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Heating of coronal loops by phase-mixing

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Abstract. A simple, self similar solution for the heating of coronal loops is presented. It is shown that the Heyvaerts-Priest model gives a good description of phase mixing in a certain class of coronal loops. In addition, under typical coronal conditions the ohmic heating, due to phase mixing, can provide magnetic energy on a timescale comparable with the coronal radiative time. Thus, it is possible that phase mixing can maintain a hot coronal loop for large Lundquist number. If the photospheric motions continually excite coronal loops, then phase mixing could contribute to a background level of coronal heating for very large Lundquist number.

Key words: MHD – Sun: corona

1. Introduction

The coronal heating mechanism remains one of the major unsolved problems in solar physics. Many mechanisms have been proposed for heating both closed and open field regions. In the open field regions of coronal holes, wave heating mechanisms remain an attractive possibility and recently Hood, Ireland and Priest (1996) presented an analytical solution to the phase mixing equations discussed in Heyvaerts and Priest (1983). Phase mixing occurs when the local Alfvén speed varies with position. Alfvén waves of the same frequency on different fieldlines travel with different speeds and hence have different wavenumbers. As they propagate they become out phase with their neighbours. This is phase mixing in space.

For standing Alfvén waves in closed field regions (such as the one described by Fig. 1), the Alfvén wavenumber is fixed by the finite size of the region. The inhomogeneous background Alfvén velocity makes the wave frequency space dependent, i.e.

$$\omega(x) = v_A(x)k, \tag{1}$$

for wavenumber $k = 2\pi n/L$, where L is the loop length. In a non-dissipative plasma, linear Alfvén waves of wavenumber k have solution $v \sim \exp(i\omega(x)t - ikz)$, which implies

$$\frac{\partial v}{\partial x} \sim v.t.\omega'(x), \tag{2}$$

that is, gradients in the x-direction increase with time, implying that the waves are phase mixed in time. The different frequency on each field line means that initially in phase wave motions move out of phase with respect to each other, causing large Alfvén wave field gradients to appear across the loop. The small lengthscales created allow dissipation to have a large effect. To this end, we examine phase mixing in a coronal loop to establish its viability as a plausible loop heating mechanism, given typical solar parameters.

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2. Basic equations and loop models

Fig. 1 describes the basic model we are considering. Footpoint motions excite linear Alfvén waves in the coronal loop cavity, which we assume to be inhomogeneous only in the x-direction.

The low β, linearised resistive MHD equations are

$$\rho\frac{\partial \mathbf{v}}{\partial t} = \frac{1}{\mu}(\nabla \times \mathbf{B}_1) \times \mathbf{B}_0, \tag{3}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) + \eta\nabla^2\mathbf{B}_1, \tag{4}$$

where the equilibrium magnetic field, $\mathbf{B}_0$, is uniform in the axial direction and the equilibrium density, ρ, is a function of position. The equilibrium velocity is zero and $\mathbf{B}_1$ and $\mathbf{v}$ are the Alfvén wave perturbations to the magnetic and velocity fields respectively.

Two symmetries are considered for the coronal loops namely the slab (or Cartesian model) and the cylindrically symmetric loop model. For the slab we assume that the density is only a function of the horizontal distance, i.e. $\rho = \rho(x)$, and that the plasma only moves in the y direction, i.e. $\mathbf{v} = v(x,t)\sin kz\mathbf{e}_y$ and $\mathbf{B}_1 = B(x,t)\cos kz\mathbf{e}_y$, where $k = \pi/L$ and L is the length of the coronal loop. Thus, we are interested in line-tied disturbances that vanish at the photospheric ends of the coronal loop. In the cylindrical case, we assume the plasma is cylindrically symmetric and that the plasma only moves in the θ direction. Thus, $\mathbf{v} = v(r,t)\sin kz\mathbf{e}_\theta$ and $\mathbf{B}_1 = B(r,t)\cos kz\mathbf{e}_\theta$. Hence, (3) and (4) reduce to the phase-mixing equation

$$\frac{\partial^2 B}{\partial t^2} = -k^2 v_A^2(x)B + \eta\nabla^2\frac{\partial B}{\partial t}. \tag{5}$$

