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The Hipparcos Catalogue as a realisation of the extragalactic reference system

J. Kovalevsky[1], L. Lindegren[2], M.A.C. Perryman[3], P.D. Hemenway[4], K.J. Johnston[5], V.S. Kislyuk[6], J.F. Lestrade[7], L.V. Morrison[8], I. Platais[9], S. Röser[10], E. Schilbach[11], H.-J. Tucholke[12], C. de Vegt[13], J. Vondrák[14], F. Arias[15], A.M. Gontier[16], F. Arenou[7], P. Brosche[12], D.R. Florkowski[5], S.T. Garrington[17], R.A. Preston[18], C. Ron[14], S.P. Rybka[6], R.-D. Scholz[11], and N. Zacharias[5]

[1] CERGA, Observatoire de la Côte d'Azur, Av. Copernic, F-06130 Grasse, France
[2] Lund Observatory, Box 43, S-22100 Lund, Sweden
[3] Astrophysics Division, European Space Agency, ESTEC, Noordwijk 2200 AG, The Netherlands
[4] Astronomy Dept., University of Texas, RLM 15308, Austin, TX 78712 1083, USA
[5] US Naval Observatory, 3450 Massachusetts Ave NW, Washington DC, 20392, USA
[6] Main Astron. Observatory Ukrainian Ac. Sc., Goloseevo, 252127 Kiev, Ukraine
[7] Observatoire de Paris, Section de Meudon, F-92195 Meudon Principal Cedex, France
[8] Royal Greenwich Observatory, Madingley Road, Cambridge CB3 0EZ, UK
[9] Yale University Observatory, PO Box 208101, New Haven CT 06520-8101, USA
[10] Astronomisches Rechen-Institut, Mönchhofstrasse 12–14, D-69120 Heidelberg, Germany
[11] WIP, Universität Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany
[12] Universitäts Sternwarte, Auf dem Hügel 71, D-53121 Bonn, Germany
[13] Hamburger Sternwarte, Gojensbergsweg 112, D-21029 Hamburg, Germany
[14] Astronomical Institute, Boční II 1401, 14131 Prague 4, Czechien
[15] Observatorio Astronomico, Paseo del Bosque, S/N, 1900 La Plata, Argentina
[16] IERS, Observatoire de Paris, 61, avenue de l'Observatoire, F-75014 Paris, France
[17] University of Manchester, NRAL, Jodrell Bank, Macclesfield, Cheshire SK11 9DL, UK
[18] Jet Propulsion Laboratory, 4800 Oak Grove Drive, Pasadena, CA 91109, USA

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Abstract. The paper describes the methods and observations by which the Hipparcos Catalogue was linked to the International Celestial Reference System (ICRS). The contributions of several groups represented in the authorship of this paper, using a variety of techniques, were synthesised in order to determine the global orientation and rotation (spin) of the coordinate frame defined by the Hipparcos data with respect to extragalactic sources. The following link techniques were used: interferometric observations of radio stars by VLBI networks, MERLIN and VLA; observations of quasars relative to Hipparcos stars by means of CCDs and photographic plates, and by the Hubble Space Telescope; photographic programmes to determine stellar proper motions with respect to extragalactic objects (Bonn, Kiev, Lick, Potsdam, Yale/San Juan); and comparison of Earth orientation parameters obtained by VLBI and by ground-based optical observations of Hipparcos stars. Although vastly different in terms of instruments, observational methods and objects involved, the various techniques generally agree to within 10 mas (milliarcsec) in the orientation and 1 mas/yr in the spin of the system. Two different numerical methods are described for the systematic comparison and synthesis of the link observations. The methods give very similar solutions, and a mean value was adopted for the definition of the system of positions and proper motions in the Hipparcos Catalogue. As a result, the coordinate axes defined by the published catalogue are believed to be aligned with the extragalactic radio frame to within ±0.6 mas at the epoch 1991.25, and non-rotating with respect to distant extragalactic objects to within ±0.25 mas/yr.

Key words: astrometry – catalogs – reference systems

1. Introduction

The Hipparcos and Tycho Catalogues are now completed and will be generally available to the astronomical community in mid-1997 (ESA 1997). They are the result of an immense effort not least on the part of the various scientific 'consortia' charged with the preparation of the observing programme and the derivation of astrometric and photometric parameters from the satellite data: the Input Catalogue Consortium, INCA (Turon et al.

1992), and the data reduction consortia FAST, NDAC and TDAC (Kovalevsky et al. 1992, Lindegren et al. 1992, Høg et al. 1992). Linking the satellite results to an extragalactic reference system represents an additional and considerable effort, involving a wide range of observational techniques both from the ground and from space. It is the purpose of this paper to describe these techniques and their main results, and to explain how the results were synthesised in order to define the coordinate axes for the system of positions and proper motions in the Hipparcos and Tycho Catalogues.

Hipparcos was able to measure very accurately the angles between objects on its observing list. From these angles, and their variation in time, the positions and proper motions of the stars could be calculated in a single coordinate system covering the whole sky, and their absolute trigonometric parallaxes were obtained at the same time. However, because practically no extragalactic objects were observed (due to the limiting magnitude of $V \sim 12.3$), the resulting coordinate system was essentially free to spin slowly with respect to distant galaxies, and the precise orientation of its axes at a given instant was also undefined by the observations. The data reductions thus resulted in preliminary versions of the Hipparcos and Tycho catalogues, in which the positions and proper motions were expressed in a very well-defined, but to some extent arbitrary, reference frame. This provisional frame was in practice fixed by the special procedure by which the main FAST and NDAC astrometric results were merged into a single provisional catalogue called H37C. The positions and proper motions in the Hipparcos Catalogue were obtained by applying a rigid-body rotation of the coordinate frame in H37C. This rotation was defined by six numbers: three of them represent the orientation offset of H37C about the principal equatorial axes at the catalogue epoch J1991.25 (ε_{0x}, ε_{0y}, ε_{0z}); the other three give the rates of change in time of the orientation offset, or the spin of H37C with respect to the final frame ($\omega_x, \omega_y, \omega_z$). The Tycho Catalogue was later aligned with the Hipparcos Catalogue by a similar process, based on a direct comparison of the positions and proper motions of the common stars (which included nearly all the Hipparcos stars).

The precise definition of the orientation and spin vectors were given in a previous paper (Lindegren & Kovalevsky 1995). The equations of condition for determining these vectors from different kinds of link observations were also given. Following the main principles and conventions of that paper, the link observations were analysed with respect to the positions and proper motions in H37C by the various groups represented in the authorship of the present paper and further described in Sect. 3. Each group thus provided an estimate of the orientation and/or spin of H37C with respect to the extragalactic frame, and these estimates were then compared and synthesised as described in Sect. 4 and 5. The end result of this process was an adopted set of orientation and spin components, which was applied to H37C in order to produce the reference frame of the published Hipparcos Catalogue.

The actual values of the orientation and spin components found by the various groups and by the synthesis methods are not given in this paper, as they merely reflect the arbitrary rotational state of the provisional data in H37C. Instead, this paper gives consistently the *differences* between these values and the final set of adopted values. The rotation parameters reported below consequently describe the results of the link observations as if they had been analysed with respect to the final Hipparcos Catalogue. Also, the adopted results of the synthesis, as given in Sect. 5, are identically zero.

The purpose of the link is to put the Hipparcos and Tycho Catalogues on a reference frame which corresponds as closely as possible to the extragalactic frame realised over the past decades through the extremely accurate VLBI observations of radio sources.

The importance of this link was stressed already in the planning of the observing programme for Hipparcos. Extensive preparations were made by the INCA Consortium, within the framework of major collaborative programmes, to initiate and collect relevant ground-based observations of radio stars and stars in the fields of compact extragalactic radio sources, and to ensure that suitable link stars were included on the observing list. Various techniques for link observations were considered (radio observations by VLBI, MERLIN and VLA; ground-based astrometry; speckle interferometry; Hubble observations with the FGS) and have been described in some detail in Argue (1989; see also Argue 1991 for an update). In parallel, the suitability of the selected link stars for measurement by Hipparcos and for consistency with the radio model were performed. Link conditions were first simulated by Frœschlé and Kovalevsky (1992) and then other tests were made using simulations of the mission and, in some cases, the observing programme was modified in order to obtain a satisfactory observing time on each of the radio and Hubble link stars (Turon et al. 1992).

Meanwhile, the IAU adopted a resolution (Bergeron 1992) stating that the next celestial reference system should be based upon positions of extragalactic radio sources, but that it will come into effect only when there is a realisation of the system in the optical domain. It was then understood that this realisation should be the Hipparcos Catalogue, given its expected high precision and extension to more than a hundred thousand stars.

