

The huge enhancement of Spectral-Spatial Fluctuations (SSF) in the Cosmic Background Radiation (CBR)

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Abstract. The luminescence process for proto-objects moving with a peculiar velocity at high redshift is considered. It is shown that some primordial molecules could produce low-frequency photons (which correspond to the rotational lines) by decaying high-frequency rovibration photons from the CBR. Due to the rather different numbers of these photons, a huge enhancement (relative to the pure reflection mechanism) of the emission line intensity from the proto-objects could occur at the appropriate wavelengths.

Key words: cosmic microwave background – cosmology: theory – molecular processes

1. Introduction

The Big Bang model of the Universe's evolution is now the preferred one, after the success of the COBE mission. Strong evidence against alternative models is provided by COBE-FIRAS (Mather et al., 1990) and recent COBRA (Gush et al., 1990) results. Their spectral measurements of the CBR have set stringent upper limits on any spectral deviations from a pure Planckian curve. This means that there was no substantial energy emission after the epoch of annihilation of the electrons and positrons at redshift $z = 10^9$ as well as the latest reionization ($z \leq 70$). So, we are led to the conclusion that the appropriate scenario for the evolution of the Universe may be the simplest one, and the pure Big Bang model should be used as the standard. The COBE mission provides us with the answer to one of the most fundamental questions - were do we live? - we live in an expanding Universe with a Big Bang at the beginning!

This conclusion is confirmed by the results of measurements of the spatial fluctuations of CBR temperature in the other part of the COBE program. Some evidence in favor of a Zel'dovich-Harrison spectrum of primordial large-scale structure is apparent in the spatial fluctuations and this should be kept in mind for future considerations.

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So, now we can try to investigate the next level of observational effects which are provided by proto-objects in the post recombination epoch. One of the most probable classes of such effects will be considered here. They are the so-called SSF - Spectral Spatial Fluctuations (Dubrovich, 1994). Actually, they are the proto-objects at high redshift - $10 \leq z \leq 300$, which must contain some amount of molecules and which have a peculiar velocity V_p relative to the CBR. There are necessary and sufficient conditions for the SSF to be produced. The theory of this process was discussed by Dubrovich (1977, 1983, 1994) and by Maoli et al. (1994). Some experiments were described by De Bernardis (1992) and some plans by Signore et al. (1993).

All this work is based on the simplest mechanism of the SSF formation - pure reflection of the CBR photons due to the opacity of a proto-object in narrow spectral lines and the Doppler shift in frequency due to its peculiar velocity. In this paper another mechanism will be considered.

2. Luminescence of primordial molecules

As mentioned previously, opacity (in the narrow molecular lines, for example) and peculiar motion of the protoclouds results in CBR disturbances. This can be more easily seen in the rest frame of the proto-object. In this frame the CBR becomes non-isotropic and out of thermal equilibrium. From the side towards which the protocloud moves, the temperature of the CBR will be higher than average and from the back side it will be lower. After reflection, the photons are distributed isotropically in this frame. This leads to non-isotropic distribution in our frame. That is the explanation of the principal role of opacity. But the amplitude of the effect depends on the peculiar velocity and spectral index of the reflecting radiation (Dubrovich, 1977, Maoli et al., 1994). This effect corresponds to the elastic scattering between the molecules and photons, i.e. the total number of the photons does not change. But in fact all molecules have a quite complicated energy level structure. This allows for the possibility of a non-elastic process. It is the well-known luminescence process which, for example, plays an important role in the formation of radiation from a reflection nebula. In our case a "hot spot" in the CBR (in the rest frame of the cloud) plays

the role of a “star” for the reflection nebula. This consideration is just a phenomenological one. For the full description of this process, the equations of the photon transfer must be solved. Here we won’t do this, but will only make simple estimates.

Let us consider this process in detail. Taking into account only those chemical elements (the most abundant) which are predicted by the pure Big Bang model – H, He, D, ^3He , Li, and their ions, we can list the most probable molecules in the primordial matter at $z = 100\text{--}200$: H_2 , H_2^+ , HD, HD^+ , HeH^+ , LiH , LiH^+ , H_2D^+ , $^3\text{He}^4\text{He}^+$. All other molecules should be considered more critically, because their appearance is caused by some non-standard circumstances: non-equilibrium nuclear synthesis at the early times ($z=10^9$), or star formation at $z = 200\text{--}300$, etc. But, as was mentioned before, the pure model of the Universe is the most probable one. So, we won’t consider any other molecules here.

