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Acceleration mechanism in compact objects

A. Marcowith, G. Pelletier, and G. Henri[1]

Groupe d'Astrophysique de Grenoble, Université J. Fourier, BP 53X, F-38041 Grenoble Cédex, France

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Abstract. We investigate the kinetic theory of a mixing of relativistic pair plasma in a conventional electron-proton plasma and an intense anisotropic Compton radiation field as those found in the vicinity of a compact object, a neutron star or a black hole.

In a two-flow configuration we discriminate two regimes: the first one is realized when the mass energy density of the pair plasma is lower than the ambient (electron-proton) plasma one, whereas the second one is characterized by a more massive relativistic pair plasma. The first stage, called "non relativistic regime", is entirely treated within the scheme of weak turbulence theory, and leads us to calculate the bulk Lorentz factor, the internal energy and the distribution function of the relativistic particles distribution function. We propose a new scenario to get a power law distribution. For the second stage, called the "relativistic regime", we mostly calculated an unusual instability triggered by the ambient protons that amplifies the left circularly polarized Alfvén wave of the pair plasma at synchrotron resonance. This instability plays an important role to couple the beam with the ambient plasma, and governs energy and momentum exchanges.

The kinetic theory is applied to different astrophysical sources such as Blazar and galactic "micro-Quasar" jets. We calculate the internal Lorentz factor and find it of order of 10^4 for extragalactic black holes, and 10^2 for galactic black holes. The resulting bulk Lorentz factor derived in term of the compactness of the soft photon source is of order of 10 for extragalactic sources of order of 5 for galactic sources.

Key words: acceleration of particles – instabilities – radiation mechanisms: non-thermal – turbulence – galaxies: active – galaxies:jets

1. Introduction

Since its launch in 1991, CGRO (Compton Gamma-Ray Observatory) has observed numerous extragalactic radio sources above 100 MeV (see von Montigny et al. (1995)). Among the sources detected more than 40 are classified as radio-loud objects (BL Lac objects and flat radio spectrum Quasars) all with the same characteristics: intense high energy emission, strong variability in all wavelengths, and superluminal motions. More recently, galactic sources exhibiting multi-wavelength Quasar spectra, superluminal motions at small scales, and called for these reasons "micro-Quasars", have been detected by SIGMA (Finoguenov et al. (1994), Gilvanov et al. (1994)) and CGRO (Harmon et al. (1995)).

To explain the strong and highly variable high energy emission of these objects many authors have considered a well-collimated relativistic jet in bulk motion characterized by Lorentz factors γ_b between 2 and 20. In this hypothesis, the high energy emission is beamed in the forward direction into a cone of opening angle $\sim 1/\gamma_b$. Such approach removes the important problem of the gamma-ray absorption by pair creation in a static and homogeneous source (see Maraschi et al. (1992) for discussions).

The origin of the high energy emission is mainly considered in two different ways. A first class of models involve a cascade of gamma-rays due to meson production by energetic protons (with an energy $\sim 10^{10} GeV$) as proposed by Mannheim & Bierman (1992), or associated with energetic neutrons (Mastichiadis & Protheroe 1990). These models are referred to be hadronic as opposed to the leptonic models in which gamma-rays are mostly produced by Inverse Compton (IC) scattering of low energy photons by relativistic electrons. The soft photons are coming either from synchrotron radiation (Synchrotron self-Compton process), see for example Ghisellini et al. (1991), or from an accretion disk. In the last case the disk radiation can either be reprocessed by surrounding clouds as proposed by Sikora et al. (1994), Blandford & Levinson (1995), and Levinson & Blandford (1996) for "micro-Quasars" objects, or interact directly with the relativistic particles (Dermer & Schlickeiser (1993)).

This is also our viewpoint (Henri & Pelletier (1991) thereafter: HP), and Henri, Pelletier & Roland (1993)). We have developed a model treating the high energy emission of radio-loud sources (Marcowith, Henri & Pelletier (1995); MHP thereafter). The aim of the MHP paper was to reproduce the global feature of radio-loud Quasar spectra, and particularly the spectral break observed in the MeV region by COMPTEL. For that purpose

Send offprint requests to: A. Marcowith

we first compute the anisotropic IC emitted power by an ultra-relativistic electron and the resulting anisotropic IC spectrum by a population of relativistic particles. With these tools, we were able to compute a self-consistent formation of the relativistic pair plasma, including both pair creation and annihilation. Note that the anisotropy of the high energy emission is a consequence of both disk radiation and particle distribution anisotropy in the disk frame. In this picture the spectral break is a consequence of a differential energy absorption of gamma-rays along the jet due to pair production. Such multiple emission zone model first proposed by Pelletier, Henri & Roland (1992), and Blandford (1993) allow us to obtain a good fit over ten decades of energy of the two best quasi-simultaneous observations of Radio-loud Quasar and Blazar: 3C273 (Lichti et al. (1995)) and 3C279 (Hartmann et al. (1992), Collmar et al. (1992)). In this mostly hydro-dynamical model, we focussed on the treatment of the radiation transfer along the jet, assuming a power law relativistic pair distribution continuously re-heated to balance IC losses. But a kinetic theory is necessary to constrain the acceleration mechanisms at the origin of the particle distribution. In a first approach HP have proposed the formation of a relativistic beam by pair cascade on the photons coming from an accretion disk, and argue that a turbulent heating by Alfvén waves could easily explain final bulk Lorentz factors of order of 10 observed in VLBI. In this model, all the turbulence is supported by a sub-relativistic electron-proton jet launched by an accretion disk. The relativistic beam confined by the magnetohydrodynamical accretion ejection flow can then be re-accelerated up to distances where the Compton drag becomes inefficient. This two flow configuration first proposed by Pelletier (1985) is presented in more details in Sol et al. (1989).

Other detailed theories of the particle heating mechanism in the inner region of compact objects have only been recently considered. Mastichiadis & Kirk (1995) have developed a diffusive shock acceleration model for a relativistic proton population in the accreting motion in the vicinity of a massive black hole. Dermer et al. (1996) have also considered a stochastic acceleration process, where both proton and electron populations are heated by plasma turbulence generated in black hole magnetospheres.

Accounting for all these astrophysical problems, the present paper tackles in a formal way the kinetic theory of the acceleration mechanism of a relativistic pair plasma coupled with a sub-relativistic MHD plasma and moving with a relativistic bulk speed $\beta_b c \sim c$ in an intense soft photon radiation field. The pair plasma is submitted to strong anisotropic Inverse Compton losses in the soft photon source frame, and in turn produces X and gamma-rays.

Two distinct regimes may appear depending on whether the ambient plasma is the more massive, and supports the turbulence or not. In the first stage, the Compton radiation field accelerates a tenuous beam of electron-positron that pervades the cold electron-proton ambient plasma. Here we understand "cold" as non relativistic temperature ($\leq 100ev$); so we call this stage "non relativistic" regime. A second stage takes place when the electron-positron beam has been so heated and densified by pair creation that it becomes more massive than the ambient one. It is therefore convenient to analyze the micro-turbulence in its rest frame, the ambient medium being considered as a perturbing back-stream. So we called this second stage the "relativistic" regime. Each regime has a specific kinetic theory that provides a self consistent calculation of the internal energy and bulk Lorentz factors.

This general picture can be applied to astrophysical objects emitting high energy radiations (X and gamma-ray): active galactic nuclei, galactic microquasars, gamma-ray bursters, and perhaps X-ray binaries. The internal energy and the bulk Lorentz factor of the pair beam can be derived in term of the soft photon source compactness.

This paper is organized as follows. The Sect. 2 recalls the main effects of the anisotropic Compton emission from the relativistic particles. Sections 3 to 6 present the non relativistic regime. In Sect. 3 we explain why the theory of weak turbulence applies, why we need a non-linear theory to calculate the turbulent spectrum, and how the spectrum is used to calculate the coefficients of the Fokker-Planck equation that governs the evolution of the pair distribution. The Sect. 4 is devoted to the contribution of Langmuir turbulence, whereas Sect. 5 is devoted to the contribution of the Alfvén turbulence: linear growth rates, and main non-linear mode couplings. The stationary solutions of a Fokker-Planck equation are derived in Sect. 6. A power-law solution appears naturally as a consequence of the cooling of high energy particles in the intense soft radiation field.

The next section (Sect. 7) is devoted to the linear kinetic theory of the unusual physical situation in the relativistic regime. In particular, an interesting micro-instability due to cold proton streaming in a relativistic plasma is derived. The non-linear effects are also discussed. The Sect. 8 is devoted to the astrophysical applications of the kinetic theory to high energy extragalactic and galactic sources. In each non relativistic and relativistic regime the mean Lorentz factor and the bulk Lorentz factor of the pair plasma are estimated before concluding in Sect. 9.

Thereafter the prim denotes quantities expressed in the relativistic pair frame.

2. Anisotropic Compton losses

As made precise in the introduction the acceleration of the relativistic particles strongly competes with the Inverse Compton (IC) cooling process of these pairs on soft photons coming from an accretion disk or from synchrotron radiation. We consider here only the former case as developed in MHP. The reader should refer to this paper for a complete treatment of the anisotropic Inverse Compton process in the Thomson regime. However, we sum up here the main physical results.

In the Thomson regime, in an incident soft radiation field given by $I_{\varepsilon_s}(\Omega_s)$, an electron with a Lorentz factor γ must emit a power (Blumenthal & Gould (1970)):

$$P_{tot}(\widehat{\Omega}_e) = \sigma_T \gamma^2 \int \int_{4\pi} I_{\varepsilon_s}(\widehat{\Omega}_s)(1 - \beta \cos\psi)^2 d\Omega_s d\varepsilon_s. \qquad (1)$$

Where ψ is the direction between the electron and the soft photon. The solid angle subtended by an unit vector in the direction of the electron (of the photon) is $\widehat{\Omega}_e$ ($\widehat{\Omega}_s$). The factor ε_s is the soft photon energy in $m_e c^2$ unit. Whatever the source of soft photons is, we can characterize the soft radiation field by its Eddington parameters:

$$J = \frac{1}{4\pi}\int I_{\varepsilon_s}(\Omega_s) d\Omega_s d\varepsilon_s$$

$$H = \frac{1}{4\pi}\int \mu_s I_{\varepsilon_s}(\Omega_s) d\Omega_s d\varepsilon_s \quad (2)$$

$$K = \frac{1}{4\pi}\int \mu_s^2 I_{\varepsilon_s}(\Omega_s) d\Omega_s d\varepsilon_s .$$

For a relativistic particle the IC power per unit of solid angle is emitted in a sharp cone of opening angle $\sim \gamma^{-1}$, and takes the form

$$P_{IC} = 2\pi\sigma_T\gamma^2[(3J - K) - 4H\mu_e + (3K - J)\mu_e^2]. \quad (3)$$

We have defined $\mu_e = cos\alpha_e$ (and $\mu_s = cos\alpha_s$) the cosine of the angle between the electron (the soft photon) direction and the jet axis.

