Astron. Astrophys. 322, 982–984 (1997)

ASTRONOMY AND ASTROPHYSICS

Comments on the "analytic theory of p modes" by Dzhalilov and Staude

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Received 1 March 1996 / Accepted 2 December 1996

Abstract. Dzhalilov & Staude (1995; hereinafter DS) have proposed a new "analytic theory" of p modes for the atmospheres of the Sun and Sun-like stars, which explains the p-mode spectrum in terms of "resonant transmission" of acoustic waves. It is shown that this result is mistaken because of an incorrect fit of the piecewise analytic solutions at the boundaries between the diverse layers of the atmosphere. Apart from this incorrect treatment of wave propagation, the interpretation in terms of resonant transmission appears to arise from a mix-up of resonance and interference phenomena. There is also some confusion about the concepts of tunnel effect and turning point.

Key words: hydrodynamics – Sun: atmosphere – Sun: oscillations – stars: oscillations

1. Introduction

Since the discovery of Deubner (1975) of the fine structure in the spectrum of the solar 5-min oscillations it has been generally accepted that this is the manifestation of acoustic waves trapped in the solar convection zone, as it had been predicted by Ulrich (1970). In contrast to this, DS claim that "it has been demonstrated that the mechanism of resonant transmission of acoustic waves through the considered layers is sufficient to explain the main features of the observed frequency spectrum of the solar p modes, including the 5-min oscillations and the peaks in the high-frequency tail". This would mean that the observed spectrum can be explained by resonant transmission without taking into account the trapping of p modes in the solar interior. Indeed, DS state that "the consideration of a lower reflecting boundary is not necessary in such a study of resonant transmission; it is an investigation of trapping of atmospheric waves rather than of global oscillations in the 'classical' sense, though both aspects are closely related to each other". This claim is not correct, because wave trapping is possible only if there are two reflecting boundaries. DS do not explain the difference between trapping of atmospheric waves and global oscillations.

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In the present comments we point out how to treat acoustic waves in a model atmosphere which permits piecewise analytic solutions, and discuss problems of wave propagation in a stratified atmosphere, especially in connection with resonant transmission and interference. A discussion of turning points and the tunnel effect is also included because DS tried to give a new interpretation of p-mode tunneling through the temperature minimum, which is very different from the classical one (Zhugzhda 1972), and also tried to define the position of the turning point, which is not possible in a unique manner.

2. Resonant transmission of waves

Resonant transmission of waves manifests itself as a maximum or a number of maxima in the transmission coefficient of an atmospheric layer, or of an optical or electronic device. The effect has been considered, for example, in the context of Alfvén wave propagation in coronal arches (Zhugzhda & Locans 1982) and for 3-min oscillations in a sunspot chromosphere (Zhugzhda et al. 1987a,b).

The interference filter is a well-known example of an optical device based on resonant transmission. Transmission occurs when the waves that are reflected from two surfaces within the filter are in opposite phase. The frequencies of these waves are equal to the resonance frequencies of the layer between the two surfaces. For atmospheric waves resonant transmission appears if there are two potential barriers of finite height, with a resonance layer in between. Waves incident from outside on one of the barriers can penetrate through both barriers if their frequency is equal to the frequency of the resonator. DS propose to apply this effect to the p modes in the photospheric and sub-photospheric layers. It is true that there is a resonance layer below the photosphere, but the waves are evanescent below that resonator and neither incident nor reflected waves occur. Moreover, the p modes are generated within the layer where they are trapped. Thus resonant transmission cannot exist, because there is no transmission of waves through the sub-photospheric resonator.

Resonant transmission of acoustic waves from the photosphere to the corona through the chromosphere is possible for waves having one of the resonance frequencies of the chromospheric resonator. For the chromosphere above sunspots this effect has been considered by Zhugzhda et al (1987a,b). As explained, resonant transmission does not work for p modes in the sub-photospheric resonator.

The correct way of finding resonant transmission of waves is to compute the transmission coefficient (Zhugzhda & Locans 1982, Zhugzhda et al. 1987a,b). Resonant transmission manifests itself in the appearance of maxima of this coefficient, which correspond to resonance frequencies of the "interference filter", if such a resonator exists in the atmosphere. The transmission coefficient of the DS model does not show maxima (their Fig. 5). It seems that the maxima which should appear due to the chromospheric resonator are absent because of the wrong fitting of the solution at the temperature minimum, which removes wave reflection there. The treatment of resonant transmission by DS differs from the regular procedure. They calculate the wave power at fixed levels in the atmosphere as a function of wave frequency (their Figs. 10 – 12). They consider the maxima as a manifestation of resonant transmission and treat their result as a power spectrum of the solar p modes. This is not correct. The maxima are just a consequence of the change with frequency of the levels where constructive interference occurs. Thus the "resonance frequencies" are different for the different layers of the atmosphere. The conclusion that the p modes are a manifestation of resonant transmission is therefore incorrect, because the interference pattern is confused with the phenomenon of resonance.

