Astron. Astrophys. 323, 382–386 (1997)

ASTRONOMY
AND
ASTROPHYSICS

Bondi flows on compact objects revisited

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Received 15 October 1996 / Accepted 10 December 1996

Abstract. We revisit Bondi flows on black holes, naked singularities and neutron stars keeping in mind recent progress in understanding of the flow behaviour near compact objects. In an *adiabatic spherical accretion* on black holes and naked singularities, standing shock waves do not form, but shocks on neutron stars may or may not form depending on the boundary conditions. For low accretion rates (hard state), these objects may behave similarly. However, for high accretion rates (soft state), the converging flow on a black hole will produce a weak hard tail of energy spectral slope of ~ 1.5 (apart from the soft bump) while the flow on a neutron star or a naked singularity will have no hard tail. The soft bumps of neutron star candidates may generally be at higher energies than those of naked singularities.

Key words: accretion, accretion disks – black hole physics – stars: neutron – hydrodynamics

1. Introduction

Spherical flows have been studied in the past four decades rather extensively (Bondi, 1952; Shapiro, 1973; Shapiro & Salpeter, 1975; Park & Ostriker, 1989; see Chakrabarti, 1996a for a recent review). These zero angular momentum flows were not found to be very efficient radiators since they carry the total energy along with them, except at the boundary layer of objects with hard surfaces. Therefore, it was difficult to explain, for example, the quasar luminosity and the soft states of galactic and extra-galactic black hole candidates using spherical flow models around black holes. The Keplerian accretion disks (Shakura & Sunyaev, 1973), on the other hand, are very efficient radiators. They locally dissipate the entire amount of heat that is generated by viscosity. Generalized adiabatic Bondi flows which contained angular momentum (Liang & Thompson, 1980; Chakrabarti, 1989; Chakrabarti 1990a, hereafter C90a) could also be highly inefficient emitters. However, such flows with viscosity and cooling effects which may join a Keplerian or sub-Keplerian flow far away can have intermediate efficiencies (C90a; Chakrabarti, 1990b hereafter C90b; Chakrabarti 1996ab, hereafter C96ab). In a black hole accretion, they partly carry along energy through the event horizon (see, Fig. 8a of C90a) and partly radiate the dissipated heat and thus successfully bridge the gap between a classical Bondi flow and a classical Shakura-Sunyaev type Keplerian disk. Shocks may also form just outside the horizon if the flow can pass through two sonic points. In a neutron star accretion, the flow dissipates the energy carried along in the boundary layer just outside the neutron star surface (Chakrabarti 1989; C96b). These disks which may contain both Keplerian and sub-Keplerian flows can explain most of the stationary and non-stationary spectral features of black hole and neutron star candidates as explained in detail in Chakrabarti & Titarchuk (1995, hereafter CT95); C96b; Molteni et al. (1996); Ryu et al. (1997).

In the present paper, we take a closer look at spherical accretion flows, not only on black holes (BH), but also on other compact objects, such as weakly magnetized and slowly rotating neutron stars (NS) and naked singularities (NSing). Naked singularities are not as widely considered to be astrophysically relevant as black holes and neutron stars, though recently their theoretical existence have been discussed, and serious studies of using them to explain astrophysical phenomena are being considered (Penrose, 1974; Ori & Piran 1990; Nakamura et al. 1993; Chakrabarti & Joshi, 1994). The adiabatic accretion flow onto black holes and naked singularities has to pass through a sonic surface and enters the horizon or the singularity supersonically, while on an unmagnetized neutron stars, the flow may or may not have standing shock waves as it rapidly lands on the surface. We discuss the fundamental differences between these solutions and present some possible spectral signatures by which these objects could be distinguished. This review of the subject was essential in view of the recognition that the quasi-spherical sub-Keplerian flows could be useful in explaining the quiescent states of black holes (Ebisawa et al, 1996), hard states of black holes and neutron stars and more importantly weak hard tails seen in the soft states of black hole candidates (CT95).

In the next Section, we present the basic equations, and the procedure to solve them. In §3, we present the solutions of these

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equations. In §4, we briefly discuss the spectral properties of these solutions. Finally, in §5, we discuss importance of angular momentum and its effect on the results and make concluding remarks.

