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The generation order parameters and the γ-ray spectra of pulsars*

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Abstract. Based on the Ruderman-Sutherland (or shortly R-S) model, the mechanisms of γ-ray emission from pulsars are discussed. We mainly consider the influences of magnetic fields on photon absorption. The concept of generation order parameters for pulsars is refined. The generation order parameters are proven to be monotonically and smoothly related to the spectral index and the mean photon energy. The agreement between the calculated results and the observational data is good. The radiation conversion efficiency is also introduced. Based on these concepts, observational quantities such as luminosities, fluxes, mean photon energies and others are calculated and listed in tables. Finally, we discuss the influence of both electric and magnetic fields on the photon absorption.

Key words: magnetic fields – stars: neutron – pulsars: general – gamma rays: theory

1. Introduction

Before the launch of the Compton GRO Satellite, only Crab and Vela were known to be γ-ray pulsars. After that, PSR1706$-$44 (Thompson et al. 1992), Geminga (Bertsch et al. 1992), PSR1509$-$58 (Ulmer et al. 1993) and PSR1055$-$52 (Fierro et al. 1993) have also been discovered to be γ-ray pulsars. Recently, the seventh γ-ray pulsar PSR1951+32 has been discovered (Ramanamurthy et al. 1995; see also Li et al. 1987; Li et al. 1989). Among these, only PSR1509$-$58 has been observed by the low energy instruments BATSE and OSSE, while all other six have been observed by the high energy equipment EGRET. We will call these six the EGRET pulsars.

Usually two mechanisms based on the vacuum gap concept are used to explain the pulsed γ-ray emission from pulsars. One is the inner gap model, known as the Ruderman-Sutherland (1975) (or shortly R-S) model, based on the gap in the polar cap region of the neutron star surface. The other is the outer gap model, known as the Cheng-Ho-Ruderman (1986) (or shortly CHR) model, based on the gap far from the surface of a neutron star. This gap begins at the "null charge surface" ($\mathbf{\Omega} \cdot \mathbf{B} = 0$), and continues along the open field line, near the boundary of the closed field line region, to the light cylinder (here $\mathbf{\Omega}$ and $\mathbf{B}$ are the angular velocity and the surface magnetic field of a neutron star respectively). Within gaps, the parallel component of the electric field does not vanish ($\mathbf{E} \cdot \mathbf{B} \neq 0$), so positrons/electrons can be accelerated to very high energies and emit γ-rays. Usually, the γ-rays thus emitted are energetic enough to be further converted into positron-electron (or e^+e^-) pairs in electric or magnetic fields. Then cascade processes (Daugherty & Harding 1982; Zhao et al. 1989; Lu & Shi 1990) lead to strong γ-ray intensities. Although both kinds of gaps can exist in the magnetosphere of a neutron star, their γ-rays are usually beamed in quite different directions.

Since the discovery of several new γ-ray pulsars, a lot of work has been developed based on pulsar inner gap and outer gap emission models (Arons & Scharlemann 1979; Arons 1983; Chiang & Romani 1992; Dermer & Sturner 1994; Sturner et al. 1995; Daugherty & Harding 1994; Chiang & Romani 1994; Romani & Yadigaroglu 1995). In particular, Dermer & Sturner (1994) proposed that the cascade processes are initiated by magnetic Compton scattering rather than by the conventional curvature radiation. However, determining which of these two processes is more important is not easy since these processes are very sensitive to the magnetic field strength, the temperature of neutron star surface, and the geometry of emitting region. So far these are uncertain. Therefore, much more work should be done to determine which process is more efficient. For simplicity, here we suppose that the pair cascade processes are initiated by curvature radiation (Daugherty & Harding 1982, 1994; Chiang & Romani 1992). In addition, we note that even though the magnetic Compton scattering might be more efficient than curvature radiation, our discussion will still hold qualitatively. As e^+e^- pairs of second and even higher generation usually move

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with angles relative to the magnetic field, they will lose their energy mostly by synchrotron radiation due to a very strong magnetic field. Therefore in this paper we will base our discussion on the inner gap model, but we are aware that there might be some γ-ray pulsars of the CHR type.

