Astron. Astrophys. 323, 151–157 (1997)

ASTRONOMY
AND
ASTROPHYSICS

On the cycle periods of stellar dynamos

A.J.H. Ossendrijver

SRON Laboratorium voor Ruimteonderzoek, Sorbonnelaan 2, 3584 CA Utrecht, The Netherlands

Received 11 June 1996 / Accepted 6 December 1996

Abstract. We show that the cycle periods ($P_{\rm cyc}$) of slowly rotating lower main-sequence stars with *well-defined* periodic chromospheric activity can be parametrized by the rotation period $P_{\rm rot}$ and the convective turnover time $\tau_{\rm c}$ through a relation of the form $P_{\rm cyc} \propto P_{\rm rot}^b \tau_{\rm c}^c$, with $b = 2.0 \pm 0.3$ and $c = -2.0 \pm 0.3$. This suggests a common dynamo mechanism for slowly rotating stars lower main-sequence stars. Using a simple linear mean-field dynamo model, we are able to reproduce the observed relation if $\Delta\Omega$, the total difference in angular velocity along the radial direction, scales as $\Delta\Omega \propto P_{\rm rot}^p$ with $p = 1.1 \pm 0.2$, and if the α-coefficient scales as $\alpha \propto {\rm Ro}^q$ with $q = -5.1 \pm 0.6$. This would suggest that, with increasing rotation rate, differential rotation decreases while $|\alpha|$ rapidly increases.

Key words: stars: magnetic fields – stars: activity – stars: late-type – MHD

1. Introduction

The flux that is emitted in the Ca II H and K lines serves as a diagnostic of stellar magnetic activity (Wilson 1978, Saar & Baliunas 1992, Baliunas et al. 1995). These emission lines are formed in the chromosphere by non-thermal heating related to magnetic fields. Direct confirmation of the magnetic nature of Ca II H and K emission is observed on the Sun, where the strength of these lines is correlated with the magnitude of, and the area covered by magnetic fields (Schrijver et al. 1989). On the main sequence, chromospheric activity is observed for $B-V \gtrsim 0.4$, a limit which roughly coincides with the onset of a convective envelope. This suggests that stellar magnetic fields are produced by dynamos and raises the question whether various features of stellar magnetic activity can be parametrized by quantities related to dynamo theory.

The overall chromospheric activity level of lower main-sequence stars is well parametrized by an *empirical* Rossby number, ${\rm Ro}_{\rm e} = P_{\rm rot}/\tau_{\rm e}$ (Noyes et al. 1984a, Stępień 1994). Here $P_{\rm rot}$ is the stellar rotation period and $\tau_{\rm e}(B-V)$ is the *empirical* convective turnover time, whose functional dependence on $B-V$ is determined from the data themselves by minimizing the scatter in the relation between activity and ${\rm Ro}_{\rm e}$. The resulting parametrization indicates that stellar activity increases with decreasing ${\rm Ro}_{\rm e}$ (more rapid rotation). For $B-V \lesssim 0.8$ the empirical $\tau_{\rm e}$ closely resembles $\tau_{\rm c}$, the *theoretical* turnover time near the base of the convection zone, (Gilman 1980, Gilliland 1985, Kim & Demarque 1996). For $B-V \gtrsim 0.8$, however, the activity level depends only on $P_{\rm rot}$ and not, or only mildly, on $B-V$ (Stępień 1989). Hence the empirical turnover time $\tau_{\rm e}$ is essentially constant for $B-V \gtrsim 0.8$. This is not in disagreement with the results of Gilman, since his calculations did not extend beyond $B-V = 0.8$, but the calculations by Gilliland and Kim & Demarque reveal a further increase of $\tau_{\rm c}$ with $B-V$. It follows that for $B-V \gtrsim 0.8$ the Rossby number is not a useful indicator of the chromospheric activity level.

Four main categories of chromospheric activity are identified by Baliunas et al. 1995), namely stars with a constant activity level (13%), long-term variations (13%), irregular variations (24%) and periodic variations (50%). Here the percentages indicate the fraction of stars within each category, as estimated by Baliunas et al. In this paper we focus on stars with periodic variations. The assignment of a star to this category rather than to that of the irregular variations or long-term trends depends on the confidence level at which one requires the periods to be determined. In fact, the percentage of stars that have *well-defined* cyclic variations, i.e. those with periods rated *good* or *excellent* by Baliunas et al., is 15%. Furthermore, it is hard to distinguish between stars with cyclic variations on timescales longer than about 20 years and stars with long-term variations or a constant activity level within the time interval (about 25 years), spanned by the observations. Only continued observations can resolve these issues by increasing the reliability of the period determinations and by allowing longer periods to be detected.

As a result, the search for possible trends in the cycle length $P_{\rm cyc}$ has yielded mixed results. Noyes et al. (1984b) found a correlation between $P_{\rm cyc}$ and ${\rm Ro}_{\rm e}$ for a small sample of stars with clear periodic variations. In the mean time, the sample of stars with periodic activity variations has grown, and it has been claimed that there is no longer any evidence of a correlation between cycle length and rotation period or Rossby number for the extended sample (Soderblom 1988, Saar & Baliunas 1992).

