5400	5300
		5196			
76151	G3V	5600	5680	5900	5800
		5727			
90508	G1V	5727	5780	6000	6000
		5929			
115617	G6V	5600	5550	5600	5700
131156	G7V	5478	5490	5600	5600
		5538			

bility of line data used as an input. Line parameters of the line data had been checked by comparing synthetic spectra for solar and Arcturus models with the high resolution observed spectra of these objects. The line parameters were adjusted for those lines which show significant difference from the observed behaviour. There are 43,921 line species involved, of which 13,403 lines are predicted by the solar models. The main sources of line opacity are Mg, MgH and Fe. Gulati et al. (1993) had used this library for investigation of consistency of synthetic spectral indices with those derived from observations by the Lick group (Faber et al., 1985). For this analysis, we have taken a subset of the synthetic spectra which are representative of G-K dwarfs with the parameters:  $T_{\text{eff}}$  4000-6000°K in steps of 100°K (after interpolating the original 250°K steps),  $\log g$  of 4.5 dex and  $[M/H]$  of 0.0 dex.

In the present study, these synthetic spectra (normalised to continuum flux at each wavelength point) which were available at 0.02 Å binning with 1.65 Å spectral resolution, were required to be brought to the common platform of wavelength bin and range as that of the observed spectra described in the previous section. This task was accomplished by performing the standard processes of trimming, spline fitting, and then convolving the available synthetic spectra with an appropriate Gaussian function (assuming that the instrument effect is close to a Gaussian) to bring them to a spectral resolution of 2.4 Å and 1.2 Å binning for a range of 4850-5384 Å. Thus, similar to the observed spectra, the synthetic spectra too were having 446 flux values. The final set consisted of 21 synthetic spectra with  $T_{\text{eff}}$  values from 4000 to 6000°K in 100°K steps.

## 3. Determination of effective temperature ( $T_{\text{eff}}$ ) using ANN

### 3.1. Methodology

We have used the supervised ANN methods and the metric distance minimisation ( $\chi^2$ ) for the present work. These methods are described in brief in the following paragraphs and more de-

tails can be found in our earlier communications (G94a, G94b, G96).

The supervised ANN consists of a minimum configuration of three layers with the first layer called as the input layer where the input patterns are read. A second layer, known as the hidden layer, is used for processing the information from the input layer. Finally a third layer i.e. output layer gives the corresponding output patterns. Each of these three layers has nodes which represent the number of points where information is stored or processed. These nodes are further interconnected by weights. The networks run in two stages i.e. a training stage and a testing stage.

In the training stage, the input patterns and the desired output patterns are formed and learning process of the ANN is carried out. This is an iterative procedure with the network weights getting modified at each iteration. During an iteration, the computed output and desired output are compared to give an error term which is utilised to modify the network weights for the next iteration following a back propagation algorithm (Rumelhart et al., 1986). The learning is stopped once a pre-decided error threshold is reached and at this point the network weights are considered to be frozen with the learning process of the trained patterns converged to the pre-decided threshold.

In the second or the testing stage, the test patterns are given as input to the network and using the information of the frozen weights, the outputs are determined in terms of the training classes. For the present application of ANN to the stellar spectra, the synthetic spectra described above were the training set and the test set consisted of the observed spectra.

The  $\chi^2$  minimisation method is essentially a statistical way of finding the closest member from the training set of spectra for a given test spectrum. The method computes a set of the reduced  $\chi^2_j$  values for the given test spectrum  $T_i^k(\lambda_i)$  corresponding to each spectrum  $S_i^j(\lambda_i)$  of the training set (in the ANN terminology used above). The corresponding  $T_{\text{eff}}$  value of the training spectrum which has the minimum  $\chi^2$  value amongst this set is assigned to the given test spectrum. In this manner, all the test set spectra are assigned  $T_{\text{eff}}$  values of the corresponding training set spectra in a unique way defined by the minimum  $\chi^2$  values.

The set of reduced  $\chi^2_j$  values is defined as :

$$\tilde{\chi}^2_j = \frac{\sum_{i=1}^n (S_i^j - T_i^k)^2}{p} \quad (1)$$

where  $p$  is the degrees of freedom,  $S_i^j(\lambda_i)$  is the  $j^{\text{th}}$  training spectra and  $T_i^k(\lambda_i)$  is the  $k^{\text{th}}$  test spectra,  $\lambda_i$  are the wavelength points with  $i = 1, n$  for  $n = 446$  points for each spectra.