After being accelerated to very high energies with a typical Lorentz factor of

$$\gamma_1 = 6.0 \times 10^7 P^{1/14} \dot{P}_{15}^{-1/14}, \quad (1)$$

when they go through the polar gap (Zhao et al. 1989), the e^+/e^- (first generation particles) will move along the curved magnetic field lines and emit high energy curvature radiation (first generation photons) with a characteristic photon energy E_1. Here P is the period of pulsar in units of second, and $\dot{P}_{15}$ its derivative in units of 10^{-15} s/s. If

$$E_1 > E_a, \quad (2)$$

with, according to Hardee (1977),

$$E_a = 9.5 \times 10^3 P^{1/2} \dot{P}_{15}^{-1/2} \quad \mathrm{MeV}, \quad (3)$$

these photons will usually be absorbed and converted into e^+e^- pairs (second generation particles). This is the Hardee absorption condition at the magnetic axis through an electric field. The Lorentz factor γ_2 of the second generation e^+/e^- will thus be $E_1/2mc^2$. These e^+/e^- can emit second generation photons through synchrotron radiation with a characteristic energy E_2. If $E_2 > E_a$, further e^+e^- pairs can be produced. Thus, cascade processes occur. Usually one high energy e^+ or e^- can emit a lot of synchrotron photons. Therefore, photon numbers will dramatically increase per generation, while their spectra in turn will get softer and softer. It should be emphasized that for any real pulsar, its emitted photons can only have passed through very few generations. This behaviour is rather general and might not depend upon details of the model. This makes the concept of generation very useful and fruitful. Calculations based on the concept of generation have been done in some approximations (Zhao et al. 1989; Lu & Shi 1990). The main difference between the ZLHLP scheme (Zhao et al. 1989) and the LS scheme (Lu & Shi 1990) lies in the conversion of high energy photons into e^+e^- pairs: through magnetic fields in the former scheme, and through electric fields in the latter scheme.

Originally, the concept of generation applies to each photon individually. This means that some emitted photons from a pulsar might be of second generation and some of third generation.

Recently, the *generation order parameter*, ζ, has been introduced (Lu 1993; Lu et al. 1994) to characterize *a pulsar*. In fact, this parameter, ζ, can be understood as an effective or average generation number for a pulsar's photons. Evidently, the generation order parameter, ζ, is a concept which applies to the pulsar, different from the concept of generation itself, which applies to photons. For a γ-ray pulsar, different emitted photons might be of different generation, so in general, a non-integer should be used for the generation order parameter to characterize a pulsar. A pulsar with larger ζ emits more photons with lower energies. Lu et al. (1994) calculated this parameter based on the LS scheme, in which the Hardee (1977) mechanism has been used to describe the conversion of a γ photon into an e^+e^- pair. Goldoni et al. (1995) recently used this generation order parameter in their work of multiwavelength phenomenology of isolated neutron stars. In this scheme, only the electric (not the magnetic) field is taken into account for the conversion.

The very important features of the generation order parameter are the clearness of its physical meaning and the simplicity of its expression. Arons' model has some advantage, but it is much more complicated than the R-S model. So in this paper, we will still base our discussion on the R-S model. We will discuss the more complicated case later on.

Here we will first take a different approximation from the LS case. Here we consider only the magnetic field as the cause for the conversion, similar to the ZLHLP scheme. However, though the magnetic field has been considered to be the cause for the conversion of high energy photons into e^+e^- pairs in the cascade processes, the Hardee (1977) mechanism (electric field being the cause of the conversion) was used for the critical condition for the final conversion in the ZLHLP scheme. This inconsistency will be removed here by using another critical condition instead of the Hardee condition. In section 2, this refinement is given. In section 3, the generation order parameter is given in the above approximation. In section 4, the relations between the generation order parameters and the γ-ray spectra are given, and in section 5, the concept of γ-ray conversion efficiency, η, is introduced. As η is defined as the efficiency per generation, a combination of ζ and η could be used to determine the γ-ray luminosity of a pulsar. In section 6, we calculate the luminosity and flux of the six EGRET pulsars and some other pulsars. In section 7, we discuss the general case, considering both electric and magnetic fields. Finally, some conclusions will be given.

2. Theoretical model

According to Daugherty and Lerche (1975), the absorption coefficient of a photon in electric and magnetic fields is expressed as

$$\mu = 0.23 \frac{2\pi\alpha}{\lambda_e} \frac{2mc^2}{E_\gamma} \chi exp(-\frac{4}{3\chi}), \quad (4)$$

$$\chi = \frac{E_\gamma}{2mc^2} \frac{B}{B_c} [(\eta_x - \frac{E}{B})^2 + \eta_y^2 (1 - \frac{E^2}{B^2})]^{1/2} \quad (5)$$

with α denoting the fine structure constant, λ_e the Compton wave length, $B_c = m^2c^3/e\hbar = 4.4 \times 10^{13}$ G the critical magnetic field, (η_x, η_y, η_z) the direction cosine of photon motion, and E_γ the photon energy. The magnetic field direction is defined as the z-axis and the electric field direction as the y-axis. Supposing that photons move in the yz-plane and make an angle θ with magnetic field, the absorption coefficient of the conversion into e^+e^- pairs of a photon can be written as

$$\mu = 0.23 \frac{2\pi\alpha}{\lambda_e} \frac{(E^2 + B_\perp^2)^{1/2}}{B_c} exp[-4/(3 \frac{E_\gamma}{2mc^2} \frac{(E^2 + B_\perp^2)^{1/2}}{B_c})]$$

