Astron. Astrophys. 323, 524–528 (1997)

ASTRONOMY
AND
ASTROPHYSICS

Research Note

On the NLTE plane-parallel and spherically symmetric model atmospheres of helium rich central stars of planetary nebulae

Jiří Kubát

Astronomický ústav, Akademie věd České republiky, 251 65 Ondřejov, Czech Republic

Received 19 September 1996 / Accepted 15 January 1997

Abstract. The assumption of plane-parallel geometry is tested for the case of NLTE model atmospheres of central stars of planetary nebulae. As a test example, two hot hydrogen deficient central stars, namely LoTr4 and K1-27 were used. It was found that for atmospheric parameters of the star K1-27, namely $T_{\rm eff}$ = 100000K, gravity $\log g$ = 6.5, stellar mass $0.52 M_{\odot}$, and abundance of hydrogen $n_{\rm H}/n_{\rm He}$ = 0.2, the sphericity effects are very small. On the other hand, atmospheric parameters of LoTr4, namely $T_{\rm eff}$ = 120000K, gravity $\log g$ = 5.5, stellar mass $0.65 M_{\odot}$, and abundance of hydrogen $n_{\rm H}/n_{\rm He}$ = 0.5, yield much greater extension and remarkable sphericity effects.

Key words: stars: atmospheres – line: profiles – radiative transfer – planetary nebulae: general

1. Introduction

The central stars of planetary nebulae have been studied by means of NLTE model atmospheres by several groups of scientists. Static plane-parallel NLTE model atmospheres have been used for the analysis for some central stars by Rauch et al. (1994, 1996), and Saurer et al. (1996). On the other hand, several other groups applied NLTE expanding model atmospheres for analysis of central stars of planetary nebulae with winds (Kudritzki 1996), like e.g. Wolf-Rayet central stars (for a review see Hamann 1996).

In this paper we would like to show that the applicability of plane-parallel static model atmospheres is limited not only to atmospheres without any macroscopic motion, but also to really very thin atmospheres where the thickness of the atmosphere is lower than 10^{-3} of a stellar radius. As a source of suitable stellar parameters for our study we used results of static plane-parallel modelling of two helium rich central stars of planetary nebulae, namely LoTr4 and K1-27.

The hydrogen deficient central stars of planetary nebulae K1-27 and LoTr4 were studied in great detail by Rauch et al. (1994, 1996), respectively. The authors used for their analysis detailed *static plane-parallel* NLTE model atmospheres calculated with a help of the ALI method. Rauch et al. (1994) analyzed the central star of K1-27. They found that this star has the effective temperature $T_{\rm eff}$ = 100000 $\pm$ 15000K and gravity $\log g$ = 6.5 $\pm$ 0.5. They also determined the upper limit of hydrogen abundance to be $n_{\rm H}/n_{\rm He} < 0.2$. The central star of LoTr4 was analyzed in great detail by Rauch et al. (1996) and the authors determined its effective temperature and gravity to (120000 $\pm$ 15000)K and 5.5 $\pm$ 0.3, respectively. The abundance ratio of hydrogen to helium was found to be about $n_{\rm H}/n_{\rm He}$ = 0.5.

The sphericity effects, i.e. the effects arising from the difference between plane-parallel and spherically symmetric atmospheres, have been studied for the cool stars assuming LTE by a number of authors (e.g. Watanabe & Kodaira 1978, Schmid-Burgk et al. 1981, Scholz & Tsuji 1984, Jørgensen et al. 1992). Castor (1974) studied the sphericity effects for Of and Wolf-Rayet stars also in LTE. First studies of the sphericity effects for the NLTE case were done by Mihalas & Hummer (1974) and Kunasz et al. (1975) for a sample of static extended model atmospheres of hydrogen rich O stars and central stars of planetary nebulae. A few years later, Gruschinske & Kudritzki (1979) studied the sphericity effects for an sdO star with $T_{\rm eff}$ = $45000K$ and $\log g$ = 4.0. However, they used instead of the plane-parallel model a spherically symmetric one with large mass of $50 M_{\odot}$. It was shown by Kubát (1996a) that such large-mass spherically symmetric model still has slightly different properties from the plane parallel one.