In 1993 the Hipparcos Science Team appointed a working group for the determination of the extragalactic link, continuing and extending the preparatory work of the INCA Consortium. The composition of the working group has varied somewhat, but when the final link results were computed in early 1996, the 14 first authors of this paper had the main responsibility for the different link techniques and for the evaluation and synthesis of the results.

The quality of the final link is a function of three factors:

1. The uncertainties of the proper motions of the Hipparcos Catalogue. Because the results for double stars are subject to greater errors, only apparently single stars were used for the link with the exception of VLBI for which a double star model was provided for two radio-sources. The average standard errors of these stars, if brighter than $V = 9$ mag, are 0.8 mas and 0.9 mas/yr for the components of position (at epoch J1991.25) and proper motion; for $11 \leq V \leq 12$ the typical standard errors are, respectively, 2.1 mas and 2.6 mas/yr.

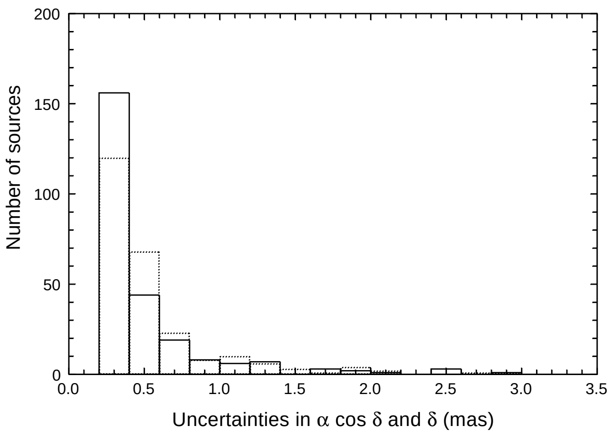

Fig. 1. Histogram of uncertainties (standard errors) achieved for the 253 radio positions in ICRF for the defining radio-sources and all sources used for Hipparcos link. Full line: right ascensions; dotted line: declinations

2. The uncertainties of the extragalactic reference frame used for setting the origin of the catalogue. This frame is discussed in the next section.

3. The actual errors and uncertainties of other references used and of the link observations connecting the two reference frames. These are different for each method, as presented in Sect. 3.

• In practice we shall show that the limiting factors presently affecting the reference frame link are the number and accuracy of the positions and proper motions of the non-Hipparcos observations. These may be improved in the future with acquisition of new observations of Hipparcos stars in the extragalactic reference frame.

2. The international celestial reference system

Since 1988, the International Earth Rotation Service (IERS) has implemented and maintained an extragalactic reference frame containing an increasing number of extragalactic radio sources observed by several VLBI networks throughout the world. At the request of the IAU working group on reference frames, IERS finalised this iterative process and provided a definitive list of objects and coordinates in October 1995.

The final catalogue is the International Celestial Reference Frame (ICRF) of 610 sources uniformily distributed on the sky. The J2000.0 VLBI coordinates of these sources materialize the International Celestial Reference System (ICRS, Arias et al., 1995). In order to provide radio-source coordinates to link Hipparcos reference frame to the ICRS, a subset of this catalogue that contains 212 defining sources and all sources used for Hipparcos link was issued. Fig. 1 gives the distribution of accuracies achieved on these subset of 253 sources. More details are to be found in Ma et al. (1997). The coordinates of these sources are given in IERS (1996).

As a result of the present link, all the coordinates published in the Hipparcos and Tycho Catalogues are referred to the ICRS. It is expected that in 1997 the IAU will approve ICRS as the new reference system replacing the FK5 system. The Hipparcos and Tycho Catalogues will be its first realisation for optical astronomy.

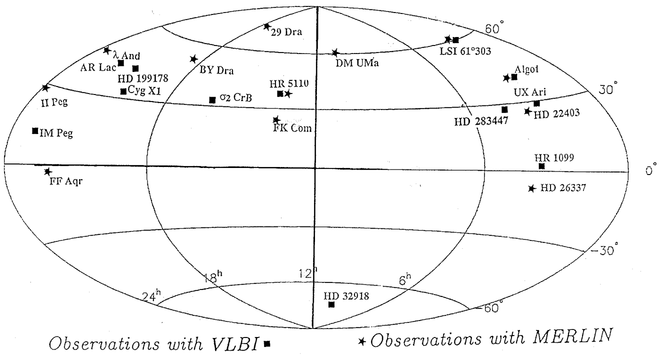

Fig. 2. Sky distribution of radio stars used for the link by VLBI (squares) and MERLIN (asterisks)

It should be noted that the orientation of the Hipparcos Catalogue is fixed to the ICRF using direct links between the positions of Hipparcos stars at the catalogue epoch J1991.25 and radio sources in the ICRF itself. The time-dependent rotation (spin), on the other hand, is fixed by linking the Hipparcos proper motions to any external sources, such as galaxies, which are not necessarily included in the ICRF.

3. Individual link solutions

In the following subsections, the various methods used for the link are described individually. They are presented in the following order:

1. Radio and optical techniques providing high-precision positional links to a small number of Hipparcos stars. These define the orientation parameters ε_0 very accurately, but contribute less to the determination of the spin (ω) due to the small number of objects and the relatively short time span of the observations.
2. Use of proper motion surveys where the motions of large numbers of stars are measured relative to galaxies. These only contribute to the determination of ω.
3. Special photographic link programmes.
4. Use of Earth orientation parameters.

The numerical results of the individual link solutions, expressed as residuals with respect to the adopted global solutions, are given in Table 3.

3.1. VLBI observations

Multi-epoch VLBI observations were conducted between 1984 and 1994 to determine the positions, proper motions and parallaxes of 12 radio-emitting stars. Their positions on the sky are shown by the squares in Fig. 2.

The observations were conducted on the US VLBI Network, NASA Deep Space Network, NRAO Very Large Baseline Array (VLBA) and European VLBI Network. The data processing and analysis is described in Lestrade et al. (1995). All the VLBI observations for each star were phase-referenced to an angularly nearby extragalactic radio source on the ICRF list. The resulting uncertainties in the astrometric parameters are presented in Table 1. The standard errors of the relative positions between extragalactic reference sources and the radio stars are all smaller than 1 mas, but uncertainties in the IERS coordinates of the reference sources make the standard errors in absolute position for the stars usually larger than 1 mas.

The six components of ε_0 and $\boldsymbol{\omega}$ were simultaneously solved by a least squares fit as described in Lestrade et al. (1995), using weights based on the combined VLBI and Hipparcos *a priori* measurement uncertainties. This led to a chi-square goodness of fit of 56, with 42 degrees of freedom. Subsequently, two objects were down-weighted by increasing their positional uncertainties by a factor of 3: for HIP 12 469 (LSI 61°303) because of its jet structure on a 10 mas scale, and for HIP 19 762 (HD 283 447) because of its known duplicity on a scale of $\simeq 0.1$ arcsec (Ghez et al. 1993) which is difficult for Hipparcos. No modifications had to be applied on the *a priori* proper motion uncertainties. After this adjustment of the weights, the chi-square goodness of fit became a satisfactory 36, with 37 degrees of freedom.

Further tests were done by splitting the 12 stars in various subsets. This showed that the fit is quite robust: for instance, the differences between the fits of two independent subsets of six stars each were within the combined uncertainties (less than 1 mas for the orientation components and less than 0.6 mas/yr for the spin components). The formal standard errors are very similar in all three axes, viz. 0.5 mas for the components of ε_0 and 0.3 mas/yr for the components of $\boldsymbol{\omega}$. The rms post-fit residuals are, respectively, 1.72 mas and 0.84 mas/yr.

J.F. Lestrade has led the project in close collaboration with R.A. Preston, D.L. Jones (JPL) and R.B. Phillips (Haystack) for the northern stars, and J. Reynolds, D. Jauncey (CSIRO) and J.C. Guirado (JPL) for the southern hemisphere.

3.2. Observations with MERLIN

MERLIN is a real-time radio-linked radio interferometer array with a maximum baseline of 217 km, giving a resolution of approximately 40 mas at 5 GHz. A general description of MERLIN is given by Thomasson (1986) and a detailed discussion of the observations and procedures used here is being presented elsewhere (Garrington et al. 1997).

As in the VLBI observations described above, the positions of weak radio stars were obtained by using ICRF sources as phase calibrators. Observations were made at 4993 MHz with a bandwidth of 15 MHz. Typically, the star-calibrator separation was 5° and the cycle time was 5 to 10 min. Four of the brighter radio stars common to the list observed by VLBI were observed in 1992. These were: HIP 12 469 (LSI 61°303), HIP 14 576 (Algol), HIP 66 257 (HR 5110) and HIP 79 607 (σ^2 CrB). In order to select an independent set, a survey of 25 RS CVn and chromospherically active stars with a known history of radio flaring was carried out in 1995. Astrometric observations were then made of nine stars which were detected: HIP 16 879 (HD 22 403), HIP 19 431 (HD 26 337), HIP 53 425 (DM UMa), HIP 65 915 (FK Com), HIP 85 852 (29 Dra), HIP 91 009 (BY Dra), HIP 108 644 (FF Aqr), HIP 116 584 (λ And) and HIP 117 915 (II Peg). These stars have flux densities of between 2 and 12 mJy which is weaker than those used for VLBI observations. Their positions are shown by the asterisks in Fig. 2.