(6)

with $B_\perp = B\sin\theta$.

Here we suppose the magnetic fields of a neutron star to be of standard dipole type. Then, when a photon initially moves along the magnetic field, namely $B_\perp = 0$, as the field itself is curved, $B_\perp$ will deviate from zero and have its maximum value

$$B_{\perp max} = 0.08 B_0 (\frac{R_0}{r_0})^3 \theta_0 \tag{7}$$

after the photon travelled a distance of about $r_0/3$. Here B_0 is the magnetic field strength at neutron star surface, R_0 the radius of the neutron star (typically $\approx 10^6$ cm), and (r_0, θ_0) the photon's initial polar coordinates. Near the neutron star surface, $r_0 \approx R_0$. Let $a = \theta_0/\theta_p$, here

$$\theta_p = (\frac{R_0 \Omega}{c})^{1/2} = 1.45 \times 10^{-2} P^{-1/2}$$

is the angle subtended by the polar cap of a neutron star. Obviously, $0 \le a \le 1$ for an open field line region. Hence, equation (7) can be simply written as

$$B_{\perp max} = 1.16 \times 10^{-3} B_0 a P^{-1/2}. \tag{8}$$

Evidently, when $a = 1$, $B_{\perp max}$ will reach its maximum value. From equation (6) we see, both electric and magnetic fields will influence the photon absorption. To accurately calculate the photon absorption, one should consider both electric and magnetic fields. In the previous paper (Lu et al. 1994), we took the approximation in which only the electric field effect was considered. In this paper, we will take another approximation, considering the photon absorption mainly by the magnetic field. The advantage of this approximation will become clear later. As the electric field is neglected here, for simplicity, we will take the maximum magnetic field, by taking $a = 1$.

According to R-S model, the condition for a photon being absorbed and converted into an e^+e^- pair by electric and magnetic fields is

$$\chi = \frac{E_\gamma}{2mc^2} \frac{(E^2 + B_\perp^2)^{1/2}}{B_c} \approx \frac{1}{15}. \tag{9}$$

For the present case, neglecting the influence of electric fields, and using Eq.(8), we obtain the critical energy for a photon to be absorbed by a magnetic field as follows:

$$E_a = 2.6 \times 10^3 B_{0,12}^{-1} P^{1/2} = 2.6 \times 10^3 \dot{P}_{15}^{-1/2} \quad \text{MeV}. \tag{10}$$

This is just the critical energy to replace the Hardee energy in the ZLHLP scheme. In the numerical calculation, the relation of the magnetic dipole model

$$B_{0,12} \approx P^{1/2} \dot{P}_{15}^{1/2}$$

has been used, where $B_{0,12}$ denotes B_0 in units of 10^{12} G. This means that a photon with energy $E_\gamma > E_a$ will be absorbed and converted into an e^+e^- pair by the magnetic field, a photon with energy $E_\gamma < E_a$ will penetrate the magnetosphere without being absorbed.

Similar to the ZLHLP and LS schemes, after being accelerated in the polar gap, the e^+/e^- emit curvature radiation (first generation photons) with characteristic photon energy

$$E_1 = E_c = 1.6 \times 10^4 P^{-4/14} \dot{P}_{15}^{-3/14} \quad \text{MeV}. \tag{11}$$

For $E_1 > E_a$, these first generation photons will be absorbed and converted into e^+e^- pairs (second generation particles). We assume these e^+/e^- have same energy, so their Lorentz factor $\gamma = E_1/2mc^2$. As the condition for magnetic absorption is

$$\gamma B_\perp / B_c = 1/15, \tag{12}$$

these e^+/e^- will emit synchrotron radiation (second generation photons) with characteristic photon energy

$$E_2 = \frac{3}{2}\frac{B_\perp}{B_c} mc^2 \gamma^2 = \frac{1}{20} E_1 \tag{13}$$

For $E_2 > E_a$, these second generation photons will be absorbed by the magnetic field and converted into e^+e^- pairs (third generation particles). Then third generation photons with energy

$$E_3 = \frac{1}{20} E_2 = (\frac{1}{20})^2 E_1 \tag{14}$$

will be emitted as synchrotron radiation. Generally, the nth generation photon energy can be expressed as

$$E_n = \frac{1}{20} E_{n-1} = (\frac{1}{20})^{n-1} E_1. \tag{15}$$

These cascade processes will also continue until the photon energy becomes less than the critical energy E_a.