For obtaining the greatest interaction between molecules and photons, two values are important – the cross-section for scattering and the concentration (or relative abundance) of this molecule. The first parameter depends on the specific quantum structure of the molecule – its symmetry and charge. The second one depends on the abundance of the chemical elements of which it is composed and on the rate of the appropriate chemical reactions. According to these constraints, we should take into consideration only those molecules which have a large enough dipole moment and relatively high abundance. These are: HD^+ , HeH^+ , LiH , H_2D^+ , $^3\text{He}^4\text{He}^+$. The molecules H_2 and H_2^+ have no dipole moment, while HD has a rather small dipole moment and the abundance of D is not high enough. LiH^+ has a very low potential of dissociation and so its abundance in a hot Universe is very small. So, we can expect only a small number of molecules to be visible from the early Universe: HD^+ , HeH^+ , LiH , H_2D^+ , $^3\text{He}^4\text{He}^+$.

3. Consequences of CBR fluctuations

It has been shown previously (see Sunyaev and Zel’dovich, 1970, Dubrovich, 1977, Maoli et al., 1994) what the magnitude is of the temperature fluctuations, $\Delta T/T$ of the CBR due to the pure reflection of photons by the moving object. In this case the effect depends on its peculiar velocity V_p and optical depth τ ,

$$\frac{\Delta T}{T} = \frac{V_p}{c} \tau \quad (1)$$

here V_p is the component of the peculiar velocity along the line of sight and c is the speed of light. It should be pointed out, that if this fluctuation is caused by the interaction with a resonant system, it must occur only at the corresponding wavelength and, what is very important, there should be no influences from the one resonance to any other. In our case it means that this effect could be at the wavelength corresponding to the rotational and the rovibration transitions, but the amplitudes of the $\Delta T/T$ from the separate transitions are fully independent. It is one of the fundamental properties of pure reflection.

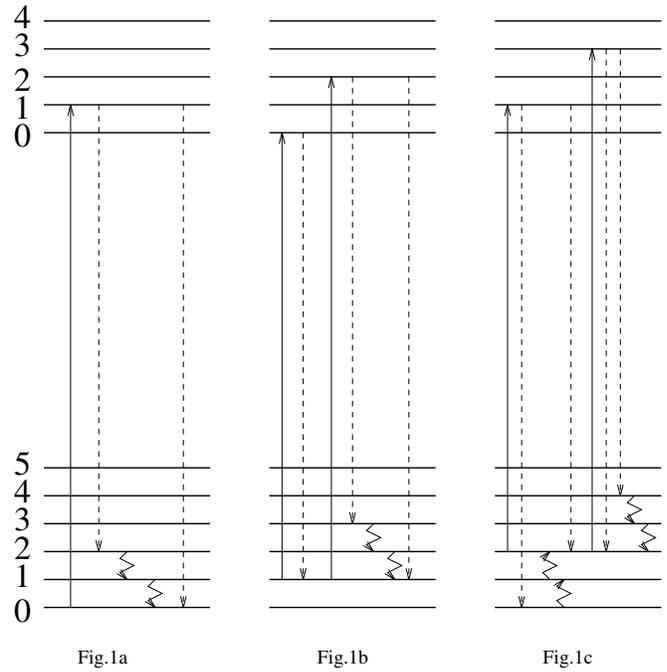


Fig. 1a–c. The scheme of some rovibration transitions in diatomic molecules

On the contrary, the luminescence process causes the appearance of some photons at one wavelength due to the absorption of the appropriate photons at another wavelength. This new property of the process of interaction of matter and radiation leads us to new possibilities for SSF formation.

In order to see what consequences might follow from this new effect let us take into account that there is no luminescence due to the transitions between different rotational levels only (these are forbidden in the dipole order by the law of momentum conservation). So, the first allowed mechanism which leads to luminescence is: the absorption of a rovibration photon followed by the emission of a new rovibration photon and two new rotational quanta (Fig1).