Table 1. Uncertainties of the absolute positions (at epoch J1991.25), proper motions and trigonometric parallaxes of the 12 link stars as determined by VLBI observations

Hipparcos number (HIP)	Star name	Standard errors Pos. (mas)	P.M. (mas/yr)	Par. (mas)
12 469	LSI 61°303*	3.0	0.30	0.62
14 576	Algol	0.61	0.18	0.59
16 042	UX Ari	2.1	0.20	0.39
16 846	HR 1099	0.48	0.31	0.47
19 762	HD 283 447	3.0	0.28	0.25
23 106	HD 32 918	1.5	1.00	0.80
66 257	HR 5110	1.28	0.16	0.45
79 607	σ^2 CrB	0.29	0.05	0.10
98 298	Cyg X1**	1.50	0.14	0.30
103 144	HD 199 178	1.95	0.43	0.33
109 303	AR Lac	0.94	0.19	0.37
112 997	IM Peg	1.42	0.47	0.68

* V615 Cas ** V1357 Cyg

The positions of individual stars, relative to the ICRF sources, are estimated to have individual errors of approximately 4 mas. This includes 'noise' in the determination of the centroid of the weak radio star image and various calibration errors, which are proportional to the separation of the star and calibrator and were estimated using observations of pairs of calibrators. The error estimates were confirmed by splitting the data into subsets of sources wherever possible.

Two observations of HIP 16 879 (HD 22 403) were rejected because they were of poorer quality than the third; all the observations of HIP 85 852 (29 Dra) were also rejected because they showed a relatively bad goodness of fit in the Hipparcos reductions. The double star HIP 79 607 (σ CrB) was not included because the Hipparcos observations required special treatment and a final position was not made available to the MERLIN group at the time of the analysis. So, in total, observations of 11 radio stars by MERLIN were used to link Hipparcos to the ICRF. Three of these stars are common to the VLBI list (HIP 12 469, 14 576, 66 257).

Algol is a triple system, AB–C, where the separation of AB is ~ 4 mas and AB–C is ~ 95 mas. The Hipparcos position is the centre of mass of the AB–C system. The radio emission comes from AB, so the MERLIN position was reduced to the centre of mass using the orbital elements and mass ratios given by Pan et al. (1993). However, there is evidence from the Hipparcos observations that the line of nodes in Pan et al. is wrong by 180°. The subsequent residuals for the MERLIN position of Algol in this paper confirm this. With this alteration, the computed corrections from AB to the centre of mass at the epoch of the MERLIN observation (1993.00) were found to be +12.9 mas and -10.0 mas in right ascension and declination, respectively.

The Hipparcos proper motions and parallaxes were used to reduce the MERLIN geocentric positions to the barycentre and catalogue epoch of Hipparcos, which is J1991.25. For the 1995 observations, the uncertainties from the Hipparcos proper motions are comparable to the MERLIN position errors. The MERLIN and Hipparcos positions were subtracted and the usual unweighted observational equations (see, for example, Lindegren & Kovalevsky 1995) for the three orientation angles were solved by least-squares.

The solution for ε gives standard errors of 2.2 to 2.6 mas in the components. Compared with the VLBI solution which is virtually independent, there is a close agreement which lends confidence to the stability of the link which could have been distorted by significant offsets between the optical and radio emission of some of the binary stars.

The team which worked on this project comprised S.T. Garrington and R.J. Davis (NRAL, Jodrell Bank), L.V. Morrison, R.W. Argyle (RGO), and A.N. Argue (IoA, Cambridge).

3.3. Observations with VLA

The procedures outlined in Florkowski et al. (1985) were followed. Observations of radio emitting stars were made with the Very Large Array (VLA) operated by the National Radio Astronomy Observatory. The astrometric measurements were made when the VLA was in the high resolution A configuration where the baseline lengths ranged from 1–36 km. A typical synthesised (half-power) beam was about 0.4 arcsec. The observations were made between March 1982 and August 1995. As the stellar continuum flux density is weak, typically only a few mJy, the observations were made at C Band (6 cm) with a bandpass of 100 MHz. The observations were made in a differential fashion with respect to an unresolved extragalactic radio source. In order to minimise atmospheric noise, the phase reference source was always within 15° of the star. Observations were made over a number of widely spaced hour angles in order to minimise atmospheric effects. The data from the quasar were employed to normalise the antenna gains and phases for the stellar radio source. The quasar 1328+307 (3C286) was used as a primary flux density calibrator.

Images of the sky near the stellar radio emission were created using the Astronomical Image Processing System (AIPS). They were made with a cell size of 0.1 arcsec and in order to maximise the angular resolution of the image, uniform weighing and no taper were applied to the visibility data. The maps were cleaned using the array processor version of CLEAN devised by Clark (1980). The images of the region around the stars were not very complicated, and the enhanced sidelobes did not cause any difficulties in cleaning the images. Since we are interested in the absolute phase centre of the map, self-calibration, commonly used to improve maps, was not used here. A two-dimensional gaussian function was fitted to the stellar emission, and the centre of the gaussian was taken to be the star's position.

The stellar positions (in the extragalactic reference frame) were moved to the epoch J1991.25 by means of the proper motions and parallaxes in H37C. The differences between the radio and Hipparcos positions were then used to solve for the orientation vector ε_0. The standard error in each component of the vector was about 5 mas.

3.4. Optical positions of compact sources

The Hamburg/USNO reference frame programme has been described in Johnston et al. (1995), Ma et al. (1990), and Zacharias & de Vegt (1995), and the reader is referred to these publications for details. The programme is aimed at the determination of precise optical and radio positions of about 400 to 500 selected compact radio sources which display optical counterparts, mostly QSOs and BL Lac's, within a visual magnitude range of 12 to 21. Optical positions in the Hipparcos reference frame are obtained via a system of secondary reference stars in the magnitude range 12 to 14. The procedure thus requires two steps: first the establishing of the secondary reference positions by means of astrograph plates, and then the observation of the radio sources with respect to the secondary frame by means of larger telescopes.

The secondary frame was established using wide field ($\sim$ 5°) astrographs in both hemispheres: the yellow lens of the USNO 8-inch twin astrograph (South), the Hamburg astrograph, and the yellow lens of the Lick Carnegie astrograph with $3^\circ \times 3^\circ$ plates. All radio source fields were taken with Kodak 103aG emulsion and a Schott filter for short-wavelength cutoff. The spectral range used was only 600 to 800 Å wide (centred at about 5400 Å), thus providing a minimum influence of atmospheric dispersion and residual colour aberrations of the optics. For each field, four plates centred on the source position were taken and measured in two orientations on the CCD-camera based Hamburg astrometric measuring machine. The measurements included all Hipparcos stars in the whole plate field (typically 50 to 100 stars), and secondary reference stars selected from the HST Guide Star Catalog in the central 1° field. The resulting precision of the secondary reference star catalogue is 60 to 80 mas for the individual positions based on four plates each. The astrograph plate solutions, based on the Hipparcos positions, reached a typical mean unit weight error of 60 to 70 mas, or 0.6 to 0.7 μm on the plate. Formally, the Hipparcos reference frame could therefore be transferred locally to each radio source field with a precision better than 10 mas.

Optical source positions were then obtained using plates from Schmidt telescopes and the prime focus of large telescopes in both hemispheres; recently also CCD frames from KPNO and CTIO 90 cm telescopes were obtained (Zacharias & de Vegt 1995; de Vegt & Gehlich 1982). Plate or CCD solutions were obtained using the secondary reference star catalogue. The precision of the optical source positions based on several plates and/or CCD frames is better than 30 mas in each case. The programme therefore provides optical positions of the extragalactic reference frame sources in the Hipparcos frame, and the determination of a precisely defined rotation between both systems is feasible. The presented preliminary solution is based entirely on CCD frames of 78 globally selected sources at mean epoch 1988.5. The full program will be based on about 400 sources and will determine the orientation parameters on the 1 mas accuracy level.

3.5. Observations with the Hubble Space Telescope

The Fine Guidance Sensors (FGSs) of the Hubble Space Telescope (HST) have been used to measure the angular separation of Hipparcos stars from extragalactic objects. If the positions of the extragalactic sources are known from VLBI observations, the orientation of the Hipparcos frame can be determined from a set of such measurements. If the measurements are spread over a sufficient interval in time, also the spin of the Hipparcos frame can be determined.