2. Basic equations

We simplify our hydrodynamic calculations around a black hole and a neutron star by choosing the Paczyński-Wiita (1980) pseudo-potential,

$$\phi = -\frac{1}{(x-2)} \qquad (1a)$$

which mimics the geometry around a compact star quite well. Around the naked singularity we choose an ordinary Newtonian potential

$$\phi = -\frac{1}{x}, \qquad (1b)$$

allowing matter to flow arbitrarily close to $x = 0$. The main conclusions drawn in this paper are not affected by these simplifying assumptions. However, intuitively, we do assume that the inner boundary condition on accretion flows into a naked singularity is supersonic, which is valid if the flow does not violate causality (velocity of sound less than the velocity of light).

In an adiabatic Bondi flow, the conserved specific energy is given by (C90b)

$$\mathscr{E} = \frac{1}{2}u^2 + na^2 - \frac{(1-C)}{(x-x_0)} = n{a_\infty}^2 \qquad (2)$$

using $G = M_{BH} = c = 1$, where G is the gravitational constant, M_{BH} is the mass of the compact object, and c is the velocity of light, so that the units of mass, length and time are M_{BH}, $\frac{GM_{BH}}{c^2}$ and $\frac{GM_{BH}}{c^3}$ respectively. Here u is the radial velocity, x is the spherical radial coordinate, a is the adiabatic sound speed, $a^2 = \gamma P/\rho$ and γ is the adiabatic index, P is the pressure, ρ is the mass density, $n = (\gamma-1)^{-1}$, and x_0 is either 2 or 0 depending on the nature of the compact object. Equation (2) is obtained by integrating the radial momentum equation. Note that we have weakened the gravitational potential by $C/(x-x_0)$ which essentially represents the outward radiative force $C/(x-x_0)^2$. Here, C is chosen to be independent of x, for convenience. Roughly speaking, on a neutron star accretion, C scales with accretion rate: $C \sim \dot{M}/\dot{M}_{Edd}$. In a black hole or naked singularity accretion, radiation could be partly trapped and drawn in radially, and therefore there is no simple relation between C and the accretion rate. On the right hand side we have a_∞ which is the sound speed at a large distance. If the flow originates from a Keplerian flow (C90ab, C96ab), it may have to be pre-heated (either by radiation or by magnetic dissipation) to make $\mathscr{E} > 0$, in order that it passes through a sonic point at a large distance. The mass flux is obtained as,

$$\dot{M} = 4\pi\rho u x^2, \qquad (3a)$$

which could be re-written in terms of 'entropy-accretion' flux

$$\dot{\mathscr{M}} = \dot{M}(\gamma K)^n/4\pi = a^{2n}ux^2. \qquad (3b)$$

Here, $K = P/\rho^\gamma$ is a constant and a measure of entropy and therefore K can change only if the flow passes through a shock. Thus, $\dot{\mathscr{M}}$ is a measure of entropy and mass flux. From Eqs. 2 and 3,

$$\frac{du}{dx} = \frac{\frac{(1-C)}{(x-x_0)^2} - \frac{2a^2}{x}}{\frac{a^2}{u} - u}. \qquad (4)$$

At the *sonic surface*, where numerator and denominator vanish, one must have,

$$u_c = a_c, \qquad (5a)$$

and,

$$x_c = x_0 + \frac{(1-C)}{4a_c^2} + [\frac{x_0(1-C)}{2a_c^2} + \frac{(1-C)^2}{16a_c^4}]^{1/2}. \qquad (5b)$$

We have ignored the other positive root for x_c, as it is nearly zero and therefore is inside the horizon or the star surface. Flow can be sonic at $x_c = 0$ for a naked singularity ($x_0 = 0$).

Since two conditions (5a) and (5b) are to be satisfied, while only one extra unknown (namely, x_c) is introduced, clearly, both the specific energy $\mathscr{E}$ and entropy-accretion flux $\dot{\mathscr{M}}$ cannot be independent. Indeed, for a given energy $\mathscr{E}$, the critical entropy-accretion flux is,

$$\dot{\mathscr{M}}_c = [\frac{2}{3}(\mathscr{E} + \frac{1-C}{x_c - x_0})]^{n+1/2}x_c^2. \qquad (6)$$