However, a trend may be concealed by a large spurious scatter that is caused by stars with ill-defined cycle periods. Several

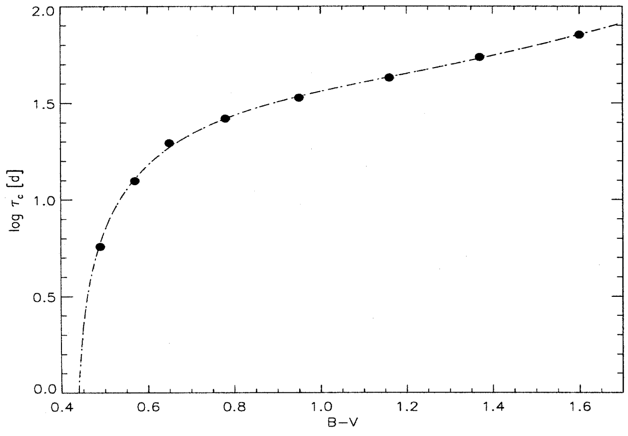

Fig. 1. Logarithm of the convective turnover time log τ_c (in days) versus $B-V$, for lower main-sequence stars. The data points represent model calculations by Kim & Demarque (1996); the dotted line is a cubic fit (Eq. 2).

further arguments can be given to justify a renewed investigation of possible trends in the cycle length. First, we now have at our disposal estimates of the convective turnover time for lower main-sequence stars up to $B-V \approx 1.6$ (Kim & Demarque 1996). Most of the stars studied by Noyes et al. (1984b) are in the range $0.8 < B-V < 1.4$, for which no estimates of τ_c - apart from the empirical τ_e - were available at that time. Second, the accuracy of the measured cycle periods has increased due to the longer span of observations. Third, for several stars new measurements of the rotation periods are available, that differ from previous values, or replace values that were predicted by means of the observed correlation between the Ca II flux and Ro_e.

In Sect. 2 we examine the available cycle periods and look for trends in terms of dynamo parameters. In Sect. 3 we compare the observed trends with a simple linear mean-field dynamo model. Sect. 4 contains our conclusions.

2. Activity-cycle length, rotation period and color

The stars of the HK-project can be divided into two groups according to their rotation rate: rapidly rotating young stars with a high activity level and slowly rotating old stars with a low activity level (Baliunas et al. 1995, 1996b). Activity variations with well-defined periods are observed predominantly in older stars. Younger stars tend to display stronger, more irregular activity variations. We focus on the slowly rotating stars, which we define as those stars having a Rossby number larger than 0.9, and we exclude the rapidly rotating stars from our analysis. The resulting subset is similar to the group of solar-type stars examined by Soon et al. (1994) and Baliunas & Soon (1995).

In Table 1 we summarize the relevant properties of all known lower main-sequence stars with well-defined activity cycles. The third column gives cycle periods, compiled from Baliunas et al. (1995). We have included only stars with well-defined cycles (those rated "good" or "excellent"), as well as

Table 1. Stars with periodic chromospheric activity

HD No.	$B-V$	P_{cyc} [a] [yr]	P_{rot} [b] [d]	τ_c [c] [d]	Ro
Sun	0.66	11.0	25	19	1.32
3651	0.85	13.8	44	30	1.48
4628	0.88	8.4	38	31	1.22
10476	0.84	9.6	39	29	1.33
16160	0.98	13.2	48	35	1.35
26965	0.82	10.1	43	28	1.52
32147	1.06	11.1	47	39	1.21
81809	0.89[d]	8.2	41	32	1.30
103095	0.75	7.3	31	25	1.26
160346	0.96	7.0	37	35	1.07
166620	0.89	15.8	43	32	1.36
219834A	0.80	21.0	42	27	1.54
219834B	0.91	10.0	43	32	1.32
9562[e]	0.64	> 20	29	18	1.62
10700[e]	0.72	> 20	34	23	1.48
141004[e]	0.60	> 20	26	15	1.72
143761[e]	0.60	> 20	17	15	1.13
115404[f]	0.93	12.4	18	33	0.54
152391[f]	0.76	10.9	11	25	0.44
156026[f]	1.16	21.0	21	43	0.42
201091[f]	1.18	7.3	35	44	0.80
201092[f]	1.37	11.7	38	54	0.71

[a] Cycle periods taken from Baliunas et al. (1995).
[b] Rotation periods taken from Baliunas et al. (1996a).
[c] Convective turnover times taken from Kim & Demarque (1996).
[d] This star is a binary. We estimated $B-V$ assuming that the chromospheric activity can be attributed to a star of type K0V (cf. Baliunas et al. 1995).
[e] These stars are possibly in a Maunder-minimum phase according to Baliunas & Soon (1995).
[f] These stars rotate rapidly (Ro $\lesssim 0.9$) and have a high level of activity. HD 201091 is an intermediate case.

four stars that may be in the equivalent of a Maunder minimum or, alternatively, have cycle periods longer than about 20 years (Baliunas & Soon 1995). There are no stars with well-defined periods shorter than 7 years. The intervals between consecutive maxima in the sunspot record, as measured since the beginning of the 18th century, have a mean length of 11 years, and a standard deviation of about 2 years, i.e. 18%. This is taken to be indicative for the variability of stellar cycles. We thus estimate the deviation of the measured cycle period from its mean value as

$$\sigma_P \approx 0.18 P_{cyc}/\sqrt{25/P_{cyc}}, \quad (1)$$

where $25/P_{cyc}$ is the number of cycles covered by 25 years of observations.