The FGSs measure relative positions of targets within the instrumental reference frame to a precision of a few milliarcsec (Benedict et al. 1992). Because of inaccuracies of the guide stars, and uncertainties of the locations of the instruments on board HST with respect to each other, the absolute orientation of the spacecraft on the sky is unknown at the milliarcsec level within the Astrometric FGS field of view. Therefore, the separation of two objects is the most accurate datum available for the frame tie work. 78 separations of 46 Hipparcos stars next to 34 extragalactic objects were measured from April 1993 through December 1995. GaussFit, a non-linear least-squares package (Jefferys et al. 1988), was used to determine the orientation and spin parameters from these data.

$O - C$ residuals for the separations (with ‘observed’ data O coming from the HST, and ‘computed’ data C from the Hipparcos positions for the epoch of the HST observations combined with the ICRF positions) showed an initial rms of 30 mas. Application of a time-variable field distortion calibration to the FGSs brought the rms down to 10 mas. Application of the accurate Hipparcos parallax values brought the rms to 8 mas. The GaussFit program allowed rigorous application of the generalised least-squares method, requiring the Hipparcos data, the VLBI data, and the HST data all to be treated as observables with their own attendant errors. The major sources of error are the HST data (estimated at 3 to 4 mas for a single separation measurement) and the propagation of the Hipparcos proper motion errors to the epochs of the HST observations. One source pair with marginally significant residuals was dominating the solution for one of the axis rotations because of its particular location with respect to the solution, and was rejected. Several other HST observations were rejected for various reasons including large telescope drift, large observational scatter, and inability to lock onto an extragalactic object because it was extended. A time-variable scale factor in the FGS was also found from our data, and was fitted in the solution. The details will be published by Hemenway and his co-investigators.

Based on observations with the NASA/ESA Hubble Space Telescope, this work was carried out by many people, but the data collection and analysis was the result of continued efforts of P.D. Hemenway, E.P. Bozyan, R.L. Duncombe, A. Lalich, B. MacArthur, E. Nelan and the Hubble Space Telescope Team.

3.6. Use of the Lick proper motion programme

The published Part 1 of the Lick Observatory Northern Proper Motion program (NPM), also known as NPM1 (Klemola et al. 1987 and 1994), contains 149 000 stars from 899 NPM fields north of $\delta = -23°$ for which the proper motions have been determined relative to background galaxies. The mean number of galaxies per field is 80. The typical accuracy of the NPM1 absolute proper motions is 5 mas/yr.

In total 13 455 stars are common between the NPM1 catalogue and the Hipparcos Catalogue. Preliminary comparisons of proper motions in H37C and the NPM1 indicated a linear magnitude equation of about 1 mas yr^{-1} mag^{-1} in the NPM1 data essentially down to the magnitude limit of the Hipparcos Catalogue. The magnitude equation is coordinate-independent, although in declination it shows a different slope for stars north and south of $\delta = -2.5°$. It should be noted that due to the lack of measurable multiple grating images in the same exposure, it is impossible to eliminate the magnitude equation *internally*. Furthermore, there are no absolute proper motions available which could readily be used to correct the magnitude equation *externally*. Since the rotation parameters are correlated with the magnitude equation, the Hipparcos data cannot be used to correct the magnitude equation as part of the link solution.

Whatever the reason may be for the magnitude equation for the fairly bright Hipparcos stars (e.g. grating image blending, instrumental effects), extensive tests have shown that the NPM1 catalogue is relatively free of systematic errors for the stars with $m_B > 12$ mag on which the galactic structure and kinematic studies are based.

Two different groups have independently undertaken the task to use the NPM1 catalogue in an attempt to contribute to the link of the Hipparcos Catalogue to the extragalactic reference system. Because the two groups obtained significantly different results, their separate reports are presented below. Both groups have benefited from discussions with R.B. Hanson and A.R. Klemola of the Lick Northern Proper Motion programme.

Fig. 3 gives the distribution of link stars in the Lick as well as the Yale/San Juan (SPM) programmes.

3.6.1. The Yale solution

With regard to the magnitude equation described above, affecting the bright NPM1 stars, three possible solutions to the problem emerge. Firstly, one may use in the solution only the relatively faint stars (e.g. $m_B > 10.5$ mag), anticipating that the magnitude equation vanishes at some magnitude. A set of link calculations at different magnitude cutoffs showed that the solution improves when it is limited to fainter stars. However, it cannot be concluded with confidence at which magnitude the effect disappears. Secondly, despite the correlations it seems feasible to supplement the conventional link equations with a

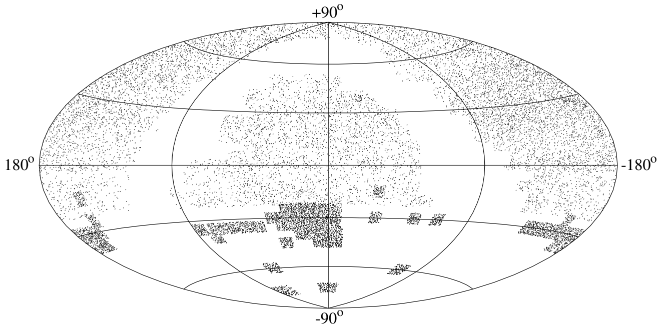

Fig. 3. Distribution of Lick NPM1 fields (light grey) and of the Yale/San Juan SPM fields used in the link solutions (dark grey)

linear magnitude equation, of the form $d_0 + (m - 9)d_1$, where m is the star's magnitude. Indeed, a substantial improvement of the solution was noted after this modification. This procedure would be ideal except that the correlations between the spin components and the magnitude equation prevents unambiguous disentangling of the effect. For that reason a third way of handling the magnitude equation was proposed.

A careful inspection of how the magnitude equation affects the spin components indicates the significance of the star distribution over the sky. The key to success in minimising the effect of a *coordinate-independent* magnitude equation is to seek for a well-balanced distribution of the stars. In other words, for the spin components ω_x and ω_y the stars must be distributed such that the sums of the corresponding geometrical weighting factors in the link equation [see Eq. (27) in Lindegren & Kovalevsky 1995] are close to zero. In the presence of a magnitude equation in right ascension the z component is so strongly correlated with the magnitude equation that ω_z cannot be accurately determined. A linear magnitude term in right ascension was nevertheless included in the solution, in order to remove a possible bias in x and y, but the result for ω_z was not used in the synthesis, as discussed in Sect. 4. In practice, the distribution of the NPM1 stars used in the solution was balanced by introducing a fictitious 44°-width 'zone of avoidance' perpendicular to the galactic plane. In addition, the stars with $\delta < -2.5°$ (only in the declination solution) and $m_B < 10$ mag were deleted from the sample in order to reduce further the effect of magnitude equation.

3.6.2. The Heidelberg solution

In total, 9357 stars were available for the comparison performed in Heidelberg. As a first step, the Lick proper motions were precessed from the B1950 equator to the J2000 equator using the IAU 1976 constant of precession. Note that this does not correspond to a transition from the FK4 to the FK5 system: it is simply a re-orientation of the equatorial axes affecting the position angles (but not the absolute values) of the Lick proper motions.

In the Lick programme, the stars common to Hipparcos are bright compared to the faint ($m_B \lesssim 15$ mag) galaxies. Therefore, an investigation was performed to test a possible dependence of the spin components on the brightness of the stars. Table 2 shows the results obtained for samples of different brightness.

Table 2. Results of the Heidelberg solutions for the components of the spin vector $\boldsymbol{\omega}$ (in mas/yr) using the Lick NPM1 proper motions. The first line gives the solution without magnitude limits for the selection of stars; subsequent lines give the results for stars in certain intervals of the Lick (m_B) or Hipparcos (Hp) magnitude. The last line gives the formal standard errors for the solution with 1135 stars

Magnitude range	Number of stars	Spin components ω_x	ω_y	ω_z
(no limit)	9236	−0.70	−0.27	−2.14
$10.5 < m_B < 11.5$	2616	−0.76	+0.17	−0.85
$10.9 < m_B$	2220	−0.72	+0.02	+0.10
$11.9 < m_B$	510	−0.25	−0.12	+0.84
$10.0 < Hp < 12.2$	2535	−0.81	+0.11	−0.25
$10.6 < Hp < 12.2$	1135	−0.85	+0.16	+0.60
Formal errors:	1135	0.25	0.20	0.20

It turns out that the dependence on magnitude is relatively unimportant for the rotation around the x and y axes. The case in which only stars with $m_B > 11.9$ were considered is not representative, because only some 500 stars of the original sample were retained. ω_z, on the other hand, shows a strong dependence on magnitude. Moreover, there seems to be no asymptotic behaviour when going to fainter and fainter magnitudes. The conclusion is that ω_z cannot be reliably determined from the Lick proper motions. These findings are confirmed by Hanson (1996, private communication) who reported on small systematic errors in the Lick proper motions in right ascension. To summarise, the spin components ω_x and ω_y of the Hipparcos proper motions with respect to the Lick proper motions were estimated to have the values and uncertainties given in Table 3.