It is clear that if the flow is *hot* at a large distance ($a_\infty \gtrsim 0$), i.e., $\mathscr{E} \gtrsim 0$, then the sonic surface $x = x_c$ is located at a finite distance provided the polytropic index is suitable (roughly, $\gamma < 5/3$. See, C90b). Once the flow passes through the sonic surface it will continue to remain supersonic if the central object is a black hole or a naked singularity, but the flow has to pass through a standing shock (at $x = x_s$, say) and become subsonic if the central object is a neutron star. On the other hand, a neutron star accretion can be subsonic throughout if

$$\dot{\mathscr{M}} < \dot{\mathscr{M}}_c. \qquad (7a)$$

If on the surface of the neutron star,

$$\dot{\mathscr{M}}|_{r_*} > \dot{\mathscr{M}}_c \qquad (7b)$$

then the flow must have a standing shock and the entropy generated at the shock must be such that the post-shock flow has

$$\dot{\mathscr{M}}|_{r_s} = \dot{\mathscr{M}}|_{r_*}. \qquad (8)$$

This condition along with the pressure balance condition

$$P_- + \rho_- u_-^2 = P_+ + \rho_+ u_+^2 \qquad (9)$$

determines the location of the standing shock.

The Eqs. 7a, 7b and 8 can be translated in many ways in terms of the injection speed, temperature, the gradient of velocities at the outer or inner boundary, or the location of the sonic point by employing any of the definitions given by Eq. 2, Eq. 3b, Eq. 4 or Eq. 6.

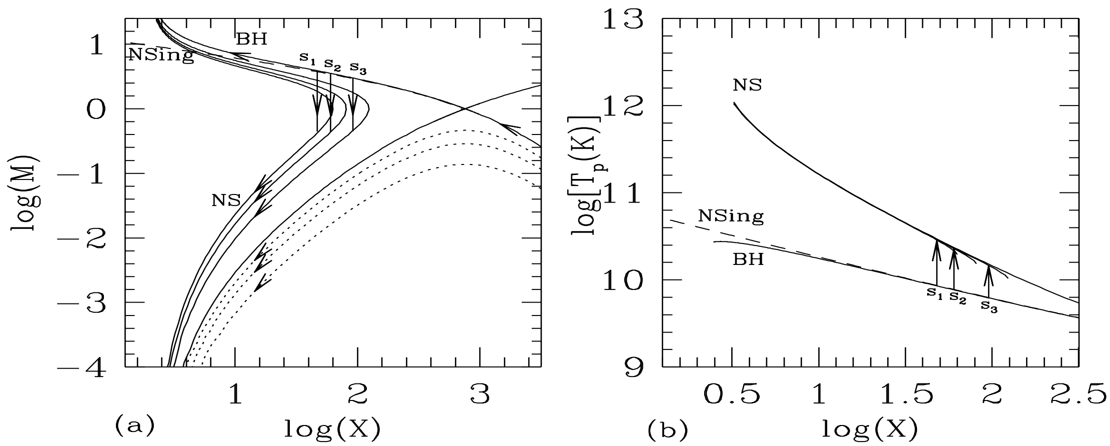

Fig. 1. a Mach number variation with radial distance. Vertical arrows are shock locations in the neutron star accretion. Different arrows are for different entropy accretion rates on neutron stars. Solutions on black holes (BH, solid), naked singularities (NSing, long dashed), and neutron stars (NS, dotted and subsonic solid) are distinguished. **b** Proton temperatures in corresponding solutions. Subsonic neutron star accretion flow is much hotter.