If the forward direction corresponds to $H > 0$, then the anisotropy of the incident radiation field favors the emission in the backward direction. The particles moving forward are submitted to softer radiation losses, and the pair plasma will then be accelerated along the jet axis. This is the so-called Compton rocket effect (O'Dell (1981)) which expresses the transfer of stochastic internal energy of the plasma in bulk motion along the jet axis. The transfer to the momentum of the relativistic particle is given by

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\sigma_{T m_e c^2}}{h\ c}\int I_{\varepsilon_s}(1 - \widehat{\beta}.\widehat{k})[\gamma^2(1 - \widehat{\beta}.\widehat{k})\widehat{\beta} - \widehat{k}] d\nu_s d\Omega_s (4)$$

Where $\widehat{k}$ and $\widehat{\beta}$ are respectively the photon and the particle directions. We can define a particular frame moving with the velocity $\beta_b c$ to the respect of the observer frame where the mean particle speed $<\beta>= 0$. In this frame the relativistic particle distribution is supposed to be isotropic. In the soft photon source frame a saturation velocity $\beta_{bs}c$ of the plasma can be obtained by the cancellation of the parallel IC force integrated over the particle distribution (O'Dell (1981)). Namely,

$$F_z' = -2\pi\sigma_T H'(\gamma_{bs})(1 - \frac{2}{3} < \gamma'^2 >) = 0\ , \quad (5)$$

or

$$H' = [(J + K)\beta_{bs} - H(\beta_{bs}^2 + 1)] = 0\ , \quad (6)$$

and where $<\gamma^2>= \int \gamma^2 f(\gamma)\gamma^2 d\Omega d\gamma$.

For $\eta = H/J = 1 - \epsilon_\eta$ and $\chi = K/J = 1 - \epsilon_\chi$, where $\epsilon_\eta \leq \epsilon_\chi \ll 1$, the saturation Lorentz factor is at the first order in ϵ_χ, and ϵ_η

$$\gamma_{bs} \simeq \sqrt{2}[2\epsilon_\eta - \epsilon_\chi]^{-1/4}\ . \quad (7)$$

The bulk Lorentz factor γ_b will then closely follow γ_{bs} until the relaxation time of γ_b to γ_{bs} equals the evolution time of γ_{bs}. The evolution of γ_{bs} with the distance above the source, and the final value of the bulk Lorentz factor both depends on the kind of the soft photon source (see MHP).

Let us now consider Eq. (3). The anisotropy leads to an IC emitted power strongly reduced in the inner part of a cone of opening angle α^*. The minimal emitted power is obtained for $\mu = 1$, namely

$$P_{ICi} = 4\pi\sigma_T\gamma^2 J[2\epsilon_\eta - \epsilon_\chi]\ . \quad (8)$$

For $\mu = \mu^* = cos\alpha^*$ the emitted power is twice P_{ICi}, and we can rewrite Eq. (3) as

$$P_{IC} = 4\pi\sigma_T\gamma^2 J[(1 - \mu^*)^2 + 2(2\epsilon_\eta - 2\epsilon_\chi)]\ . \quad (9)$$

where, at the first order, we have put $\mu = 1$ in the second member of (9).

The angle α^* is then given by

$$\alpha^* \simeq \sqrt{2}[2\epsilon_\eta - \epsilon_\chi]^{1/4}\ . \quad (10)$$

We can then defined a cone of opening angle $\alpha^* \simeq 2/\gamma_b$ where the IC losses are strongly reduced such that the internal and the external IC emitted power verify $P_{ICi} \sim \gamma_b^{-4} P_{ICe} \ll P_{ICe}$. This result is independent of the nature of the soft photon source.

3. Weak turbulence theory in the non relativistic regime

First, we consider the case where the pair plasma is not dense enough to be energetically dominant. The streaming of the relativistic pairs propelled by the radiation field triggers micro-instabilities by amplifying waves of the ambient plasma. The anisotropy due to the longitudinal (along the jet axis) magnetic field will favor the destabilization of Langmuir, and Alfvén waves with wave vectors having direction close to the magnetic field (and jet axis). For this reason, only this two kind of micro-instabilities will be studied in the following sections. We do not consider other kind of instabilities with much lower growth rates. The pair beam, close to the soft photon source, can be efficiently heated by strong Langmuir turbulence. Because the beam is hot in the sense it has high internal energy (and a mean Lorentz factor) $\overline{\gamma}' \gg 1$, these instabilities have a broad band. Consequently, they excite broad waves spectra which promote the weak turbulence theory. Indeed, a broad band spectrum allows fast phase mixing in the resonant interaction between particles and waves.

In the case of Langmuir turbulence, interactions occur at Landau-Čerenkov resonance ($\omega - k_\| v_\| = 0$), and the phase mixing time (correlation time τ_{cL}) is such that

$$\tau_{cL}^{-1} = \Delta(\omega - k_\| v_\|) \simeq \omega_{pe}\frac{\Delta k}{k}\ , \quad (11)$$

where $\omega_{pe} = 4\pi e^2 n_e/m_e$, is the plasma frequency, and n_e is the ambient electron density.

In the case of Alfvén turbulence, interactions occur at Landau-synchrotron resonance ($\omega - k_\| v_\| \pm \omega_{se} = 0$), and the corresponding correlation time τ_{cA} is such that

$$\tau_{cA}^{-1} = \Delta(\omega - k_\| v_\| \pm \omega_{se}) \simeq \omega_{se}\frac{\Delta k}{k}\ , \quad (12)$$

where for a particle of Lorentz factor γ, and a mean magnetic field B, $\omega_{se} = eB/(\gamma m_e c)$, is the plasma gyro-frequency.

The condition of a short correlation time compared to the time scale of the distribution evolution and to the growth time of the instability are the usual condition to apply the so-called quasi linear theory. However, we need to go beyond quasi linear theory in the situation we deal with, because the origin of the instability is maintained externally (by the anisotropy of the incident soft radiation field), so that a quasi linear evolution cannot remove it. Therefore we come to next order of the theory of weak turbulence to calculate the saturation spectrum. Furthermore the broad band character of the spectrum allows to validate the random phase approximation and to obtain a low level of turbulence, which justifies the perturbative expansion. In the Langmuir waves, the turbulent energy density W verifies

$$\frac{W}{n_e T_e} \ll 1 . \tag{13}$$

The quantity T_e refers to the cold electron temperature.

In the Alfvén waves, the turbulent fluctuations $< \delta B^2 >$ verifies

$$\frac{< \delta B^2 >}{B^2} \ll 1 , \tag{14}$$

The nonlinear effects couple unstable modes with damped modes which redistribute energy to the particles; however a part of the energy is carried away by an inertial cascade. Acceleration of particles comes from the damped modes (especially backward modes) that are continuously supplied by their coupling with ever-unstable modes. Thus Compton losses ensure a stationary state by balancing the stochastic acceleration of the particles. So the evolution of distribution function is governed by a Fokker-Planck equation that contains a drag term describing the Inverse Compton process and a diffusion tensor describing the effect of the turbulence on the particles, namely pitch angle scattering and stochastic acceleration. The Fokker-Planck description is valid as long as τ_{cL} and τ_{cA} are much shorter than the diffusion time; which is well satisfied for broad band spectra (see Eqs. (11) and (12)) and low turbulence level (Eq. (13)).

Despite the pitch angle scattering, the distribution will keep a strong anisotropy because of the Compton radiation field. We recall the important point that the hotter the pairs, the stronger the Compton boost (see HP). This is the key point to understand that strong boosting can exist in presence of intense IC emission since the latter is balanced by strong heating.

4. Langmuir turbulence

4.1. Growth rates

The growth rate of Langmuir waves in a magneto-active plasma can be easily derived from a linearized Vlasov equation. Expressed in CGS units the rate becomes

$$\frac{\Gamma_{\boldsymbol{k}}}{\omega_{pe}} = -2\frac{\pi^2 q_e^2}{k^2}\int \delta(\omega_k - k_\parallel v_\parallel)\boldsymbol{k}_\parallel . \frac{\partial f}{\partial \boldsymbol{p}_\parallel} d^3\boldsymbol{p} . \tag{15}$$

The condition for Čerenkov resonance in a magnetized medium is

$$\omega_{\boldsymbol{k}} = \boldsymbol{k}_\parallel . \boldsymbol{v}_\parallel = kv\mu cos\theta . \tag{16}$$

We define μ as the cosine of the pitch-angle of the particle, and θ the angle between **k** and **B**. The magnetic field modifies the oscillation frequency of Langmuir waves. The dispersion relation of the strongly magnetized Langmuir modes is

$$\omega_{\boldsymbol{k}}^2 = \omega_{pe}^2 \frac{1 - n^2 cos^2\theta}{1 - n^2} , \tag{17}$$

where we have taken $v = c$, and $n = kc/\omega_k$.

The resonance condition (Eq. (16)) leads to

$$\omega_{\boldsymbol{k}}^2 = \omega_{pe}^2 (1 - \frac{sin^2\theta}{1 - \mu^2 cos^2\theta}) + o(k^2 v_{the}^2) . \tag{18}$$

where the thermal speed of ambient electrons of temperature T_e is $v_{the} \equiv \sqrt{T_e/m_e}$. The dispersion relation in a magnetized plasma can be approximated as $\omega_k \simeq \omega_{pe} cos\theta$, and

$$\omega_{pe} cos\theta = \boldsymbol{k}_\parallel . \boldsymbol{v}_\parallel . \tag{19}$$

In spherical coordinates we have

$$\frac{\partial}{\partial p_\parallel} = \mu \frac{\partial}{\partial p} + \frac{1 - \mu^2}{p} \frac{\partial}{\partial \mu} . \tag{20}$$

Equations (19) and (20) transforms the growth rate as

$$\begin{aligned} \frac{\Gamma_{\boldsymbol{k}}}{\omega_{pe}} &= -\frac{2\pi^2 q_e^2}{k^2 c} \int \delta(k_0 cos\theta - k cos\theta\mu) \\ & \quad k cos\theta (\mu \frac{\partial f}{\partial p} + \frac{1 - \mu^2}{p} \frac{\partial f}{\partial \mu}) 2\pi p^2 dp d\mu , \end{aligned} \tag{21}$$

with the wave number k_0 defined as ω_{pe}/c. Here we assume a mono-energetic distribution for the pair plasma in the plasma frame

$$f'(p') = \frac{n'_*}{4\pi \overline{p}'^2} \delta(p' - \overline{p}') . \tag{22}$$

The Lorentz transformation of the distribution function leads to $f'(\boldsymbol{p}', \boldsymbol{r}') = f(\boldsymbol{p}, \boldsymbol{r})$. As we shall see, the real distribution function is not a delta function but the growth rate is not very sensitive to its exact shape.