In terms of the quantum numbers: J , which corresponds to the number of rotational levels ($J=0,1,2,\dots$) and v , which corresponds to the number of the rovibration levels ($v=0,1,2,\dots$) the simplest circle of the transitions can be written: $(J=0,v=0)\text{--}(J=1,v=1)\text{--}(J=2,v=0)\text{--}(J=1,v=0)\text{--}(J=0,v=0)$. Here we have used the law for the allowed dipole transitions: $\Delta v=(\pm)1$, $\Delta J=(\pm)1$. The frequencies of these transitions are: ν_1, ν_2, ν_3 and ν_4 respectively, and they must satisfy the equation (1 is the absorbing photon and 2,3,4 are emitted photons):

$$\nu_1 = \nu_2 + \nu_3 + \nu_4 \quad (2)$$

$$N_1 = N_2 = N_3 = N_4 \quad (3)$$

$$\frac{\Delta\nu_1}{\nu_1} = \frac{\Delta\nu_2}{\nu_2} = \frac{\Delta\nu_3}{\nu_3} = \frac{\Delta\nu_4}{\nu_4} \quad (4)$$

where $\Delta\nu_i$ -the line's width and N_i -the photon's number. The energy levels of the diatomic molecules could be described by:

$$E = h\omega_e(v + 1/2) + hB_eJ(J + 1) \quad (5)$$

where E is the total energy of the level, ω_e and B_e are the specific molecule constants. For several molecules the ratio ω_e/B_e is rather similar - about 50–100. But this is the ratio of the frequencies of the first rovibration and the first rotational photon, as follows from expression (5). It is very important for our estimations. The key point is that the number of photons which are reflected by pure elastic scattering in the rotational lines at the frequencies ν_3 and ν_4 corresponds to the number of the CBR photons at these frequencies, but according to (3), the number of these photons due to the luminescence will correspond to the number of the CBR photons at ν_1 . Dubrovich (1977,1994) estimated the redshift, when several molecules could appear using Saha recombination rates as $z = 300$ – 100 . At these redshifts, ν_3 and ν_4 lie in the extreme Rayleigh-Jeans wing of the CBR and ν_1 - near it's maximum! So, the luminescence causes much more intense disturbances of the CBR.

Let us calculate its quantitative value. The amplitude of the $\Delta T/T$ in the Rayleigh-Jeans wing of the CBR could be defined as the ratio of number of the new photons (N_3 or N_4) to the number of the CBR photons N_0 at the same frequency (ν_3 or ν_4) and in the same spectral interval $\Delta\nu$:

$$\frac{\Delta T(\nu_i)}{T} = \frac{N_i}{N_0(\nu_i)} \quad (6)$$

According to the previous investigations by Dubrovich (1977) and Maoli et al. (1994), and our new considerations, we obtain:

$$N_1 = \frac{dN_0(\nu_1)}{dT} T \frac{V_p}{c} \tau \xi = \frac{h\nu_1 N_0}{kT(1 - \exp(-h\nu_1/kT))} \frac{V_p}{c} \tau \xi \quad (7)$$

$$N_0 = 8\pi(\nu_1/c)^3 (\exp(h\nu_1/kT) - 1)^{-1} (\Delta\nu/\nu) \quad (8)$$

where τ is the optical depth of the elastic scattering at frequency ν_1 , and ξ is the ratio of the decaying photons to all scattered photons ($\xi \leq 1$). So, using Eqs. (1–4) and (6–8) we obtain:

$$\frac{\Delta T(\nu_i)}{T} = \frac{V_p}{c} \tau K \quad (9)$$

$$K = \left(\frac{\nu_1}{\nu_i}\right)^2 \left(\frac{h\nu_1}{kT}\right)^2 \exp\left(-\frac{h\nu_1}{kT}\right) \xi = K_0 \left(\frac{z_r}{z}\right)^2 \exp\left(-\beta \frac{z_r - z}{z}\right) \quad (10)$$

where z_r is the redshift of molecule's recombination by Saha, K_0 is derived from this equation at $z = z_r$, and $\beta = h\nu_1/kT_0 z_r$. Here we have assumed that $h\nu_1/kT \gtrsim 1$, and $h\nu_3/kT \leq 1$, $h\nu_4/kT \leq 1$. The value of the optical depth (for rotovibrational transitions here) we will estimate on the base of the expressions obtained by Dubrovich (1994) for pure rotational transitions. The accuracy of such an estimation maybe not more than one order of magnitude.