3.7. *Catalogue of faint stars (KSZ)*

A general catalogue of absolute proper motions of stars with respect to galaxies was compiled by Rybka & Yatsenko (1997), using data from 185 sky areas produced in Kiev, Moscow, Pulkovo, Shanghai and Tashkent. The catalogue includes 977 Hipparcos stars in the magnitude range 4 to 13.

Proper motion differences in right ascension and declinations, taken in the sense Hipparcos *minus* KSZ, were analysed taking into consideration the zero point offsets of the KSZ areas as well as investigations of systematic dependencies of the residuals on magnitude, colour, and position on sky. In addition, the proper motion residuals in each coordinate were represented by series developments using products of Hermite and Legendre polynomials as well as Fourier terms. No significant dependency on these variables was found: the residuals represent random errors only.

However, different results for ω_x, ω_y, ω_z were found when whole interval of stellar magnitudes was used and when only bright (≤ 9.0 mag) or faint (> 9.0 mag) stars were used. Since the stellar data in KSZ were obtained relative to faint galaxies it was assumed that the magnitude effect on the absolute proper motions is less important for the fainter stars. Thus the 'faint' KSZ solution (using only stars fainter than 9th magnitude) is the one recommended as the most reliable for the link. For that solution, 415 stars were kept from 154 areas of the sky. The standard errors obtained for the components of $\boldsymbol{\omega}$ are 0.8 mas/yr.

This link was realised at Kiev Observatory by N.V. Kharchenko, V.S. Kislyuk, S.P. Rybka and A.I. Yatsenko. See also Kislyuk et al. (1997).

3.8. The Yale/San Juan Southern Proper Motion program

The Yale/San Juan Southern Proper Motion program (SPM) is an extension of the Lick Observatory Northern Proper Motion program to the sky south of $\delta = -17^\circ$. A brief description of the observational material can be found in van Altena et al. (1990) and Platais et al. (1995). In total 63 SPM fields, containing about 4100 Hipparcos stars, have been measured and reduced. The distribution of the Hipparcos stars in these fields is shown in Fig. 3 where the SPM fields can be recognised as the more densely populated areas in the southern sky. The mean number of reference galaxies per field is 250 on blue plates and 190 on visual plates, yielding a mean uncertainty in the correction to absolute proper motions of 1.0 mas/yr for each field. Since the Hipparcos stars are represented by several images per star (ten in the most favourable case), the single proper motion accuracy in each colour (blue or visual) can be as good as 2 to 3 mas/yr. If this error were composed entirely by the random measurement and modelling errors, the precision of each spin component with the given number of the Hipparcos stars in hand could be in the range of 0.1 to 0.2 mas/yr. However, the link solutions indicate a somewhat larger scatter in the spin components when compared to this precision estimate. This may very well be due to a small systematic error remaining after the correction for the magnitude equation.

A preliminary study of the systematic errors in the SPM plates (Platais et al. 1995) clearly showed the presence of significant magnitude equation in the SPM coordinates. The bulk of the magnitude equation in coordinates and, presumably, in proper motions was removed using the grating-image offset technique formulated by Jefferys (1962) and modified by Girard et al. (1997). It has to be cautioned, however, that this technique has inherent limitations set by the small number of stars at the bright end, and by the fact that the magnitude equation may have a complicated form, too difficult to model adequately. In addition, the magnitude equation in the SPM plates is stronger and more complex in declination than in right ascension. We therefore believe that the link solution using only the proper motions in right ascension is more secure against systematics related to the magnitude effect. A detailed description of the SPM data reduction procedure and analysis will be presented in a separate paper (Girard et al. 1997). Four different solutions were computed from images of different colours (blue, visual) and components (α, δ). Only the right ascension solution, with a linear magnitude equation included, was adopted for the synthesis and is presented in Table 3.

This work as well as the Yale link solution using the Lick survey were shared by I. Platais, T.M. Girard, V. Kozhurina-Platais; H.T. MacGillivray and D.J. Yentis furnished positions and magnitudes from the COSMOS/UKST database of the southern sky and W.F. van Altena provided many useful suggestions.

3.9. The Bonn link solution

Within the photographic contributions to the Hipparcos link, the special property of the Bonn contribution consists in large epoch differences (typically 70 years and up to 100 years). This leads to a rather high accuracy of the individual results. It also ensures averaging over transient phenomena in the optical images of reference objects with time scales up to decades, like astrometric binaries not recognised as such. The plates were predominantly taken with the $f = 5$ m double refractor of the Sternwarte Bonn. The absolute proper motions of Hipparcos stars were measured with respect to optically bright extragalactic radio sources or bright galaxies with star-like features. For some fields the relative proper motions have been calibrated by measurements of large numbers of stars and galaxies on Schmidt plates and Lick astrograph plates.

The Bonn link solution uses 88 Hipparcos stars in 13 fields distributed over the northern celestial hemisphere. The median internal accuracy of the relative proper motions is 1.0 mas/yr, while the calibration to an inertial system in each of the 13 fields has a median uncertainty of 1.3 mas/yr. The spin difference $\boldsymbol{\omega}$ was obtained with formal errors of about 0.3 mas/yr in each component; the standard deviation of a single proper motion component of one star was 2.7 mas/yr. No significant correlations of the residuals from this solution with magnitude, colour, spherical coordinates or relative position within a field were found.

Complete information about this contribution to the Hipparcos extragalactic link can be found in Tucholke et al. (1997) and Geffert et al. (1997). The workload was shared by H.-J. Tucholke, P. Brosche, M. Geffert, M. Hiesgen (Münster), A. Klemola (Lick), M. Odenkirchen and J. Schmoll.

3.10. The Potsdam link solution

The Potsdam programme (Dick et al. 1987) is mainly based on MAMA and APM measurements of plates taken with the Tautenburg Schmidt telescope (134/200/400 cm). Proper motions of 360 Hipparcos stars were derived in 24 fields (each of about 10 square degrees) well distributed over the northern sky. From 200 to 2000 galaxies per field were used to link the proper motions to the extragalactic reference system. With at least two plate pairs per field and epoch differences of 20 to 40 years, an internal accuracy of 3 to 5 mas/yr was achieved for the proper motions of Hipparcos stars (Kharchenko et al. 1994). Due to the large number of galaxies in each field the formal zero point error is less than 1 mas/yr.

Previous investigations in fields with globular and open clusters (Kharchenko & Schilbach 1995, Scholz & Kharchenko 1994) showed that systematic, magnitude-dependent errors could affect the proper motions of bright stars measured on Tautenburg plates. Indeed, a simple test computing the spin parameters $\omega_x, \omega_y, \omega_z$ with bright ($m_B \leq 9.1$) and faint ($m_B > 9.1$) link stars yielded significantly different results for these groups. The influence of the magnitude effect was studied by successively omitting the bright stars from the solution. It was found that the solution became stable with stars fainter than 8.9 mag. Using 256 Hipparcos stars ($m_B \geq 9.0$) for the link, the final determination of the spin vector yields formal uncertainties of 0.5 mas/yr on the components of $\boldsymbol{\omega}$. The rms residual in the proper motions of stars is 6.9 mas/yr. The reduction procedure and data analysis applied in the Potsdam solution will be described in detail in a separate paper (Hirte et al. 1997).

3.11. Comparison of Earth Orientation Parameters

The Earth Orientation Parameters (EOP) as determined by optical astrometry and VLBI were used to obtain an indirect link of the Hipparcos coordinate system to the extragalactic system. In this method the terrestrial reference system (rigidly tied to the rotating Earth) is used as an intermediary whose orientation with respect to both the Hipparcos catalogue (by optical astrometry) and extragalactic objects (by VLBI) has been monitored quasi-simultaneously during a decade.

VLBI determines five EOP at roughly 5-day intervals; they are the components of polar motion x, y (giving the position of the celestial ephemeris pole in the terrestrial system), the celestial pole offsets $\Delta\epsilon$, $\Delta\psi \sin\epsilon$ (giving the displacement of the actual spin axis of the Earth from its ephemeris position in obliquity of the ecliptic ϵ and in longitude ψ) and the difference between Universal and Atomic time UT1–TAI (giving the angle between the actual zero meridian and its expected position assuming the uniform speed of rotation), all referred to extragalactic objects.