Injected in Eq. (21) and using the Lorentz transformations for the momentum $p' = p\gamma_b(1 - \beta_b\mu)$ the derivation over μ and p reads

$$\begin{aligned} \frac{\partial}{\partial \mu} &= -p\gamma_b\beta_b \frac{\partial}{\partial p'} \\ \frac{\partial}{\partial p} &= \gamma_b(1 - \beta_b\mu) \frac{\partial}{\partial p'} . \end{aligned} \tag{23}$$

We finally obtain

$$\frac{\Gamma_{\boldsymbol{k}}}{\omega_{pe}} = \frac{\pi}{2\overline{\gamma}'\gamma_b^2} (\frac{\omega'_{p*}}{kc})^2 \frac{cos\theta}{|cos\theta|} \frac{\mu_r - \beta_b}{(1 - \beta_b\mu_r)^3} . \tag{24}$$

Where $\mu_r \equiv k_0/k$ selects the pitch-angle of a resonant particle for a given wave number k, and $\omega'_{p*} = [4\pi q_e^2 n'_*/m_e]^{1/2}$ is the pair plasma frequency. The maximum growth rate is obtained for $k = k_0$

$$\frac{\Gamma_{\boldsymbol{k}}}{\omega_{pe}}|_{max} \simeq 2\pi \frac{n'_*}{n_e}\frac{\gamma_b^2}{\overline{\gamma}'} \; . \tag{25}$$

It is straightforward to see from Eq. (24) that the forward modes ($cos\theta \geq 0$) are destabilized ($\Gamma_{\boldsymbol{k}} > 0$) for particles moving within a cone of half opening angle $\sim 1/\gamma_b$. The backward modes ($cos\theta < 0$) are destabilized by the particles with a pitch-angle such that $\mu < \beta_b$.

4.2. Non-linear transfer

In an isothermal plasma the most important interactions mechanisms encountered by Langmuir waves plasma are the induced scattering off ions and/or electrons, and four plasmons interactions. The excitation of ion-acoustic oscillations must be added in a non-isothermal medium (Tsytovich (1977)). In the scattering process the Čerenkov resonance condition is generalized to the interaction of a particle and the beat of two Langmuir waves. The probability for scattering off electron and ion or ion waves can be derived from an expansion up to a second and third order Vlasov equation. In a field free plasma an estimate of the characteristic time for spectral transfer of energy over an interval $\sim k$ for the scattering by the electrons

$$\tau_e \simeq \omega_{pe}^{-1}\frac{n_e T_e}{W}(\frac{\omega_{pe}}{k v_{the}})^3 \; . \tag{26}$$

For a thermal population of ions and isotropic turbulence the transfer can be differential for $k \ll k_d$ (where $k_d = \omega_{pe}/v_{the}$ is the Debye wave number). In this case the transfer occurs over an interval $\ll k$. An estimate of the characteristic transfer time is

$$\tau_i \simeq \omega_{pe}^{-1}\frac{n_e T_e}{W}(\frac{\omega_{pe}}{k v_{the}})^{-2} \; . \tag{27}$$

It clearly appears that the thermal ions will dominate the scattering process for $k \leq k_d(\frac{3m_i}{m_e})^{1/5}$. This relation still holds in a magnetic field. We then focus on the ion scattering in the region of differential transfer, since it is indeed this process which is relevant in the range of resonance with particles. It contributes to the energy redistribution among particles, and to an inverse cascade. The inverse cascade leads to four waves interaction but in a wave number range outside the resonance region, and thus will be disregarded. Thereafter we will use a reduced turbulent spectrum S_k (k^{-1}) defined by

$$\int_k S_{\boldsymbol{k}}\frac{dk}{2\pi} = \frac{W}{n_e T_e} \; . \tag{28}$$

Following Tsytovich (1977), we introduce a characteristic wave number for scattering off ions

$$k_* = \frac{1}{3}k_d\sqrt{\frac{m_e}{m_i}} \; , \tag{29}$$

or in a non-isothermal plasma for scattering off ion acoustic oscillations

$$k_* = \frac{1}{3}k_d\sqrt{\frac{m_e}{m_i}\frac{T_e}{T_i}} \; . \tag{30}$$

The electron temperature can be of order of the Compton temperature $T_e = T_{Com} \simeq (m_e c^2/4) < \varepsilon_s > \; ev.$

The kinetic equation describing the evolution of the turbulent spectrum is

$$\frac{\partial S_{\boldsymbol{k}}}{\partial t} = \frac{\pi}{12}\omega_{pe}k_*^2 S_{\boldsymbol{k}}\frac{\partial S_{\boldsymbol{k}}}{\partial k} \; , \tag{31}$$

The exact treatment of the transfer in the presence of an ambient magnetic field in equipartition with the thermal plasma is rather tedious. The transfer is not differential for $cos\theta \ll 1$ and/or $\mu \ll 1$ that is for $k \sim k_d$ where the scattering is mainly due to the thermal electrons. In fact, as seen in the previous section, the growing modes are those with a wave number close to k_0. The energy redistribution in a wide range of k (and a wide angular range) is above all ensured by ion wave scattering and roughly described by Eq. (31). More precisely, the scattering in a strong magnetic field favors the almost elastic backscattering of the primary modes. Those secondary backscattered modes are thus absorbed at a rate almost equal to the growing rate of the primary modes (in absolute value). This is precisely by this means that the beam is re-heated. However the re-heating has a yield slightly reduced by the inverse cascade.

In a first approach we assume that these effects are not strong enough to modify neither well the dispersion relation nor the transfer equation. Works are in progress to treat this question in a more rigorous way.

The resulting spectrum is scale invariant. Adding the wave generation term Γ_k contribution the stationary turbulent spectrum is obtained via the kinetic equation

$$\begin{aligned}\frac{\partial S_{\boldsymbol{k}}(cos\theta)}{\partial t} &= 2\Gamma_{\boldsymbol{k}}S_{\boldsymbol{k}}(cos\theta) \\ &+ \frac{\pi}{12}\omega_{pe}k_*^2 S_k(cos\theta)\frac{\partial S_{\boldsymbol{k}}(cos\theta)}{\partial k} \; .\end{aligned} \tag{32}$$

Where

$$\int_k\int_{-1}^{1} S_{\boldsymbol{k}}(cos\theta)\frac{dk}{4\pi}dcos\theta = \frac{W_0}{n_e T_e} \; . \tag{33}$$

For an isotropic turbulence we have

$$S_{\boldsymbol{k}} = S_0 - \frac{24}{\pi k_*^2}\int dcos\theta_1\int_{k_0}^{k} dk_1\frac{\Gamma(k_1, cos\theta_1)}{\omega_{pe}} \; . \tag{34}$$

The lower limit of the θ integral is imposed by $|\mu_r| = k_0/k \leq 1$ in the resonance condition of Langmuir waves. The quantity W_0 is the transfer spectrum due to ionic diffusion for $k \leq k_0$. Replacing $\Gamma_{\boldsymbol{k}}$ in Eq. (28) by its expression obtained in equation (20) and performing the integration over k and $cos\theta$ the turbulent spectrum becomes for $k_0 \leq k \leq k_d$

$$\begin{aligned}S_{\boldsymbol{k}} &\simeq S_0 - \frac{12}{k_*^2}\frac{k_0}{\overline{\gamma}'\gamma_b^2\beta_b^2}\frac{n'_*}{n_e} \\ &\quad (1 + (1 + \beta_b^2 - 2\beta_b k_0/k)(1 - \beta_b k_0/k)^{-2})) \; .\end{aligned} \tag{35}$$

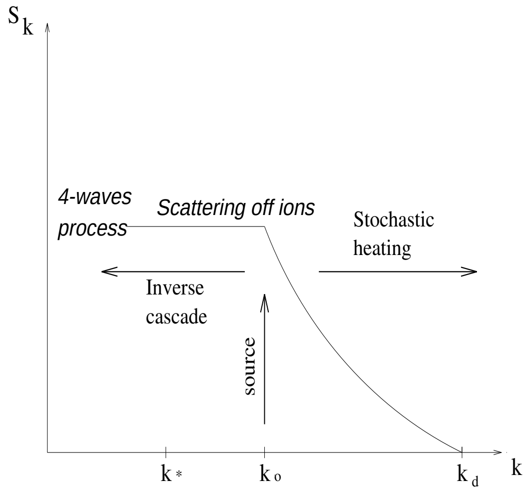

Fig. 1. The Langmuir turbulence spectrum.

In the derivation of the stationary turbulent spectra we have limited the variation of $cos\theta$ between $\mu_0 = v_{the}/c = k_d/k_0$ beyond which the waves are damped and 1.

The Langmuir modes are damped by Landau effect at the Debye scale ($k \sim k_d$). We can then write $S_{k_d} \simeq 0$, and

$$S_0 = \frac{24}{\pi k_*^2} \int dcos\theta_1 \int_{k_0}^{k_d} dk_1 \frac{\Gamma(k_1, cos\theta_1)}{\omega_{pe}} \ . \quad (36)$$

Finally the transfer spectrum is

$$S_0 \simeq \frac{36}{k_*^2} \frac{k_0}{\overline{\gamma}' \gamma_b^2} \frac{n'_*}{n_e} \ . \quad (37)$$

The wave number $k_* \leq k_0$ is the characteristic scale of the turbulent energy transfer. The reduced Langmuir stationary turbulence spectra is given by fig 1.

5. Alfvén turbulence

The electro-magnetic modes generated in the cold plasma can be excited by the relativistic particles if they fulfill the Landau-synchrotron resonance condition

$$\omega_k = s\omega_{se} + \boldsymbol{k}_{\parallel}.\boldsymbol{v}_{\parallel} \ . \quad (38)$$

Where $\omega_{se} = |q_e|B/m_e c\gamma$ is the gyro-magnetic frequency of the resonant particle. For a frequency range of $\omega \ll \omega_{ce}$ we are mainly interested with the excitation of MHD waves by kinetic effects and particularly with the Alfvén waves.

In this case, we will only retain the resonances with $|s| = 1$ and will neglect ω in regards of the two other terms. Thus resonance with Alfvén waves occurs for $p_{\parallel} > m_p V_A$. Because this threshold is high, Langmuir turbulence is essential to accelerate particles up to an energy above the threshold.