$$\tau = \tau_0 \omega \alpha_M \Omega^{-1/2} (z/z_r)^{1/2} H^{-1} (J+1) \exp(-\gamma \frac{J(J+1)}{z+1}) \quad (11)$$

$$\tau_0 = 8\pi^{5/2} d^2 z_r^{1/2} n_0 H_0^{-1} (B_e/3kT_0) \quad (12)$$

where Ω , ω are the total and the baryonic average densities of the matter relative to the critical one, α_M is the abundance of the molecule relative to the atomic hydrogen, z is the redshift of the proto-object, H is the Hubble constant, normalized to $H_0 = 75 \text{ km/s/Mpc}$, γ is the specific molecule constant, d is the dipole moment of the molecule, T_0 and n_0 are temperature of CBR and the critical density at $z = 0$.

In order to estimate the value of ξ , several transition pathways should be considered. These pathways for the first three rotational levels are displayed in Figs. 1 a, b, c - respectively. In these Figs. the solid line corresponds to the first absorption (ν_1), the dash line to the emission of the one or the three photons (ν_2, ν_3, ν_4), the wave line - to the "secondary" absorptions. The transition to the initial level corresponds to pure reflection. The transition to the level, which lies lower than the initial one (Fig.1c) actually means that we lose two photons from the CBR which must be absorbed for the initial level to be exited. This is what we mean by "secondary" absorptions and it should be compared with the emission which is due to the process in Fig.1a. The total intensity of each line should be calculated as the sum of all these parts, taking into account the optical depth dependence on J . The dependence of ξ on J , can be found from the expression of the Einstein coefficient in appropriate form:

$$A_{vJ,v'J'} = \frac{J'+1}{2J+1} A_{v,v'} \quad (13)$$

here J is the number of the rotational level at $v=1$, and J' is the number of the rotational level at $v'=0$, and $A_{v,v'}$ very slow depends on J , J' . It is easy to see that ν_2 depends slowly on the J' . So, for the levels with small J and J' , the main factor is $(J'+1)$. In this case, it is easy to calculate the value of the ξ for several pathways (the first and the second arguments of the ξ correspond to the initial and the final rotational numbers at $v=0$, respectively):

$$\xi(0, 2) = \frac{3}{4}, \xi(0, 0) = \frac{1}{4}, \xi(1, 3) = \frac{1}{2}, \xi(1, 1) = \frac{1}{2} \quad (14)$$

for $J \geq 2$:

$$\xi(J, J+2) = \frac{J+3}{4(J+1)}, \xi(J, J) = \frac{1}{2}, \xi(J, J-2) = \frac{J-1}{4(J+1)} \quad (15)$$

These expressions show that the value of ξ is about 0.5 for the different cases. Concerning the comparison of the 1a and 1c pathways, we need to have a more precise value. Using (11) and (14, 15) we can get a lower limit to the effective ξ after adding these pathways. Indeed, If we neglect in (11) the factor $\exp(-\gamma J(J+1))$, we will obtain an increase of τ by a factor of three from the level with $J=2$ relative to that with $J=0$. This means that by the first pathway there will be three times more photons captured than by the second one. But only 1/12 of the first photons give rise to "secondary" absorptions. On the other hand, due to the 1a process, 3/4 of the captured photons give emission. The result will be: $\xi_t = 3/4 - 3(1/12) = 1/2$. So, finally for all future estimations we should take $\xi = 0.5$. In Table 1

Table 1.

		LiH	HD ⁺	HeH ⁺	H ₂ D ⁺
D_e	(eV)	2,43	2,67	1,85	4,3
z_r		195	210	150	340
λ_r	(μm)	676	227	149	1920
λ_v	(μm)	7,1	3,0	3,3	4,5
λ	(cm)	13,2	4,77	2,23	65,3
π_0		7.5(7)	4.9(6)	1.8(7)	3.3(6)
K_0		2,2(3)	50	1,8	4,8(4)
β		4,3	9,2	12,0	4,2
α'		1,5(-10)	3,0(-9)	1,1(-7)	3,0(-10)
V'/c		4,5(-7)	2,0(-6)	5,5(-5)	2,1(-10)

we present the main information about the most probable and highly interacting primordial molecules and some estimate of the fundamental parameters that could be measured.

here: D_e is the dissociation potential of the molecule, λ_r is the wavelength of the first rotational transition ($\lambda_r = c/2B_e$), λ_v is the wavelength of the rovibration transition ($\lambda_v = c/\omega_e$), $\lambda = \lambda_r z_r$ refers to the highest wavelength where this molecule could now be seen, α' is the limit to the molecule abundance which could be reached if we assume that $V_p/c = 10^{-4}$ and that the observational limit which can be achieved is $\Delta T/T = 10^{-4}$, V'/c is the lower limit which could be placed on the peculiar velocity if we assume $\tau \gtrsim 1$ and an observational limit of the $\Delta T/T = 10^{-4}$. The triatomic molecule H_2D^+ is more complicated than the other molecules in Table 1.