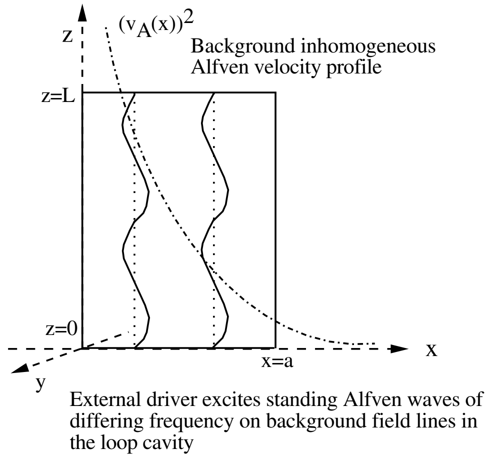

Fig. 1. Model of phase mixing in a coronal loop.

The important derivative in the dissipation term is the second order cross field derivative. The other derivatives will remain small in comparison to this term during the phase mixing process. Thus, the slab case reduces to

$$\frac{\partial^2 B}{\partial t^2} = -k^2 v_A^2(x) B + \eta \frac{\partial^2}{\partial x^2}\frac{\partial B}{\partial t}. \tag{6}$$

The cylindrical case, however, is exactly the same with x replaced by r. Thus, the two different geometries can be studied with the single Eq. (6).

The boundary conditions for the amplitude function $B(x, t)$ are taken as

$$B(x, 0) = 1, \tag{7}$$

$$B(0, t) = 0 \qquad \text{and} \qquad B(\infty, t) = 1, \tag{8}$$

to be consistent with the form of the similarity solution discussed in the next section.

In (6), $v_A^2(x) = B_0^2/\mu\rho$ is the square of the Alfvén speed and it may be expressed as

$$v_A^2(x) = v_0^2 f(x), \tag{9}$$

where v_0 is a typical Alfvén speed in the corona and $f(x)$ is a dimensionless function of the transverse direction x or equivalently the radial direction r. To reduce (6) to its simplest form we choose the following dimensionless variables,

$$t = \bar{t}\tau_A, \; x = \bar{x}a, \tag{10}$$

where a is the typical lengthscale for variations of the Alfvén speed; $\tau_A = 1/kv_0$ is the time for an Alfvén wave to propagate along the loop. Thus, (6) can be expressed as the single parameter equation

$$\frac{\partial^2 B}{\partial t^2} = -f(x) B + \delta \frac{\partial^2}{\partial x^2}\frac{\partial B}{\partial t}. \tag{11}$$

where the ‘bars’ have been dropped for convenience and

$$\delta = \frac{\eta}{a^2}\tau_A = \frac{\tau_A}{\tau_d}\left(\frac{L}{a}\right)^2. \tag{12}$$

Thus, δ is simply the ratio of the Alfvén travel time along the coronal loop and $\tau_d = L^2/\eta$, the diffusion time along the loop, multiplied by the aspect ratio squared.

It is perhaps worthwhile discussing other dissipation mechanisms at this point. In (3) viscosity has been neglected. Heyvaerts and Priest (1983) included a kinematic viscosity of the form $\rho\nu\nabla^2\mathbf{v}$ since the flow is incompressible for the Alfvén mode discussed here. The effect of kinematic viscosity is to change the dissipation coefficient in (6) from η to $\eta + \nu$. There is no qualitative change to the solutions presented here.

Alternatively, the full Braginskii (1965) viscous stress tensor can be included as in Van der Linden, Goossens and Hood (1988) on the investigation of visco-resistive ballooning modes. The dominant viscous coefficient is the term parallel to the equilibrium magnetic field but for the incompressible Alfvén mode with no velocity component parallel to $\mathbf{B}_0$ this term does not contribute. The remaining terms are small and of a similar size and form to the kinematic viscosity contribution mentioned above. Thus, viscosity can easily be included by simply changing the dissipation coefficient.

The final dissipation mechanism is thermal conduction. This requires a thermal energy balance equation and the effect of thermal conduction is to influence the perturbed pressure and density. However, in a low β plasma perturbations to these quantities do not effect the Alfvénic solutions to (3) and (4) and so we neglect thermal conduction in this analysis.