5.1. Growth rates

The previous remarks allow us a direct derivation of the growth rate of Alfvén waves from Melrose (1968):

$$\frac{\Gamma_{\boldsymbol{k}}}{kV_A} = -2\frac{\pi^3 q_e^2}{k^2 c|cos\theta|} \int \frac{1-\mu^2}{|\mu|} p^2 (\frac{cos\theta}{|cos\theta|}\frac{\partial}{\partial\mu} - \frac{V_A}{c} p \frac{\partial}{\partial p})|_{p_r(\mu)} f d\mu \ , \quad (39)$$

where $V_A = B/(4\pi m_i n_i)^{1/2}$ is the Alfvén speed. In the previous expression p must fulfill the resonance condition

$$p = p_r(\mu) = \frac{|q_e|B}{kc|cos\theta||\mu|} \ . \quad (40)$$

The derivation of the growth rate with a mono-energetic distribution function (Eq. (22)) is rather tedious, since $f(\boldsymbol{p}')$ contains a factor $\delta(p' - \overline{p}')$ in addition to the other δ factor included in the resonance condition. The detailed calculation is reported to the appendix. The final expression can be cast into the form

$$\frac{\Gamma_{\boldsymbol{k}}}{kV_A} = \frac{\pi}{|cos\theta|\overline{\gamma}'} (\frac{\omega'_{p*}}{kc})^2 (\frac{cos\theta}{|cos\theta|}\beta_b - \frac{V_A}{c})(1 + \gamma_b \beta_b \mu_0(\overline{p}')) \ , \quad (41)$$

where $\mu_0(p) = \omega_{se}/(k|cos\theta|p) \geq 0$. The forward modes ($cos\theta \geq 0$) are unstable for $v_b \geq V_A$.

5.2. Non-linear transfer

We are interested with non-linear quadratic interactions involving Alfvén waves. The second order processes are essentially scattering off ions and three modes coupling. The first process has been already studied by Kaplan and Tsytovich (1973). Akhiezer et al. (1967) have derived the plasmon collision integral involving Alfvén with fast and slow magneto-sound waves. There is no process involving three Alfvén waves since the matrix element vanishes in this case. Moreover in the isothermal plasma considered here there are no coupling with ion-acoustic oscillations.

All these processes imply a formally identical evolution equation for the reduced turbulent spectrum $S_a(k, cos\theta)$. This spectrum is normalized as

$$\int_k S_a(k, cos\theta)\frac{dk}{4\pi} dcos\theta = \frac{<\delta B^2>}{B^2} \ , \quad (42)$$

where $<\delta B^2>/B^2$ is the turbulence level.

Including the growth rate this equation takes the form

$$\frac{\partial}{\partial t} S_a(k, cos\theta) = 2\Gamma_{\boldsymbol{k}} S_a(k, cos\theta) + kV_A S_a(k, cos\theta) \int_0^\infty \int_{-1}^1 \Psi(k_1, cos\theta_1, k, cos\theta) S_a(k_1, cos\theta_1) dk_1 dcos\theta_1 \ , \quad (43)$$

and $\Psi(k_1, cos\theta_1, k, cos\theta)$ is the non-linear kernel of the interaction ($\boldsymbol{k_1}$ the wave number of the second decay plasmon is characterized by its modulus k_1, its angle relatively to the magnetic field θ_1), and is of order of unity.

These interactions do not favor any particular scale. Thus the kernel of order of 1 can be written as $\Psi(k_1/k, cos\theta_1, cos\theta)$. If we use the new variable $y = k_1/k$ in the Eq. (43), we obtain

$$k^2 V_A \int dy dcos\theta_1 \Psi(y, cos\theta_1, cos\theta) S_a(y, cos\theta_1) = -2\Gamma_{\boldsymbol{k}}(cos\theta) \quad . \tag{44}$$

Let us assume that the growth rate is a power-law function of k say

$$\Gamma_k(cos\theta) = g(cos\theta) k_1 V_A (\frac{k}{k_1})^m , \tag{45}$$

where $k_1 = \omega_{ce}/\overline{\gamma}c$. We seek for solutions of the form

$$S_a(k, cos\theta) = \frac{h(cos\theta)}{k_1} (\frac{k}{k_1})^n , \tag{46}$$

where the functions g and h represent the anisotropic contribution to both growth rate and turbulent spectrum and where $k_1 \leq k \leq k_{abs} = \omega_{ci}/V_A$. Reporting Eqs. (45) and (46) in Eq. (44) we easily find a simple relation between n and m

$$n = m - 2 . \tag{47}$$

The isotropic part of this turbulent spectrum can be cast into the following form obtained from (41)

$$S_a(k) \simeq \frac{1}{k_1 \overline{\gamma}' \gamma_b^2} (\frac{\omega'_{p*}}{(k_1 c)})^2 (k/k_1)^{-3} . \tag{48}$$

$S_a(k)$ is the instability contribution to the turbulent spectrum.

We have also to consider the inertial contribution, which must cancel the first term of the Eq. (44). The resulting transfer spectrum has a power-law form with an index p.

Note that p is not unique, see for example the solutions in k^2 and $k^{-3/2}$ derived by Mc Ivor (1977) and Achterberg (1979), the first one corresponding to a zero energy flow and the second (Kraichnan spectrum) to a constant energy flow towards the smallest scales. Like in the case of Langmuir wave, mostly backward waves contribute to accelerate particles.

6. Stochastic particle acceleration

The interactions between particles and turbulent fluctuations are characterized by an exchange of energy which can lead to the acceleration of the particles. This exchange is described by a diffusion equation of the particles in momentum space.

$$\frac{\partial f}{\partial t}|_{turb} = \frac{\partial}{\partial p_i} D_{ij} \frac{\partial f}{\partial p_j} . \tag{49}$$

The quasi-linear coefficients D_{ij} describe scattering effects and can be evaluated with the stationary turbulent spectra. For the Langmuir waves:

$$D_{ij} = \pi q_e^2 n_e T_e \int \frac{k_i k_j}{k^2} S_{\boldsymbol{k}} \delta(\omega - \boldsymbol{k}_\parallel . \boldsymbol{v}_\parallel) dcos\theta dk . \tag{50}$$

In magnetized plasma the relevant diffusion coefficient is $D_\parallel(\mu)$ parallel to the magnetic field direction.

The Alfvén waves contribute essentially to the relaxation of the anisotropic part of the distribution function and tends to isotropize the particle distribution function with a pitch-angle scattering frequency:

$$\nu_{sA}(\mu, p) = (\frac{q_e B v}{\sqrt{2} pc})^2 \int cos\theta^2 S_a(\boldsymbol{k}) \delta(\omega \pm \omega_S - k_\parallel v_\parallel) dcos\theta dk . \tag{51}$$

The acceleration by Alfvén waves is of the second order in V_A/c with respect to the pitch-angle frequency.

6.1. Derivation of the Fokker-Planck equation

In strong field regime, with an isotropic turbulence, we derive the diffusion equation in spherical coordinates. Therefore, for Langmuir turbulence we express the momentum diffusion coefficient D_{pp} and the pitch-angle scattering frequency ν_{sL} in term of the previous parallel coefficient, namely for the Langmuir turbulence

$$\begin{aligned} D_{ppL} &= \mu^2 D_\parallel , \\ \nu_{sL} &= 2\frac{1-\mu^2}{p^2} D_\parallel . \end{aligned} \tag{52}$$

For Alfvén turbulence we introduce an acceleration coefficient

$$D_{ppA}(\mu, p) = \frac{1}{2}(\frac{V_A}{c})^2 p^2 \nu_{sA}(\mu, p) . \tag{53}$$

The diffusion is balanced by the Inverse Compton losses given by the Eq. (3), which can be written as $\gamma^2 \nu_{IC}(1, \mu)$; $\nu_{IC}(1, \mu)$ describes the losses for a Lorentz factor $\gamma = 1$

$$\nu_{IC}(1, \mu) = 2\pi\sigma_T \frac{J}{m_e c^2} [(3 - \chi) - 4\eta\mu + (3\chi - 1)\mu^2] , \tag{54}$$

where $\eta = H/J$, and $\chi = K/J$.
we obtain,

$$\begin{aligned} \frac{\partial}{\partial t} f(p, \mu) \simeq & \frac{1}{p^2}\frac{\partial}{\partial p}(p^4 \nu_{IC}(\mu) f(p, \mu)) \\ & + \frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} f(p, \mu) \\ & + \frac{1}{2}\frac{\partial}{\partial \mu}(1 - \mu^2)\widetilde{\nu}_s(p, \mu)\frac{\partial}{\partial \mu} f(p, \mu) , \end{aligned} \tag{55}$$

where $\widetilde{\nu}_s \simeq \nu_{sA} + \nu_{sL}$.

6.2. Characteristic times

The previous stationary turbulent spectra straightly give the diffusion coefficients and the pitch-angle frequency.

For the Langmuir waves injecting Eq. (37) in Eq. (50) we obtain

$$D_{ppL} \simeq \frac{n'_*}{n_e} \frac{6}{\overline{\gamma}' \beta_b^2 \gamma_b^2 |\mu|} \frac{k_0^2}{k_*^2} \omega_{pe} m_e T_e . \tag{56}$$

The pitch-angle μ is in the range $\mu_0 - 1$. The exact expression of the pitch-angle frequency ν_{sA} is unknown since the core of the non-linear three waves interaction cannot be derived analytically. Thus, approximatively ν_{sA} is only derived from the growth rate $\Gamma_{\boldsymbol{k}}$ and the Eq. (51)

$$\nu_{sA} \simeq \frac{\pi}{5}\frac{\omega_{p*}^{'2}}{\omega_{ce}}\frac{\gamma}{\gamma'}\mu^2 \ . \qquad (57)$$

The ratio of the Langmuir acceleration time $\tau_{accL} = p^2/(2D_{ppL})$ to the Alfvén angular diffusion time ν_{sA}^{-1} is

$$\nu_{sA}\tau_{accL} \simeq \frac{\pi}{108}\frac{m_e}{m_p}\frac{\omega_{pe}}{\omega_{ce}}\gamma_b^2\gamma^3(\frac{c}{v_{the}})^4\mu^2|\mu| \ . \qquad (58)$$

For a non relativistic thermal plasma with a kinetic temperature of order of the Compton temperature $T_{Com} \sim 50ev$, the thermal speed of the ambient electrons is $v_{the} \simeq 10^{-2}c$. The ratio of the cyclotron to the plasma frequency cannot be greater than 10^3 in standard astrophysical conditions. The Langmuir turbulence is then inefficient at high energy, and all angular scattering and acceleration processes are dominated by Alfvén turbulence, even if the Lorentz factor γ_{lim} (or a momentum p_{lim}) which balances the two rates tends to increase with an increasing angle. Anyway, Langmuir turbulence is very efficient to accelerate particles up to Lorentz factors of order of ten to one hundred for hotter thermal electrons; which prevents an accumulation of low energy pairs (the so called "dead-end" problem) and ensures an injection above the threshold for resonant interaction with Alfvén waves.