3. Generation order parameter

We have discussed the concept of generation in the cascade processes above. It is very important to note that the photon number increases dramatically with each generation and the spectrum in turn gets softer and softer. Also, for any real pulsar, a photon can only go through very few generations. These features make the concept of generation very useful and fruitful for describing the γ-ray radiation from pulsars. However, for any one pulsar, even primary (first generation) photons will not be mono-energetic, while photons of different energies can proceed to different generations. Therefore, one should use a parameter averaging over various photons to describe a pulsar. In fact, the generation order parameter ζ has been introduced as such a parameter (Lu 1993; Lu et al. 1994).

Based on the reason similar to that shown in the papers of Lu (1993) and Lu et al. (1994), by setting

$$E_{n=\zeta} = E_a, \tag{16}$$

we can define the generation order parameter ζ:

$$\zeta = 1 + \frac{0.8 - (2/7)\log P + (2/7)\log \dot{P}_{15}}{1.3} \tag{17}$$

Here, the generation order parameter, ζ, is usually not an integer, but something like a "fractal".

This refined generation order parameter ζ differs from that of Lu (1993) and Lu et al. (1994) in the following: here we consider the photon being absorbed by the magnetic field, while in the previous papers only the electric field has been taken into account. It is worthwhile to note that in the present approximation, the generation order parameter ζ is a direct function of $P/\dot{P}$. This means that ζ is solely determined by pulsar's spin-down age $\tau = P/2\dot{P}$:

$$\zeta = 1.88 - 0.22 \log \tau_6 \qquad \text{or} \qquad \tau_6 = 10^{(1.88-\zeta)/0.22},$$

here τ_6 denotes the spin-down age of pulsar in units of 10^6 years. For many years people have been aware of the importance of τ in fitting the γ-ray data of pulsars, the generation order parameter might give a natural interpretation.

The generation order parameters ζ for the 6 EGRET pulsars and some other possible γ-ray pulsars have been calculated and are listed in Table 1 and 2.

4. Relations of γ-ray spectra with generation order parameters

The γ-ray spectral indices δ of the 6 EGRET pulsars (Fierro et al. 1993; Lu et al. 1994; Ramanamurthy et al. 1995) and their mean photon energies $\bar{E}$ are also listed in Table 1. In the calculation of the mean photon energies the photon energy range of (50 MeV — 5 GeV) has been used. Evidently, the generation order parameters ζ are strongly correlated with the spectral indices δ and the mean photon energies $\bar{E}$. In fact, as discussed above, due to cascade processes, the photon energies should dramatically decrease with higher generations.

This is the very reason for such a correlation. Statistically, from the data listed in Table 1, the correlation between ζ and δ can well be fitted linearly (see Figure 1):

$$\delta = 1.28 - 1.35\zeta \tag{18}$$

and the correlation between ζ and $\bar{E}$ can be fitted to a curve (see Figure 2):

$$\bar{E} = 6.2 \times 10^3 (0.095)^{\zeta-1} \text{MeV}, \tag{19}$$

where 6.2×10^3 and 0.095 are determined by using the method of least squares.

5. Conversion efficiency of γ-ray radiation

Now we will discuss the question: what fraction of the energy of a γ-ray photon could be converted into γ-rays in the next generation?

A high energy photon can be converted into an e^+e^- pair near a pulsar. As the pair moves with an angle relative to the magnetic field, it can emit next generation photons in the form of synchrotron radiation. However, a pair can convert only a fraction of its energy into synchrotron radiation. Assuming the electron (positron) moves in the same direction as the original

