Astron. Astrophys. 323, 801–808 (1997)

ASTRONOMY
AND
ASTROPHYSICS

Testing the stability and reliability of starspot modelling

Zs. Kővári[1] **and J. Bartus**[1,2]

[1] Konkoly Observatory of the Hungarian Academy of Sciences, P.O. Box 67, 1525 Budapest, Hungary
(kovari@buda.konkoly.hu,bartus@buda.konkoly.hu)
[2] Institut für Astronomie, Universität Wien, Türkenschanzstraße 17, A-1180 Wien, Austria (bartus@astro.ast.univie.ac.at)

Received 8 November 1995 / Accepted 15 January 1997

Abstract. Since the mid 70's different starspot modelling techniques have been used to describe the observed spot variability on active stars. Spot positions and temperatures are calculated by application of surface integration techniques or solution of analytic equations on observed photometric data. Artificial spotted light curves were generated, by use of the analytic expressions of Budding (1977), to test how the different constraints like the intrinsic scatter of the observed data or the angle of inclination affects the spot solutions. Counteractions between the different parameters like inclination, latitude and spot size were also investigated. The results of re-modelling the generated data were scrutinized statistically. It was found, that (1) $0\overset{\mathrm{m}}{.}002 - 0\overset{\mathrm{m}}{.}005$ of photometric accuracy is required to recover geometrical spot parameters within an acceptable error box; (2) even a $0\overset{\mathrm{m}}{.}03 - 0\overset{\mathrm{m}}{.}05$ error in unspotted brightness substantially affects the recovery of the original spot distribution; (3) especially at low inclination, under- or overestimation of inclination by 10° leads to an important systematic error in spot latitude and size; (4) when the angle of inclination $i \lesssim 20°$ photometric spot modelling is unable to provide satisfactory information on spot location and size.

Key words: stars: activity; imaging; late-type; starspots – methods: data analysis

1. Introduction

Stability of modelling photometric data of spotted stars has been a real problem since the advent of these kinds of techniques. Although different authors of different modelling procedures have always tried to estimate the reliability of their results, no comprehensive analysis of the stability of photometric spot modelling has been published to date (i.e. the effect of the scatter of the data, or other constraints originating from the geometry). However, studies concerning modelling procedure of different tasks have already been carried out by e.g. Vogt (1981), Strassmeier (1988), Rhodes et. al (1990), Eker (1995), Kővári (1995). Efforts have been made to combine photometric and spectroscopic data to make the results more accurate by use of entirely different techniques to solve the same problem (for interplay with Doppler imaging see e.g. Strassmeier 1991). Unfortunately, this combined effort is seldom possible, mostly for observational reasons. However, it is important to note, that a huge photometric dataset of spotted variables exists dating back sometimes for 30 years, and with APTs, it grows day by day. These photometric datasets could only be analysed by spot modelling. Thus, systematic investigation of the stability of the modelling procedure and consequently the reliability of the results is a timely problem.

To accomplish the task of testing the reliability and stability of spot modelling, synthetic light curves were generated for studying different aspects detailed in the following sections. These artificial light curves were completed using known input parameters appropriately chosen for the test in question. When re-modelling these datasets, the generating input parameters were assumed unknown, or uncertainties were planted in them and the data were handled as real. Finally, the resulting spot parameters were investigated and compared with the original ones. All of the calculations carried out for the tests in this paper were based on Budding's (1977) analytical equations.

2. Validity of the accepted two-spot scenario

Before generating spotted light curves, some preconditions should be fixed. Though the assumptions accepted in the following sections draw a highly idealized picture, this could represent a satisfactory frame for testing the modelling process of spotted stars.

The analytical equations developed by Budding (1977) assume dark circular spots on the surface of the rotating star. In most cases a one or two-spot model can follow the rotational variability considerably well. Thus, to get a general overlook on the stability and reliability of the modelling process only the case of two-circular-spot solutions was investigated.

The validity of the two-spot simplification is demonstrated by the following test. Twenty light curves were generated putting ten circular spots randomly distributed on the surface of a 'program star'. The radii of the spots varied between 10° and

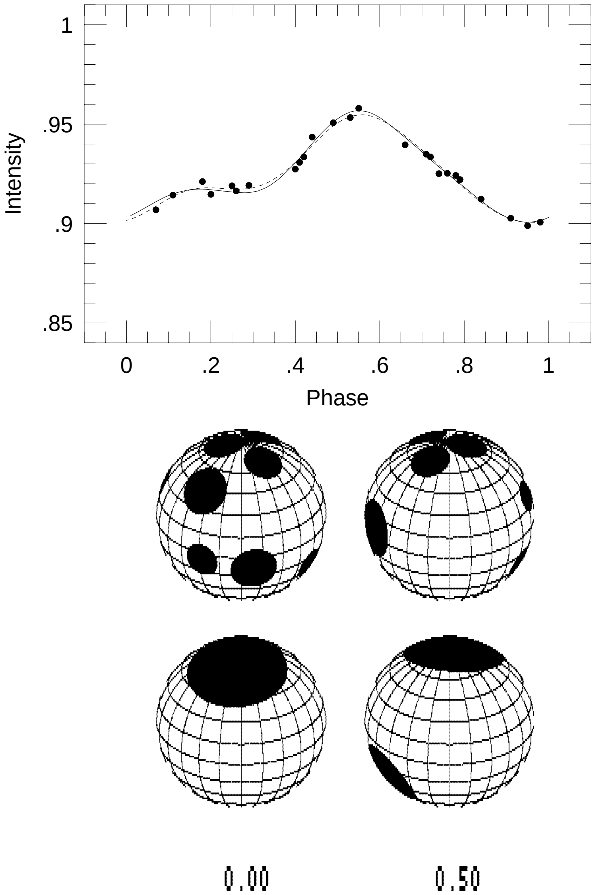

Fig. 1. An example of fitting a ten-spot curve with two spots. Upper panel shows the artificial light curve of ten random spots (solid line), dots are the scattered 'observations', while dashed line shows our two-spot model. Below the ten random spots and the replacing two-spot model can be seen in two phase values at 0.00 and 0.50. The fit to the 'observations' with our two-spot model hardly deviates from the original curve