### 3.2. Application

While applying the ANN to the training set of synthetic spectra based on model atmospheres it was found that these spectra are somewhat smooth and the learning process was very difficult in terms of convergence (for the present analysis a predecided threshold of  $9.0 \times 10^{-2}$  in about 10,000 iterations was necessary to determine the  $T_{\text{eff}}$  within the intrinsic step of  $\pm 250^\circ\text{K}$ ).

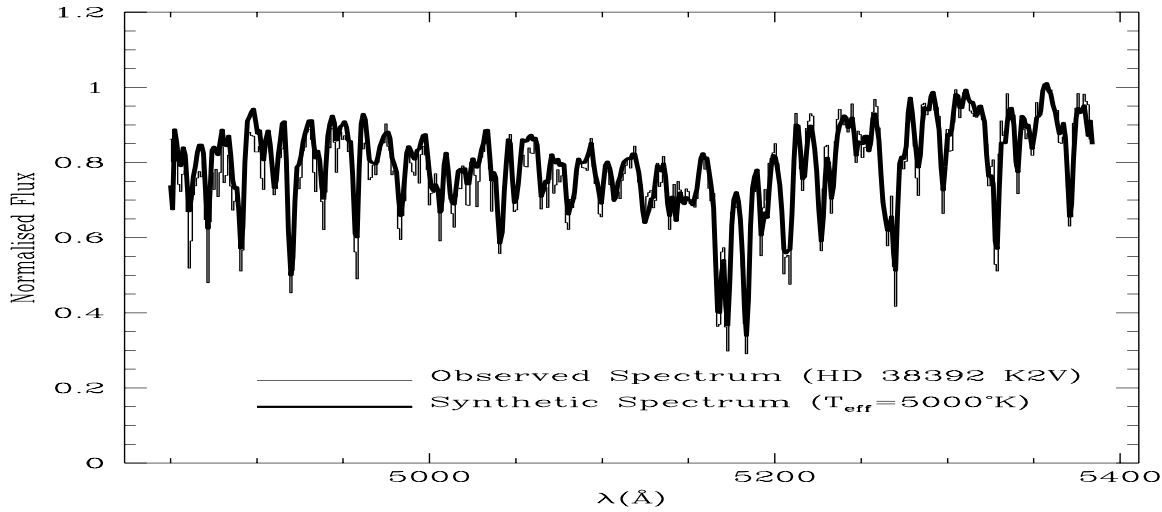
To make them more realistic and comparable to the observed spectra, random noise of the level of 10% was added to each flux value of the spectra since this was the level of estimated noise in the observed spectra. It may be noted here that by addition of random noises of 5% or 15% to the flux values of the synthetic spectra did not change the results significantly and thus a final value of 10% was taken as upper limit. After addition of the random noise, the learning of the training patterns converged well for a ANN architecture of 446 input nodes, 2 hidden layers with 256 nodes each and 21 output nodes (i.e. 446:256:256:21). Several other architectures like 446:500:21, 446:64:64:21, 446:128:21 etc. were explored but the desired errors in the  $T_{\text{eff}}$  were not within the required limits of intrinsic steps mentioned above. Fig. 1 shows the quality of matching obtained for one of the observed stars namely HD 38392 (K2V) with the synthetic spectrum of 5000°K. The bold line is the synthetic spectrum generated from a stellar atmospheric model corresponding to a  $T_{\text{eff}}$  value of 5000°K and the thin line is the observed spectrum for the star. It is interesting to note here that all the important spectral features have matched quite well.

The same training set with 10% noise level as described above was used while applying the  $\chi^2$  minimisation method to the data set.

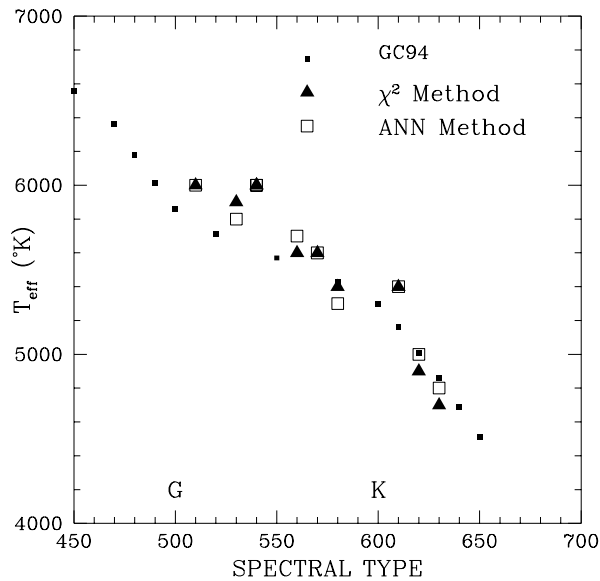
### 3.3. Comparison of results from two methods