Recently, Kubát (1995a,b, 1996a,b,c) showed that the sphericity effects can be important not only to extended atmospheres of O stars, but also to relatively thin atmospheres of white dwarfs. It is therefore natural to expect that the sphericity effects might be important also to hot central stars of planetary nebulae. The purpose of this study is to test the assumption of

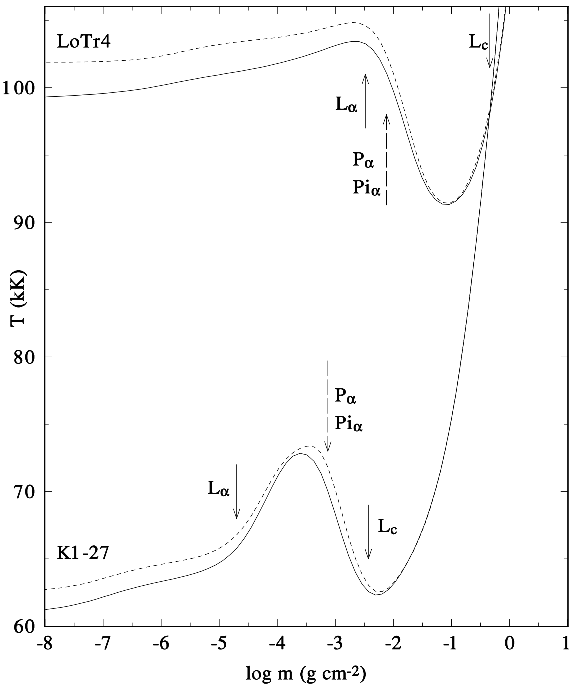

Fig. 1. The NLTE model atmospheres for the parameters listed in Table 1. Full lines are spherically symmetric models, dashed lines denote the plane-parallel models. T stands for temperature, and m for the column mass depth. Arrows indicate the depth of formation of selected He II lines.

a plane-parallel atmosphere for these stars and to estimate the effects made by neglecting the curvature of the atmosphere.

There is no doubt that spherically symmetric model atmospheres are superior to the plane parallel ones. However, during the very first analysis of every star we compare the observed spectrum with a large number of model atmospheres with various parameters to find a best fit. Since the plane-parallel models can be calculated more quickly than the spherical ones, their usage is very advantageous. Therefore it is worthwhile to know something about the error that we introduce when we apply plane parallel models to the analysis of stellar atmospheres.

2. Model atmospheres

Model atmospheres were calculated with a help of our code based on an ALI method. The code enables calculation of both spherically symmetric and plane-parallel NLTE model atmospheres and has been described in Kubát (1994, 1996a).

Basic parameters of the stars K1-27 and LoTr4 are summarized in Köppen et al. (1996). For clarity, we list our adopted parameters in Table 1. For both stars we calculated model atmospheres consisting of hydrogen (10 levels H I + H II) and helium (15 levels He II + He III) only. Due to the high temperatures, He I was not taken into account since it has no effect on atmospheric structure. Oscillator strengths were taken from Wiese et al. (1966), lines were assumed to have a Doppler profile during model calculation. Photoionization cross sections are hydrogenic using a standard formula (see e.g. Mihalas 1978). However, the effect of level dissolution of the uppermost levels on the photoionization cross section was taken into account after Hubeny et al. (1994). The collisional ionization rates for both H I and He II were evaluated using the polynomial fit of Napiwotzki (1993). The collisional excitation rates for He II were calculated after Mihalas & Stone (1968). For the collisional excitation of H I the expressions given in Mihalas (1967) were used. The higher, non-explicit levels were assumed to be in LTE with respect to the next higher ion. The ionization from these levels is taken into account by means of so-called modified free-free cross-section (Auer & Mihalas 1969). The collisional transitions between explicit and non-explicit levels are taken into account by means of a modified collisional ionization rate (see Hubeny 1988). In addition, the population numbers of both explicit and non-explicit levels were calculated using the occupation probability formalism of Hummer & Mihalas (1988) in its NLTE form (Hubeny et al. 1994).

Table 1. Basic stellar parameters for central stars of K1-27 and LoTr4.