20°, and no overlap was allowed. An arbitrarily chosen inclination angle of 60° was used. Finally, in each case, 25 data points were taken randomly from the original light curves adding some scatter (less than ±0.5%). In Fig. 1 we show, through an example, how the 'observations' of ten-spot configurations can be fitted by two spots.

As demonstrated, photometric observation (i.e. *noisy* data) of a star with ten random spots on its surface is too poor to provide sufficient information to recover the given ten-circular-spot configuration. In addition, the assumption of more than two spots, increases the number of free parameters, so that the reliability of the result becomes obviously worse, and its uniqueness, if it existed, vanishes altogether.

The fact, that a two-spot model provides good fit to the observations, does not mean that the model configuration reveals to the true one. Moreover, the interpretations of different technique observations may contradict each other. However, as the aim of this paper is not to answer the problem of the starspot structure, this simplified spot configuration is accepted, as a working assumption.

Table 1. Spot configuration used for the tests

λ_1	β_1	γ_1	λ_2	β_2	γ_2
100	20	20	220	70	30

3. Generation of spotted light curves

One of the basic questions is the physical condition of the 'program star' chosen. To match reality, a typical active cool star was taken at T_{eff} = $4500K$ with a probable spot temperature value of ΔT_{spot} = $500K$. In all, three different groups of input parameters were used to generate noisy data:

1. physical parameters: effective temperature, flux ratio between the spot and the photosphere and limb darkening;
2. geometry of the star and the spot; i.e. i and λ_j, β_j, γ_j, (inclination, and astrographic longitude, latitude and radius of the jth spot, respectively, in degrees);
3. noise of the photometric data; that means a maximum ±% value, which limits the random error of the 'observations'.

It is generally believed, that appearence and disappearence of spots at the stellar pole can cause the long term variability of the mean light level. On the other hand, rotational modulation can mostly be represented by spots placed outside the circumpolar zone. Investigating active regions on spotted stars, many authors report low-latitude spots beside polar spottedness (see e.g. Strassmeier 1990, Saar et al. 1994, Kürster et al. 1992, 1994, Vogt & Hatzes 1996, Schüssler 1996, etc.). Thus, the spot distribution used in our tests has one spot placed near the equator, and another placed to the polar zone (see Table 1). This two-spot configuration was applied to construct suitable data for different purposes. For a given input parameter set of the rotating spotted star, Budding's method (1977) was used to calculate light intensity variation. Then, from these 'mother-light curves' (see two examples in Fig. 2 at two different inclination values) 25 data points were selected randomly and with a limit to the noise with a maximum photometric error, a random scatter was put on to the chosen points. Altogether nearly five thousand artificial light curves were generated, with systematically varied input parameters listed above, to get usable data for the tests detailed in the following sections. In this way 50 light curves of the same geometry and the same limit of data noise were made for each case. Only the selected datapoints were different. Thus, these datasets of 50 curves each imitate 50 photometric observations carried out *simultaneously* and *independently* for a 'realistic' spotted star. Therefore, results from re-modelling them should be suitable for statistics.

4. Re-modelling the spotted light curves

When modelling the artificial light curves, only the spot coordinates and sizes were taken as free parameters of the Levenberg-Marquardt method (LMM) used here to minimize the value of χ^2 of data fitting.

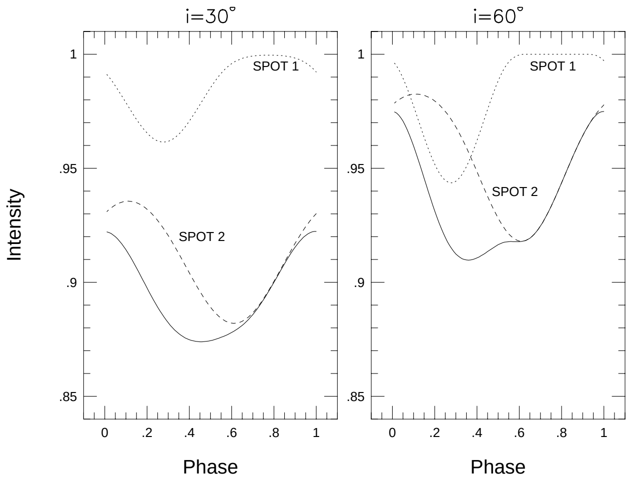

Fig. 2. 'Mother-light curves' using two different inclination values at 30° and 60°. Dotted line shows the light loss due to the low latitude spot (SPOT 1), dashed line to the polar spot (SPOT 2). Solid line is the sum of the separated light losses.

