

*Letter to the Editor***Rotational splitting effect in neutron star QPOs****L. Titarchuk<sup>1,2</sup> and A. Muslimov<sup>1</sup>**

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**Abstract.** We explain the peak spacing in the power spectra of millisecond variability detected by the Rossi X-ray Timing Explorer (RXTE) in the X-ray emission from the LMXBs (Sco X-1, 4U1728-34, 4U1608-522, 4U1636-536, etc) in terms of the rotational splitting of an intrinsic frequency (which is of order of the local Keplerian frequency) caused by an accretion disk. We calculate this effect and demonstrate that there is a striking agreement with the observational data. We show that the observed discrete frequencies ranging from 200 to 1200 Hz can be described by a whole set of overtones. For higher overtones (lower frequencies,  $\lesssim 100$  Hz) the ratio between frequencies is determined by the quantum numbers alone. We suggest that a similar phenomena should be observed in Black Hole (BH) systems for which the QPO (quasi-periodic oscillation) frequency should be inversely proportional to the mass of the compact object. For BH systems the characteristic frequency of oscillations should therefore be a factor of 5-10 less than for a neutron star system.

**Key words:** Accretion: disk–binaries: LMXBs–X-Rays: QPOs**1. Introduction**

The launch of RXTE opened up a new era in the study of quasi-periodic oscillations (QPOs). Recently, kHz QPOs have been discovered in the persistent fluxes of eight LMXBs: Sco X-1 (van der Klis et al. 1996a), 4U1728-43 (Strohmayer et al. 1996a), 4U 1608-52 (Berger et al. 1996), 4U 1636-53 (Zhang et al. 1996), 4U 0614+091 (Ford et al. 1996), 4U 1735-44 (Wijnands et al. 1996), 4U 1820-30 (Smale et al. 1996), and GX5-1 (van der Klis et al. 1996). In addition, episodic and nearly coherent oscillations have been discovered during several type-I X-ray bursts from KS 1731-260 with a frequency of 363 Hz (Strohmayer et al. 1996a), during one type-I burst from KS

1731-260 with a frequency of 524 Hz (Morgan & Smith 1996). In addition during 3 bursts from vicinity of GRO J1744-28, oscillations with a frequency of 589 Hz (Strohmayer et al. 1996a), and during 4 type-I bursts from 4U 1636-53 with a frequency of 581 Hz (Zhang et al. 1996) were seen. The presence of a pair of kHz QPOs is characteristic of almost all these observations. The centroid frequency of these QPOs ranges from 400 (4U 0614+091, Ford et al. 1996) to 1171 Hz (4U 1636-53, van der Klis et al. 1996c). Also, it is worthwhile to point out that van der Klis et al. (1997) reported that they detected, simultaneously with the kHz QPO, two additional peaks near 40 and 90 Hz.

In this *Letter* we shall demonstrate that the effect of a rotational splitting (caused by the Keplerian accretion disk) of oscillation frequencies that has been ignored in all previous theoretical studies of QPOs may be important for our understanding of a phenomenon of QPOs in binary systems with neutron stars and black holes. Our conclusion about relevance of this effect to the oscillations in the innermost region of accretion disk around a compact object is robust and does not depend on a particular model of a disk. If the QPO phenomenon is associated with such oscillations, then we must observe the rotationally split frequencies. The detailed study of this effect may provide us with very important information about the physics of accretion disks.

**2. The Effect of Rotational Splitting by an Accretion Disk**

It is well known (see e.g. Unno et al. 1979, and references therein) that in a rotating star the non-radial oscillations are split:  $\sigma_{klm} = \sigma_{kl} + m\Omega_* C_{kl}$ , where  $\sigma_{kl}$  is the frequency in the nonrotating star,  $\sigma_{klm}$  is the frequency in a rotating star (in the corotating frame),  $m$  is an azimuthal number,  $\Omega_*$  is the stellar rotation frequency, and  $C_{kl}$  is expressed in terms of an integral that depends on the stellar structure and on the eigenfunction. Thus, in a rotating star, a nonradial oscillation drifts at a rate  $m\Omega_* C_{kl}$  relative to a fixed longitude of the star. The effect of rotational splitting is due to the Coriolis force and is similar to the Zeeman effect in a magnetic field.