The same five EOP, referred to the celestial optical system tied to the stars of our Galaxy, were determined by optical astrometry following the algorithms outlined in Vondrák (1991, 1996) and Vondrák et al. (1992, 1995). Using the preliminary Hipparcos catalogue H37C, all the latitude/Universal time observations made with 46 instruments at 29 different observatories all over the world were recalculated into that reference frame. About 3.6 million observations were used to derive EOP at 5-day intervals between 1899.7 and 1992.0.

Only the last twelve years of the global solution, common with VLBI observations (877 different epochs), were then used to calculate some of the angles and rotations linking the two celestial systems. The methodology and an example is described in Vondrák (1996). The detailed inspection of the relations between the two sets of EOP (i.e. as observed by optical astrometry and VLBI) shows that only the celestial pole offsets, which are completely independent of the terrestrial reference systems used by the two techniques, are fully suitable for determining the link. Polar motion is useless for the purpose since it is fully sensitive to the terrestrial reference frame, and almost entirely insensitive to the celestial reference frame. Universal time UT1 is a special case: it is sensitive to both the celestial and terrestrial reference frames used by the two observational techniques. This makes the determination of ε_{0z} impossible, as the unknown difference $\Delta\lambda$ between the two terrestrial frames in longitude is a non-removable part of the difference in UT1 as measured by the two techniques.

For the arbitrary epoch t, the formulae for calculating the components of the orientation vector $\boldsymbol{\varepsilon}$ from the two sets of EOP read:

$$\begin{aligned} \varepsilon_x &= \Delta\epsilon_{\rm VLBI} - \Delta\epsilon_{\rm Hip} \\ \varepsilon_y &= (\Delta\psi_{\rm Hip} - \Delta\psi_{\rm VLBI}) \sin\epsilon \\ \varepsilon_z + \Delta\lambda &= 15.041({\rm UT1}_{\rm Hip} - {\rm UT1}_{\rm VLBI}) \,, \end{aligned} \qquad (1)$$

where ϵ is the obliquity of the ecliptic. Assuming a linear variation with time, the angles ε_{0x}, ε_{0y}, $\varepsilon_{0z} + \Delta\lambda$ can be determined for the reference epoch t_0 while the spin components are obtained as $\omega_i = \dot{\varepsilon_i}$ for $i = x, y, z$. Practical tests with ω_z however proved that this method yields an improbably large value for that parameter, very probably due to a non-zero value of $\Delta\dot{\lambda}$ (drift of both terrestrial reference frames in longitude). Thus only the x and y components of $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\omega}$ were determined.

A detailed description of the method and the results is given in Vondrák et al. (1997).

4. Discussion of the individual results

The problem which had to be solved was to use all the results provided by the techniques described in the previous section in order to derive the single set of rotation parameters (ε_x, ε_y, ε_z, ω_x, ω_y, ω_z) which best represents them all. The individual results are summarised in Table 3 and presented graphically in Fig. 4 and 5.

The zero points for the data in Table 3 and Fig. 4 and 5 (and also in Table 2) have been shifted by the vectors representing the adopted solution found by the synthesis. This eliminates all reference to the provisional catalogue H37C which is not relevant any more. The given values of the orientation and spin components are the residuals of the individual determinations with respect to the adopted solution. For the orientation components the residuals refer to the approximate mean epoch of observation of the link observations in question; this epoch is also given in Table 3 for the methods providing estimates of $\boldsymbol{\varepsilon}$.

Before describing the methods used for the synthesis and their results, let us make some remarks on the data provided by the various link techniques.

4.1. Radio techniques

The relative precisions provided by the three interferometric techniques in the determination of the orientation ($\boldsymbol{\varepsilon}$) are consistent with their baseline lengths and are therefore considered as realistic. The precision of the determination of the spin ($\boldsymbol{\omega}$) depends both on the basic uncertainty of the observation and of

Table 3. Results of the individual link solutions, expressed as residuals with respect to the adopted solution. The second column contains the abbreviations used to identify the solutions in Fig. 4 and 5. The standard errors supplied with the individual solutions are given in parentheses. The last column gives the approximate mean epoch of the link observations used to determine ε

Method	Label	Spin components (mas/yr)			Orientation components (mas)			Epoch 1900+
		ω_x	ω_y	ω_z	ε_x	ε_y	ε_z	
VLBI	VI	−0.16 (0.30)	−0.17 (0.26)	−0.33 (0.30)	−0.10 (0.47)	+0.08 (0.49)	+0.16 (0.50)	91.3
MERLIN	ME				+1.41(2.60)	−0.64(2.20)	+0.51(2.40)	94.0
VLA	VA				+4.27 (4.70)	−3.75 (5.30)	−5.76 (5.20)	86.3
Hamburg/USNO	HP				+3.38 (5.00)	−0.06 (4.90)	−9.20 (4.70)	88.5
HST/FGS	ST	−1.60 (2.87)	−1.92 (1.54)	+2.26 (3.42)	−6.10 (2.16)	−3.25 (1.49)	+5.42 (2.14)	94.3
NPM (Heidelberg)	LH	−0.77 (0.40)	+0.15 (0.40)	+0.23 (⋆)				
NPM (Yale)	LY	+0.09 (0.18)	−0.20 (0.18)	+1.46 (⋆)				
KSZ Kiev	KZ	−0.27 (0.80)	+0.15 (0.60)	−1.07 (0.80)				
SPM (blue, α)	YBA	+0.23 (0.13)	+0.50 (0.20)	0.00 (0.08)				
SPM (blue, δ)	YBD	+0.07 (0.15)	+0.58 (0.08)					
SPM (visual, α)	YVA	+0.44 (0.12)	+0.71 (0.18)	−0.30 (0.07)				
SPM (visual, δ)	YVD	+0.30 (0.12)	+0.76 (0.06)					
Bonn plates	BP	+0.93 (0.34)	−0.32 (0.25)	+0.17 (0.33)				
Potsdam plates	PP	+0.22 (0.52)	+0.43 (0.50)	+0.13 (0.48)				
EOP	EO	−0.93 (0.28)	−0.32 (0.28)		+2.33 (0.88)	+7.80 (0.90)		85.0

(⋆) not used in the synthesis; see Sect. 3.6.1–2

the time span. This gives a major advantage to the VLBI observations both for the orientation and spin. An attempt was made to determine $\boldsymbol{\omega}$ with MERLIN observations, but the result was not retained, the time span being notably insufficient.

4.2. Optical determination of the orientation

It was very important to be able to confirm the radio determinations of ε by optical astrometry. The USNO/Hamburg programme which consisted originally of 400 extragalactic sources was only 20% complete at the time of the link, while the Hubble Space Telescope observations started very late due to the well-known problems with the telescope. In both cases, the uncertainties are large, but the methods are promising and would have given better results with more data. However, the uncertainties of the HST results seem to be underestimated by the formal errors.

4.3. Photographic catalogues referred to galaxies

These techniques are much more sensitive to magnitude-dependent errors than the preceding two optical methods. Most of the stars measured in these surveys are faint and comparable in magnitude with the reference galaxies. But for the link one had to choose only the brightest of the survey stars, for which the effect is likely to be much larger. The magnitude dependence is not necessarily linear for these bright stars and the link results depend strongly upon either the model applied or the magnitude cut-off adopted. This is illustrated by the discussions of Sect. 3.6 to 3.8 and no result can be considered as being unbiased in this respect, although the SPM solutions have an advantage in that the magnitude equation could be calibrated internally by the grating image technique. In any case, the formal errors given by the authors of these methods are small because of the large number of stars, and cannot be considered as realistic. Additional magnitude-related biases certainly exist and this has justified a significant down-weighting of the results provided. The difference between the results obtained at Yale and Heidelberg in their analyses of the NPM1 stars also justifies this policy.

4.4. Special photographic link programmes

Relying on archival plates for the first-epoch measurements, these programmes are also prone to magnitude equation, with little possibility to control or study the effect. Because of this, their formal errors are probably underestimated. Strangely enough, there seems to have been little gain in having a very long time span. Possibly the magnitude-dependent errors are larger or more difficult to model in the old plates, offsetting the advantage of the long time baseline.

4.5. Earth Orientation Parameters

In a sense this method is less direct than the others, as it depends on an intermediate (terrestrial) reference frame, whose relations in the optical and radio domain may not be completely understood. Apart from the problem with the z components discussed in Sect. 3.11, an additional uncertainty arises from the fact that the mean epoch of observations is 1985 and that all of them were performed before Hipparcos mean epoch. In the synthesis method where only ε_0 was estimated, this led to a considerable down-weighting of the data. However, even when the strong correlation between ε_0 and $\boldsymbol{\omega}$ was taken into account, the formal errors had to be substantially increased to make sense in relation to other data.