The dependence on the initial conditions is a non-negligible question of the minimization technique actually applied. In most cases (i.e., when the solution space is far from convex shape) a good initial guess of the spot parameters is an important preliminary condition to find the original configuration among the different solutions. On the other hand, our former experiences showed that, in individual cases of the two-spot light curves of idealistic conditions (no data noise, satisfactory phase coverage), a maximum of three-four guesses of the initial spot parameters had always led us to the true solution, which generally had *significantly* better fits than the others. (The result was the same even when using grid search minimization technique, which seemed more sensitive to the initial values than the LMM, see Kővári 1995). Therefore, during the tests we start from a quasi-optimal state when a good initial guess of the parameters is already known. The spot parameters are randomly chosen from a ±10° size box around the original values, so the effect of rough initial coordinates is more or less excluded from the results in the following sections.

Before starting the search of geometrical spot parameters, random errors were implanted in the physical parameters (see Section 3) and in the value of spot temperature. These errors (limited as $\delta_{\rm phys.} = \pm 5\%$ and $\delta_{\Delta T_{\rm spot}} = \pm 100K$, respectively) try to imitate our ignorance originating from deviations of different model atmospheres (e.g. models of Buser & Kurucz 1992, or a blackbody assumption) and uncertainty resulting from spot temperature determination. As a substantial source of error, the uncertainty of unspotted brightness is tested in Section 5.2.

As a result of the searching process, six spot parameters ($\lambda_j, \beta_j, \gamma_j, j = 1, 2$) were obtained for a given light curve. 50 *independent* iterations supplied resulting spot coordinates beside given input parameters (i.e. fixed scatter limit and i value). Moreover, additional uncertainties of the iterations arose from the interplay of the included errors in the physical parameters with each other and with the data noise. The solutions are displayed and discussed.

5. Results

5.1. Counteractions between photometric data noise and resulting spot solutions

In Fig. 3 we show our results obtained by re-modelling the synthetic light curve sets generated using $i = 30°$. Panels from top to bottom show the different resulting spot parameters as the results of 50 independent models for a given photometric noise value on the horizontal axis. Original λ_j, β_j and γ_j; $(j = 1, 2)$ values are also displayed for comparison (shown as horizontal solid lines). From Fig. 3 information can be procured immediately on how the stability of modelling gets worse with decreasing photometric accuracy. One can easily notice the difference between the two columns: if the data noise, $\delta_{\rm phot.} \lesssim \pm 1.5\%$ at low latitude (SPOT 1) the spot longitude can be recovered better than for a spot near the pole (SPOT 2). However, the lower the original value of the spot latitude, the worse the stability of the resulting spot latitude and, of course, the strongly correlated spot radius (as was indicated by the linear correlation coefficients of LMM). When the noise grows over $\pm 1\%$, and even more, over $\pm 2\%$, results could become absolutely unreliable (see the dashes lying far away from the 'body' of the results).

If we look at Fig. 2, it is well seen, that at low inclination (left panel), the humps of the light curve become shallow, and the effects of the individual spots are blended. In the case of high inclination (right panel), however, the two spots on the light curve are distinguishable. Thus, separation of the light loss due to the two spots (i.e. spot parameter search) is more straightforward.

Using $i = 60°$, in the sense of the latitude values of the two spots, the same behaviour can be detected as before; see Fig. 4. However, if the photometric noise is small enough ($\lesssim \pm 0.5\%$, in the order of $0\overset{\rm m}{.}002 - 0\overset{\rm m}{.}005$), the reliability of the resulting spot parameters is generally more acceptable than in the previous ($i = 30°$) case. Close to zero photometric noise, the solutions fall into a very small error box of $2-3°$ in size. Even if the data noise increases, longitude determination remains more stable. However the other parameters become unstable. One can notice the appearance of a wide range of possible spot parameters found (especially the latitude and size of SPOT 1) and quite a few alternative 'ghost solutions' at noise values over $\pm 2\%$. Again, this picture argues against the reliability of the model obtained from using inaccurate data.

5.2. The uncertainty of unspotted brightness

In the calculations presented above a possible source of error, the misestimation of the unspotted brightness, was neglected (the reference level was set to unity). To test this source we first used our previous synthetic data with *zero* photometric noise. The light curves were multiplied by a random factor of $1 + x$, where x is a limitation of the random shift of the light curve relative to the reference level at unit value (i.e. the unspotted