As described in Sect. 2, the IC anisotropic emission of a relativistic particle in a soft photon field is strongly reduced in a small cone of opening angle $\sim 2/\gamma_b$. The particle distribution can then be divided in two different contributions depending on the pitch-angle of the particle.

The pitch-angle scattering by Alfvén turbulence is the fastest process inside the cone. Thus, the particle distribution function is weakly dependent of α, and is obtained by balancing, in the Fokker-Planck equation, the IC cooling and the acceleration contributions. Note that at low energies, the distribution function is also angle independent because the angular diffusion is dominated by Langmuir waves whose pitch-angle frequency $\nu_{sL} \propto 1/p^2$.

Outside the cone the IC cooling is the dominant energetical process and controls the shape of the distribution function.

6.3. Stationary solutions of the Fokker-Planck equation

6.3.1. Inside the cone

For $\alpha \le \alpha^*$, the distribution function in the disk frame can be described by

$$f(p,\mu) = f_0(p) + \delta f(p,\mu) \ , \qquad (59)$$

where $\delta f(p,\mu) \le f_0(p)$.

At high energies, and for $\mu \simeq cos\alpha^* \sim 1$, f_0 is solution of

$$\frac{1}{p^2}\frac{\partial}{\partial p}(p^4\nu_{ICi}f_0(p)) + \frac{1}{p^2}\frac{\partial}{\partial p}p^2 D_{ppA}\frac{\partial}{\partial p}f_0(p) = 0. \qquad (60)$$

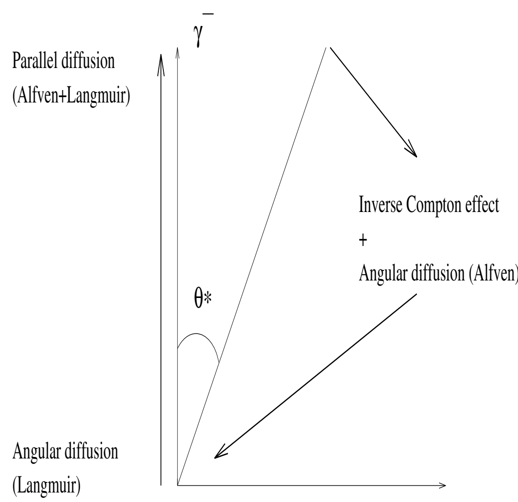

Fig. 2. Scheme of the acceleration mechanism.

The frequency ν_{ICi} obtained from Eq. (8) is

$$\nu_{ICi} = 4\pi\sigma_T J[2\epsilon_\eta - \epsilon_\chi] \ . \qquad (61)$$

For momenta in the absorption range $p_{min} < p < p_{max}$, and from Eqs. (53) and (57) D_{ppA} scales like p^3. Here p_{max} corresponds to the resonance with the mode of wave number $k_1 = \omega_{ce}/(\gamma_{max}c)$, and $p_{min} = Max(m_pV_A, p_{lim})$. The acceleration time scales as p like the cooling time, leading to a power-law stationary solution with an index given by

$$s_i = 2(\frac{c}{V_A})^2\frac{\nu_{ICi}}{\nu_s} \ . \qquad (62)$$

For $p > p_{max}$, particles resonate with the flatter transfer spectrum and thus the diffusion coefficient varies as p^r with $r < 3$. This kind of spectrum can be associated with an external source of Alfvén turbulence from the accretion disk, at large scales ($k < k_1$), and loaded by the MHD jet (see HP). In this range, the acceleration rate does not follow the variation of the IC cooling rate and the distribution function drops exponentially with p.

The maximum momentum p_{max} can be estimated by the balance between the IC cooling and the acceleration times. Moreover, as these times have the same energy dependence this balance is verified for all momenta $p_{min} < p < p_{max}$, and the spectral index $s_i = 2$. Thus, the main effect of the Langmuir turbulence at low energy and Alfvén turbulence at higher energies is to built a flat energy distribution function inside the cone, dominated by high energy pairs with a maximum Lorentz factor $\gamma_{max} = p_{max}/(m_ec)$. The high energy particles are submitted to a strong pitch-angle scattering and are injected in the outer part of the acceleration cone. The angular diffusion by the Langmuir turbulence ($\nu_{sL} \propto 1/p^2$) dominates at small energy the angular diffusion by Alfvén waves ($\upsilon_{sA} \propto p$). The low energy pairs are re-injected in the inner part of the cone leading to a self-consistent mechanism (see fig 2).

6.3.2. Outside the cone

For $\alpha \geq \alpha^*$, the injection process described above can be treated as sources terms in the Fokker-Planck equation. We have neglected the width of the acceleration cone, so the injection occurs at $\mu = 1$.

The high energy particles are injected outside the cone at p_{max}, the cooled low energy pairs are re-injected in the cone with a momentum p_{min}. Therefore, we introduce a source term for the population outside the cone:

$$S(\mu, p) \equiv \phi_0 \delta(\mu - 1)(\delta(p - p_{max}) - \delta(p - p_{min})) , \tag{63}$$

where ϕ_0 is the source flux.

The inhomogeneous Fokker-Planck equation (Eq. (56)) can be written as

$$\begin{aligned} S(\mu, p) \simeq & \frac{1}{p^2} \frac{\partial}{\partial p} (p^4 \nu_{ICe}(1, \mu) f(p, \mu)) \\ & + \frac{1}{p^2} \frac{\partial}{\partial p} D_{pp}(p, \mu) p^2 \frac{\partial}{\partial p} f(p, \mu) \\ & + \frac{\partial}{\partial \mu} (\frac{\widetilde{\nu}_s(p, \mu)}{2} (1 - \mu^2)) \frac{\partial}{\partial \mu} f(p, \mu) . \end{aligned} \tag{64}$$

The external cooling frequency ν_{ICe} is of order of $\gamma_b^4 \nu_{ICi}$. Thereafter all the coefficients are replaced by there average value over μ; ν_{ICe}, $\widetilde{\nu}_s(p)$, and $D(p)$. Expanding the distribution function in Legendre polynomials, $f(p, \mu) = \Sigma_n P_n(\mu) f_n(\mu)$, we can solve the exact solution of (64) for $p_{min} < p < p_{max}$; which gives

$$f_n(p) = A_n p^{-s_n} . \tag{65}$$

The spectral index s_n is the positive root of a second degree algebraic equation. Since the acceleration rate is smaller than the IC cooling rate in this angular region, we get the following estimate:

$$s_n = 4 + \frac{n(n+1)}{2} \frac{\nu_s}{\nu_{ICe}} . \tag{66}$$

The fundamental component (n = 0) dominates at high energy and thus we obtain a power-law distribution in p^{-4}. We therefore obtain a power-law distribution similar to that usually expected from shocks.

In a more general way, whatever are the momentum dependence of the diffusion coefficient and the pitch-angle scattering frequency, the Inverse Compton cooling process imposes the form of f_0 to be in p^{-4}.

In the general case keeping the acceleration coefficient in Eq. (64), the homogeneous solution is a pile-up distribution function

$$w(p) = exp(-\frac{1}{3}(\frac{p^3}{D})) . \tag{67}$$

For a more general acceleration coefficient $D = D_0 p^{s_1}$, the solution is

$$w(p) = exp(-\frac{1}{3 - s_1}(\frac{p}{p_{min}})^{3-s_1}) , \tag{68}$$

where $D_0 = p_{min}^{3-s_1}/\nu_{ICe}$. The index s_1 must be lower than 3, in order to have an IC cooling time ($\propto 1/p$) dominant in the outer part of the cone.

Moreover we can develop the source term as

$$\delta(\mu - 1) = \sum_{n=0}^{\infty} \frac{2n+1}{2} P_n(\mu) . \tag{69}$$

The inhomogeneous distribution function can be cast into the form

$$\begin{aligned} & f_0(p) = \phi_1 w(p) + \\ & \frac{\phi_0 w(p)}{2D_0} \int \frac{p'^{-(2+s_1)}}{w(p')} [H(p' - p_{max}) - H(p' - p_{min})] dp' , \end{aligned} \tag{70}$$

where the heaviside function $H(x - x_0) = 1$ for $x > x_0$ and, $H(x - x_0) = 0$ for $x \leq x_0$.

For $p_{min} < p < p_{max}$, using the new variable $x' = [1/(3 - s_1)](p'/p_1)^{3-s_1}$, the stationary solution is a power-law function with an index equals to 4. Namely,

$$f_0(p) \simeq \phi_1 w(p) + \frac{\phi_0}{2D_0}(\frac{p}{p_{min}})^{-4} . \tag{71}$$

We can write the explicit value of the zero order coefficient A_0 of the previous expansion as

$$A_0 = \frac{\phi_0}{2D_0} . \tag{72}$$

Therefore, for $p_{min} < p < p_{max}$, the particle energy density distribution varies as, p^{-2}.

7. Relativistic regime

The heating of the pair plasma is leading to the situation where most of the energy is supported by the relativistic pair plasma. Here again the instability generating mechanism is the streaming of a beam in the plasma. The unusual streaming of a cold beam (the ambient medium) in a hot pair plasma needs a particular treatment of both growth rates and non-linear transfer. In a first step, we derive the dispersion relation of electrostatic, and electro-magnetic waves, and consider their destabilization by the backward streaming cold electron-proton beam.

All quantities are derived in the relativistic plasma frame where, for convenience, the primes are omitted.

7.1. Pair plasma electrostatic waves

The general dispersion relation for longitudinal modes in a magneto-active plasma is given by:

$$\varepsilon_l(\omega, \boldsymbol{k}) = 1 + (\frac{\omega_{p*}}{k_\parallel})^2 \frac{m_e}{n_*} \int \frac{1}{\omega - \boldsymbol{k}_\parallel . \boldsymbol{v}_\parallel} \boldsymbol{k}_\parallel \cdot \frac{\partial f}{\partial \boldsymbol{p}_\parallel} d\boldsymbol{p} . \tag{73}$$

For a mono-energetic isotropic distribution function

$$f(\boldsymbol{p}) = \frac{n_*}{4\pi \overline{p}^2} \delta(p - \overline{p}) , \tag{74}$$

adding an imaginary part to ε_l to account for the Landau damping we easily obtain after integrating over μ

$$\varepsilon_l(\omega, k_\parallel) = 1 - \frac{\omega_{p*}^2}{2\omega^2}\frac{\omega}{k_\parallel \overline{v}}\left(\frac{\omega}{\omega - k_\parallel \overline{v} + i\eta} - \frac{\omega}{\omega + k_\parallel \overline{v} + i\eta}\right) . \quad (75)$$

where η is an arbitrary small positive quantity introduced to treat the pole according to causality. This quantity describes the treatment of the resonant pole in Eq. (73).

