The galactic double-mode Cepheids

II. Properties of the generalized phase differences

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Abstract. By considering the least-squares fits of the double-mode Cepheid light curves discussed in Paper I we defined their properties by their Fourier parameters and generalized phase differences $G_{i,j}$. When plotting the latter quantities as a function of the order, the second order terms are confined in the region just below $3\pi/2$; the third order terms have $\pi/2 < G_{i,j} < \pi$, the fourth order ones cluster around 2π (or 0), the fifth order ones seem to have $\pi < G_{i,j} < 3\pi/2$. The mean $G_{i,j}$ values are also regularly spaced. The progression of the $G_{i,j}$ values as function of period was investigated and the signature of a possible resonance near 6.0 d was found.

Key words: methods: data analysis – stars: oscillations – Cepheids – Galaxy: stellar content

1. Introduction

Pardo & Poretti (1997, hereinafter Paper I) made a frequency analysis of the available photometry of galactic double-mode Cepheids (DMCs) and obtained a very reliable set of Fourier parameters for each star. However, a fully comprehensive and synthetic (as can be done by few well-defined parameters) observational description of the DMC light curves is still lacking. In this work, directly originating from the previous one, we try to determine the common characteristics of the DMC light curves, searching for the boundary values of the Fourier parameters and regularities in their progression.

2. The generalized phase differences

In Paper I we fitted the V magnitudes by means of the formula

$$V(t) = V_o + \sum_{z} A_z \cos[2\pi f_z(t - T_o) + \phi_z] \qquad (1)$$

where f_z is the generic frequency, which can be an independent frequency (f_1 and f_2), a harmonic or a cross coupling term. Our previous analysis demonstrated that each component in the

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DMC light curves can be defined as a combination of two basic frequencies f_1 and f_2; by defining $z = (i, j)$ with i and j integers, we have $f_z = f_{i,j} = if_1 + jf_2$. Some examples: for (i, j)=2,0 we have the harmonic $2f_1$; for (i, j)=1,1 the $f_1 + f_2$ term; for (i, j)=-1,1 the $f_2 - f_1$ term; for (i, j)=3,2 the $3f_1 + 2f_2$ and so on.

In order to define the properties of the Fourier parameters of the DMC light curves it is very useful to recall the *generalized phase differences* introduced by Antonello (1994b), here noted as $G_{i,j}$. They are a linear combination of the phases of each term $f_{i,j}$ and of the phases Φ_1 and Φ_2 of the independent frequencies f_1 and f_2 (Ω_1 and Ω_2 in Antonello's notation). Their expression is given by

$$G_{i,j} = \phi_{i,j} - i\Phi_1 - j\Phi_2 + 2k\pi \qquad (2)$$

The numerical application to the TU Cas fit provides some examples (the integer k values have to be selected so that $G_{i,j} \in [0, 2\pi]$):

$$G_{1,1} = \phi_{1,1} - \Phi_1 - \Phi_2 + 2k\pi =$$
$$= 4.13 - 4.31 - 2.19 + 2\pi = 3.91$$

$$G_{-1,1} = \phi_{-1,1} + \Phi_1 - \Phi_2 + 2k\pi =$$
$$= 1.92 + 4.31 - 2.19 = 4.04$$

$$G_{3,2} = \phi_{3,2} - 3\Phi_1 - 2\Phi_2 + 2k\pi =$$
$$= 2.11 - 3 \cdot 4.31 - 2 \cdot 2.19 + 6\pi = 3.64$$

Table 1 lists the $G_{i,j}$ values for each star's $f_{i,j}$'s, as calculated on the basis of the phase values listed in Tab. 3, 4 and 5 of Paper I; we remind the reader that the real significance of a component was established star by star and hence we are dealing here with very reliable values. The reported errors are calculated in a conservative way, i.e. summing the formal errors of the involved terms. Table 1 supplies the first synthetic description of the DMC light curves, as obtained from the available photometry.