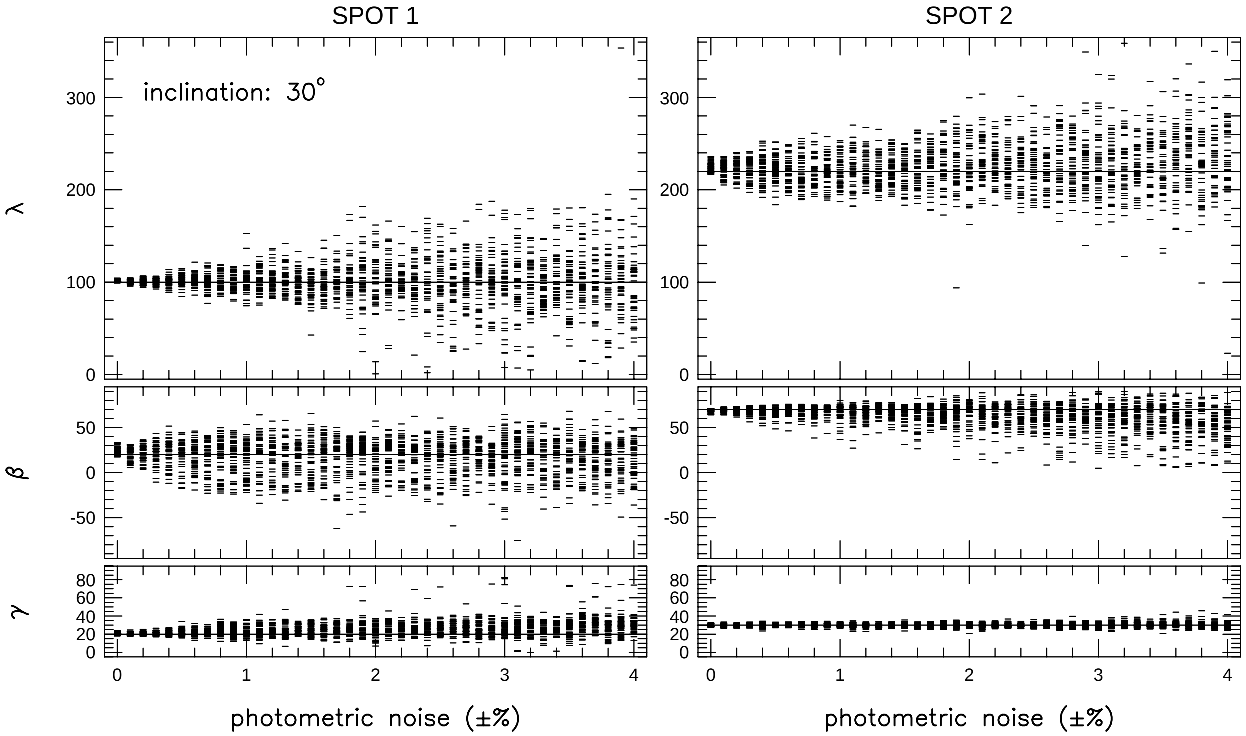

Fig. 3. Re-modelling of the artificial datasets generated using 30° inclination. 50 model parameter sets ($\lambda_j, \beta_j, \gamma_j; j = 1, 2$) were obtained for each noise value (shown as horizontal dashes). The first column corresponds to the low latitude spot (SPOT 1), and the second to the high latitude one (SPOT 2). Original spot parameters are marked by horizontal lines.

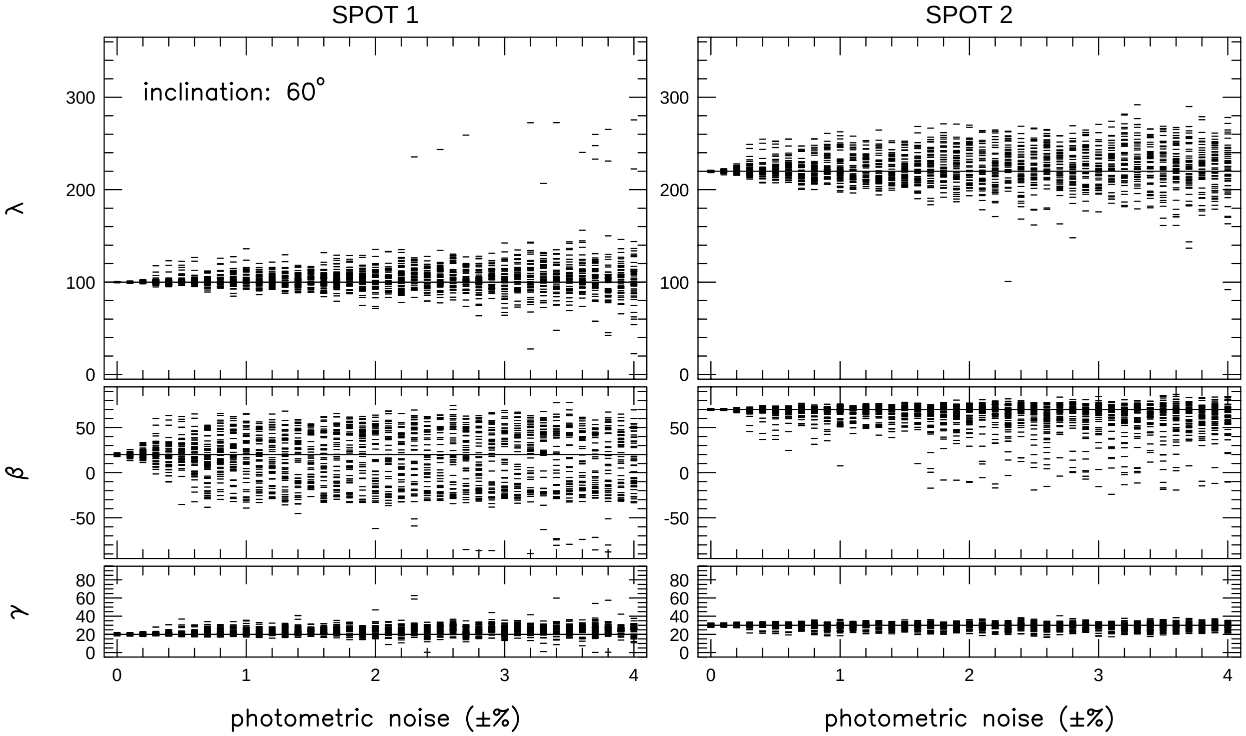

Fig. 4. Re-modelling of the artificial datasets generated using 60° inclination (symbols as in Fig. 3)

Table 2. Comparing the calculated and the iterated (average) values of β_j using over- and underestimated inclinations (original values are in bold face)