Now, we can write the expression for $\Delta T/T$ in a more simple form:

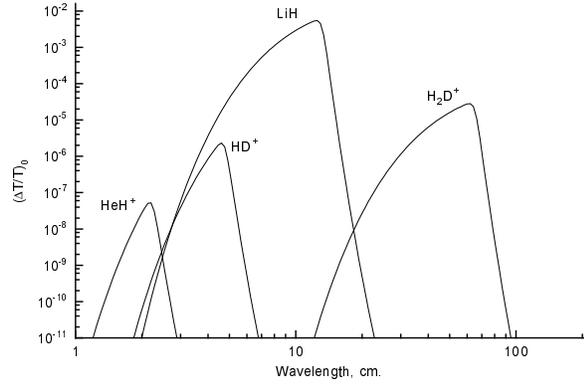
$$\Delta T/T = (\Delta T/T)_0 (\alpha/10^{-10}) (V_p/30 \text{ km s}^{-1}) (\omega/0.1) \quad (16)$$

In order to search for these molecules, the most auspicious wavelength regions (for the first rotational lines) can be found in Fig.2. The expected values of $(\Delta T/T)_0$ are shown as a function of wavelength for each molecule and correspond to the value of $\alpha = 10^{-9}$. The red wings of these curves are actually due to the rate of recombination of each molecule, assuming Saha recombination rates. The blue wings are described by expression (8). The second rotational line of each molecule has a factor of two higher frequency and a value of K which is four times lower than the first one.

Here are some comments to Table 1.

LiH: This is a very important molecule, because it consists of primordial Li. Its abundance is a good test for the epoch of nuclear synthesis in the early Universe. Its large dipole moment and relatively low frequency of the rotational and rovibration transitions lead to the high value of K . But unfortunately, its small abundance and some difficulties with the chemical processes of forming this molecule lead to a non-optimistic prediction for $\alpha \leq 10^{-11}$. Even so, this value of α , eqn. 16 and the peak value of $(\Delta T/T)_0$ from Fig. 2 lead to predicted values as high as about $\Delta T/T = 10^{-4}$ for $\omega = 0.1$.

HD⁺: This is also an important molecule, due to the presence of primordial deuterium, D. The abundance of D is about 5 orders of magnitude larger, than that of Li. But HD⁺ has a dipole

**Fig. 2.** The value of $(\Delta T/T)_0$ for several molecules

moment about 10 times less than LiH and a cross-section which is 100 times smaller. Another small factor is the abundance of H^+ at redshift $z = 200$, which might be about $10^{-3} - 10^{-4}$ relative to that of neutral hydrogen. Due to the relatively high frequency of the rotational and rovibration transitions, the resulting value of K is not very large. But, if high sensitivity were reached, this molecule might be seen.

HeH⁺: This molecule does not have any low abundance constituents. There are only two small factors which lead to a low abundance: a high rate coefficient for destruction (by electron recombination and collisions with the neutral atoms of hydrogen) compared with the rate coefficient of formation, and a small abundance of H^+ at high redshift. But it might be the most likely molecule to be searched for.

H₂D⁺: This is the simplest triatomic molecule with a high dipole moment. It contains primordial deuterium. Due to the presence in its spectrum of very low frequency transitions, the value of K can be very high. In Table 1 the value K_0 corresponds to $\xi = 1$. It is very important that the redshift of the recombination H_2D^+ be relatively high.

The expected abundances of these molecules in the early Universe are discussed by many authors (Lepp and Shull, 1984, Puy et al, 1993, Palla et al, 1995, Maoli et al, 1996, Stancil et al, 1996a) and more recent results by Stancil et al. (1996b).