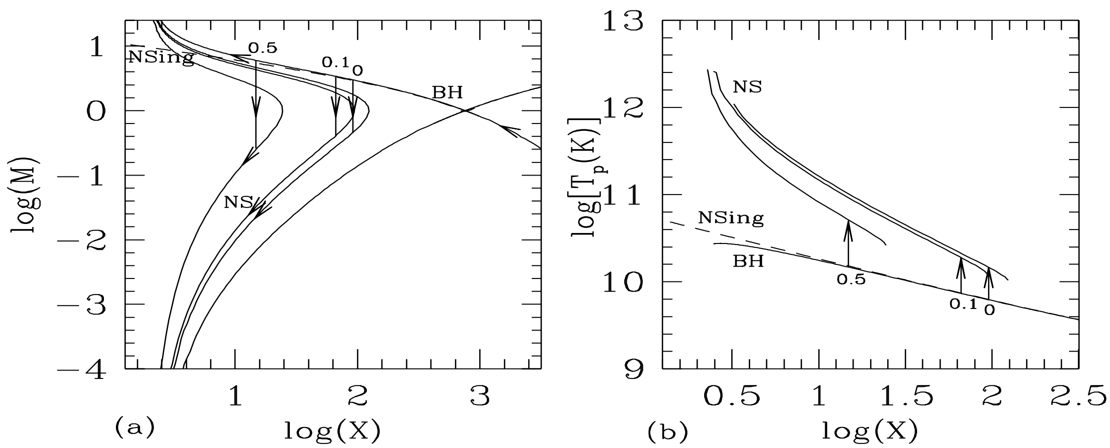

Fig. 2. a Similar to Fig. 1, except that the radiation pressure (parametrized by $C = 0,\ 0.1,\ 0.5$ marked on curves) on the neutron star surface is varied. **b** Proton temperatures in corresponding solutions.

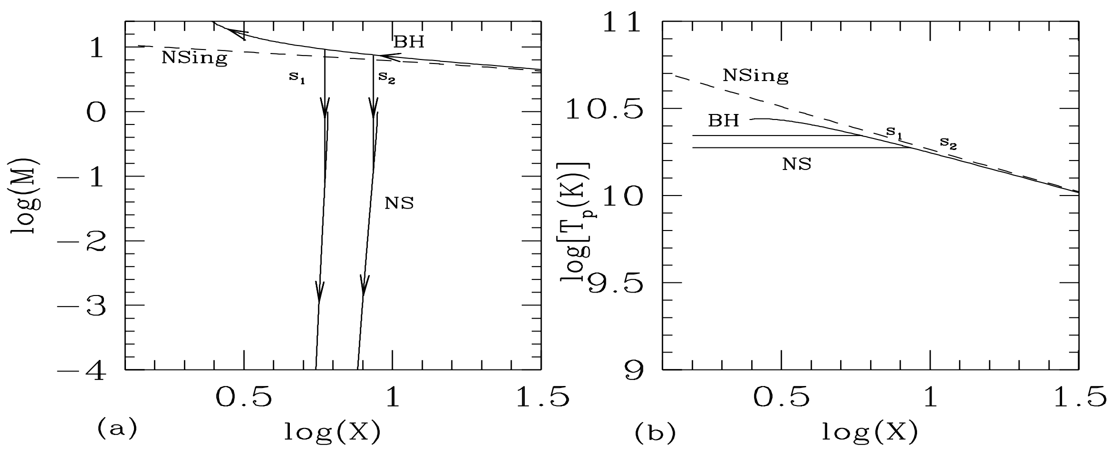

Fig. 3. a Similar to Fig. 1, except that the post-shock region is cooler and isothermal due to dissipation. **b** Proton temperatures in corresponding solutions.

3. Solutions of the basic equations

We now present the complete set of solutions obtained using fourth order Runge-Kutta method. Fig. 1a shows the variation of the Mach number with radial distance (in logarithmic coordinates in both directions). The solid curves intersecting at the sonic point x_c are the well known Bondi solutions. This has $\dot{\mathscr{M}}_c = 4.315 \times 10^{-6}$. The arrowed branch which becomes supersonic on the horizon represents the black hole accretion. The long dashed curve (marked NSing) is drawn with $x_0 = 0$ (with same $\dot{\mathscr{M}}_c$) and represents an accretion on a naked singularity. The accretion flows on neutron star surfaces must be subsonic. The exact subsonic branch depends on the inner boundary condition, i.e., the way matter lands on the staller surface (say, the derivative of the velocity on the surface.). The solid arrowed curves leading to the neutron star surface have more entropy than the curve passing through the critical point ($\dot{\mathscr{M}}|_{r_*} > \dot{\mathscr{M}}_c$). The difference in entropy $\dot{\mathscr{M}}|_{r_*} - \dot{\mathscr{M}}_c$ must be generated at the standing shocks located at s_1, s_2 and s_3 respectively. In the present example, $\dot{\mathscr{M}}_{r_*} = 3.4315 \times 10^{-5}$, 2.4315×10^{-5} and 1.4315×10^{-5} (for the solid curves drawn in the order inside to outside). The corresponding shock locations are $s_1 = 46.77$, $s_2 = 60.0$ and $s_3 = 91.6$ respectively. Here, $\gamma = 4/3$ (relativistic flow) and $C = 0$ (no excess radiation pressure) is chosen. These correspond to low accretion rate solutions. The short dashed arrowed curves leading to a neutron star have even less entropy than $\dot{\mathscr{M}}_c$. In this case, they are drawn for $\dot{\mathscr{M}}_{r_*} = 10^{-6},\ 2 \times 10^{-6}$ and 3×10^{-6} (from bottom to top) respectively. In Fig. 1b, the proton temperatures T_p (in degrees Kelvin) for the corresponding solutions are plotted on a logarithmic scale as a function of the logarithmic radial distance. The black hole and naked singularity solutions have lower temperatures since they are supersonic, while the adiabatic subsonic neutron star solutions have higher temperature, and the distributions are almost independent of the branch that is chosen. As the entropy of the post-shock flow is increased, the shock location comes closer to the neutron star surface and the post-shock temperature is also increased.