Mainly we have three distinct regimes: $\omega \ll k_\parallel c$, $\omega \gg k_\parallel c$, and $\omega \simeq k_\parallel c$.

i. $\omega \gg k_\parallel c$:
By expanding to the lowest order in $k_\parallel c/\omega$ the dispersion relation becomes for $\overline{v} = c$

$$\varepsilon_l(\omega, k_\parallel) = 1 - \frac{\omega_{p*}^2}{\omega^2}\frac{1}{\overline{\gamma}}(1 + o(\frac{k_\parallel^2 c^2}{\omega^2})) . \quad (76)$$

The beam instability implies $k_\parallel \beta_b c \simeq \omega_{p*}/\sqrt{\overline{\gamma}}$. Thus the resonant wave number $k_\parallel \sim \omega_{p*}/(\beta_b c\sqrt{\overline{\gamma}})$ implies $\beta_b \gg 1$ and then no allowed resonance for such ω values.

ii. $\omega \ll k_\parallel c$:
Always by expanding to the lowest order in $\omega/k_\parallel c$ the dispersion relation becomes

$$\varepsilon_l(\omega, k_\parallel) = 1 + \frac{\omega_{p*}^2}{k_\parallel^2 c^2 \overline{\gamma}}(1 + \frac{\omega^2}{k_\parallel^2 c^2}) . \quad (77)$$

The waves are evanescent in this ω regime.

iii. $\omega \simeq k_\parallel c$:
In his case we obtain

$$\varepsilon_l(\omega, k_\parallel) = 1 - \frac{\omega_{p*}^2}{\overline{\gamma}(\omega^2 - k_\parallel^2 c^2)} , \quad (78)$$

which leads to

$$\omega^2 = \frac{\omega_{p*}^2}{\overline{\gamma}} + k_\parallel^2 c^2 . \quad (79)$$

For this regime, electrostatic waves can spontaneously propagate in the relativistic plasma. The cold perturbative medium contributes to the dispersion relation and may destabilize the waves. Injecting a mono-energetic distribution

$$f_{ei}(\boldsymbol{p}) = n_b \delta^{(2)}(p_\perp)\delta(p - p_b) , \quad (80)$$

where $p_b = -m v_b \gamma_b$, and n_b is the beam density in the pair plasma frame we obtain

$$\varepsilon_{lb}(\omega, k_\parallel) = \frac{n_b}{n_*}\frac{\omega_{p*}^2}{\gamma_b(\omega + k_\parallel v_b)^2} . \quad (81)$$

For the waves propagating backward with respect to the direction of motion of the beam, the solutions can be search with $\omega = k_\parallel c(1 + Z)$ where Z is complex and $\ll 1$. By expanding to the lowest order in Z this leads to a second order equation with a positive discriminant. No waves are destabilized in this direction.

For forward propagating waves $\omega = -k_\parallel c(1+Z)$, the dispersion relation is of the third degree in Z. Neglecting the factors with $\overline{\gamma}^{-1} \ll 1$, the ratio η of the cold beam to the pair density n_b/n_* must verify the following condition to give rise to an instability:

$$\eta < \frac{4}{3\gamma_b^3}\frac{k_\parallel^2 c^2}{\omega_{p*}^2} . \quad (82)$$

For $\omega_{p*} \sim k_\parallel c$ the condition is not relevant. Even in this case there is no way for destabilizing electrostatic Langmuir modes.

This may be explain by the fact that the phase speed of electrostatic modes generated in a relativistic electron-positron beam is $\geq c$, ruling out any resonance with non-relativistic particles.

7.2. Pair plasma electro-magnetic waves

7.2.1. Dispersion relation and absorption

In a magneto-active plasma, the general form of the dispersion relation of both right and left electro-magnetic modes is given for any distribution function $f(\boldsymbol{p}, \boldsymbol{r})$:

$$D_\pm = \omega^2 - k_\parallel^2 c^2 + \frac{\omega_{p*}}{2}\frac{m_e}{n_*} \sum_a \int_0^\infty 2\pi p_\perp dp_\perp \int_{Re} dp_\parallel \frac{v_\perp \Psi_a}{\omega - k_\parallel v_\parallel \mp \varepsilon_a \omega_{sa}} , \quad (83)$$

a denotes the different species present in the plasma.

$$\Psi_a = (\omega - k_\parallel v_\parallel)\frac{\partial}{\partial p_\perp} f(\boldsymbol{p}) + k_\parallel v_\perp \frac{\partial}{\partial p_\parallel} f(\boldsymbol{p}) , \quad (84)$$

with $\varepsilon_a = \mathrm{sgn}\ q_a$.

For Alfvén modes $\omega \ll \overline{\omega}_{sa}$. The dispersion relation for the relativistic medium expressed in spherical coordinates in momentum space is

$$D_\pm = \omega^2 - k_\parallel^2 c^2 - \sum_a \frac{\omega_{p*}^2}{2\overline{\gamma}} \int (1 - \mu^2)\left(\frac{\omega}{\omega \mp \epsilon_a \overline{\omega}_{sa} - k_\parallel c\mu} \mp \epsilon_a \frac{\overline{\omega}_{sa}}{\omega}\frac{\omega^2}{(\omega \mp \epsilon_a \overline{\omega}_{sa} - k_\parallel c\mu)^2}\right) d\mu . \quad (85)$$

Where $\overline{\omega}_{sa} = |q_a|B/\overline{\gamma} m_e c$. After integrating over μ, for both electron and positron contributions, the principal value leads to the dispersion relation

$$D_\pm = \omega^2 - k_\parallel^2 c^2 - \frac{8}{3}\frac{\omega_{p*}^2}{\overline{\gamma}}(\frac{\omega}{\overline{\omega}_{se}})^2 = 0 . \quad (86)$$

Which leads to the modified Alfvén speed of the relativistic medium:

$$\frac{\omega}{k} = V_* = \frac{c}{\sqrt{1 + \frac{8}{3}\frac{\omega_{p*}^2}{\overline{\gamma}\omega_{se}^2}}} . \quad (87)$$

The treatment of the poles allows to examine the absorption of these waves. It depends very sensitively to the following characteristic wave number $k_{abs} \equiv \overline{\omega}_{se}/c$. At $k \sim k_{abs}$, the waves are over-damped by synchrotron absorption. However, the damping rate vanishes rapidly when k is sufficiently smaller than k_{abs} (exponentially for an exponentially decreasing distribution, like a power law for a power law distribution). Thus, the dissipation range is restricted to k smaller but close to k_{abs}.

7.2.2. The proton back-streaming instability

We now turn to the contribution of the cold $e^- - p^+$ beam to the dispersion relation. For $\omega \ll \overline{\omega}_{sa}$

$$D^{th}_{\pm} = -\sum_a \frac{\omega^2_{pa}}{\gamma_b} \frac{\omega + k_{\|} v_b}{(\omega + k_{\|} v_b) \mp \varepsilon_a \omega_{ca}/\gamma_b} \ . \quad (88)$$

The general dispersion relation regardless the Landau damping effects is:

$$D_{\pm} = \omega^2 - k_{\|} V^2_* - \sum_a \frac{\omega^2_{pa}}{\gamma_b} \frac{\omega + k_{\|} v_b}{(\omega + k_{\|} v_b) \mp \epsilon_a \omega_{ca}/\gamma_b} \ . \quad (89)$$

Frequency regimes with $\omega \ll \omega_{c*}/\overline{\gamma}$ give a fortiori $\omega \ll \omega_{ca}/\gamma_b$, and ω can be neglected in the numerator of the cold plasma dispersion relation.

The left polarized modes are destabilized by the ions. In this case the resonant wave number is $k_r = -\omega_{ci}/\gamma_b(v_b - V_*) < 0$ for $v_b > V_*$. Then a backward propagating mode with $k_{\|} = k_r(1+\delta)$ (δ real and $\ll 1$) and with a frequency $\omega = -k_{\|} v_*(1+Z)$ (Z complex, and $|Z| \ll 1$) leads to a second order equation

$$Z^2 - Z\delta(\frac{v_b}{V_*} + 1) + \frac{\omega^2_{pi}}{2k^2_r V^2_* \gamma_b}(\frac{v_b}{V_*} - 1) = 0 \ . \quad (90)$$

A negative discriminant can be obtained for $v_b > V_*$, as long as

$$|\delta| \leq \sqrt{2\gamma_b} \frac{\omega_{pi}}{\omega_{ci}} (\frac{v_b - V_*}{V_*})^{1/2} \ , \quad (91)$$

in the relativistic case $\omega_{pi} \leq \omega_{ci}$ and $|v_b - V_*| \ll c$ leads to $|\delta| \ll 1$. Therefore an instability is found for Alfvén waves that resonate with the proton back-stream. The maximum growth rate of pair Alfvén waves is

$$\Gamma_k \simeq \frac{\omega_{pi}}{\gamma_b} (\frac{v_b - V_*}{2V_*})^{1/2}. \quad (92)$$

This instability exists only if $k_r < k_a$, which puts a threshold that we will discuss later on.

The backward propagating right polarized Alfvén modes are not destabilized by the cold electron beam, since the discriminant of the corresponding second order equation is always positive.

The above mentioned condition $k_r < k_a$ leads to an upper limit for the mean Lorentz factor of the relativistic population $\overline{\gamma} < \overline{\gamma}_{sat} \equiv \gamma_b(m_i/m_e)$. The factor $\gamma_b(m_i/m_e)$ is a saturation value for the internal energy of the pair distribution due to their interaction with the cold protonic plasma. Below $\overline{\gamma}_{sat}$ the interaction with the cold ions contributes to increase the internal energy of the pair population, while above this value the pairs cool by Compton radiation.

7.3. Non-linear transfer

This particular instability does not evolve according to the quasi linear relaxation followed by random phase mode coupling. Indeed, the excited waves have a narrow band spectrum Δk about k_r, namely $\Delta k/k_r < 1$ (Eq. (91)). A quasi monochromatic wave can grow and can probably trap the protons. In fact whatever the protons are trapped or not, a self modulation instability of the monochromatic wave will develop building non-linear wave packets (possibly solitons). The shortest wavelength components of these wave packets will undergo Landau-synchrotron damping on the pair plasma. Therefore, the ambient protons couple to the pair plasma through this non-linear process whose theory would deserve detailed investigations. This process should necessarily lead to a driving of the ambient protons and to the heating (by Landau-synchrotron absorption) of the pair plasma. As long as no other process is at work, a regulation mechanism brings the internal energy of the pair plasma to a saturation value. When this internal energy is smaller, the interaction with the ambient protons heats the pair plasma, and when the internal energy is larger, the interaction no more holds and the radiation losses cool the pair plasma. Further heating can be provided by the turbulence carried by the MHD jet (see HP).