The fifth column contains the convective turnover times τ_c near the bottom of the convection zone. These are based on calculations by Kim & Demarque (1996) of the local turnover time (in fact, half the *global* turnover time) for lower main-sequence stars in the mass range $0.5 \leq M/M_\odot \leq 1.2$. Their

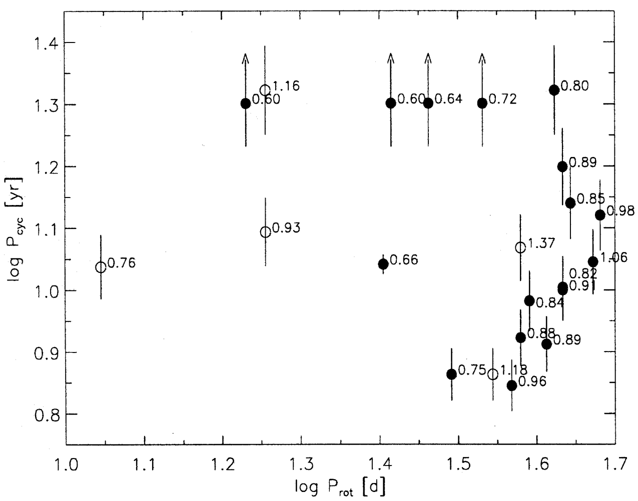

Fig. 2. $\log P_{\rm cyc}$ versus $\log P_{\rm rot}$, for all the stars in Table 1. Open symbols denote the rapidly rotating stars; arrows denote the four 'Maunder-minimum stars'. The labels indicate $B-V$.

results indicate that $\tau_{\rm c}$ is roughly independent of age on the main sequence, depending only on stellar mass, i.e. on $B-V$. We chose a stellar age of 2 Gyr and we performed a cubic fit in terms of $x \equiv B-V$ to their data,

$$\tau_{\rm c} = -68.3 + 224.8x - 177.2x^2 + 57.0x^3, \quad (2)$$

where $\tau_{\rm c}$ is in days. The resulting curve is shown in Fig. 1. We reiterate that, unlike $\tau_{\rm e}$, the increase of $\tau_{\rm c}$ with $B-V$ does not come to a halt or slow down significantly beyond $B-V = 0.8$.

In trying to identify trends in the cycle periods of slowly rotating stars we proceed along lines similar to those followed by Noyes et al. (1984b). As a first step we consider the dependence on the rotation period. In Fig. 2 we plot the cycle period $\log P_{\rm cyc}$ versus $\log P_{\rm rot}$ - the slowly rotating stars are indicated by the full circles. For stars with similar values of $B-V$ we observe an increase of $P_{\rm cyc}$ with increasing $P_{\rm rot}$. But stars of different spectral type seem to be located on different curves, that are shifted to the left by an amount that increases with decreasing $B-V$. Hence the cycle period cannot be parametrized by the rotation period alone and must also involve a color-dependent term, which may be provided by the convective turnover time. We assume that $P_{\rm cyc}$ depends on $P_{\rm rot}$ and $\tau_{\rm c}$ through a powerlaw, i.e.

$$P_{\rm cyc} = 10^a P_{\rm rot}^b \tau_{\rm c}^c, \quad (3)$$

where $P_{\rm cyc}$ is measured in years, and $P_{\rm rot}$ and $\tau_{\rm c}$ in days. We determine a, b and c by performing a least square fit of the form $\log P_{\rm cyc} = a + b\log P_{\rm rot} + c\log\tau_{\rm c}$. The uncertainty in the values of $\log P_{\rm cyc}$ is $\sigma' = \sigma_{\rm P}/(P_{\rm cyc}\ln 10)$, and we assign a weight $1/\sigma'^2$ to each data point. The best fit has $a = 0.84 \pm 0.15$, $b = 1.99 \pm 0.3$ and $c = -2.03 \pm 0.3$, with correlation coefficients $C_{ab} = -0.49$, $C_{ac} = 0.21$ and $C_{bc} = -0.96$. This yields $c/b = -1.02 \pm 0.05$ (note the effect of the strong anticorrelation between b and c).