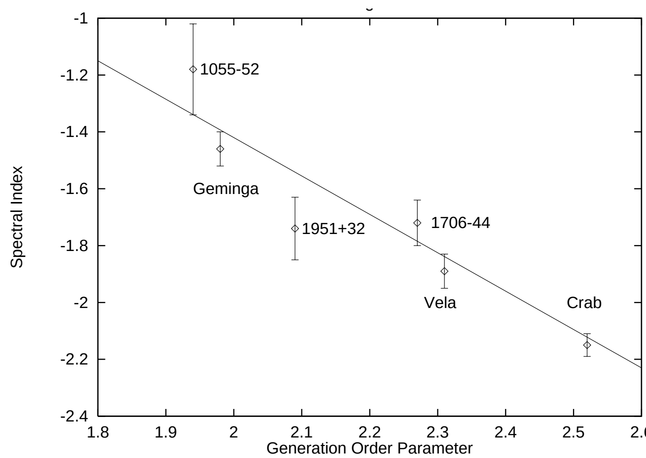

Fig. 1. Correlation between generation order parameters ζ and spectral indices δ

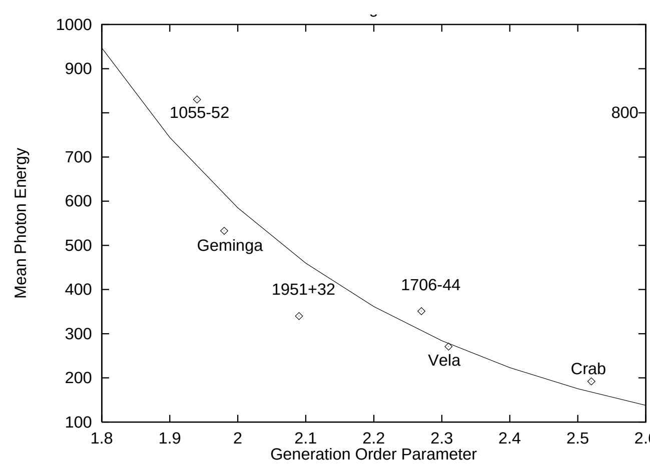

Fig. 2. Correlation between generation order parameters ζ and mean photon energies $\bar{E}$

photon, then it also has an angle θ, the same as the photon, to the magnetic field. Take

$$\gamma_\perp = \sqrt{(p_\perp/mc)^2 + 1} \approx \sqrt{(\gamma \sin\theta)^2 + 1}, \tag{20}$$

here $p_\perp$ is the momentum component perpendicular to the magnetic field. It has been proven that

$$\gamma_{||} = \gamma/\gamma_\perp \tag{21}$$

will remain constant even though most of the electron energy γ is quickly lost to synchrotron energy (Cheng et al. 1986). As $\gamma_\perp$ drops to 1, no further synchrotron radiation can be emitted. According to equation (21), $\gamma_{||}$ is just the Lorentz factor of the electron at $\gamma_\perp \to 1$. This leads to the conclusion that only the energy of $(\gamma - \gamma_{||})mc^2$ can be emitted in the form of synchrotron radiation by an electron with Lorentz factor γ at an angle θ to the magnetic field. Hence, we can define

$$\eta = \frac{(\gamma - \gamma_{||})mc^2}{\gamma mc^2} = \frac{\gamma_\perp - 1}{\gamma_\perp} \tag{22}$$

Table 1. Characteristic quantities of the 6 EGRET γ-ray pulsars

Characteristic quantities of the 6 EGRET γ-ray pulsars

PSR	ζ	η	δ	$\bar{E}$ MeV	L'_γ erg/s	F_γ $\mathrm{cm^{-2}s^{-1}}$
0531+21(Crab)	2.52	0.21	-2.15 ± 0.04	1.9×10^2	1.9×10^{34}	1.1×10^{-6}
J0633+1746(Geminga)	1.98	0.52	-1.46 ± 0.06	5.3×10^2	1.4×10^{33}	1.3×10^{-5}
0833−45(Vela)	2.31	0.25	-1.89 ± 0.06	2.7×10^2	4.6×10^{33}	4.8×10^{-6}
1055−52	1.94	0.65	-1.18 ± 0.16	8.3×10^2	1.7×10^{33}	1.0×10^{-7}
1706−44	2.27	0.28	-1.72 ± 0.08	3.5×10^2	4.1×10^{33}	2.7×10^{-7}
1951+32	2.09	0.84	-1.74 ± 0.11	3.4×10^2	2.1×10^{34}	4.7×10^{-7}

Note: For J0633+1746(Geminga), the distance has been taken to be 150 pc.

as the γ-ray conversion efficiency per generation. Note, for the case of magnetic field dominating the photon absorption (see Eq.(12)),

$$\gamma_\perp = \gamma \sin\theta = \frac{1}{15}\frac{B_c}{B}, \tag{23}$$

the γ-ray conversion efficiency η of equation (22) can be written as

$$\eta = \frac{[(B_c/15B)^2 + 1]^{1/2} - 1}{[(B_c/15B)^2 + 1]^{1/2}}. \tag{24}$$

It is very interesting to note that the efficiency η depends only upon magnetic field. *The stronger the magnetic field is, the lower the efficiency η will be*.