8. Astrophysical applications

8.1. Cooling processes

In relativistic astrophysical leptonic jets, synchrotron and Inverse Compton processes are the two main sources of cooling mechanism. The total IC power emitted by a relativistic particle with a Lorentz factor γ is given by the Eq. (3). This expression is general and does not depend on the nature of the soft photon source; an accretion disk, synchrotron radiation in synchro-Compton process, or clouds diffusing primary radiations. This power equals the opposite of the energy lost per second by an ultra-relativistic particle $\dot{\gamma} m_e c^2$. The corresponding IC cooling time is (with the notation associated with the Eq. (54))

$$\tau_{IC} = |\frac{\dot{\gamma}}{\gamma}| \simeq (\gamma \nu_{IC}(1, \mu))^{-1}. \quad (93)$$

The same relativistic particle with a pitch-angle ϑ in a magnetic field B will emit synchrotron radiation with a total power given by

$$P_{syn} = 2\sigma_T c \gamma^2 sin^2\vartheta \frac{B^2}{8\pi} \ , \quad (94)$$

The resulting synchrotron cooling time is

$$\tau_{syn} = \frac{8\pi}{2 sin^2\vartheta \sigma_T c} \frac{m_e c^2}{B^2 \gamma} \ . \quad (95)$$

For a longitudinal magnetic field ($cos\vartheta = \mu_e$) the condition the synchrotron process dominates if

$$\frac{U_{ph}}{U_B} \frac{2 + \varepsilon_\chi - 4(1+\mu)\varepsilon_\eta + (2 - 3\varepsilon_\chi)\mu^2}{2(1-\mu^2)} \leq 1 \ . \quad (96)$$

where $\chi = K/J = 1-\varepsilon_\chi, \eta = H/J = 1-\varepsilon_\eta, U_B = B^2/8\pi$, and $U_{ph} = 2\pi J/c$. At a pitch-angle $\alpha^* \simeq 2/\gamma_b$ where the IC losses are strongly reduced (see Eq. (8)) the condition (96) simplifies as

$$\frac{U_{ph}}{U_B} \leq \frac{2\gamma_b^2}{3} . \quad (97)$$

As explained in the previous sections, the nature of the kinetic theory of the relativistic beam depends on the pair beam density. The evolution of the pair population has already been described in MHP in the framework of two flow model (Sol et al. (1989)); a pair beam confined in the magnetic structure of a MHD jet launched by a magnetized accretion disk. One of the most important result is the existence of a compact region where the pair beam density reaches a maximum value. The localization of this region above the central object (at a distance z_0) is set by a double condition on the opacity to pair production and the Thomson opacity of the pair beam

$$\begin{aligned}\tau_{\gamma\gamma}(z_0) &= 1\\ \tau_T(z_0) &= n'_*(z_0)\sigma_T r(z_0) = 1 ,\end{aligned} \quad (98)$$

where $r(z_0)$ is the beam width at z_0. From the condition fulfilled by the Thomson opacity, for a jet width of $10r_g$ the maximal pair density scales as

$$n'_*(z_0) \simeq 5\frac{10^{17}}{M/M_\odot}\mathrm{cm}^{-3} . \quad (99)$$

For an Eddington accretion rate $\dot{M} \simeq 3\ 10^{-8}(M/M_\odot)M_\odot/yr$, the distance of the compact region derived from Eq. (78) in MHP, for a power law energy distribution with an index s is

$$\begin{aligned}\frac{z_0}{r_g} &\simeq 70[\ 3^{(s+5)/(s+17)}][10^{8(s-3)/(s+17)}]\\ &\quad (\frac{M}{M_\odot})^{(3-s)/(s+17)} .\end{aligned} \quad (100)$$

Extragalactic sources have an index $2 \leq s \leq 3$, and black hole mass of order $10^8 M_\odot$ leading to a maximal pair density of order of $10^9\mathrm{cm}^{-3}$ and a compact zone located at $\sim 100-200r_g$. Galactic objects with sometimes steeper index up to 4, and black hole with solar mass have maximal pair density of order $10^{18}\mathrm{cm}^{-3}$ and a compact zone located at $\sim 200r_g$. The different values of z_0 indicate the extension of the non-relativistic zone; the relativistic regime typically dominates for $z > z_0$.

Let us return now to the comparison of the different cooling processes. The soft disk photon energy density scales as

$$U_{ph} \simeq 2\frac{10^{15}}{M/M_\odot}(\frac{r_g}{z})^2\mathrm{erg/cm}^3 . \quad (101)$$

This estimation have been made for a disk emitting at an Eddington luminosity $L_d = L_{Edd} = 10^{38}M/M_\odot$erg/s, and a distance above the central object z is measured in Schwarzschild radius unit $r_g \simeq 3\ 10^5 M/M_\odot$cm. The condition (97) then transforms as

$$B \geq [6\ 10^{16}(\frac{r_g}{z})^{5/2}(M/M_\odot)^{-1}]^{1/2}Gauss . \quad (102)$$

At distances $\sim 100-200r_g$, the condition (97) is fulfilled for magnetic fields $\sim 10^2 Gauss$ for extragalactic black holes, and for $10^6 Gauss$ for galactic black holes, but due to re-absorption, the real magnetic values are surely underestimated. From Eq. (96) if the magnetic energy density increases, the opening angle of the reduced losses cone decreases as $\sqrt{U_{ph}/(\gamma_b U_B)}$. The synchrotron losses then contribute to sharpen the cone, but do not question its existence.

In the same way, an external source of photon (apart from the disk) cannot be considered as a dominant source of cooling in the compact region. As showed by MHP, the Compton power emitted by the relativistic particles due to their interaction with soft photons diffused by surrounding clouds is not efficient before $500-1000r_g$, far from the regions involved here.

Thereafter, we will uniquely consider the role of soft photons from the accretion disk in the cooling processes of the relativistic pair plasma.

8.2. The non relativistic regime

We now turn to the kinetic theory developed in full details in the previous sections. As described before, unstable modes arises by the streaming of the relativistic population in the ambient electron-proton plasma. In this scheme, the plasma waves are supported by the medium with the greater mass energy density. For tenuous pair population, the ambient plasma energy density overcomes the pair one. All the theory is derived in the MHD jet frame.

8.2.1. Conditions for the non relativistic regime

The non relativistic regime is achieved when the pair energy density is smaller than the MHD plasma one. This gives

$$n'_*\overline{\gamma}' m_e c^2 < n_i m_i c^2 . \quad (103)$$

Let us give an estimate of n_i.

Sol et al. (1989), and Rosso & Pelletier (1994), argued that the MHD jet at VLBI scales has an Alfvén velocity in the range

$$0.1c < V_A < c . \quad (104)$$

So defining n_0 such that $V_A = c$ we obtain in the jet

$$n_0 < n_i < 10^2 n_0 . \quad (105)$$

The proton density n_{0d} on the disk surface corresponding to an Alfvén speed $V_{Ad} = c$ for $B = B_{Edd} = 4\ 10^8/\sqrt{M/M_\odot G}$ is of order of

$$n_{0d} = \frac{10^{19}}{M/M_\odot}\mathrm{cm}^{-3} . \quad (106)$$

The proton density n_0 typically drops by a factor of 10^3 for an isothermal disk (Ferreira & Pelletier (1995)), leading to a proton density in the MHD jet of order of $10^8 cm^{-3}$ for extragalactic sources and $10^{16}\mathrm{cm}^{-3}$ for galactic sources. Note that because of the electro-neutrality in the ambient medium, the cold electrons have a density $n_e = n_i$. In fact, the Bohm diffusion of the protons from the MHD flow to the pair plasma region implies a diffusion time much longer than the transit time. The proton density in the beam region is surely lower than the previous estimates.

8.2.2. Astrophysical signatures

We now emphasize on the astrophysical consequences of the kinetic theory derived previously.

The highly anisotropic Compton cooling (Eq. (3)) determines a cone of half angle $\alpha^* \sim 2/\gamma_b$ of strongly reduced IC losses. Inside the cone, acceleration by Langmuir waves prevails at low energy, but for Lorentz factors $\gamma \geq \gamma_{lim}$ the Alfvén turbulence dominates. This factor γ_{lim} is given by the Eq. (58) with $\mu \sim 1$

$$\gamma_{\rm lim} \simeq [\frac{108}{\gamma_b^2}\frac{m_p}{m_e}(\frac{v_{the}}{c})^4\frac{\omega_{ce}}{\omega_{pe}}]^{1/3} . \tag{107}$$

For extragalactic sources the cold electron Compton temperature $\sim 50ev$, leading to thermal speeds $v_{the} \sim 10^{-2}c$. In galactic objects the Compton temperature is higher; $\sim 1KeV$ and $v_{the} \sim 4\ 10^{-2}c$. The ratio of the cyclotron to the plasma frequency is $\leq 10^3$, leading to Lorentz factors $\gamma_{lim} \leq 10$.

The low energy pairs are then accelerated to Lorentz factors of order of few and feed the high energetic component of the distribution function. The balance between acceleration by Alfvén waves and the IC cooling process leads to the formation of a power-law distribution with an index s_i and with $\gamma_{lim} \leq \gamma \leq \gamma_{max}$.

Let us have an estimate of γ_{max}.

The Inverse Compton cooling time of a relativistic particle (with a Lorentz factor γ) is given by

$$\tau_{IC} = (\gamma \nu_{IC}(1,\mu))^{-1} , \tag{108}$$

where

$$\nu_{IC}(1,\mu) = 2\pi\sigma_T\frac{J}{m_e c^2}[(3-\chi) - 4\eta\mu + (3\chi-1)\mu^2] , \tag{109}$$

we recall that $\eta = H/J = 1-\varepsilon_\eta$ and $\chi = K/J = 1-\varepsilon_\chi$, such that $2\varepsilon_\eta - \varepsilon_\chi \simeq \gamma_b^4$ (see Sect. 2).