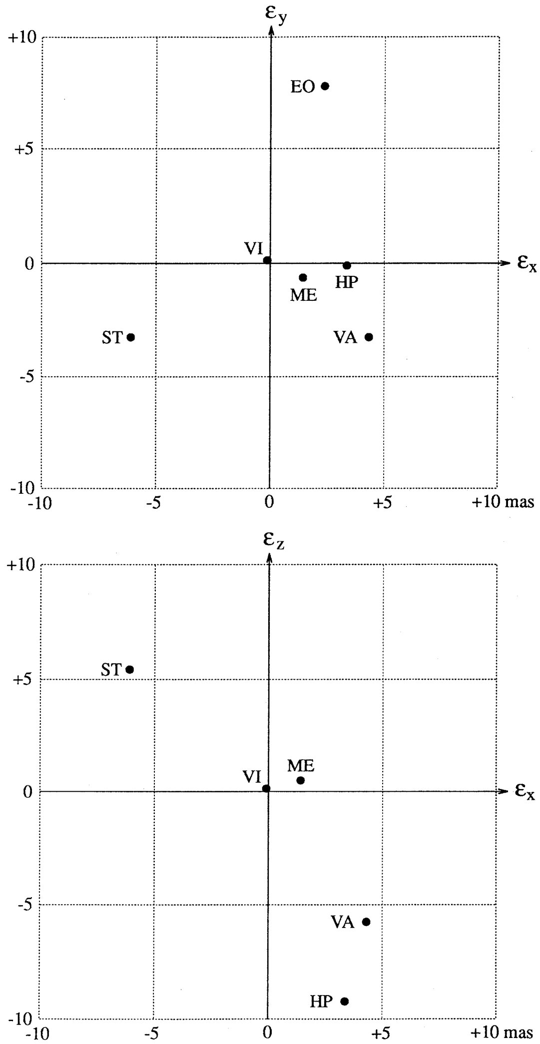

Fig. 4. Projections onto the xy and xz planes of the individual solutions for the orientation vector ε (residuals with respect to the adopted vector at the mean epoch of each solution). The components are expressed in mas. The solutions are labelled as in Table 3

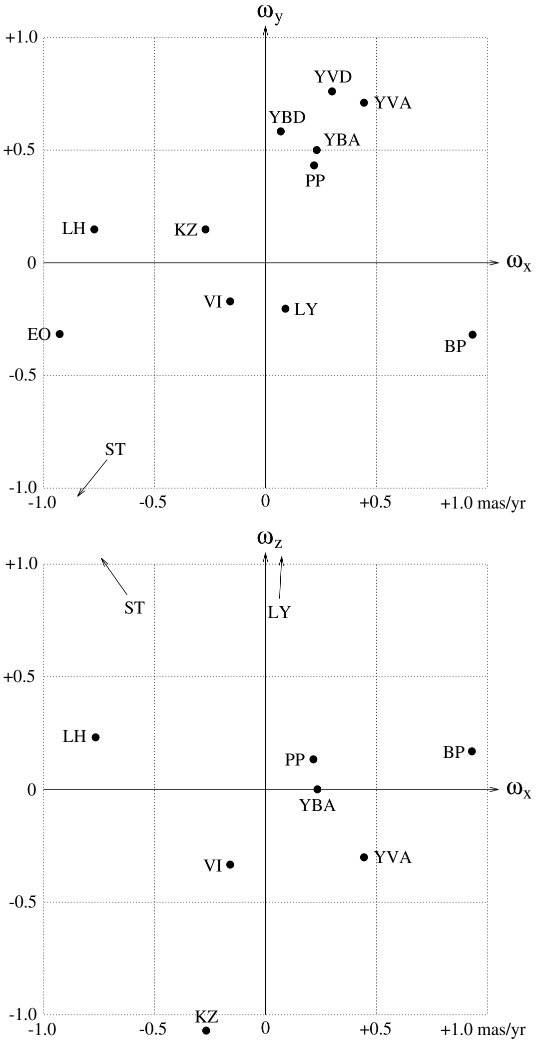

Fig. 5. Projections onto the xy and xz planes of the individual solutions for the spin vector ω (actually residuals with respect to the adopted vector). The components are expressed in mas/yr. The solutions are labelled as in Table 3

5. General synthesis

It was decided that the two first authors would proceed independently to synthesise the observations by different methods (A, B) and then to compare the results in order to make and justify the final choice.

5.1. Individual link data

The results obtained by various individual solutions all included the values of the components of one or both of the vectors ε_0 and ω, together with their estimated standard errors and the associated correlation matrix. These data (except the correlations) are summarised in Table 3 in the form of the residuals with respect to the finally adopted solution.

In the synthesis it was found that the individual solutions are quite incompatible if the given standard errors were taken at face value. Consequently it was necessary to reduce the weights of at least some solutions. Different strategies were tried for this down-weighting, including a semi-automatic procedure described in Sect. 5.2 (Method A). For Method B (Sect. 5.3) the down-weighting was essentially based upon the considerations given in Sect. 4, moderated by the examination of how the modifications of the weights affected the goodness-of-fit of the synthesised solution and the individual residuals. So, while the weights deduced from the rms provided by the authors represent essentially formal errors, down weighting takes, at least roughly, account of external errors.

5.2. Method A (L. Lindegren)

This method is described in detail in Lindegren & Kovalevsky (1995) and was strictly followed. As shown in that paper, it is possible to cast the results of each individual link solution (j) in the form of an information array $[\ \boldsymbol{N}_j\ \boldsymbol{h}_j\ ]$ representing the normal equations $\boldsymbol{N}_j \boldsymbol{s} = \boldsymbol{h}_j$ for the six-dimensional state vector $\boldsymbol{s} = [\ \varepsilon_{0x}\ \varepsilon_{0y}\ \varepsilon_{0y}\ \omega_x\ \omega_y\ \omega_y\ ]'$. The full set of correlations among the six parameters were taken into account; in particular the correlations between the orientation and spin components are important for a uniform treatment of the different mean observation epochs shown in Table 3.

The Heidelberg and Yale analyses of the NPM1 do not represent independent determinations. They were therefore averaged prior to the synthesis, and the more pessimistic standard errors from the Heidelberg analysis were adopted for this average. Similarly the blue and visual SPM solutions were averaged, but in this case giving more weight to the blue solutions. Thus, effectively 12 different solutions were considered in the synthesis. The main problem was then to assign a weight $w_j \leq 1$ to each of the solutions (corresponding to an increase of its formal errors by the factor $w_j^{-1/2}$); this process is described below. After that, the synthesised solution $\boldsymbol{s}$, including an estimate of its covariance, could be obtained by solving the normal equations resulting from the sum of the 12 weighted information arrays.

The determination of the weights w_j was based on a comparison of the goodness-of-fit statistics of the individual solutions, $q_j = \boldsymbol{d}_j' \boldsymbol{D}_j^+ \boldsymbol{d}_j$. Here, $\boldsymbol{d}_j = w_j \boldsymbol{h}_j - w_j \boldsymbol{N}_j \boldsymbol{s}$ is the weighted residual right-hand-side of the normal equations for solution j and $\boldsymbol{D}_j$ its covariance; see Eq. (44) in Lindegren & Kovalevsky (1995). In the absence of biases and if all the weights are correctly assigned, the expectation of this quantity is $r_j = \text{rank}(\boldsymbol{D}_j)$, which varies between 2 and 6 for the different link techniques. Note that the generalised inverse $\boldsymbol{D}_j^+$ was used in the expression for q_j in order to handle the techniques where less than all six rotation parameters were determined. The total goodness-of-fit $Q = \Sigma_j\, q_j$ was also computed; its expectation in the ideal situation is $R = \Sigma_j\, r_j = 44$.

Starting from some *a priori* set of weights, the statistics q_j and Q were computed. Typically this gave a much too high value of Q due to unrealistically small standard errors in some of the individual solutions. The most discrepant solution was identified by comparing the normalised statistics q_j/r_j, the weight of that solution was halved, and a new synthesised solution was computed with revised q_j and Q. This process was iterated until $Q \simeq R$ and all $q_j \simeq r_j$.

It is not obvious that this semi-automatic process of down-weighting converges to a unique result. Indeed, slightly different weights were obtained depending on whether the formal standard errors were taken as the starting point (i.e. $w_j = 1$ for all j initially) or some *a priori* judgement of the relative weights were first applied. However, the synthesised solutions resulting from such experiments rarely differed by more than 0.1 mas and 0.1 mas/yr, and the final result was not very sensitive to additional changes in the weights. Independent of the starting point, it was found that the solutions from the SPM programme and the Earth orientation parameters had to be severely down-weighted, and the Bonn and HST solutions slightly down-weighted, but otherwise the given standard errors were roughly consistent with the overall solution.