6. Luminosity and flux

The picture how the pulsar emits radiation in the R-S model can be described as follows: after an electron or positron has been accelerated to a very high energy, it will emit high energy photons in the form of curvature radiation, which in turn can usually be converted into e^+e^- pairs. They will again emit high energy photons in the form of synchrotron radiation and thus lead to cascade processes. Every generation in the cascade process photons get softer and softer in energy, and larger and larger in number. However, each generation only a fraction of the energy is kept in the form of high energy photons; the other part of the energy is emitted as low energy curvature radiation and/or ejected in the form of e^+/e^- winds.

According to Zhao et al. (1989) and Lu & Shi (1990), the γ-ray luminosity of a pulsar can approximately be written as

$$\mathrm{L}_\gamma = 1.17 \times 10^{32} P^{-10/7} \dot{P}_{15}^{3/7} \quad \mathrm{erg/s}. \tag{25}$$

The angular half width of γ-ray radiation is

$$\Delta\phi = \frac{3}{2}\,\theta_p = 2.2 \times 10^{-2}\ P^{-1/2}, \tag{26}$$

here θ_p denotes the angle subtended by the polar cap area. As we have assumed that the magnetic field near the surface of a neutron star is of standard dipole type, the angle between the magnetic field and the magnetic axis should be larger than the polar angle θ by a factor of about 3/2. However, the conversion efficiency has not yet been taken into account in this calculation. Taking this into account, we should multiply the luminosity by a factor of $\eta^{\zeta-1}$, which means

$$L_\gamma \longrightarrow L'_\gamma = \eta^{\zeta-1} L_\gamma. \tag{27}$$

Here, L'_γ is the real γ-ray luminosity of a pulsar. It should be noted that although the factor $\eta^{\zeta-1}$ decreases with increasing magnetic field, L'_γ will usually still increase, because L_γ increases. The luminosities of some pulsars with high generations and/or shorter distances have been calculated and listed in Table 1 and 2. We now see the importance of the approximation of the magnetic field domination, in which the combination of the generation order parameter and the γ-ray conversion efficiency can simply give a main factor to the γ-ray luminosity of a pulsar.

Table 2. Characteristic quantities of some predicted possible gamma-ray pulsars

Characteristic quantities of some possible γ-ray pulsars

PSR	ζ	η	$\bar{E}$ MeV	L'_γ erg/s	F_γ $\mathrm{cm^{-2}s^{-1}}$
0114+58	2.00	0.75	5.9×10^2	5.0×10^{33}	1.4×10^{-7}
0450+55	1.80	0.70	9.4×10^2	6.0×10^{32}	1.5×10^{-7}
0656+14	2.09	0.16	4.8×10^2	3.3×10^{32}	1.8×10^{-7}
0740−28	2.06	0.50	5.1×10^2	2.4×10^{33}	1.3×10^{-7}
0950+08	1.61	0.92	1.5×10^3	4.2×10^{32}	2.0×10^{-6}
1133+16	1.73	0.41	1.1×10^3	8.4×10^{31}	2.7×10^{-7}
1929+10	1.77	0.83	1.0×10^3	9.0×10^{32}	3.5×10^{-6}

In order to calculate the photon flux, we should know the mean energy of the emitted spectrum. Here we use equation (19) to estimate the mean energy. Then, the photon flux can be calculated according to

$$F_\gamma = \frac{L'_\gamma}{4\pi d^2 \bar{E} \Delta\phi} \tag{28}$$

with d denoting the distance of a pulsar (Taylor et al. 1993). The fluxes thus calculated are also listed in Table 1 and 2 (only pulsars with fluxes larger than $1.0 \times 10^{-7}\ \mathrm{cm^{-2}\ s^{-1}}$ are listed).

7. Discussion

We have discussed the situation of photon absorption by electric fields (Lu et al. 1994) and by magnetic fields (this paper) respectively. But we know that both electric field and magnetic field should exist in the magnetosphere of a neutron star, so in principle, one should take both electric field and magnetic field into account to calculate the conversion of high energy photons into e^+e^- pairs. However, the geometry of the emitting region near the neutron star surface is very complicated, and in addition, the electron density distribution over the polar cap region is not known exactly, so we can only roughly estimate the influence of both electric field and magnetic field on photon absorption.