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The oscillations of a rotating disk must split as well. The rotational splitting of the disk oscillations is particularly important for those modes whose frequencies are comparable to the local rotation frequency of a disk relative to a distant observer. The detailed theory for a star is given by Ledoux & Walraven (1958) and by Unno et al. (1979). Here we present calculations of the splitting effect for an oscillation frequency due to the Coriolis force in a disk geometry.

The general formula for the correction to the oscillation frequency of a star/disk produced by the Coriolis force reads (cf. Unno et al. 1979, equation [18.28])

$$\sigma^{(1)} = \frac{-i \int_V [\mathbf{\Omega} \times \boldsymbol{\xi}_{k,m}^{(0)}] \boldsymbol{\xi}_{k,m}^{(0)*} dV}{\int_V \boldsymbol{\xi}_{k,m}^{(0)} \boldsymbol{\xi}_{k,m}^{(0)*} dV}. \quad (1)$$

Here  $\mathbf{\Omega}$  is the local angular velocity in a disk in the laboratory frame of reference,  $\boldsymbol{\xi}_{(k,m)}^{(0)}(t, r, \varphi, z)$  is the  $(k,m)$ -displacement component, and the integration is over the volume of a disk/ring-like configuration.

Let us consider the oscillations localized in some annulus of a disk at radius  $R$ . We assume that the amplitude of these oscillations decreases toward the innermost edge of a disk. Then, the component of a displacement vector can be calculated by using the  $(k, m)$ -harmonics of a complete set of eigenfunctions for a disk/ring-like configuration  $\{U_{k,m} = ru_{k,m}(r, \varphi, z, t)\}$ , where  $r, \varphi, z$  are cylindrical coordinates, and  $t$  is the time in a laboratory frame of reference. In equation (1) the integration has to be taken over the volume of a ring of radius  $R$ , width  $\Delta R$ , and half-thickness  $H$ . The vectors  $\boldsymbol{\xi}_{(k,m)}^{(0)}(t, r, \varphi, z)$  are usually presented in such a form where their components are proportional to the corresponding components of a gradient of a potential  $U_{k,m} = ru_{k,m}$ . In cylindrical coordinates they can be written as (cf. Unno et al. 1979, equation [18.28])

$$\boldsymbol{\xi}_{k,m}^{(0)} = \left[ \xi_r, \xi_\varphi \frac{\partial}{\partial \varphi}, \xi_z r \frac{\partial}{\partial z} \right] u_{km}. \quad (2)$$

The functions  $u_{k,m}$  satisfy the free-boundary conditions at  $z = \pm H$  (see e.g. Morse & Feshbach 1953):

$$\frac{\partial u_{k,m}}{\partial z}(t, H, \varphi) = \frac{\partial u_{k,m}}{\partial z}(t, -H, \varphi) = 0. \quad (3)$$

Also, these functions must be symmetric about the disk plane

$$u_{k,m}(t, z, \varphi) = u_{k,m}(t, -z, \varphi), \quad (4)$$

where  $0 \leq z \leq H$ .

In the case of uniform rotation, the temporal and azimuthal dependences of eigenfunctions  $u_{k,m}$  are given by  $\exp[i(m\varphi - \Omega_0 t)]$ , where  $\Omega_0$  is the angular eigenfrequency of the oscillation (see Unno et al. 1979), thus, the component  $u_{k,m}$  can be written as

$$u_{k,m} = e^{im\varphi} \cos(\pi kz/H) e^{-i\Omega_0 t}, \quad (5)$$

where  $m = \pm 1, \pm 2, \dots$ , and  $k = 1, 2, \dots$ .