3. Self-similar solution

In a manner similar to Hood, Ireland and Priest (1996) we look for a similarity variable involving x and t. For this type of solution the Alfvén speed profile must be restricted to a particular form. However, the main property of the Alfvén speed is that it is monotonic and varies over a typical lengthscale a. To progress, we take the similarity variable in Eq. (11) as

$$s = t/x^2, \tag{13}$$

in terms of the dimensionless variables t and x. Substituting (13) into the basic equation shows that

$$f(x) = 1/x^4$$

and that the Alfvén speed profile must be

$$v_A^2 = v_0^2 \frac{a^4}{x^4}. \tag{14}$$

While the profile is obviously unphysical at $x = 0$, it does have the correct behaviour for $x > 0$. This is not too bad for the cylindrical case where the radius has to be positive.

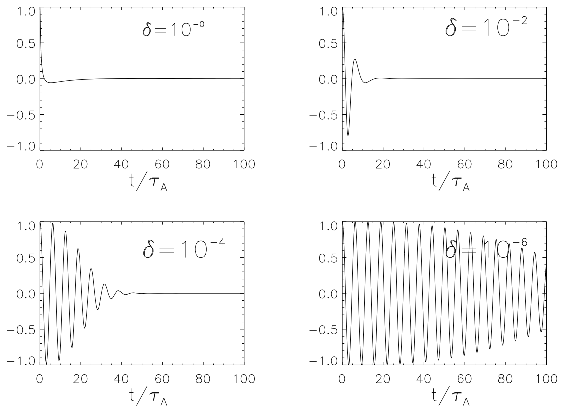

Fig. 2. The pertubed magnetic field as a function of time for various values of δ.

Now the basic equation (6) reduces to a single ordinary differential equation of third order when $B(x, z, t) = F(s)e^{ikz}$. Hence,

$$F'' = -F + \delta \left(4s^2 F''' + 14sF'' + 6F'\right), \tag{15}$$

where $'$ denotes differentiation with respect to s. Eq. (15) is solved numerically for different values of the resistivity parameter δ. Fig. 2 shows the time evolution of the magnetic field perturbations at a fixed value of x. For large δ the perturbations are readily damped and for small δ there is very little damping. This is similar to the results of the coronal hole, open field configuration considered by Hood, Ireland and Priest (1996). Unlike the Hood, Ireland and Priest (1996) case, this similarity solution cannot be written in a closed form. However, comparing the numerical solution with an Heyvaerts and Priest (1983) type of approximation, namely

$$F = \cos(s)e^{-2\delta s^3/3}, \tag{16}$$

we find excellent agreement when δ is small.

The important quantity for a suitable heating model to heat a coronal loop is the Ohmic dissipation, j^2/σ. This is readily calculated from the numerical solution to (15). Since

$$B(x, z, t) = F(s)e^{ikz},$$

the Ohmic dissipation,

$$\frac{|j|^2}{\sigma} = \frac{1}{\sigma\mu^2}\left(\frac{1}{a^2}\left|\frac{\partial B}{\partial x}\right|^2 + \left|\frac{\partial B}{\partial z}\right|^2\right), \tag{17}$$

becomes

$$\frac{|j|^2}{\sigma} = \frac{1}{a^2x^2\sigma\mu^2}\left((ka)^2x^2|F(s)|^2 + 4s^2|F'|^2\right). \tag{18}$$

Fig. 3 shows the ohmic heating as a function of s for $\delta = 10^{-4}$. It is evident the ohmic heating combines a periodic term modulated by a single peaked envelope. At small δ, the ohmic heating envelope can be approximated by

$$\frac{|j|^2}{\sigma} \propto \left[(ka)^2x^2 + 4s^2\left(1 + 4\delta^2 s^4\right)\right]\exp(-4\delta s^3/3). \tag{19}$$

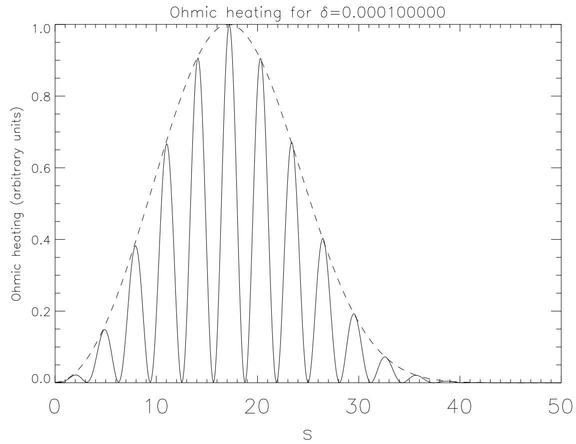

Fig. 3. The ohmic heating as a function of time for $\delta = 10^{-4}$, calculated using Eq. (16) with $x = 1$. The dashed curve indicates the envelope function in (19).