Table 1. Amplitude ratios (first two lines; unit: 10^{-3}) and generalized phase differences (unit: 10^{-2} rad) for all the galactic DMCs. Except for the unique $1O/2O$ pulsator CO Aur, the stars are listed in order of increasing period

	TU Cas	U TrA	VX Pup	AS Cas	AP Vel	BK Cen	UZ Cen
$R_{21}(f_1)$	346±4	323±4	184±6	305±10	283±4	263±8	326±11
$R_{21}(f_2)$	123±9	99±20	118±7	139±15	117±15	102±19	122±37
$2f_1$	415±3	415±3	414±8	415±5	417±4	423±5	420±6
f_1+f_2	391±4	404±3	414±5	412±4	396±5	427±7	416±11
f_2-f_1	404±6	374±3	450±9	409±7	441±6	445±11	393±20
$2f_2$	433±12	439±10	447±13	434±11	445±13	488±23	449±39
$3f_1$	207±6	208±6	231±33	219±15	231±9	208±11	216±10
$2f_1+f_2$	189±6	246±5	206±13	210±8	222±9	214±9	226±14
f_1+2f_2	178±11	181±9	228±17	243±12	202±17	230±23	
$2f_2-f_1$	165±21			238±25	219±18		
$4f_1$	604±13	588±13					642±26
$3f_1+f_2$	608±9	623±8	668±34	629±19	588±27	621±24	634±21
$2f_1+2f_2$	586±13						
$3f_1-f_2$							687±52
$3f_1+2f_2$	364±21						
$4f_1+f_2$	386±15	407±15					

	Y Car	AX Vel	GZ Car	BQ Ser	EW Sct	V367 Sct	CO Aur
$R_{21}(f_1)$	298±12	104±19	156±14	175±6	164±12	153±6	179±6
$R_{21}(f_2)$	103±26	77±7	46±23	45±9	24±9	120±9	
$2f_1$	418±6	417±9	433±10	421±5	440±7	450±7	409±8
f_1+f_2	427±7	436±5	453±10	450±4	469±8	402±9	452±24
f_2-f_1	458±7	466±9	428±14	456±6	469±10	499±11	473±28
$2f_2$	433±26	456±10	470±51	523±21	300±34	362±12	
$3f_1$	226±13			230±19	255±53	218±36	183±29
$2f_1+f_2$	224±17	283±18		234±13	234±33		
f_1+2f_2	227±34						
$2f_2-f_1$			165±25				
$4f_1$							
$3f_1+f_2$	638±30						

2.1. Separations between different order terms

It is quite interesting to plot the $G_{i,j}$ values against their order defined as $\mid i \mid + \mid j \mid$. Light curves of DMCs are often quoted as an example of erratic behaviour and cycle-to-cycle variations, both in amplitude and in phase. In Paper I we already proved that these light curves seem to be much more stable than often reported and that a frequency locked fit yields a satisfactory representation. Only in the cases of U TrA and EW Sct we found some slight evidence of frequency or amplitude variations.

The suspicion that the DMC light curves have a predictable behaviour is confirmed by the natural upper and lower limits that can be easily observed in Fig. 1. The second order terms are confined in the region just below $3\pi/2$; the third order terms have $\pi/2 < G_{i,j} < \pi$, the fourth order ones cluster around 2π (or 0), the fifth order ones seem to have $\pi < G_{i,j} < 3\pi/2$.

The mean $G_{i,j}$ values are 4.30±0.34 rad for the second order (i.e. $\mid i \mid + \mid j \mid =2$), 2.20±0.23 rad for the third one, 6.24±0.31

for the fourth one, 3.85±0.21 for the fifth one. These mean values are roughly equispaced, with a slight tendency to increase: indeed, the differences between the mean $G_{i,j}$ of adjacent orders are 2.10, 2.24, 2.39 rad, respectively. The latter result and the boundary values established above yield an experimental confirmation of the conjectures first expressed by Antonello (1994b) about the extension to DMCs of the rule of uniformity of phase differences in monoperiodic Cepheids. However, the observed separation ($\sim$2.2 rad) is a bit larger than expected ($\pi/2$) in the case of adiabatic pulsations in a one-zone model.