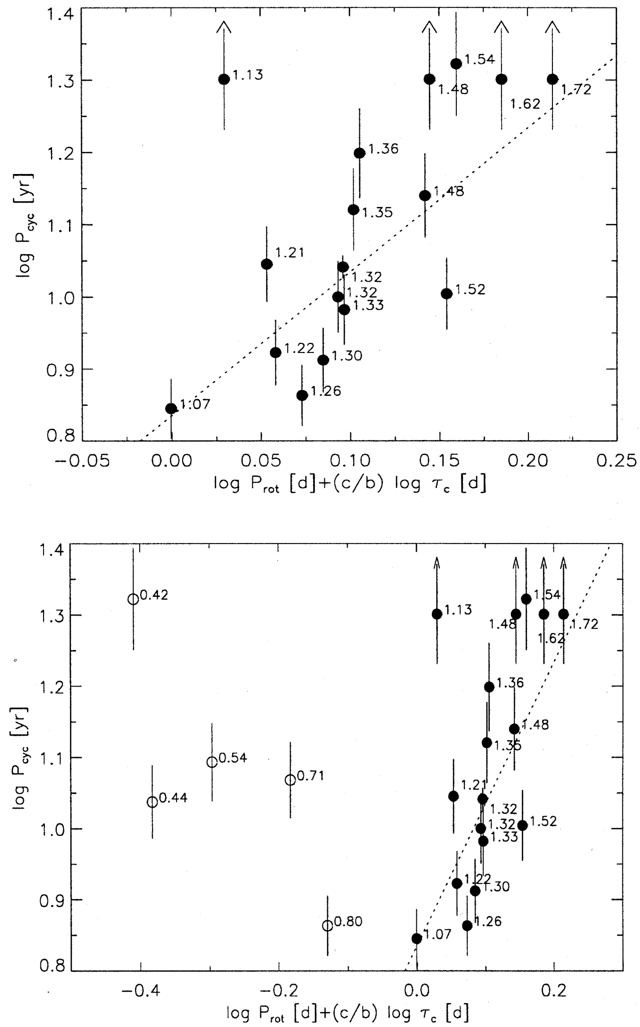

Fig. 3. $\log P_{\rm cyc}$ versus $\log P_{\rm rot} + (c/b)\log\tau_{\rm c}$, with $c/b = -1.02$ (the uncertainties in a, b and c are not indicated in this plot). The dotted line, defined by $y = a + bx$, results from a least square fit applied to the slowly rotating stars (excluding the four 'Maunder-minimum stars'). The symbols have the same meaning as in Fig. 2. Labels indicate the Rossby number. Top (a): slowly rotating stars. Bottom (b): the complete sample of stars.

Thus our result is in agreement with a parametrisation $P_{\rm cyc} \propto {\rm Ro}^{2.0\pm0.3}$, where Ro is the Rossby number,

$${\rm Ro} = P_{\rm rot}/\tau_{\rm c}. \quad (4)$$

In Fig. 3 we plot $\log P_{\rm cyc}$ versus $\log P_{\rm rot} + (c/b)\log\tau_{\rm c}$, so that all the data points are shifted along the horizontal axis by the amount $(c/b)\log\tau_{\rm c}$, relative to their positions in Fig. 2.

Some of the measured cycle periods show significant deviations from the best fit, for which their may be two reasons. First, our estimates of $\sigma_{\rm P}$ are based on the assumption that the level of variability in $P_{\rm cyc}$ (cf. Eq. 1) is the same for all the stars in our sample. Perhaps we underestimate the variability in some cases. Second, we have ignored the uncertainty in the rotation periods. No reliable estimates of this uncertainty are available, but it may contribute to the scatter.

Although the four "Maunder-minimum stars" were not included in the least square fit because their cycle periods are (as yet) unknown, the resulting shift puts three of them at a location in Fig. 3, that is roughly in agreement with a cycle period of about 20 years. This suggests that their chromospheric activity may prove to be periodic in the future.

The cycle periods of rapidly rotating stars, indicated in Fig. 3b by the open circles, are much longer than what would be expected on the basis of the relation for the cycle periods of slowly rotating stars, and the deviation appears to increase with decreasing Rossby number. Hence the powerlaw as derived for the slowly rotating (old) stars does not hold for rapidly rotating (young) stars.

3. Dynamo model

The existence of a correlation between $P_{\rm cyc}$, $P_{\rm rot}$ and $\tau_{\rm c}$ points to a common dynamo mechanism for the stars in the sample under consideration. We compare the observed correlation with the predictions of a simple model, based on linear mean-field dynamo theory. Although the validity of the linear approach is open to debate (cf. Noyes et al. 1984b, Jennings & Weiss 1991, Rüdiger & Arlt 1996), it may be justified by the slow rotation rate and low activity level of the selected stars.

3.1. Geometry and equations

The dynamo model that we employ was proposed by Parker (1993) for the Sun. It consists of two plane parallel layers: the overshoot layer (region 1) and the convection zone (region 2), with thicknesses d_1 and d_2 respectively. The main motivation for the model arises from the presence of strong magnetic fields ($B \approx 10^5$ G) in a thin layer under the convection zone, as is suggested by observations and theoretical considerations (Hughes 1992, Schüssler et al. 1994). The strong fields give rise to the suppression of turbulence, so that α and β are reduced. Helioseismology suggests that differential rotation is concentrated near the same layer (Goode 1995). Hence differential rotation and the α-effect are possibly spatially separated, the former being restricted to the overshoot layer and the latter to the convection zone. Some turbulent diffusion is required in the overshoot layer in order to provide communication with the convection zone.

We assume that this model applies for all the stars in our sample. We use x for the radial, y for the azimuthal, and z for the latitudinal coordinates, and consider only axisymmetric solutions ($\partial/\partial y = 0$). The overshoot layer is located at $-d_1 \le x \le 0$ and the convection zone at $0 \le x \le d_2$.