The  $T_{\text{eff}}$  determined by both the methods for the observed stars on the basis of their spectral types are plotted in Fig. 2 along with the values from GC94 (see Table 2 of GC94). The small filled squares represent the GC94 calibration, large open squares represent the ANN determined results for the 10 stars and filled triangles represent the  $\chi^2$  determined results. It is worth noting that both methods produce the  $T_{\text{eff}}$  values well within the basic errors of  $\pm 250^\circ\text{K}$  (the intrinsic  $T_{\text{eff}}$  steps of the synthetic generated spectra) and follow closely the GC94 values with no gross errors in  $T_{\text{eff}}$  determinations. These errors are similar to those mentioned in GC94 where the  $T_{\text{eff}}$  scatter is comparable to the resolution of the model atmosphere grid ( $250^\circ\text{K}$ ). However, there is a tendency of the  $T_{\text{eff}}$  values determined by our methods to be somewhat higher than those of GC94 at the earlier spectral types (G1-G4), and this can be due to the fact that GC94 uses blue part of the spectrum where many hydrogen lines contribute to the opacity. Further, from this table it is evident that the ANN determined  $T_{\text{eff}}$  values are somewhat better than the  $\chi^2$  determined values thereby indicating that ANN is an improvement on the  $\chi^2$  method. One must also remember that there are errors present on the abscissa too since the spectral type determinations are ultimately taken from various sources in the literature.

Table 1 shows the programme star HD identification in the 1st column and corresponding spectral type (the catalogue spectral type) taken from Cayrel et al. (1992) in the 2nd column. The  $T_{\text{eff}}^1$  are values from the literature quoted in Cayrel et al. (1992);  $T_{\text{eff}}^2$  are the GC94 values taken (or interpolated) from their Table 2 and finally  $T_{\text{eff}}^3$  and  $T_{\text{eff}}^4$  give our determinations of  $T_{\text{eff}}$  from  $\chi^2$  minimisation and ANN methods respectively.



**Fig. 1.** Matching of observed spectrum of star HD 38392 (K2V) with the synthetic spectrum of 5000°K. The bold line represents the synthetic spectrum and the thin line represents the observed spectrum.



**Fig. 2.** Calibration of spectral type versus  $T_{\text{eff}}$  for dwarf G and K stars. The small filled squares are the GC94 calibration values (taken from the Table 2 of GC94); the large open squares are the  $T_{\text{eff}}$  values for our programme stars determined by the ANN technique and the filled triangles represent the  $T_{\text{eff}}$  values determined from  $\chi^2$  minimisation method for the same programme stars.

Comparative performance of both methods was carried out by a correlation analysis. Table 2 summarises the parameters of this analysis viz.: standard deviation  $\sigma$  (in units of °K) and the correlation coefficient  $r$ . The table suggests that the ANN performs better than the  $\chi^2$  method and that these methods are highly correlated between themselves. It is worth mentioning that the ANN method is not limited due to the small set of test spectra since even larger test set can be classified with the same

**Table 2.** Comparative performance

Parameter	$\chi^2$ vs. Catalog	ANN vs. Catalog	ANN vs. $\chi^2$
$\sigma$	181.478	124.770	59.040
$r$	0.910	0.951	0.989

set of network weights (which get generated after the training session), while the  $\chi^2$  method requires each new test spectra to go through the process of minimisation.

#### 4. Conclusion

The results depicted in Fig. 2 clearly show that the new methods can successfully reproduce the calibration values of GC94 and that for the observed dwarf stars, one can determine the  $T_{\text{eff}}$  values to an accuracy of  $\pm 250^\circ\text{K}$ . The new methods and particularly the ANN method is extremely efficient for large data sets as one has to train an exhaustive synthetic library (which covers fully the stellar atmospheric parameter space) only once and use it for any large number of observed test spectra. We plan to extend the synthetic library to a larger wavelength range, early to late type stars and all luminosity classes so that one can determine  $T_{\text{eff}}$ ,  $\log g$ ,  $[\text{Fe}/\text{H}]$  etc. for all type of stellar spectra. It is further proposed to finally use this extensive library for studying integrated light spectra from galaxies and globular clusters.

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