Inside the cone the IC cooling frequency is

$$\nu_{IC}(\gamma,\mu=1) \simeq 2\pi\sigma_T\gamma\frac{J}{m_e c^2}\gamma_b^{-4} . \tag{110}$$

As specified in Sect. (6) the maximum Lorentz factor of the pair population is obtained for balanced acceleration and cooling times leading to a spectral index $s_i = 2$ (Eq. (61)). The energy distribution function inside the cone is flat and $\overline{\gamma} \sim \gamma_{max}/2$. For $\mu \sim 1$, the pitch-angle scattering is

$$\nu_{sA} \simeq \frac{\pi}{5}\frac{\omega_{p*}^{'2}}{\omega_{ce}}\frac{\gamma}{\overline{\gamma}'} . \tag{111}$$

In the case of a Schwarzshild black hole with a mass M and a standard accretion disk emitting at the Eddington limit $L_d = L_{Edd} = 10^{38}M/M_\odot$erg/s (with a compactness $\ell_g = (L_d\sigma_T)/(4\pi m_e c^3 r_g) \sim 10^3$) the mean Lorentz factor of the particle distribution takes the form

$$\overline{\gamma}' \simeq (\frac{1}{10}\frac{V_A}{c})^2(\frac{\omega'_{p*}}{\omega_{ce}})^2\frac{\omega_{ce}r_g}{c\ell_g}(\frac{z}{r_g})^2 . \tag{112}$$

The relativistic pressure is dominated by the particles inside the cone since outside the pair distribution drops as a power law and displays an accumulation of low energy particles. The relativistic pressure is reduced by a factor γ_b^2 compared to the isotropic case

$$P_r \sim n'_*\frac{\overline{\gamma}'}{\gamma_b^2}m_e c^2 . \tag{113}$$

We assume the equipartition with the magnetic pressure; then

$$\frac{\omega'_{p*}}{\omega_{ce}} \simeq [\frac{\gamma_b^2}{\overline{\gamma}'}]^{1/2} . \tag{114}$$

Thus,

$$\overline{\gamma}' \simeq [\frac{1}{10}(\frac{V_A}{c})^2\frac{\omega_{ce}r_g}{c\ell_g}]^{1/2}(\frac{z}{r_g})^{5/4} . \tag{115}$$

In term of the black hole mass we obtain

$$\overline{\gamma}' \simeq 10^3[(\frac{V_A}{c})^2\frac{B}{B_{Edd}}]^{1/2}(\frac{z}{r_g})^{5/4}(\frac{M}{M_\odot})^{1/4} . \tag{116}$$

The compact region (of maximum of pair creation) is of order of $100r_g$ in MHP, leading to an extension of the non relativistic regime between 10 and $100r_g$. Moreover, the magnetic field typically drops by a factor of 10^2 between the disk surface and the compact zone. We then estimate the maximum Lorentz factor of the particle distribution in the non relativistic regime to be of order of 10^4-10^5 for extragalactic black holes and 10^2-10^3 for galactic black holes. The resulting bulk Lorentz factors, for a soft photon source of compactness ℓ, cannot be greater than $(l\gamma_{max}^2)^{1/7}$. For $\ell \sim 10^3$, this gives for extragalactic sources $\gamma_b \leq 10-12$ and for galactic sources $\gamma_b \leq 5-6$.

8.3. The relativistic regime

As stated before, this second case may occur in regions where the pair density n'_* approaches its maximum values and supports the instabilities. We can then examine the conditions for resonance between back-stream particles and pair plasma modes in this regime.

We consider the synchrotron resonance with electromagnetic waves $\omega' - k'_\parallel v_\parallel \pm \omega_{ca}/\gamma_b$, where ω_{ca} is the cyclotron frequency of the cold electrons ($a = e$) or protons ($a = i$).

The electron back-stream triggers a synchrotron maser instability of the right handed circularly polarized resonant mode. This would contribute to an interesting coherent emission in the radio band probably with a fast variability. We report the astrophysical investigation of this phenomenon to future works.

The proton back-stream is responsible for the destabilization of left handed circularly polarized Alfvén waves. The growth rate is slower than the electronic one, but has a much stronger dynamical effect on the flow.

However, this Alfvén wave is unstable only if its wavelength is larger than Larmor radius of the energetic pairs. In other words, we can define a saturation energy level for those pairs corresponding to the equality of the wavelength with the Larmor radius, namely

$$\overline{\gamma}'_{sat} = \frac{m_i}{m_e}\gamma_b \ , \tag{117}$$

such that the instability grows for $\overline{\gamma}' < \overline{\gamma}'_{sat}$.

The theory of the non-linear evolution remains to be done for this unusual scheme. Nevertheless we can state that the subsequent heating of the relativistic plasma leads to a saturation value of $\overline{\gamma}'$ given by Eq. (117).

Inversely, if the pair plasma would have a $\overline{\gamma}'$ larger than $\overline{\gamma}'_{sat}$, because of radiation cooling, $\overline{\gamma}'$ would decrease to $\overline{\gamma}'_{sat}$ as long as no other turbulent heating is at work. We then expect that the relativistic regime built up a distribution function of pairs such that $\overline{\gamma}' \sim \overline{\gamma}'_{sat}$. Bearing in mind that, even coupled, the proton component is less massive, the dynamics of pairs in the Compton radiation field is not significantly modified and therefore we still have:

$$\gamma_b \simeq (\ell\overline{\gamma}'^2)^{1/7} \tag{118}$$

Combining the Eqs. (12) and (13)

$$\gamma_b \leq \ell^{1/5}(\frac{m_p}{m_e})^{2/5} \tag{119}$$

and

$$\overline{\gamma}' \leq \ell^{1/5}(\frac{m_p}{m_e})^{7/5} \tag{120}$$

For a soft photon source compactness of radius $10r_g$ ($\ell \sim 200$) the Eqs. (119) and (120) give typical upper values of the bulk Lorentz factor and the internal energy of the pair plasma of order of

$$\begin{aligned}\overline{\gamma}' &\leq 10^5 \\ \gamma_b &\leq 60\ .\end{aligned} \tag{121}$$

These results are interesting in the sense that they do not depend on all the details of the instability and the turbulence. They are directly expressed in term of an observable quantity (the compactness of the source ℓ). It is worth mentioning that if we consider a power supply by the turbulence carried by the MHD jet then the internal energy of the relativistic pairs can exceed the saturation value $\overline{\gamma}'_{sat}$. In this case, the protons are no more coupled with the pair plasma through this process. Only rare Coulomb collisions can do this work.

The non-linear theory is supposed to provide the spectrum of the Alfvén waves that must heat the pairs. Except some peculiar spectrum, the distribution function is unlikely a power law, but more likely a quasi-monoenergetic distribution characterized by the internal energy derived above. A power law in γ'^{-2} could be obtain by taking into account the inhomogeneity effect together with the Compton cooling. The index of the distribution must be steeper if the pair creation process is considered.

9. Discussion and conclusion

In this paper, we derived the main basic plasma physics rules that govern the interaction of the relativistic pair beam with an MHD plasma. We applied this theory to understand the high energy Inverse Compton emission of both Quasars and micro-Quasars. We considered the Inverse Compton process in the Thomson regime. However, some modifications due to the angular dependence of the Klein-Nishina limit could be relevant in the case of micro-Quasars.

The main difference with the previous works is that the pair beam has a highly relativistic internal energy. The kinetic theory is necessary to analyze the effect of the anisotropic radiation field that prevails between $10r_G$ and 10^3r_G where the Compton rocket effect do work, as discussed in Sect. 8. Particle acceleration can be maintained by the turbulence supplied by the MHD jet (like in Henri & Pelletier 1991), but also by the radiation field itself which was the main topic of this paper.

The interaction between the relativistic beam and the jet undergoes two stages. In the first stage (the non relativistic regime), the beam pervades a more massive ambient medium and the interaction is described by the standard weak turbulence theory: quasi linear growth of both Alfvén and Langmuir waves of the cold electron-proton plasma, non-linear mode couplings, and a resolution of a Fokker-Planck equation describing the pair heating processes. The entire acceleration mechanism is self-consistent: The pairs are accelerated by Langmuir turbulence at low energy and Alfvén turbulence at higher energy in the inner part of a cone of half opening angle of $\sim \alpha^*$ where the IC cooling process is less efficient than outside. Although, the Langmuir turbulence is not sufficient to accelerate the particles at the higher energies, it is nevertheless unavoidable and necessary to inject electrons and positrons above the threshold for interaction with Alfvén waves.

The high energy pairs are injected in the outer part of the cone, where the pair distribution function tends to be isotropized by Alfvén turbulence. In this angular zone the pairs are submitted to strong IC cooling leading to the formation of a power-law energy distribution with an index of 2. The cooled pairs are then re-injected in the cone by pitch-angle scattering on Langmuir waves. We derived analytically the solution with consistent approximation. However, a Fokker-Planck numerical code would be useful to improve the solution especially to take into account the pair creation.

The second stage is rather unusual; this is the relativistic regime where the beam is heavier than the ambient medium. In this case the longitudinal waves are not destabilized, and only the Alfvén waves are amplified at synchrotron resonance by the ambient protons. We did a detailed investigation of this instability and gave some suggestions of its possible non-linear development. The detailed theory of this non-linear evolution is postponed for a future work. Anyway the interesting saturation effect derived from the linear theory has simple and straightforward implications for astrophysical objects such as extragalactic Quasar and galactic "micro-Quasar".

Appendix A: anisotropic growth rate for Alfvén waves in the non relativistic regime

The combination of a mono-energetic pair distribution function (Eq. (22)) the resonant momentum (Eq. (40)) leads to a growth rate (Melrose (1968))

$$\frac{\Gamma_k}{kV_A} = \frac{2\pi^3 q_e^2}{k^2 c|cos\theta|}\frac{n'_*}{4\pi\overline{p}'^2}\int\int d\mu dp p^2 \frac{1-\mu^2}{|\mu|}\delta(p-p_R(\mu)) \\ (\frac{cos\theta}{|cos\theta|}\frac{\partial}{\partial\mu}+\frac{V_A p}{c}\frac{\partial}{\partial p})\frac{\delta(p-\overline{p})}{\gamma_b^3(1-\beta_b\mu)^3} \ . \quad (A1)$$

where we recall the resonant momentum $p_r(\mu) = |q_e|B/(kc|cos\theta||\mu|)$. The next step is done by expressing the function $p - p_r(\mu)$ as an explicit function of μ and by transforming the δ function of p to $\delta(|\mu| - \mu_0(p))$, with $\mu_0(p) = \gamma m_e \omega_{se}/(k|cos\theta|p) \geq 0$. Therefore we can integrate successively over μ and over p. Using the Eq. (1) transforming the derivatives over μ and p to derivatives over p', the integral reads as

$$\int \frac{1-\mu^2}{|\mu|}\delta(|\mu|-\mu_0(p))(\frac{V_A}{c}\gamma_b(1-\beta_b\mu)-\frac{cos\theta}{|cos\theta|}\gamma_b\beta_b) \\ p^2\frac{\partial}{\partial p'}\delta(p'-\overline{p}')dpd\mu \ . \quad (A2)$$

After the integration over μ, p' and p are linked by the relation $p' = \gamma_b p(1 - \beta_b\mu_0(p))$. This leads to complete expression

$$\frac{\Gamma_k}{kV_A} = \frac{\pi}{|cos\theta|\overline{\gamma}'}(\frac{\omega'_{p*}}{kc})^2(\frac{cos\theta}{|cos\theta|}\beta_b - \frac{V_A}{c})(1+\gamma_b\beta_b\mu_0(\overline{p}')) \ . \quad (A3)$$

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