We model the results of helioseismology for the equatorial region in a schematic way by adopting the following large-scale velocity field $\boldsymbol{u}_0$:

$$\boldsymbol{u}_0 = u_0(x)\boldsymbol{e}_y, \qquad \partial_x u_0 = \begin{cases} a & (-d_1 \le x \le 0) \\ 0 & (0 \le x \le d_2). \end{cases} \tag{5}$$

Here the constant a denotes the radial velocity gradient.

The mean magnetic field can be written as the sum of a toroidal and a poloidal component, i.e. $\boldsymbol{B}_0 = T\boldsymbol{e}_y + \nabla \times P\boldsymbol{e}_y$. It is governed by the following equations (Parker 1993, Ossendrijver & Hoyng 1996):

$$\left.\begin{array}{rcl} (\partial_t - \beta_1\nabla^2)\,P &=& 0 \\ (\partial_t - \beta_1\nabla^2)\,T &=& -a\partial_z P \end{array}\right\} \quad (-d_1 \le x \le 0),$$
$$\left.\begin{array}{rcl} (\partial_t - \beta_2\nabla^2)\,P &=& \alpha T \\ (\partial_t - \beta_2\nabla^2)\,T &=& 0 \end{array}\right\} \quad (0 \le x \le d_2). \tag{6}$$

Here the α-coefficient and the turbulent diffusivity in the convection zone are given by

$$\alpha = -\frac{1}{3}\tau_{\rm c}\langle \boldsymbol{u}_1 \cdot (\nabla \times \boldsymbol{u}_1)\rangle; \qquad \beta_2 = \frac{1}{3}\tau_{\rm c}\langle u_1^2\rangle, \tag{7}$$

where $\boldsymbol{u}_1 = \boldsymbol{u} - \boldsymbol{u}_0$ denotes the turbulent velocity field, having a correlation time $\tau_{\rm c}$. The suppression of the turbulent diffusivity by strong magnetic fields in the overshoot layer is parametrized by a factor

$$f_\beta = \frac{\beta_1}{\beta_2} \ll 1. \tag{8}$$

We seek solutions of the form $P = p(x)\exp({\rm i}k_z z + \lambda t)$, and similarly for T. Here k_z is the wave vector in the latitudinal direction. The boundary conditions are as follows (cf. Ossendrijver & Hoyng 1996):

$$\begin{array}{ll} P = \partial_x T = 0 & \text{at } x = -d_1, \\ [\![P]\!] = [\![T]\!] = [\![\partial_x P]\!] = [\![\beta\partial_x T]\!] = 0 & \text{at } x = 0, \\ (\partial_x + k_z)\,P = T = 0 & \text{at } x = d_2. \end{array} \tag{9}$$

Here $[\![\cdots]\!]$ denotes the jump at the boundary. In the second line, β assumes the value β_1 for $x < 0$ and β_2 for $x > 0$. We separate λ in complex and real parts according to

$$\lambda = {\rm i}\omega + \gamma, \tag{10}$$

where $\omega = \pi/P_{\rm cyc}$ is the dynamo frequency and γ is the mean-field growth rate. We solve the dispersion relation numerically and we identify the fundamental mode, i.e. that with the largest growth rate. The diffusivities β_1 and β_2 are tuned in such a way, that the fundamental mode is slightly subcritical, having a decay time $\tau_{\rm dec} = 1/|\gamma|$ of about 20 cycle periods. For a motivation of this requirement, and for analytical solutions of Eq. (6) we refer to Ossendrijver & Hoyng (1996).

3.2. Dynamo parameters of lower main-sequence stars

In this section we present expressions for the parameters that occur in the dynamo equation, in terms of the stellar structure and the rotation rate. Some of these expressions relate stellar parameters to solar parameters, which are treated in Sect. 3.4.

Apart from $\tau_{\rm c}$ (Eq. 2), the parameters related to stellar structure are derived from a set of models by Copeland et al. (1970).

Table 2. Parameters of lower main-sequence stars

$M/M_{\odot}$	0.7	0.8	0.9	1.0	1.1
$B-V$ [a]	1.16	0.95	0.78	0.65	0.57
$R_{\rm c}$ [10^{10}cm] [b]	3.16	3.61	4.14	4.97	6.04
$H_{\rm p}$ [10^{9}cm] [b]	4.40	4.73	4.99	4.91	4.30
d_2 [10^{10}cm] [b]	1.51	1.62	1.71	1.60	1.30

[a] $B-V$ taken from Kim & Demarque (1996), assuming a stellar age of 2 Gyr.
[b] Compiled from Copeland et al. (1970).