In order to observe SSF due to all these molecules, let us give some simple estimates of their main parameters. These are diffuse, extended objects, which will have only the narrow emission lines with the low brightness temperature in these lines. The width $\Delta\nu/\nu$ of these lines depends on the object's size (linear - L or angular - θ) (Dubrovich, 1982):

$$\frac{\Delta\nu}{\nu} = 2^{3/2} \Omega^{1/2} z^{1/2} \theta \quad (17)$$

$$\theta = 10'' (M/10^{12} M_{\odot})^{1/3} (100/z)^{1/2} \omega^{-1/3} \Omega^{1/2} \quad (18)$$

Here M is the mass of the proto-object, M_{\odot} is the mass of the Sun. At the redshift $z = 100$, if $\omega = 0.1$ then a protogalaxy with the mass $M = 10^{12} M_{\odot}$ has

$$\theta = 20'' = 10^{-4} \quad (19)$$

$$\Delta\nu/\nu = 3.010^{-3} \quad (20)$$

A proto-object with the mass of an ordinary cluster of galaxies, $M = 10^{14} M_{\odot}$, will have an angular size:

$$\theta = 2' = 5.010^{-4} \quad (21)$$

$$\Delta\nu/\nu = 1.510^{-2} \quad (22)$$

The value of the peculiar velocity at the redshift z might be:

$$V_p/c = 2.010^{-3} z^{-1/2} \quad (23)$$

These parameters would be the most probable for the standard model of the Universe.

4. Observational aspects

Let us consider for example that the low limit of LiH abundance could be reached by an appropriate radiotelescope. From (16) and Fig.2 one can see that now $\Delta T = 6.510^{-3} T = 2.010^{-2} K$ if $\alpha = 10^{-10}$, $V_p=30$ km/s, $\omega = 0.1$. This value of ΔT corresponds to the proto-objects at $z=200$ which can be seen now at $\lambda=13$ cm. In order to estimate the observational time Δt for 3σ level one should use the standard equation

$$\Delta T = \frac{3T_n}{\sqrt{\Delta\nu\Delta t}} \quad (24)$$

where T_n is a noise temperature. The IRAM observation (de Bernardis et al., 1993) has the $T_n=1000$ K. At $\lambda = 13$ cm one can has $T_n=100$ K. Taking into account 100 times' less $\Delta\nu$ here one could see that absolute value of ΔT measured in both cases are equal. But the ΔT which is predicted at $\lambda=13$ cm is about three order of magnitude more than at $\lambda= 1.3$ mm. There is another problem for observations at low frequency - it is that we need to have a full aperture radiotelescope with an appropriate angular resolution. For proto-objects with $1'$ size it must be about 600 m diameter. For this case $\Delta\nu = 0.01\nu = 20$ MHz and Δt from (24) is

$$\Delta t = 1.110^5 (10^{-11}/\alpha)^2 s \quad (25)$$

5. Conclusion

The peculiar motion of the proto-objects relative to the CBR causes one new mechanism for the formation of SSF's: decay of the high energy photons to some new ones. In this case the number of new photons at low frequency becomes equal to the number of the photons at high frequency, which are much more abundant. This process leads to the effective enhancement of the SSF in the centimeter and decimeter wavelength region by a factor of about 1000. It provides us with the possibility of searching for proto-objects at $z = 100-300$ with the primordial abundance of the chemical elements at the level which is predicted by a pure Big Bang model.

These objects should look like extended diffused clouds with the emission in narrow lines, corresponding to the rotational

transitions in these molecules. The width of these lines depends on the size of the proto-object and can be about 1% for standard objects in our Universe. But the most probable wavelength (and corresponding redshift) interval, where each molecule can be seen is not very broad (Fig. 2). So it is possible to investigate several different epochs of the early Universe by observations of the appropriate molecule. For some other possible molecules (like H_2He^+ , $^3He^4He^+$, H_2Li^+ , ...) needs special calculations of the factor ξ , using the method by Dubrovich and Lipovka (1995).

The observations at a level of $\Delta T/T = 10^{-4}$, and angular scales of about $1'$ with the frequency resolution $\Delta\nu/\nu = 10^{-2}$, can provide us with non-trivial information about the early Universe. Here we have described the process, which is rather model-independent and we have not considered any chemical element heavier than Li. So, the value of all fluctuations estimated here must be considered as the lower limits to the amplitude of SSF's.

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