3. Acoustic waves in a "piecewise atmosphere"

The propagation of adiabatic acoustic waves in a stellar atmosphere is described by second-order differential equations. The height profile of temperature T in the solar atmosphere can be approximated by piecewise linear and nonlinear functions for the different regions (in short, a "piecewise atmosphere"). This permits to express the solution for each layer in terms of Bessel and hypergeometric functions. If these solutions are correctly fitted at the boundaries between the layers, one obtains a solution for the entire atmosphere, which subsequently can be used for further investigation.

The fit is not carried out correctly by DS. We briefly outline the correct procedure here. Consider an ordinary differential equation of second order

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + a_1(x)\frac{\mathrm{d}y}{\mathrm{d}x} + a_2(x)y = 0, \tag{1}$$

which is valid for the range $-\infty < x < \infty$ of the independent variable x. Let us consider a piecewise atmosphere, consisting of two layers $x \le x_0$ and $x \ge x_0$ with different dependencies of the coefficients $a_1(x)$ and $a_2(x)$ on x. The general solutions for these two layers are

$$y(x) = C_1y_1(x) + C_2y_2(x) \ \text{ for } \ x \le x_0, \tag{2}$$

$$y(x) = \tilde{C}_1\tilde{y}_1(x) + \tilde{C}_2\tilde{y}_2(x) \ \text{ for } \ x \ge x_0, \tag{3}$$

where C_1, C_2, $\tilde{C}_1$, $\tilde{C}_2$ are arbitrary constants and y_1, y_2, $\tilde{y}_1$, $\tilde{y}_2$ are partial solutions. In special cases when, for example, the dependence of the coefficients a_1, a_2 on x is linear or hyperbolic, the partial solutions can be expressed in terms of known analytic functions, commonly hypergeometric and Bessel functions. The idea of a piecewise atmosphere makes sense if the solution of the considered differential equation can be expressed in terms of known analytic functions for every layer of the atmosphere. If this is possible, as in the model of DS, the general solutions for the diverse layers must be fitted onto each other at the boundaries between the layers. In our simple case of a two-layer atmosphere the solutions (2) and (3) have to be fitted at $x = x_0$. The fit yields relations between the constants C_1, C_2, $\tilde{C}_1$, $\tilde{C}_2$, which make the general solution and its first derivative continuous at x_0. The fitting conditions are trivial and read

$$C_1y_1(x_0) + C_2y_2(x_0) = \tilde{C}_1\tilde{y}_1(x_0) + \tilde{C}_2\tilde{y}_2(x_0), \tag{4}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(C_1y_1(x) + C_2y_2(x))\Big|_{x=x_0} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\tilde{C}_1\tilde{y}_1(x) + \tilde{C}_2\tilde{y}_2(x)\right)\Big|_{x=x_0} \tag{5}$$

If the coefficients a_i are continuous at x_0, the second derivative of the solution is continuous as well, because the solutions for both layers satisfy equation (1).

The fitting conditions relate the two arbitrary constants of one layer to the two arbitrary constants of the second layer. After the fitting the general solution for the entire atmosphere contains just two arbitrary constants as it should be for the solution of a second-order differential equation.

In their piecewise model DS use only one fitting condition, namely $\{\delta p\} = 0$ (their Sect. 4.5). The problem of obtaining two further relations for their arbitrary constants is solved by a *separate* fit of the partial solutions for the two layers,

$$C_1y_1(x_0) = \tilde{C}_1\tilde{y}_1(x_0), \quad C_2y_2(x_0) = \tilde{C}_2\tilde{y}_2(x_0). \tag{6}$$

This "fit" leads to jumps in the first and second derivatives of the solution at $x = x_0$. The solutions (2) and (3) fitted in such an incomplete way do not constitute a general solution of Eq. (1), valid for the entire atmosphere $-\infty < x < \infty$. Consequently the DS treatment of waves in their piecewise model is incorrect from a mathematical point of view.

However, even a correct mathematical fit may still be wrong from a physical point of view. The basic set of hydrodynamic equations can be reduced to a second-order equation for any of the physical variables: pressure, density, temperature, velocity components, entropy, and so on. A continuous fit of one of these variables may lead to a jump in other variables. In the case of linear acoustic oscillations the continuity of pressure and mass conservation over the boundary should be ensured (continuity of the vertical velocity then follows). In this case a jump of the temperature disturbance can appear, which is however not essential in the case of adiabatic oscillations.