Out of the models presented by these authors we have chosen the series with a composition given by $X = 0.7$, $Y = 0.27$, $Z = 0.03$, and with a mixing-length parameter $\alpha_{\rm ML} = l_{\rm ML}/H_{\rm p} = 1.5$ (Table 2). We employ the pressure scale height at the bottom of the convection zone, $H_{\rm p} = kT/\mu m_{\rm H} g$, to define the thickness d_1 of the overshoot layer (Skaley & Stix 1991):

$$d_1 = 0.4\, H_{\rm p}. \tag{11}$$

For the Sun, this amounts to about 2×10^4 km (Table 2). We assume that the total difference $\Delta\Omega$ in angular velocity accross the overshoot layer depends on the rotation rate in the following manner:

$$\Delta\Omega = \Big(\frac{P_{\rm rot}}{P_{{\rm rot},\odot}}\Big)^{p} \Delta\Omega_{\odot}. \tag{12}$$

No reliable estimate is known for p, but there are theoretical indications that p is *positive*, so that differential rotation decreases with increasing rotation rate (Kitchatinov & Rüdiger 1995). The corresponding velocity gradient in the overshoot layer can be expressed as

$$a = \frac{R_{\rm c}\Delta\Omega}{d_1} = \frac{R_{\rm c}\Delta\Omega_{\odot}}{d_1}\Big(\frac{P_{\rm rot}}{P_{{\rm rot},\odot}}\Big)^{p}, \tag{13}$$

where $R_{\rm c}$ denotes the distance from the origin to the bottom of the convection zone (Table 2).

Following Eq. (7), we estimate α as $\alpha \approx -\frac{1}{3} H\tau_{\rm c}\langle u_1^2\rangle/l \approx -\frac{1}{3} H H_{\rm p}/\tau_{\rm c}$, where $l \approx H_{\rm p} \approx u_{1,{\rm rms}}\tau_{\rm c}$ is the typical length scale of the convective cell and H denotes the normalised helicity coefficient,

$$H = \frac{\langle \boldsymbol{u}_1 \cdot (\nabla \times \boldsymbol{u}_1)\rangle}{\sqrt{\langle u_1^2\rangle\langle|\nabla \times \boldsymbol{u}_1|^2\rangle}}. \tag{14}$$

This coefficient measures the correlation between $\boldsymbol{u}_1$ and the vorticity $\nabla \times \boldsymbol{u}_1$. It is non-vanishing due to the effect of rotation on the convective motions, which suggests a dependence on the Rossby number. Since $|H| \leq 1$, the correlation must saturate for small values of Ro (rapid rotation), but if rotation is slow, we may assume $H = H(1){\rm Ro}^q$, with $q < 0$. Here $H(1)$, the normalised helicity for Ro = 1, is taken to be the same for all the stars in our sample. Its value is fixed by the solar calibration model (Sect. 3.4). The α-coefficient now becomes

$$\alpha = -\frac{1}{3} H(1)\, {\rm Ro}^q H_{\rm p}/\tau_{\rm c}. \tag{15}$$

Here the convective turnover time is obtained from $B-V$ (Table 2) with the help of expression (2).

We assume that all the stars have activity belts similar to those of the Sun. These activity belts originate at a latitude of about 35° during the activity minimum and migrate toward the equator in the course of one cycle period, after which new belts of opposite polarity appear. The wave vector in the latitudinal direction that is associated with this equatorward migration is estimated as

$$k_z = \frac{360}{70}\frac{1}{R_{\rm c}}. \tag{16}$$

The turbulent diffusivities are determined in the following way. We determine the ratio $f_\beta = \beta_1/\beta_2$ by means of the solar calibration model in Sect. 3.4 and we assume that it is the same for all the stars. The value of β_2 (and of $\beta_1 = f_\beta\beta_2$) is fixed by a condition on the growth rate of the mean magnetic field, see Sect. 3.5. However, we shall demonstrate in Sect. 3.4 that, to good approximation, the cycle period depends on β_1 and β_2 only through their ratio f_β, so that $P_{\rm cyc}$ is not affected by this condition.

3.3. Theoretical cycle periods

The exact cycle periods of stellar dynamos are found by numerically solving the dispersion relation that results from Eq. (6). In order to gain insight in the parameter dependence of $P_{\rm cyc}$ we employ an approximative expression. This allows us to compare in a simple manner the observed relation between $P_{\rm cyc}$, $P_{\rm rot}$ and $\tau_{\rm c}$ with the predictions of our model. Ossendrijver & Hoyng (1996) derived the following expression, using an approximative dispersion relation which is valid if $f_\beta \ll 1$:

$$P_{\rm cyc} \approx \frac{\pi\sqrt{8}}{\beta_2 k_z^2}\big\{-1 + \sqrt{1 + C^2}\big\}^{-1/2}, \tag{17}$$

where C is the dynamo number, given by

$$C = \frac{a\alpha f_\beta}{\beta_2^2 k_z^3}. \tag{18}$$

A useful approximation is obtained for large dynamo numbers ($|C| \gg 1$):

$$P_{\rm cyc} \approx \frac{\pi\sqrt{8}}{\beta_2 k_z^2}|C|^{-1/2} = \pi\sqrt{8}\,\big(a|\alpha|k_z f_\beta\big)^{-1/2}. \tag{19}$$

Notice that $P_{\rm cyc}$ now depends on the turbulent diffusivities only through their ratio f_β, which we take as a constant. Hence we do not require for the moment any further knowledge of β_1 or β_2, the values of which will be determined in Sect. 3.5. Since the dynamo number (Eq. 18) does depend on β_1 and β_2, we shall