We have shown that in both cases, the spectral index δ is nearly linearly correlated with the generation order parameter ζ; only the slope is different: in the former case the slope is 0.7, while in the latter case it is 1.3. As a rough estimate it is reasonable to think that the spectral index δ will also be nearly linearly related to the generation order parameter ζ, even though both the electric and magnetic field have been taken into account. The slope should lie between 0.7 and 1.3, and may be about 1. As an example, we assume that the first generation e^+e^- pairs distribute uniformly over the polar cap area, and find that the slope is nearly 1, just as expected. It is very interesting that the spectra get steeper with the index decreasing at a rate of 1 per generation. Here we will give a rough explanation for this relation.

First let us calculate the distribution of an electron or positron produced in a pair from a photon with energy E_γ. Its Lorentz factor should be

$$\gamma = E_\gamma/2mc^2. \tag{29}$$

In case I (the electric field dominates the photon absorption), the LS scheme is used; the angle of electron (or positron) motion relative to the magnetic field is taken as

$$\theta \approx 1/\gamma \tag{30}$$

or

$$\gamma \cdot \theta \approx 1, \tag{31}$$

while in case II (the magnetic field dominates the photon absorption), the ZLHLP scheme is used, and this relation is determined by the condition

$$\chi \approx \frac{E_\gamma}{2mc^2}\frac{B\theta}{B_c} \approx 1/15 \tag{32}$$

or

$$\gamma \cdot \theta \approx B_c/15B. \tag{33}$$

Eq.(31) and Eq.(33) can be combined into

$$\gamma \cdot \theta \approx \lambda \tag{34}$$

with $\lambda = 1$ for case I and $\lambda = B_c/15B$ for case II. From Eq.(29) and Eq.(34), we can generally express the distribution of an electron produced by a photon with energy E_γ as

$$f(\gamma,\theta)d\gamma d\Omega = A\gamma^\alpha\theta^\beta\delta(\gamma - E_\gamma/2mc^2)\delta(\theta - \lambda/\gamma)d\gamma d\Omega, \tag{35}$$

where A is a normalization constant. These two δ-functions guarantee the above conditions Eq.(29) and Eq.(34). Using the normalization condition, we have

$$\int\int f(\gamma,\theta)d\gamma d\Omega = 2\pi A\lambda^{\beta+1}(E_\gamma/2mc^2)^{\alpha-\beta-1} = 1. \tag{36}$$

The normalization condition should not depend upon photon energy E_γ, so we have

$$2\pi A\lambda^{\beta+1} = 1, \tag{37}$$

$$\alpha - \beta - 1 = 0. \tag{38}$$

Thus the electron distribution should read

$$f(\gamma,\theta)d\gamma d\Omega =$$
$$\frac{1}{2\pi}(\frac{\gamma\theta}{\lambda})^\alpha\theta^{-1}\delta(\gamma - E_\gamma/2mc^2)\delta(\theta - \lambda/\gamma)d\gamma d\Omega. \tag{39}$$

This is the electron distribution of one generation expressed in its former generation photon.

Assume the spectrum of nth generation photons to be

$$N_n(E_\gamma)dE_\gamma = k_n E_\gamma^{-\alpha_n}dE_\gamma, \tag{40}$$

then these photons will be converted into $(n+1)$th generation electrons (positrons) with a spectrum

$$D_{n+1}(\gamma,\theta)d\gamma d\Omega = \int N_n(E_\gamma)dE_\gamma f(\gamma,\theta)d\gamma d\Omega \tag{41}$$

or

$$D_{n+1}(\gamma,\theta)d\gamma d\Omega =$$
$$\frac{1}{2\pi}k_n(\frac{\gamma\theta}{\lambda})^\alpha\theta^{-1}\gamma^{-\alpha_n}(2mc^2)^{1-\alpha_n}\delta(\theta - \frac{\lambda}{\gamma})d\gamma d\Omega. \tag{42}$$