	$T_{\rm eff}(K)$	$\log g(gcm^{-2})$	$M(M_\odot)$	$n_{\rm H}/n_{\rm He}$
K1-27	100000K	6.5	0.52	0.2
LoTr4	120000K	5.5	0.65	0.5

For both combinations of basic parameters we calculated spherically symmetric and plane-parallel model atmospheres. The stellar mass was used for the calculation of the stellar radius in spherical models. We stopped our calculations when the relative error of iterations was lower than 10^{-3}. This was sufficient, since it yielded the conservation of total flux with a accuracy better than 0.5%.

The results of our calculations are plotted in Fig. 1. In order to obtain better insight into the process of the line formation in our atmospheres we have indicated depths of formation of He II Lyman continuum, L_α, P_α, and Pi_α lines with the help of arrows. Notice a remarkable difference in the temperature structure not only in the outer layers for the model of LoTr4, but also in the region of formation of P_α and Pi_α lines. On the other hand, a significant difference in the model for K1-27 is present only in the outermost layers that are optically thin. The difference in the line forming region is small. This result is consistent with the extension of atmospheres under consideration. The extension was calculated as a ratio of radius at $\tau_{\rm R} = 10^{-5}$ to the radius at $\tau_{\rm R} = 2/3$. This number is 1.028 for LoTr4, and 1.0035 for K1-27.

3. Continuum flux

The continuum flux for both sets of models is plotted in Fig. 2. The reference radius where the frequency integrated flux for the spherically symmetric models equals $\sigma T_{\rm eff}^4/(4\pi)$ is $R = r(\tau_{\rm R} = 2/3)$. In order to eliminate the difference caused by the r^{-2} dependence of the emergent flux in spherical geometry, we divided the plane-parallel emergent flux by a factor r_1^2/R^2,

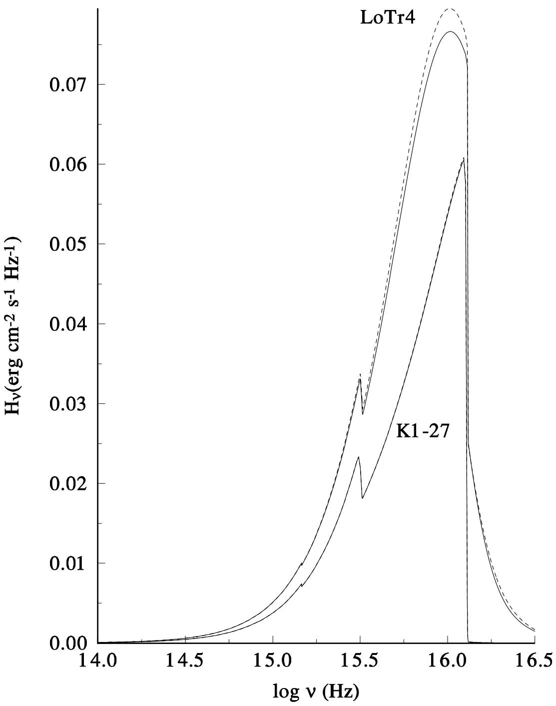

Fig. 2. The emergent continuum flux at the top of the atmosphere for NLTE hydrogen-helium model atmospheres for the parameters listed in Table 1. Full lines are calculations for spherically symmetric models, dashed lines denote the plane-parallel models.

where r_1 is the radius of the uppermost depth point of our spherical model. The emergent flux is lower for all spherical models due to the extension of the atmosphere. However, for the models of K1-27 the differences are negligible. On the other hand, the differences for the model of LoTr4 are quite large. For the latter model the assumption of plane-parallel geometry becomes questionable.

4. Line profiles

For each model atmosphere we have plotted emergent line profiles for the He II Paschen and Pickering lines (see Figs.3 and 4). The emergent flux was calculated using an approximate formula for Stark broadening after Hubeny et al. (1994).

All lines are deeper for spherical geometry. Differences between line profiles are present especially for lower series members and affect the line core, whereas the wings are affected only marginally. For higher series members the differences tend to disappear. They are the largest for the Pi_α line (7% of the continuum flux in the line core) in the more extended model (LoTr4). Differences of this size are easily detectable by contemporary observational techniques. However, for the P_α line (4686 Å) we find a difference of 3% in the line core. High signal-to-noise observations also allow the detection of such relatively small effects. On the other hand, the differences for the more compact atmosphere (K1-27) are very small with the exception of Pi_α line. The difference in the core of He II 4686 Å line is below 1%, however.