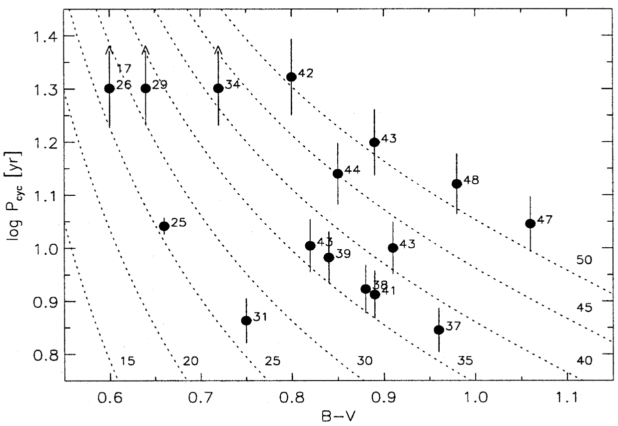

Fig. 4. The solid points denote the measured cycle periods of slowly rotating lower main-sequence stars (Table 1). The dotted curves represent Eq. (20) for various rotation rates. The labels indicate the rotation period in days.

verify the validity of Eq. (19) also in Sect. 3.5, and assume for the moment that $|C| \gg 1$. Inserting Eqs. (11–16) we may write

$$P_{\rm cyc} \approx \frac{7.08\, P_{\rm rot,\odot}^{0.5p}}{\sqrt{H(1) f_\beta \Delta\Omega_\odot}} P_{\rm rot}^{-0.5(p+q)} \tau_{\rm c}^{0.5(q+1)}, \qquad (20)$$

where $P_{\rm cyc}$ expressed in years, and $P_{\rm rot}$ and $\tau_{\rm c}$ in days. We point out that, according to this approximation, $P_{\rm cyc}$ depends on stellar structure only through $\tau_{\rm c}$. This allows us to compare Eq. (20) with Eq. (3), which describes the observations, and we conclude that these two equations are equivalent if

$$\begin{cases} b = -0.5(p+q) \\ c = 0.5(q+1) \end{cases} \Leftrightarrow \begin{cases} p = 1 - 2(b+c) = 1.1 \pm 0.2 \\ q = 2c - 1 = -5.1 \pm 0.6 \end{cases} \qquad (21)$$

The positive value of p indicates that differential rotation *decreases* with *increasing* rotation rate (Eq. 12). Note the effect of the strong anticorrelation between b and c on the uncertainty in p. The rather large negative value of q indicates a strong dependence of α on rotation.

Fig. 4 shows the observed cycle periods of the slowly rotating stars from Table 1 as a function of $B-V$, as well as the curves predicted by Eq. (20) for various rotation rates. There is reasonable agreement on the whole between the theoretical curves and the measured cycle periods.

3.4. The solar calibration model

The calibration of the stellar dynamo models is based on the solar parameters that are shown in Table 3. The total difference in angular velocity across the overshoot layer near the equator is estimated from Goode (1995). The constant $H(1)$ is determined from Eq. (15) by adopting a value for α, namely -25 cm s^{-1}. The choice of α is somewhat arbitrary. We determine β_1 and β_2 following an iterative method, i.e. by solving $P_{\rm cyc}$ and $\tau_{\rm dec}$ from the (exact) dispersion relation and applying corrections to β_1 and β_2, until $P_{\rm cyc}$ = 11 years and $\tau_{\rm dec} = 20 P_{\rm cyc}$.

Table 3. Parameters of the solar calibration model

$\Delta\Omega_\odot$	1.57×10^{-7} s^{-1}
α	-25 cm s^{-1}
$H(1)$	0.099
β_1	4.1×10^{10} cm^2 s^{-1}
β_2	5.8×10^{11} cm^2 s^{-1}
f_β	0.071
$P_{\rm cyc}$	11 yr
$\tau_{\rm dec}$	220 yr

3.5. Validity of expression (20)

In order to verify whether the dynamo number is sufficiently large for expression (20) to be valid, we return to Eq. (6) and solve the full dispersion relation (cf. Ossendrijver & Hoyng 1996) for a series of stellar models (Table 2). For each model, we calculate the dynamo parameters as indicated in Sect. 3.2, employing the values of p and q derived in Sect. 3.3. We choose stellar masses in the range $0.7 \leq M/M_\odot \leq 1.1$ and rotation periods in the range $15 \leq P_{\rm rot} \leq 50$ days, providing complete overlap with all the slowly rotating stars in Table 1. For a given stellar mass and rotation rate we let β_2 (or $\beta_1 = f_\beta \beta_2$) assume the value at which the fundamental mode of Eqs. (6) satisfies $\tau_{\rm dec} = 20 P_{\rm cyc}$. This is achieved by starting with an initial value for β_2, solving the dispersion relation, estimating the required correction to β_2, and repeating this, while keeping f_β constant, until β_2 has converged. The smallest dynamo number that occurs in any one of these models is $|C| \approx 7$, and the resulting difference between exact and approximative values of $P_{\rm cyc}$ is typically a few percent. Given the large uncertainty in the observed cycle periods (typically $\sigma_{\rm P}/P_{\rm cyc} \approx 15\%$), we ignore this effect, and we conclude that expression (20) is a valid approximation for the cycle periods of slowly rotating lower main-sequence stars.