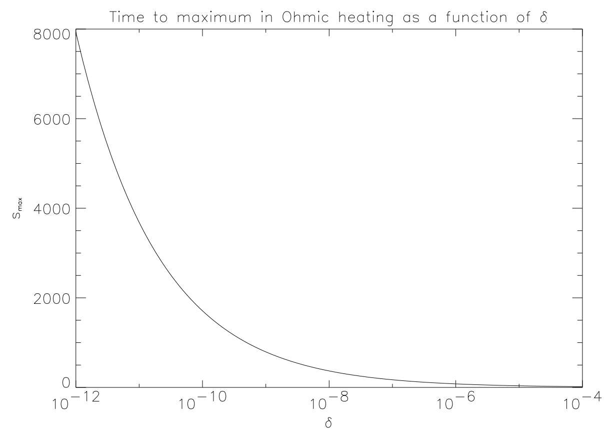

Fig. 4. The time for the maximum heating due to Ohmic dissipation as a function of the parameter δ for various radii within the cylindrical loop, calculated using Eq. (19)

(19) provides a good approximation to the Ohmic heating for small values of δ, as found above.

The results are shown in Fig. 3. The maximum heating is obtained only after many oscillation periods. For $\delta = 10^{-4}$ the maximum heating is at $s \approx 17$. The behaviour of the time to the maximum heating is estimated by locating the maximum of (19). This is given by the roots of the cubic equation

$$4\delta s^3 + \delta(ka)^2x^2 s - 2 = 0, \tag{20}$$

assuming $\delta s^2 \ll 1$. For a radius of order a, i.e. x of order unity, this gives,

$$s_{max} \approx (2\delta)^{-1/3}. \tag{21}$$

Fig. 4 shows how this maximum heating time depends on δ and the radius of the loop through the choice of x. As the Alfvén speed drops off, for larger x, so the time to reach the maximum Ohmic dissipation also is reduced.

4. Discussion

To be a viable heating mechanism, the time to maximum heating must be less than typical radiation cooling timescales (about 3,000 seconds). Following Hood, Ireland and Priest (1996), we can write δ as

$$\delta = \frac{1}{S}\left(\frac{L}{a}\right)^2 \tag{22}$$

where S is the Lundquist number and L is the loop length, which we will take as ranging from 10^6 to 10^8m. If we substitute this into the expression for s_{max} assuming further that $L/a \approx 10$ and $v_{A0} = 2 \times 10^6\mathrm{ms}^{-1}$ (Karpen et al. 1994) then the time to maximum Ohmic dissipation is

$$t_{max} \approx S^{1/3} \times \left(10^{-2} \rightarrow 1\right) \text{ seconds} \tag{23}$$

For $S = 10^{12}$, we obtain $10^2\mathrm{s} \leq t_{max} \leq 10^4\mathrm{s}$ which indicates that phase mixing can supply heating at large Lundquist number at timescales shorter than or comparable with the radiative cooling timescale. However, this is the result for one initial disturbance.

A further test for phase mixing will be to include the ohmic heating term into the time dependent simulations of coronal loops of Walsh et al. (1995) to see if repeated pulses can maintain a hot corona. Two possibilities may occur. Firstly, it may be that phase mixing is the main mechanism responsible for keeping coronal loops hot, provided that the disturbances are repeated every 1,000 seconds or so. One pulse is not enough. Walsh et al. (1995) will be able to provide an estimate of the period of the pulses. Secondly, it may be that the time to reach the maximum of the ohmic dissipation, while comparable to the coronal radiative timescale is too long to maintain a hot corona and so some other heating mechanism will be required. Even in this case, it is likely that phase mixing will contribute to a background level of heating.

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