In Figs. 2a and b, we include the effect of the radiative force to study the flow properties of the solutions on neutron stars. In Fig. 2a, we plot three solid curves with arrows leading to the neutron star surface. We choose $\dot{\mathscr{M}}_{r_*} = 1.4315 \times 10^{-5}$ as the post-shock entropy accretion rate on the surface. The vertical arrows are drawn at shock locations and at each shock, the value of C (such as 0, 0.1, and 0.5) is marked. The value of C is made different from zero only in the post-shock region, since the subsonic flow would be maximally affected by the radiation pressure effects. The corresponding shock locations are 91.54 (same as in Fig. 1a), 66.1 and 14.8 respectively. In Fig. 2b, temperature distributions are plotted. Note that as C is increased, the temperature of post-shock region becomes smaller. This is because when the radiation pressure is present, one does not require thermal pressure very much. In the pre-shock region, the ram pressure must increase sufficiently to balance the net pressure. As a result, the shock is located at a place closer to the neutron star surface.

In Fig. 3a and b, we consider the case when the post-shock region is isothermal. In this case, the shock will be in pressure equilibrium only if the neutron star surface is much cooler than the cases mentioned above. The surface temperature of the neutron star (which is also the post-shock temperature of the flow) determines the shock location. Two horizontal lines have sound speeds $a = 0.06$ (lower) and $a = 0.065$ respectively. When the pressure balance condition is satisfied at the shocks, these sound speeds determine the post-shock Mach number variation which go to zero very rapidly close to the surface. As the surface temperature of the star is lowered, the location of the shock, i.e, the width of the terminal boundary layer is also increased since the pre-shock flow temperature is monotonic.

4. Spectral signatures of compact objects

In Fig. 4a, we schematically show our present understanding of the accretion flows onto compact objects. The Keplerian or sub-Keplerian flows at the outer boundary become increasingly sub-Keplerian close to the inner boundary. On the black hole horizon, and presumably at the naked singularity, the radial velocity approaches the velocity of light (C90b; Chakrabarti 1996c) and since even in the extreme case the sound speed is less than this, the flow must be supersonic at the inner boundary. In the case of accretion onto a neutron star however, the flow is subsonic on the inner boundary. CT95 and Titarchuk et al. (1996) pointed out that in the limit of high accretion rates (where the Thompson scattering opacity $\tau \gtrsim 1$) the eigenvalue of the corresponding radiative transfer equation in presence of convergent fluid flow is given by,

$$\lambda^2 = \frac{3}{2} + \frac{3}{4}\frac{1-C}{1+C} X_b^{1/2}$$

where, X_b is the location of the inner boundary in units of the Schwarzschild radius. In terms of this eigen value, the energy spectral index α [$F(\nu) \sim \nu^{-\alpha}$] is found out to be,