Note that the functions  $u_{k,m}$  are eigenfunctions of a two-dimensional Laplace operator. They satisfy the free-boundary and symmetry conditions over a  $z$ -coordinate (see equations [3],[4]) and the periodicity condition over a  $\varphi$ -coordinate. It is important that these eigenfunctions form a complete set of functions (a fundamental system) which can be used as a basis for the expansion (see e.g. Morse & Feshbach 1953) of any arbitrary displacement  $\boldsymbol{\xi}^{(0)}$ .

The numerator and denominator in formula (1) are given by

$$I_1 = -2\pi HR \Delta R (2\Omega m \hat{\xi}_r \hat{\xi}_\varphi), \quad (6)$$

and

$$I_2 = 2\pi HR \Delta R [\hat{\xi}_r^2 + (\hat{\xi}_z R/H)^2 \pi^2 k^2 + m^2 \xi_\varphi^2], \quad (7)$$

respectively. Here  $\hat{\xi}_r, \hat{\xi}_\varphi$ , and  $\hat{\xi}_z$  are the average-weighted displacements for a disk/ring-like configuration in radial, azimuthal, and vertical directions, respectively. To illustrate the effect of rotational splitting for a disk/ring-like configuration we may justifiably assume that  $\hat{\xi}_r \sim \hat{\xi}_\varphi$ , then we arrive at

$$\sigma^{(1)} = -\frac{2m\Omega}{s\pi^2 k^2 + m^2 + 1}, \quad (8)$$

where  $s(R/H) = (\hat{\xi}_z R/\hat{\xi}_r H)^2$  is a function determined by the vertical structure of a disk. For more or less realistic structure of a disk  $s \lesssim 1$  and depends on the ratio  $R/H$ , which is either  $\gtrsim 1$  (near a compact object) or  $\gg 1$  (far away from a compact object). We must note that the relationship between  $\hat{\xi}_r, \hat{\xi}_\varphi$ , and  $\hat{\xi}_z$  depends also on the azimuthal and vertical mode numbers. For example, consider an individual oscillation mode with displacement  $\boldsymbol{\xi}_{(k,m)}^{(0)}$  in an incompressible fluid. Then  $\nabla \cdot \boldsymbol{\xi}_{(k,m)}^{(0)} = 0$ , and we get a linear partial differential equation that determines a relationship between  $\hat{\xi}_r, \hat{\xi}_\varphi$ , and  $\hat{\xi}_z$ . This relationship, according to expression (2), should depend on the azimuthal and vertical mode numbers,  $m$  and  $k$ , respectively.

Thus, the frequency of the oscillations detected by a distant observer is given by (cf. Unno et al. 1979, equation [18.33])

$$\begin{aligned} \Omega_{k,m} &\equiv \Omega_0 + m\Omega + \sigma^{(1)} = \\ &= \Omega_0 + m\Omega \left( 1 - \frac{2}{s\pi^2 k^2 + m^2 + 1} \right), \end{aligned} \quad (9)$$

where  $m$  and  $k$  are the azimuthal and vertical mode numbers, respectively.

### 3. Explanation of QPO Frequencies in Terms of Rotational Splitting

To estimate the frequency splits, we may adopt  $\Omega_0 = \Omega$ , which is a very good approximation for a nearly Keplerian accretion disk. In this case, for the lowest-order modes with  $m = 0, -1$  and  $k = 1, 2, 3, 4$ , and 5 the oscillation frequencies can be calculated as

$$\Omega_{k,0} = \Omega_0,$$

$$\Omega_{1,-1} = 2\Omega_0/(s\pi^2 + 2),$$

$$\Omega_{2,-1} = 2\Omega_0/(s4\pi^2 + 2),$$

$$\Omega_{3,-1} = 2\Omega_0/(s9\pi^2 + 2),$$

$$\Omega_{4,-1} = 2\Omega_0/(s16\pi^2 + 2),$$

and

$$\Omega_{5,-1} = 2\Omega_0/(s25\pi^2 + 2),$$

respectively.