This indicates the limit of using visual He II lines for determination of stellar parameters with the help of plane-parallel model atmospheres. One can safely use the higher series members, whereas using lower series members such as the He II 4686 Å line may cause inaccuracies rising with the extension of the atmosphere.

5. Conclusions

In this paper we pointed our attention to the effects caused by the replacement of the assumption of static plane-parallel atmosphere by the assumption of static spherically symmetric atmosphere. We did not study the effects caused by atmospheric motions. Our conclusions can be summarized as follows.

1. Differences in temperature structure between our static plane-parallel and spherically symmetric model atmospheres are present. They are more pronounced for a star with lower gravity, and, consequently, larger extension. The temperature is lower for spherically symmetric model atmospheres.
2. Continuum flux is lower for spherical atmospheres. The difference is larger for LoTr4, i.e. a star with more extended atmosphere, and almost insignificant for K1-27.
3. Paschen and Pickering He II lines are deeper for spherically symmetric model atmospheres. These differences are quite large (several per cent) for α lines, and they decrease towards higher series members. *One should avoid using lower series members (e.g. He II 4686 Å) for determination of atmospheric parameters from plane-parallel atmospheres.*
4. Using plane-parallel model atmospheres instead of spherically symmetric ones introduces *a systematic error* into results. This error is present also in highly sophisticated NLTE line blanketed models.

These conclusions underline the need to test every plane-parallel modelling by a comparison with spherically symmetric models. If we find that the difference is almost negligible (like for K1-27 in this paper), we can trust the plane-parallel analysis. On the other hand, larger difference should challenge us to reconsider the analysis if we did not use "safe" lines (i.e. lines not affected by the sphericity effects).

However, it is necessary to emphasize that our models assume *static* atmospheres. The effect of velocity fields on line profiles for stars close to the Eddington limit may be very important and probably larger than the sphericity effects described in this paper. The self-consistent study of the effects caused by atmospheric motions remains a task for future.

Acknowledgements. The author would like to thank Drs. David Holmgren, Rolf-Peter Kudritzki, Ivan Hubeny, and Klaus Werner for their comments to the manuscript. A part of the calculations mentioned in this paper was carried out on the Cray YMP-EL computer at the Institute of Physics of the Academy of Sciences of the Czech Republic. This work was supported by internal grants of Academy of Sciences of the Czech Republic No.303401 and C3003601 and by grants of the Grant