$$\alpha = 2\lambda^2 - 3.$$

In the case of a black hole accretion: $X_b = 1$ and $C = 0$; and one obtains $\alpha = 1.5$, very similar to what is observed in black hole candidates (Sunyaev et al. 1994; Ebisawa et al, 1996). If $X_b = 1.5$ is chosen (which corresponds to the last photon orbit and is probably more physical) the corresponding value is $\alpha \sim 1.83$. In the case of high accretion rates on neutron stars, $C \sim 1$ giving rise to $\alpha = 0$. The spectrum at higher energies is thus completely flat. In the case of a naked singularity, $X_b = 0$, and the spectrum in higher energies is also flat, independent of the accretion rate as long as the post-Keplerian flow (with or without shock waves) has $\tau \gtrsim 1$. Naked singularities may be similar to black holes in the hard states, when the accretion rates are smaller, while similar to neutron stars in soft states although the soft bump is cooler unless the neutron star accretion itself is isothermal (as is possible when shocks are present; see, Shapiro & Salpeter, 1975). These aspects of spectral properties are shown qualitatively in Fig. 4b.

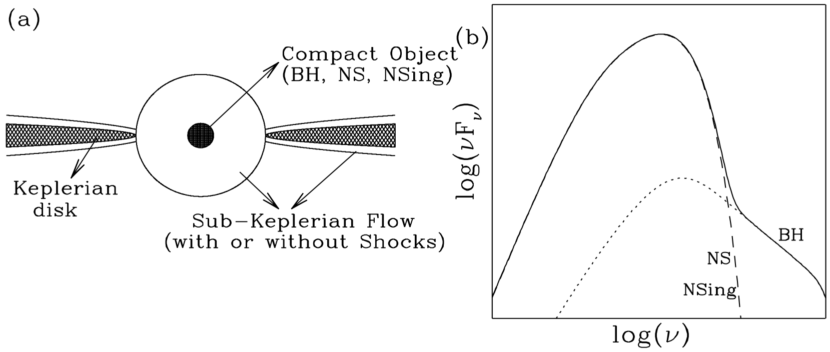

Fig. 4. **a** Schematic view of generalized solutions on compact objects where flows close to the objects are sub-Keplerian Bondi type (with or without shocks). **b** Variation of the soft state spectra in three types of compact objects. In neutron stars (NS) and naked singularities (NSing) the soft state spectra should not have the weak hard tail, while the black holes should have this feature.

As far as the total energy of radiation is concerned, one could distinguish these objects as well. Since the flow has to dissipate its energy at the hard surface of a neutron star, the luminosity of neutron stars would still be proportional to the accretion rates provided magnetic field is weak enough; non-linear interaction with magnetic fields (Illarionov, & Sunyaev, 1975) may change this conclusion. On the contrary, in a black hole accretion, luminosity could be very small since the flow disappears through the horizon (for instance, constant energy flows of C89 have, strictly speaking, zero luminosity). In a naked singularity, there is no horizon, and extremely hot matter very close to the origin may cause thermonuclear flashes and matter should be at least partially luminous also. However, the detailed physics is not very well understood since one must take into account quantum effects.

5. Concluding remarks

Our understanding of the accretion processes on compact objects is still far from complete. In the present paper, we demonstrated that the outcome of solutions of spherical hydrodynamic accretion processes in terms of spectral properties can differ very much depending upon the behavior of matter at the inner boundary. We have discussed the possibility that the weak hard tail in the soft states of a black hole is a result of the quasi-spherical flows and the hard tail should be absent in neutron stars and naked singularities. We also showed (Fig. 1b) that on account of their subsonic inner boundary condition, neutron stars produce much hotter radiation that the black holes or naked singularities. It is interesting to note that although the spherical flows are radiatively inefficient, they definitely have to dissipate their energy at the boundary layer, namely, in the post-shock region if the central object is a neutron star. The quasi-periodic oscillation from neutron stars could very well be due to the oscillation of this post-shock region (Molteni et al 1996). This is true even when some magnetic field is present on the neutron star surface as the radial flows may penetrate fields easier. For a black hole accretion such restriction on efficiency is not present as the flow can carry along most of its energy through the horizon.

Although our study is strictly applicable only for spherical flows, should some angular momentum be present, the solution would look similar to what is shown in Fig. 4a, which is a combination of Keplerian and sub-Keplerian flows (with or without shocks) as discussed in Chakrabarti & Titarchuk (1995) and Chakrabarti (1996b). Close to the inner boundary, the circular orbits are unstable anyway, and thus the flow would become quasi-spherical even in presence of rotation.

Acknowledgements. SS acknowledges the hospitality of Tata Institute VSRP programme in the summer of 1996 during which this work was carried out.

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