$\boldsymbol{i}$	$i_{\rm estim.}$	$\boldsymbol{\beta_1}$	$\beta_1^{\rm calc.}$	$\beta_1^{\rm iter.}$	$\boldsymbol{\beta_2}$	$\beta_2^{\rm calc.}$	$\beta_2^{\rm iter.}$
30°	20°	**20°**	13°	19°	**70°**	60°	49°
	40°		28°	36°		76°	73°
60°	50°	**20°**	14°	13°	**70°**	62°	64°
	70°		30°	33°		77°	72°

brightness of the star). The value of x varied from 0% to ±10% ($\approx \pm0\overset{\rm m}{.}1$). After that, the spot coordinates were iterated and displayed in Figs. 5-6, using two inclination angles (i = 30° and 60°, respectively). As shown in Fig. 5, the smaller inclination leads to more or less constant error, independent of the shifting limit, x. A large error in the iterated spot parameters appears at i = 60° (Fig. 6) when the uncertainty of the unspotted brightness $x \gtrsim \pm3-5\%$ ($\gtrsim \pm0\overset{\rm m}{.}03 - 0\overset{\rm m}{.}05$).

Figs. 7-8 reflect a more realistic case. The photometric noise is varying from 0% to ±4% while x is fixed at ±5%. The uncertainty of the unspotted brightness makes the error of the recovered parameters considerably larger, as can be seen by comparison of the corresponding panels of Figs. 3-4 (where x was set to zero). Even a $0\overset{\rm m}{.}05$ error in the unspotted brigtness can be the most important error source in recovering the original spot parameters. The inclination slightly affects the results as described in connection with Figs. 5-6.

5.3. *The effect of bad estimation of the inclination*

During this test we took the same datasets as above, but when researching the geometrical spot parameters, a ±10° systematic deviation was assumed in the value of inclination. Therefore, the re-modelling process was carried out using fixed bad inclinations: 20° and 40° instead of $i_{\rm orig.}$ = 30°; and 50° and 70° instead of 60°. We summarize the results as follows:

a) Case of $i_{\rm orig.}$ = 30°.
Assuming 20° instead of $i_{\rm orig}$ = 30°, serious systematic deviation of β and γ can be found. Spot latitudes tend to be lower, while the corresponding size of SPOT 1 increases. Similar, but less and contrary divergent parameters can be obtained with an overestimated inclination value of i = 40°. Because of the low inclination, the wide range of the results obtained points towards a big uncertainty of the modelling process.

b) Case of $i_{\rm orig.}$ = 60°.
It is shown by the simulations, when increasing i to 60°, that the effect of under- or overestimation of inclination by 10° leads to smaller systematic errors than in the low inclination case (deviations of the average parameter values decrease under about $\pm5-10°$ in either β or γ).

Nevertheless, over ±2% of data noise, the error of the determination of spot parameters due to badly chosen inclination merges into the instability caused by data noise.

By means of the known formula for the angle Ψ between the line of sight and the normal vector drawn into the centre of the spot, an approximate value of expected deviation between the original ($\beta_{orig.}$) and the iterated latitude can be deduced in advance. Let us start from the term cos Ψ, which is the function of i (inclination), β (spot latitude) and of the phase ϕ:

$$\cos\Psi(i,\beta,\phi) = \sin\beta\cos i + \cos\beta\sin i\cos\phi. \tag{1}$$

For a rough estimate we can neglect the limb darkening, and the spherical shape of the spot (take a small one with fixed size!). Restricting ourselves to small changes in i and β eq. (1) leads to the following convenient term:

$$\frac{\tan i}{\tan\beta} \approx \text{const.} \tag{2}$$

Table 2 compares the results of using this approximation with the average latitude values obtained by the simulations. Differences between estimate and iteration can be detected, which are possibly due to the simplifications applied above (mostly in spot size).

5.4. *The effect of inclination on the stability of the modelling process*

Although some experiences of the dependence of process stability on inclination could be deduced from the tests presented above, for this purpose a more suitable test was carried out, too. An additional set of light curves were made, based on the following considerations: the angle of inclination was changed from 5° to 85° by 5° steps. For each step 50 light curves were generated, supposing a fixed ±0.5% photometric error (this case the unspotted level is at unit value again).

In Fig. 9 resulting spot parameters versus inclination are displayed. When $i \lesssim 20°$ it seems to be impossible to get useful information about the position of a starspot. Longitude determination shows similar inaccuracy for both spots, mainly at low inclinations, while latitude determination of SPOT 1 remains rather inaccurate ($\delta_\beta \approx \pm30-40°$) independent of i. Contrary to the high latitude spot, parameters of the spot placed close to the stellar equator cannot be determined at a satisfacting level of accuracy, though the photometric noise, $\delta_{\rm phot.} \approx \pm0\overset{\rm m}{.}005$.

6. Conclusions

Through a simplified two-spot model, the aim of the present paper was to investigate the effect of photometric accuracy on determining spot parameters, to see how the uncertainty of the unspotted brightness effects the recovery of the spot parameters, to test the effect of bad estimation of the inclination and to test the stability of modelling spot parameters versus varying inclination. Summarizing the results, it was found that:

1. Although the two-spot solution tested here is a necessary simplification of starspot modelling, as is apparent from simulations, for general realistic conditions, phased light curves do not contain sufficient information to recover the original spot distribution.