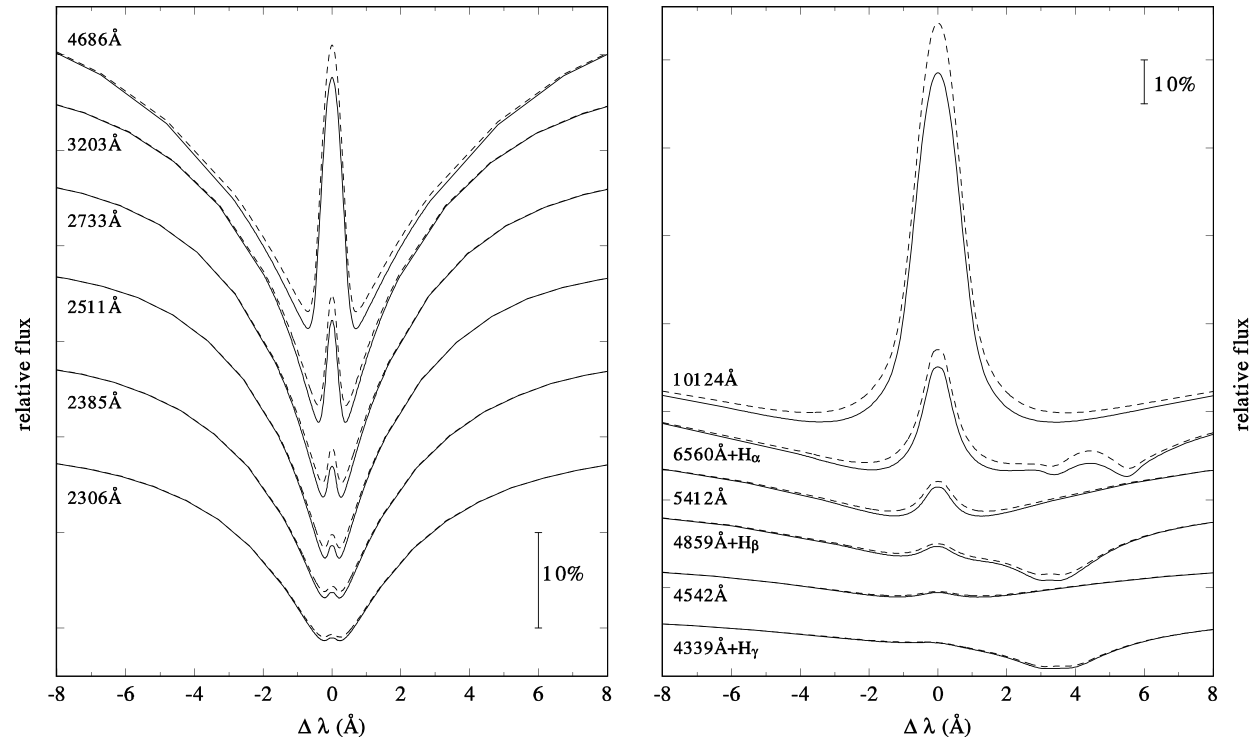

Fig. 3. The line profiles of the He II Paschen (left) and Pickering (right) series for NLTE model atmospheres with T_{eff} = 120000K, log g = 5.5, $M = 0.65 M_{\odot}$, and $n_{\text{H}}/n_{\text{He}}$ = 0.5 (LoTr4). Full lines are calculations for spherically symmetric models, dashed lines denote the plane-parallel models. Notice that even lines of the Pickering series are contaminated by hydrogen Balmer lines.