The results of Method A are summarised by the following rotation parameters (with standard errors in parentheses) and correlation matrix, referred to the epoch J1991.25:

$$\left.\begin{array}{l} \varepsilon_{0x} = +0.01\ (0.46)\ \text{mas} \\ \varepsilon_{0y} = -0.20\ (0.47)\ \text{mas} \\ \varepsilon_{0z} = +0.12\ (0.49)\ \text{mas} \\ \omega_x = +0.06\ (0.16)\ \text{mas/yr} \\ \omega_y = -0.05\ (0.15)\ \text{mas/yr} \\ \omega_z = +0.00\ (0.14)\ \text{mas/yr} \end{array}\right\} , \tag{2}$$

$$\boldsymbol{R} = \begin{pmatrix} 1 & +0.25 & +0.05 & -0.04 & -0.01 & +0.00 \\ +0.25 & 1 & -0.13 & +0.01 & -0.11 & +0.01 \\ +0.05 & -0.13 & 1 & +0.01 & +0.01 & -0.06 \\ -0.04 & +0.01 & +0.01 & 1 & +0.05 & -0.16 \\ -0.01 & -0.11 & +0.01 & +0.05 & 1 & -0.16 \\ +0.00 & +0.01 & -0.06 & -0.16 & -0.16 & 1 \end{pmatrix} . \tag{3}$$

The method allows to compute the relative contributions of the various link techniques to the synthesised solution. It turns out that for the orientation parameters, the VLBI observations dominate strongly. For the determination of the spin the contributions are more evenly spread among the several techniques, but with the SPM programme and the VLBI observations together contributing about half of the total weight. It should be noted that in this method the MERLIN, VLA, Hamburg/USNO and HST/FGS links contribute to the determination of the spin components in Eq. (2) by virtue of their spread in observational epochs, rather than through their individual determinations of the spin.

5.3. Method B (J. Kovalevsky)

This method is based upon the fundamental assumption that the errors obtained by every task are gaussian. This means that the probability density function (PDF) is given in its most general form for n variables by

$$f(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^{n/2} |\boldsymbol{V}|^{1/2}} \times \\ \times \exp\left[-\frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{n} \left[\boldsymbol{V}^{-1}\right]_{ik} (x_i - \overline{x}_i)(x_k - \overline{x}_k) \right] . \tag{4}$$

where $\boldsymbol{V}$ is the variance-covariance matrix of the variables and $\overline{x}_i$ their mean values.

This PDF can be computed from the data provided by each individual link task (j), namely the estimated variables with their standard errors and the correlation matrix. Now, the joint PDF of J gaussian distributions is the product of the PDFs of these distributions,

$$\varphi = \prod_{j=1}^{J} f_j(x_1, x_2, \ldots, x_n) , \tag{5}$$

which is easily computed, and has exactly the same form as Eq. (4) since the quantities in the exponential add. One can therefore compute back the variance-covariance matrix corresponding to φ and derive the standard errors and the correlations of the merged solution. This approach explicitly assumes gaussian error distributions and it would not be correct to apply it to other distributions. However, in the particular case to which it is applied, no indication of a non-gaussian behaviour was given by the individual link solutions which all made the same assumption.

Although formulated in a probabilistic framework, this method is in principle equivalent to a weighted least-squares and should yield similar results as Method A. However, the practical implementations differ, and Method B was also applied separately to the spin and orientation components, which may add some insight into the properties of the individual solutions. As in Method A, a main problem is to adjust the relative weights of the contributing solutions.

To begin with, the four solutions obtained from the Yale SPM programme were reduced into a single one by taking a weighted mean. The MERLIN and HST results for $\boldsymbol{\omega}$ were not used because of their large uncertainties.

In a first approximation, an unweighted solution for $\boldsymbol{\varepsilon}_0$ and another solution for $\boldsymbol{\omega}$ were computed neglecting the correlations between these quantities. This is justified because they are close to zero in the case of the most accurate method (VLBI), but less justified for the EOP method in which the correlations are about 0.89, but in this case the uncertainties are also much larger.

In further iterations weights were modified progressively in order to reduce the largest residuals and the overall goodness-of-fit, as measured by the χ^2 statistic. No systematic procedure was used to modify the weights, but rather a successive approximation technique with steps of 0.2 in the weights.

Essentially, the Lick results were strongly down-weighted as actually it was suggested already by the authors, the formal errors being abnormally small. Both HST and Earth Rotation were also strongly down-weighted, and Bonn results also somewhat down-weighted while both Potsdam and Kiev results had their weight slightly increased. Globally, this confirms the weights obtained in method A.

Then, a global solution taking as unknowns all the six parameters of $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\omega}$ and starting with the weights obtained in the preceding solutions. This did not change significantly the results, as can be seen from the following summary of the different solutions.

B1. Solution using only $\boldsymbol{\varepsilon}_0$ (referred to J1991.25) as the unknown: the weighted rms residual was 0.8 mas. The solution vector (with standard errors in parentheses) and correlation matrix were

$$\left.\begin{array}{l} \varepsilon_{0x} = 0.00\ (0.51)\ \mathrm{mas} \\ \varepsilon_{0y} = -0.04\ (0.51)\ \mathrm{mas} \\ \varepsilon_{0z} = +0.16\ (0.53)\ \mathrm{mas} \end{array}\right\} , \tag{6}$$

$$\boldsymbol{R} = \begin{pmatrix} 1 & +0.28 & -0.01 \\ +0.28 & 1 & -0.14 \\ -0.01 & -0.14 & 1 \end{pmatrix} . \tag{7}$$

B2. Solution using only $\boldsymbol{\omega}$ as the unknown: the weighted rms residual was 0.3 mas/yr. The solution vector and correlation matrix were

$$\left.\begin{array}{l} \omega_x = -0.01\ (0.13)\ \mathrm{mas/yr} \\ \omega_y = +0.08\ (0.13)\ \mathrm{mas/yr} \\ \omega_z = -0.05\ (0.18)\ \mathrm{mas/yr} \end{array}\right\} , \tag{8}$$

$$\boldsymbol{R} = \begin{pmatrix} 1 & +0.04 & -0.11 \\ +0.04 & 1 & -0.14 \\ -0.11 & -0.14 & 1 \end{pmatrix} . \tag{9}$$

B3. Solution using both $\boldsymbol{\varepsilon}_0$ (referred to J1991.25) and $\boldsymbol{\omega}$ as unknowns: the weighted rms residual was 1.2 mas for the orientation components and 0.4 mas/yr for the spin components. The solution vectors and correlation matrix were

$$\left.\begin{array}{l} \varepsilon_{0x} = +0.16\ (0.41)\ \mathrm{mas} \\ \varepsilon_{0y} = +0.18\ (0.43)\ \mathrm{mas} \\ \varepsilon_{0z} = -0.06\ (0.46)\ \mathrm{mas} \\ \omega_x = -0.06\ (0.08)\ \mathrm{mas/yr} \\ \omega_y = +0.05\ (0.09)\ \mathrm{mas/yr} \\ \omega_z = 0.00\ (0.14)\ \mathrm{mas/yr} \end{array}\right\} , \tag{10}$$

$$\boldsymbol{R} = \begin{pmatrix} 1 & +0.05 & +0.00 & +0.24 & +0.08 & -0.01 \\ +0.05 & 1 & -0.13 & +0.06 & +0.10 & -0.01 \\ +0.00 & -0.13 & 1 & -0.02 & -0.02 & +0.00 \\ +0.24 & +0.06 & -0.02 & 1 & -0.13 & -0.05 \\ +0.08 & +0.10 & -0.02 & -0.13 & 1 & -0.12 \\ -0.01 & -0.01 & +0.00 & -0.05 & -0.12 & 1 \end{pmatrix} . \tag{11}$$

The correlations obtained in solutions B1 and B2 agree rather well with those obtained from Method A, while the correlations in B3 deviate somewhat. However, the correlations are in all cases small or only moderately large, showing that the combination of link solutions gives a well-conditioned determination of all the parameters at the central epoch of the Hipparcos Catalogue.

6. Final results

After a comparison of these results and their discussion, it appeared that a mean value of the two methods should be considered as the final solution for the link. The adopted orientation and rotation vectors correspond to all parameters equal to zero, which is close to a mean value of A and B3. The standard errors of the parameters were estimated to be 0.6 mas in each of the components of $\boldsymbol{\varepsilon}_0$, and 0.25 mas/yr in the components of $\boldsymbol{\omega}$. These numbers were obtained by a conservative rounding of the formal errors resulting from the synthesis, taking into account also the spread of values obtained in the different synthesis solutions and the uncertainty in the relative weights of the different link techniques. It is to be noted that this link will become worse as time goes on. It is important that, in the future, new ground-based observations be organised to maintain and possibly improve the quality of the link.

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