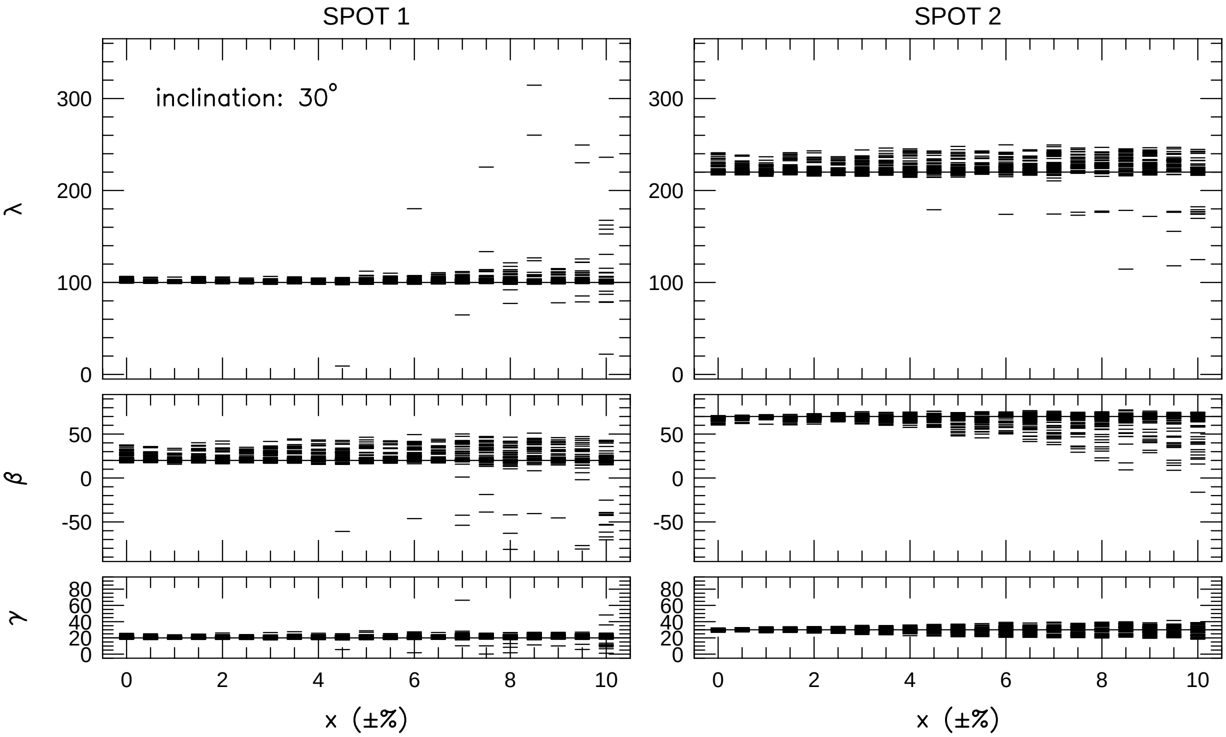

Fig. 5. The recovered spot parameters versus x, the uncertainty of unspotted brightness using i = 30°. In this case the photometric noise was set to zero.