The spectrum of synchrotron radiation emitted by a single electron in magnetic field is

$$P(E)dE = \frac{\sqrt{3}}{2\pi}\frac{e^2}{\hbar c}\omega_B\sin\theta(\frac{E}{E_c})dE\int_{E/E_c}^{\infty}K_{5/3}(x)dx, \tag{43}$$

here $\omega_B = eB/mc$, and $E_c = (3/2)\hbar\omega_B\gamma^2\sin\theta$. Then the spectrum of the $(n+1)$th generation photons produced by the $(n+1)$th generation electrons by the synchrotron mechanism can be written as

$$N_{n+1}(E)dE = \int_{\gamma_1}^{\gamma_2}\int_{\Delta\Omega} D_{n+1}(\gamma,\theta)\frac{P(E)}{E}d\gamma d\Omega dE\,, \tag{44}$$

where γ_1 and γ_2 refer to the lower and upper limit of integration. Inserting Eq.(43) into the above equation, we have

$$N_{n+1}(E)dE$$

$$\propto \int\int (\frac{\gamma\theta}{\lambda})^{\alpha}\gamma^{-\alpha_n}\delta(\theta-\frac{\lambda}{\gamma})E_c^{-1}$$

$$\int_{E/E_c}^{\infty} K_{5/3}(x)dxd\gamma d\Omega dE \quad (45)$$

Note that $E_c = (3/2)\hbar\omega_B\gamma^2\sin\theta \propto \gamma^2\theta$, so

$$N_{n+1}(E)dE$$

$$\propto \int\int (\frac{\gamma\theta}{\lambda})^{\alpha}\gamma^{-\alpha_n}\delta(\theta-\frac{\lambda}{\gamma})\gamma^{-2}d\theta d\gamma$$

$$\int_{E/E_c}^{\infty} K_{5/3}(x)dxdE. \quad (46)$$

Set $t = E/E_c$, as $\theta \propto 1/\gamma$, then $\gamma \propto E/t$ and $d\gamma \propto (E/t^2)dt$, we have

$$N_{n+1}(E)\ dE\ \propto \int (\frac{E}{t})^{-\alpha_n-2}\int_t^{\infty} K_{5/3}(x)dx\frac{E}{t^2}dtdE$$
$$\propto E^{-\alpha_n-1}\{\int_{t_1}^{t_2} t^{\alpha_n}dt\int_t^{\infty} K_{5/3}(x)dx\}dE \quad (47)$$

Since $t_1 \ll t \ll t_2$, the integral within $\{\}$ in the above equation should not depend upon E. Thus, we obtain

$$N_{n+1}(E)dE \propto E^{-(\alpha_n+1)}dE. \quad (48)$$

This leads to

$$\alpha_{n+1} = \alpha_n + 1, \quad (49)$$

just as we expected.

8. Conclusion

In this paper, we have discussed the cascade processes after e^+/e^- being accelerated through the polar gap in the R-S model. The key concept used here is the generation order parameter, ζ, which was introduced in the papers of Lu (1993) and Lu et al. (1994), and is further refined in this paper.

Though this parameter can only be calculated very roughly at present (because the physics of magnetosphere is rather complicated), its physical meaning is very clear. It can be related to the γ-ray spectrum. Combined with the γ-ray conversion efficiency η, introduced in this paper (see also Song & Lu 1994), it can give an important factor, $\eta^{\zeta-1}$, of the γ-ray luminosity (see Eq.[27]). Two factors, $\eta^{\zeta-1}$ and L_γ, of the real luminosity L'_γ of a pulsar compete, and depend upon magnetic fields oppositely. This may lead to interesting results.

The generation order parameter has been proved to be very closely related to the amount of emitted photon number and to its spectrum. As γ-ray detection is based on counting techniques, it is easier to observe pulsed γ-rays from a pulsar with a large generation order parameter. Indeed, among nearly six hundred radio pulsars, more than 90% have $\zeta < 2$, and only two of the pulsars with ζ close to 2 (Geminga and PSR1055−52) are γ-ray pulsars. Less than 10% have $\zeta > 2$, five of which (Crab, Vela, PSR1509−58, 1706−44 and 1951+32) have been observed to be γ-ray pulsars!

However, PSR1509−58 is not listed in Table 1 and 2, as its conversion efficiency η is too low. This means that it can not be observed as a γ-ray pulsar in the high energy band. Indeed, EGRET *has not* observed it. It is only a low energy γ-ray pulsar, and *has been* observed by BATSE and OSSE. Recently, Ramanamurthy et al. (1996) claimed to have observed PSR0656+14 to be a γ-ray pulsar with a spectral index of -2.8 ± 0.3. This falls outside the relation seen in Figure 1. There might be two possible reasons for this discrepancy: as the involved photon number is very small, the spectral index -2.8 ± 0.3 can not be regarded as a well determined one; if the index -2.8 ± 0.3 would be correct, then the γ-rays observed should have been emitted from the outer gap of PSR 0656+14 rather than from its polar gap.

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