For  $m = -2$  and  $k = 1$  we have

$$|\Omega_{1,-2}| = \Omega_0[1 - 4/(s\pi^2 + 5)].$$

Let us assume that  $\nu_0 \equiv \Omega_0/2\pi = 1200$  Hz. Then, for the most plausible range of  $s \approx 0.7 - 1$  we get the following frequencies (arranged in ascending order, in Hz):

$$\nu_{5,-1} \approx 10 - 14 ,$$

$$\nu_{4,-1} \approx 15 - 20 ,$$

$$\nu_{3,-1} \approx 30 - 40 ,$$

$$\nu_{2,-1} \approx 60 - 80 ,$$

$$\nu_{1,-1} \approx 200 - 300 ,$$

and

$$\nu_{1,-2} \approx 800 - 880 .$$

These frequencies perfectly match the observed QPO frequencies in LMXBs. It is important that for the modes with a low azimuthal number  $m = -1$  and high  $k$ -numbers (high overtones in the vertical direction) the ratio between frequencies does not depend on the function  $s$ , rather it is solely determined by a quantum number  $k$ :  $\nu_{k,-1}/\nu_{k+1,-1} \approx (k+1)^2/k^2$  ( $k \geq 2$ ). The frequencies of modes with  $m = -2$  and  $k \geq 2$  concentrate to  $\nu_{k,-2} \approx \nu_0$ . It must be pointed out that the modes with  $m = 1, 2, 3, \dots$  and  $k \geq 1$  have frequencies concentrating to the frequency  $(m+1)\nu_0$ . Perhaps, these modes are not allowed or, for some reason, are not excited. In this regard, it would be interesting to investigate the selection rule for the oscillations in the innermost part of accretion disk.

It is important to note that a contribution of each individual  $(k, m)$ -component to the power spectrum of oscillations is determined by the smoothness of a function characterizing the perturbed surface. It is well-known from the Fourier analysis that the power of the  $l$ -component is  $\sim l^{-2N}$ , where  $N$  is the order of highest existing derivative of a function describing the perturbed surface with respect to the corresponding coordinate. In the case of the oscillations of the innermost part of accretion disk,  $l = m$  or  $l = k$  for the azimuthal or vertical perturbations respectively. It is very likely that the azimuthal perturbations are smooth, so that we may expect the presence of only the lowest order modes with  $|m| = 0, 1, 2$  in the power spectrum. On the contrary, the vertical perturbations are essentially discontinuous, and their Fourier spectra should contain a large number of modes with  $k > 1$ , with the contribution of each of these modes to the power spectrum  $\sim k^{-2}$ .

The occurrence of the localized oscillations in the disk may indicate that there is a local release of (e.g. gravitational) energy

in addition to the energy of viscous dissipation. If so, then the local plasma temperature should be  $T \sim 10 - 20$  keV, which is much higher than the temperature in a standard viscous disk (see Shakura & Sunyaev 1973). The possibility of such oscillations, their energetics, high quality  $Q$ , and substantial modulation of the observed X-ray luminosity due to these oscillations, are discussed by Titarchuk et al. (1997) in the context of the effect of a centrifugal barrier which is thought to take place in a boundary region surrounding a neutron star.

#### 4. Summary

We suggest that in the QPO phenomenon we observe the effect of a rotational splitting of the oscillation frequency. For a Keplerian disk this effect should be most pronounced because the characteristic frequency of the oscillations of a centrifugal-barrier region is of the order of the Keplerian frequency.

We conclude that the observed three frequencies ranging from 300 to 1170 Hz can be naturally interpreted in terms of a rotational splitting of the main frequency: the lowest frequency,  $\sim 200$ -300 Hz, corresponds to a mode with  $m = -1$  and  $k = 1$ ; the higher frequency,  $\sim 800$  Hz, corresponds to a mode with  $m = -2$  and  $k = 1$ , and the highest frequency is the main frequency of the oscillations. The issue of the excitation of these modes and emission mechanism will be addressed elsewhere.

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