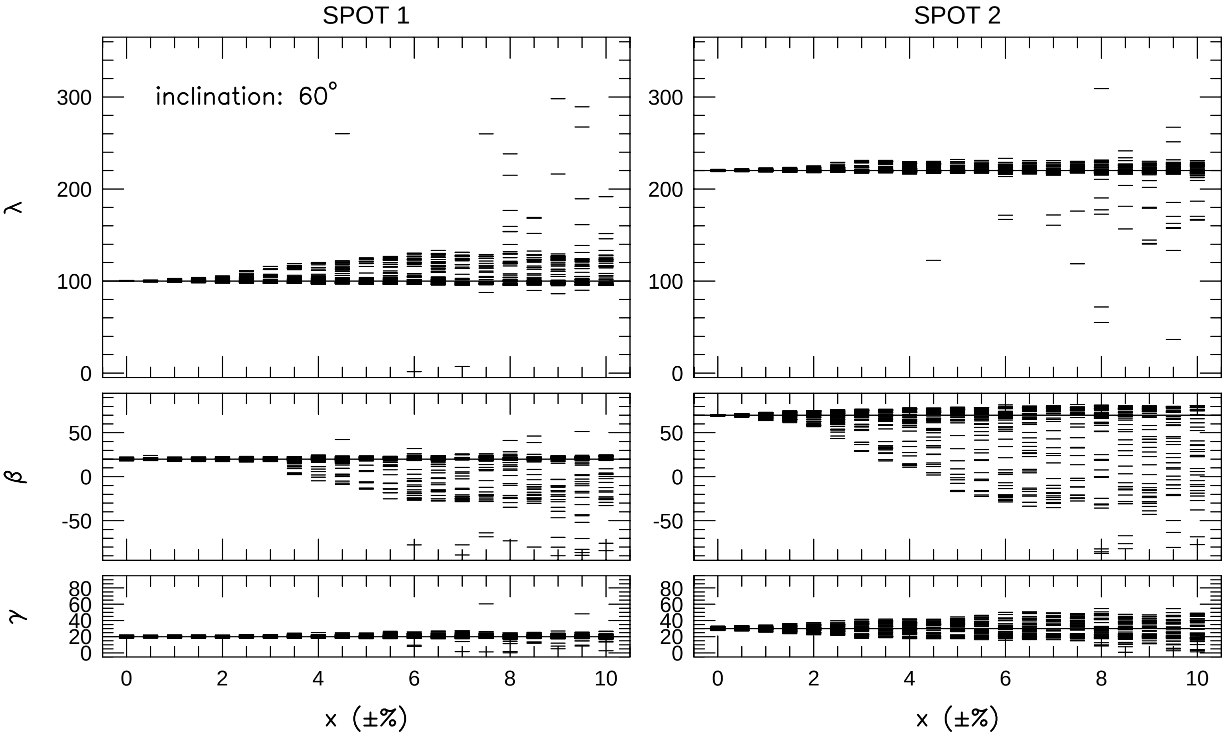

Fig. 6. As in Fig. 5 with i = 60°

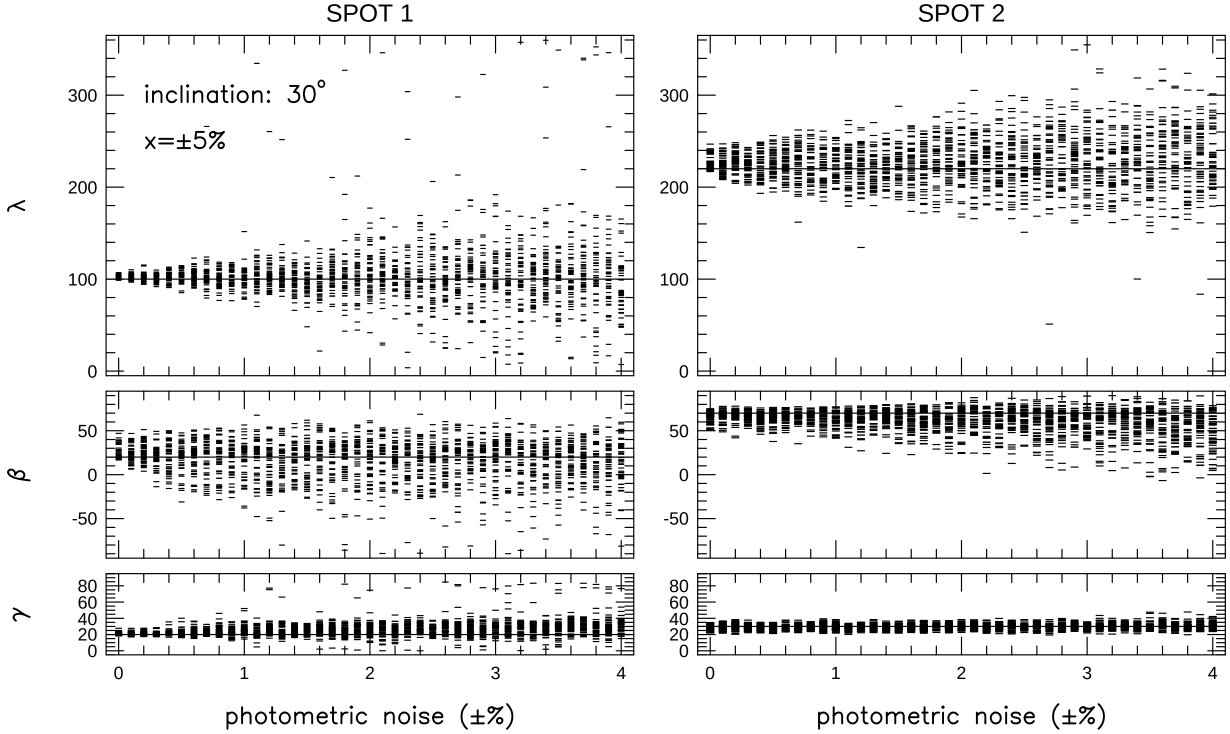

Fig. 7. As in Fig. 3 with the error box of the unspotted brightness x fixed at ±5%.