DS use only the condition of pressure continuity and do not use the mass conservation condition, except at the boundary between the chromosphere and corona. Thus, "artificial" jumps of the vertical velocity appear (their Fig. 3). The model is incorrect from a physical point of view, because it violates mass conservation, in addition to the incorrect mathematical procedure.

4. Wave propagation in a stratified atmosphere

The incorrect fit indicates some misunderstanding of wave propagation theory in a stratified atmosphere. DS tried to separate the upgoing and downgoing waves and to fit them separately. Besides the fact that such separate fitting is incorrect we want to point out that there is no unique way of separating incident and reflected waves in a stratified, non-isothermal atmosphere. The reason for this is the partial reflection of waves at all levels in the atmosphere where the sound speed is a function of depth. The incident and reflected waves are coupled together, except in the cases of a uniform or an isothermal atmosphere where reflection can occur only at boundaries between layers with different sound velocities.

In the case of a weak stratification (approximation of geometrical optics), where the wavelength is small in comparison with the characteristic scale of stratification, the reflection of waves is a small effect, and asymptotic expansions can be used in order to separate the incident and reflected waves. But this approximation can be used only in special cases. In the case of p modes the asymptotic expansions for running waves cannot be used for the entire atmosphere, because the p modes become evanescent in some of the atmospheric layers, which forbids the use of the methods of geometrical optics.

In a correct treatment of wave propagation it is neither possible nor required to separate the partial solutions which correspond to incident and reflected waves. It is only necessary to substitute the general solutions into the fitting conditions (4) and (5), written for the right variables from the physical point of view.

5. Where is the turning point of p modes?

There is no unique definition of the so-called turning point. As mentioned above, the problem of p-mode propagation can be reduced to a second-order differential equation for any of the physical variables. For a stratified non-isothermal atmosphere these equations differ from each other. Each can be reduced to a form without a first derivative term, as it was done by DS for the equation for the vertical velocity component (their Eqs. (12) and (13)). Such a form makes it possible to consider an acoustic potential and to define the turning point as the point where the solution becomes non-periodic. However, the acoustic potentials for the diverse physical variables are different in a non-isothermal stratified atmosphere. The diverse turning points do not coincide. The p modes become evanescent at different heights in the solar atmosphere, depending whether oscillations of the pressure, the temperature, or of the velocity are considered. There is no way to choose the "true" acoustic potential and to determine a unique turning point.

Only in the context of observations it is reasonable to discuss the position of turning points. If Doppler shifts are observed, then the turning point for the vertical velocity should be considered as the boundary between running and evanescent waves. If observations of brightness oscillations are discussed, the turning point for the temperature disturbances must be considered. The result that the turning point for the pressure fluctuation lies 500 km below the temperature minimum appears to be less interesting.

6. Tunnel effect and wakes

For the solar 5-min oscillations the tunnel effect has been explained by Zhugzhda (1972); it is well known, so we do not explain it again. The discussion of the tunnel effect by DS (their Sect. 6) is based on the wake phenomenon, which has no connection with tunneling. In solar physics, wakes are discussed in the context of shocks in the atmosphere. Wakes are related to transient phenomena in systems that have a low-frequency cutoff. Thus, the statement "though the solution inside the barrier has no longer an oscillatory nature the wake of running waves penetrates up to the base of chromosphere, where $T_0(z)$ has a steep increase, and is reflected from there" is conceptually not meaningful.

7. Discussion and conclusions

Dzalilov and Staude discuss resonance frequencies and line widths in their "spectrum" of p modes, while the procedure can only yield an *envelope* of the p-mode line spectrum. Their work might have been an improvement of a calculation of Worrall (1991), who had treated the problem correctly except for some unessential details. Worrall has called attention to an influence of interference of waves reflected from the photosphere upon the envelope of the p-mode line spectrum. Unfortunately his model of the solar atmosphere was too crude, and his results hardly can be compared with observations.

Although the idea of Worrall seems reasonable it appears that only a numerical integration of the oscillation equations would provide the details required for a comparison with observation. In any case a model like the one of DS should be considered as semi-analytical because, if carried out correctly, it reduces to very cumbersome transcendental expressions for the transmission coefficient or the dispersion relation, which can be treated only numerically. A true analytical model should reduce the problem to simple formulas which are easily analyzed. In the present case a numerical calculation for a realistic solar atmosphere appears more promising than a multilayer model.

Acknowledgements. This work has been partly supported by the grant INTAS-93-4. Y. Zh. is thankful to the Alexander von Humboldt Foundation for the support of his visit to the Kiepenheuer-Institut für Sonnenphysik, where the paper was finished.

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