4. Summary and conclusions

We have demonstrated that for slowly rotating (Ro $\gtrsim 0.9$) lower main-sequence stars with well-defined periodic chromospheric activity the cycle period $P_{\rm cyc}$ can be parametrised according to $P_{\rm cyc} \propto P_{\rm rot}^b \tau_{\rm c}^c$, with $b = 2.0 \pm 0.3$ and $c = -2.0 \pm 0.3$. The existence of such a relation points to a common dynamo mechanism for slowly rotating stars. Cycle periods of rapidly rotating stars (Ro $\lesssim 0.9$) do not match this relation.

We used linear mean-field dynamo theory to explain the cycle periods of slowly rotating lower main-sequence stars, assuming that the geometry of the dynamo in all these stars, including the Sun, is that of Parker's surface-wave model (Parker 1993). We assumed that the differential rotation and the α-coefficient depend on the rotation rate according to $\Delta\Omega \propto P_{\rm rot}^p$ and $\alpha \propto {\rm Ro}^q$. The other parameters were taken to be independent of $P_{\rm rot}$. In estimating k_z we assumed that the activity belts extend over 35° of latitude on either side of the equator. This is probably a reasonable assumption for slowly rotating stars, since model calculations have shown that starspot activity occurs near the poles only for rapidly rotating stars (for a star of one solar mass if $P_{\rm rot} \lesssim 0.1 P_{\rm rot,\odot}$, Schüssler et al. 1996).

The observed correlation between $P_{\rm cyc}$, $P_{\rm rot}$ and $\tau_{\rm c}$ is reproduced if p = 1.1 ± 0.2 and q = -5.1 ± 0.6. The positive sign of p suggests that differential rotation decreases with increasing rotation rate. Such a trend is supported by calculations of Kitchatinov & Rüdiger (1995), who found a similar dependence on rotation, with $p \approx 0.4$. The negative value of q indicates that the α-coefficient increases if Ro decreases, in accordance with the common assumption that $|\alpha|$ increases with increasing rotation rate. Our result also implies that for constant $P_{\rm rot}$, $|\alpha|$ increases with increasing $\tau_{\rm c}$, i.e. with increasing $B{-}V$ (Eq. 15). This is due to the fact that convective cells with longer turnover times are more strongly influenced by rotation.

References

Baliunas, S.L., Donahue, R.A., Soon, W.H. et al. 1995, ApJ 438, 269
Baliunas, S. and Soon, W. 1995, ApJ 450, 896
Baliunas, S.L., Sokoloff, D. and Soon, W.H. 1996, ApJ 457, L99
Baliunas, S.L., Nesme-Ribes, E., Sokoloff, D. and Soon, W.H. 1996, ApJ 460, 848
Copeland, H., Jensen, J.O. and Jørgensen, H.E. 1970, A&A 5, 12
Gilliland, R.L. 1985, ApJ 299, 286
Gilman, P. 1980, in *Stellar Turbulence*, eds. D.F. Gray and J.L. Linsky, Springer (Berlin), p. 19
Goode, P.R. 1995, in: *Fourth Soho Workshop: Helioseismology*, eds. J.T. Hoeksema, V. Domingo, B. Fleck and B. Battrick, ESA SP-376, Vol. 1, p. 121
Hughes, D.W. 1992, in *Sunspots: Theory and Observations*, eds. J.H Thomas and N.O. Weiss, Kluwer (Dordrecht), p. 371
Jennings, R.L. and Weiss, N.O. 1991, MNRAS 252, 249
Kim, Y.-C. and Demarque, P. 1996, ApJ 457, 340
Kitchatinov, L.L. and Rüdiger, G. 1995, A&A 299, 446
Noyes, R.W., Hartmann, L.W., Baliunas, S.L., Duncan, D.K. and Vaughan, A.H. 1984, ApJ 279, 763
Noyes, R.W., Weiss, N.O. and Vaughan, A.H. 1984, ApJ 287, 769
Ossendrijver, A.J.H. and Hoyng, P. 1996, A&A, submitted
Parker, E.N. 1993, ApJ 408, 707
Rüdiger, G. and Arlt, R. 1996, Preprint Series Astrophysikalisches Institut Potsdam 96-19
Saar, S.H. and Baliunas, S.L. 1992, in *The Solar Cycle*, ed. K.L. Harvey, ASP Conference Series, Vol. 27, p. 150
Schrijver, C.J., Coté J., Zwaan, C. and Saar, S.H. 1989, ApJ 337, 964
Schüssler, M., Caligari, P., Ferriz-Mas, A. and Moreno-Insertis, F. 1994, A&A 281, L69
Schüssler, M., Caligari, P., Ferriz-Mas, A., Solanki, S.K. and Stix, M. 1996, A&A 314, 513
Skaley, D. and Stix, M. 1991, A&A 241, 227
Soderblom, D.R. 1988, in *Secular Solar and Geomagnetic Variations in the Last 10,000 Years*, eds. F.R. Stephenson and A.W. Wolfendale, Kluwer (Dordrecht), NATO ASI Series, Vol. 236, p. 25
Soon, W.H., Baliunas, S.L. and Zhang, Q. 1994, Sol. Phys. 154, 385
Stępień, K. 1989, A&A 210, 273
Stępień, K. 1994, A&A 292, 191
Wilson, O.C. 1978, ApJ 138, 832

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.