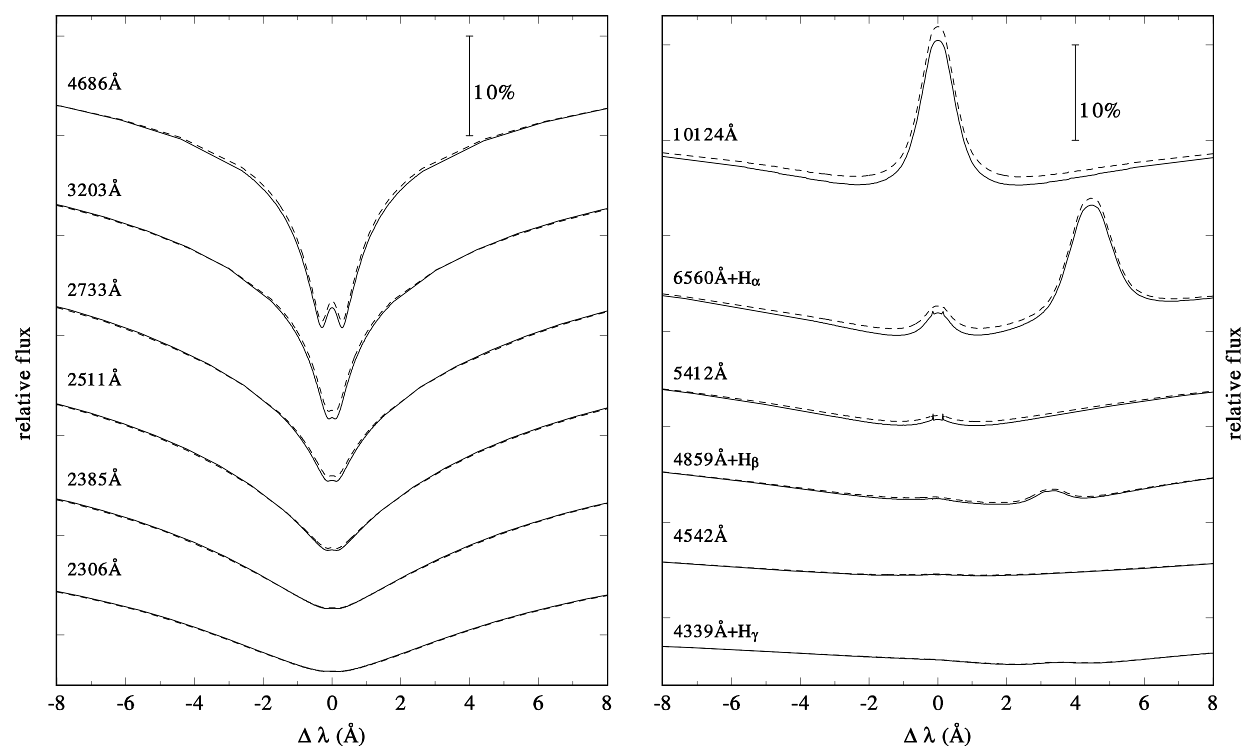

Fig. 4. The same as Fig. 3 for NLTE model atmospheres with T_{eff} = 100000K, log g = 6.5, $M = 0.52 M_{\odot}$, and $n_{\text{H}}/n_{\text{He}}$ = 0.2 (K1-27).

Agency of the Czech Republic (GA ČR) No.0025 and 205/96/1198, and by projects K1-003-601/4 and K1-043-601.

References

Auer L.H., Mihalas D., 1969, ApJ, 156, 157
Castor J.I., 1974, ApJ, 189, 273
Gruschinske J., Kudritzki R.P., 1979, A&A, 77, 341
Hamann W.-R., 1996, in Planetary Nebulae, IAU Symp. No.180, H.J.Habing & H.J.G.L.M.Lamers eds., Kluwer Acad. Publ., in press
Hubeny I., 1988, Comput. Phys. Commun., 52, 103
Hubeny I., Hummer D.G., Lanz T., 1994, A&A, 282, 151
Hummer D.G., Mihalas D., 1988, ApJ, 331, 794
Jørgensen U.G., Johnson H.R., Nordlund Å., 1992, A&A, 261, 263
Köppen J., Rauch T., Werner K., 1996, in Hydrogen Deficient Stars, C.S.Jeffery & U.Heber eds., ASP Conf. Ser., Vol.96, p.205
Kubát J., 1994, A&A, 287, 179
Kubát J., 1995a, in White Dwarfs, D.Koester & K.Werner eds., Lecture Notes in Physics, Vol.443, Springer Verlag Berlin, p.133
Kubát J., 1995b, A&A, 299, 803
Kubát J., 1996a, A&A, 305, 255
Kubát J., 1996b, in Hydrogen Deficient Stars, C.S.Jeffery & U.Heber eds., ASP Conf. Ser., Vol.96, p.266
Kubát J., 1996c, in Model Atmospheres and Spectrum Synthesis, S.J.Adelman, F.Kupka & W.W.Weiss eds., ASP Conf. Ser., Vol.108, p.165
Kudritzki R.-P., 1996, in Planetary Nebulae, IAU Symp. No.180, H.J.Habing & H.J.G.L.M.Lamers eds., Kluwer Acad. Publ., in press
Kunasz P.B., Hummer D.G., Mihalas D., 1975, ApJ, 202, 92
Mihalas D., 1967, ApJ, 149, 169
Mihalas D., 1978, Stellar Atmospheres, 2nd ed., W.H.Freeman & Comp., San Francisco
Mihalas D., Hummer D.G., 1974, ApJS, 28, 343
Mihalas D., Stone M.E., 1968, ApJ, 151, 293
Napiwotzki R., 1993, Ph.D. thesis, Universität Kiel
Rauch T., Köppen J., Werner K., 1994, A&A 286, 543
Rauch T., Köppen J., Werner K., 1996, A&A 310, 613
Saurer W., Werner K., Weinberger R., 1996, in Planetary Nebulae, IAU Symp. No.180, H.J.Habing & H.J.G.L.M.Lamers eds., Kluwer Acad. Publ., in press
Schmid-Burgk J., Scholz M., Wehrse R., 1981, MNRAS, 194, 383
Scholz M., Tsuji T., 1984, A&A, 130, 11
Watanabe T., Kodaira K., 1978, PASJ, 30, 21
Wiese W.L., Smith M.W., Glennon B.M., 1966, Atomic Transition Probabilities, Vol.I., Hydrogen Through Neon, NSRDS-NBS 4, Washington D.C.

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.