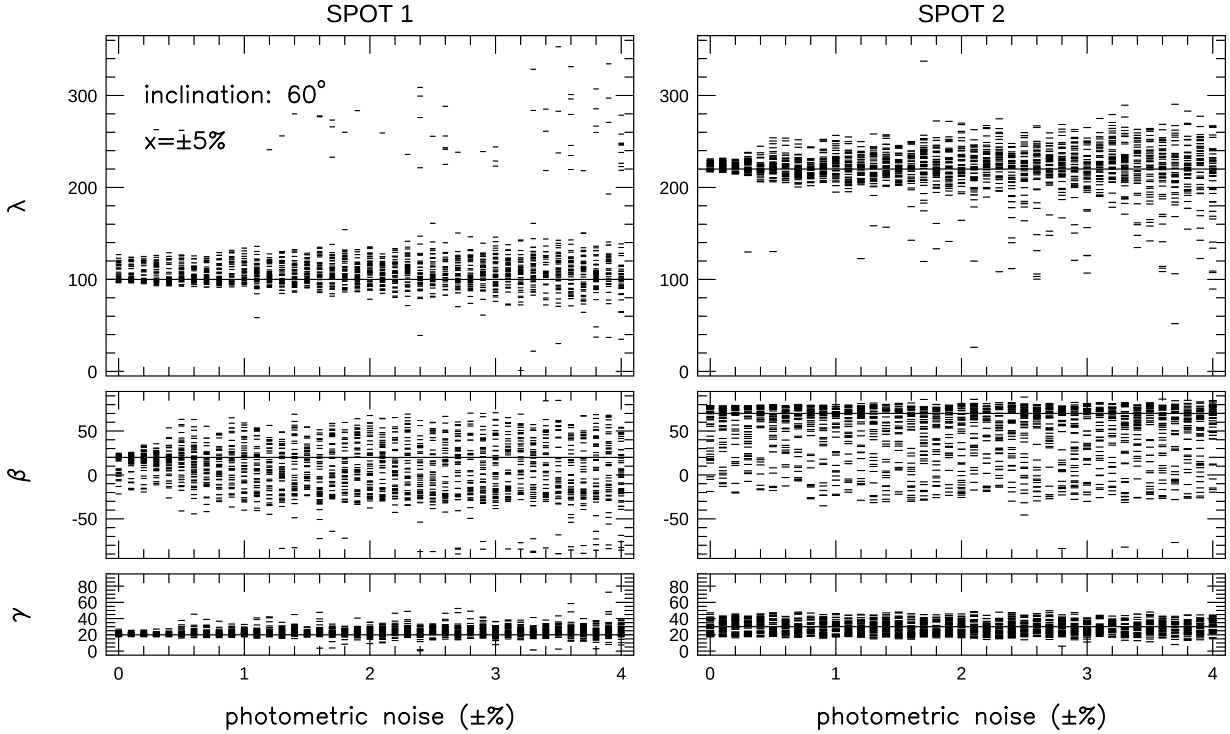

Fig. 8. As in Fig. 4 with the error box of the unspotted brightness x fixed at ±5%.

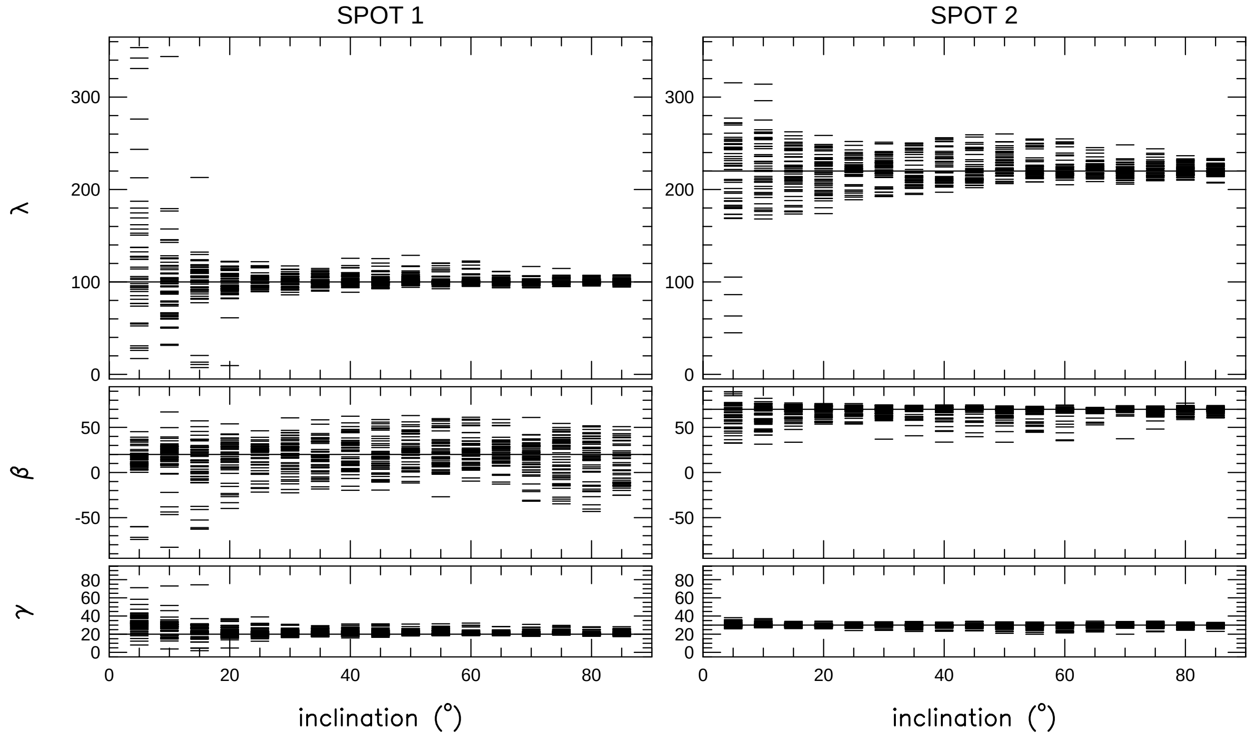

Fig. 9. Iterated spot parameters versus inclination. The polar spot (SPOT 2) can be modelled at a higher accuracy level than the equatorial one.

2. The LMM used for the iterations showed a strong correlation among the spot parameters even in case of two spots.
3. In the case of low inclination ($i \approx 30^\circ$), even using accurate photometric data, determination of spot latitude and size shows a rather big uncertainty, especially at lower stellar latitudes.
4. Changing i to 60°, with accurate data ($\delta_{\rm phot.} \approx \pm 0\overset{\rm m}{.}002 - 0\overset{\rm m}{.}005$), spot parameters could be recovered within a few degrees. However, with increased photometric noise over $\pm 1\%$, say $\pm 0\overset{\rm m}{.}01$, spot parameter determination (mainly spot latitude) becomes unstable.
5. Even a few hundredths of a magnitude error in the unspotted brigtness can be the most important error source.
6. The lower the original value of inclination, the more important the effect of misestimation of inclination on spot parameter determination.
7. In the case of $i \lesssim 20^\circ$ two-spot modelling based on photometry cannot give usable information about spot location and size, because of the big uncertainty originating from geometry.

Acknowledgements. The authors wish to thank Dr. Katalin Oláh and Dr. Edwin Budding for their kind attention and useful advice. Suggestions of our referee, Dr. Jean-Francois Donati helped to improve this paper considerably. We thank Dr. Christopher A. Tout for the careful reading of the manuscript. Supports from the Hungarian Research Grant OTKA T-015759 and OTKA F-019642 are acknowledged by ZsK. Support from the Austrian Science Foundation S7301 is acknowledged by JB.

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