2.2. The $G_{i,j}$ progressions

The second order $G_{i,j}$ values ($2f_1$, $f_1 + f_2$, $f_2 - f_1$, $2f_2$) range from 3.00 to 5.23 rad; it was expected to see a little spread of the $G_{0,2}$ values owing to the resonance at 3.0 d. Indeed the two extrema are just related to the $2f_2$ components of BQ Ser (the DMC approaching resonance from the shorter periods) and EW
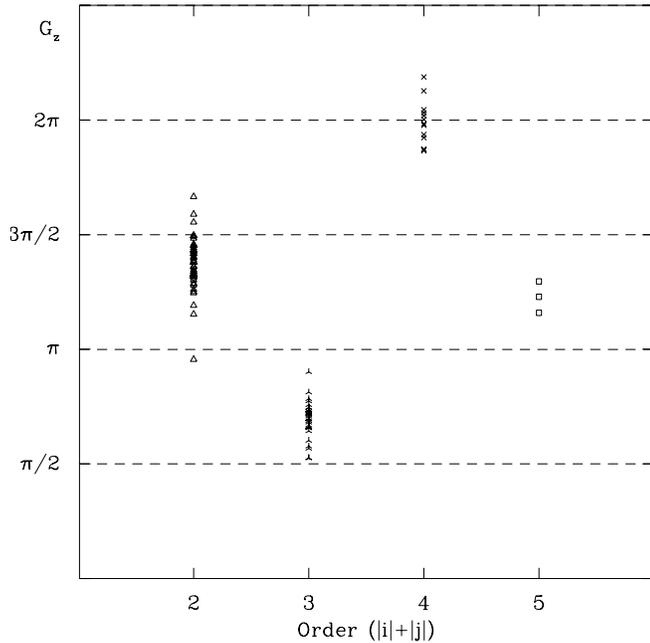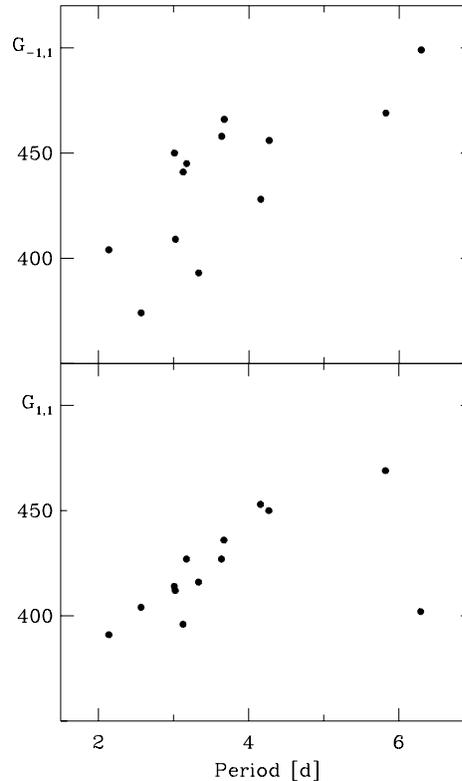
Fig. 1. A well defined separation is obtained when plotting the generalized phase values $G_{i,j}$ as function of order

Fig. 2. Generalized phase differences of the most important cross-coupling terms ($f_1 + f_2$ and $f_2 - f_1$) of DMC light curves shown as function of pulsation period. While the $G_{-1,1}$ progression does not show any particularity, the $G_{1,1}$ progression seems to be interrupted at $P \simeq 6.0$ d, as indicated by the last point (V367 Sct)

Sct (the DMC approaching resonance from the longer periods) light curves. Antonello (1994a) reported another possible resonance between the third overtone and the $f_1 + f_2$ term near 6.5 d. Later, Antonello (1994b) discussed a preliminary progression of the $G_{1,1}$ values and stressed the importance of verifying the position of the points related to V367 Sct and EW Sct light curves. This can now be carefully done as shown in Fig. 2. In the lower panel the last point (4.02±0.09 rad, V367 Sct, P=6.293 d) is clearly out of the progression followed by the other points (the last is at 4.69±0.08 rad, EW Sct, P= 5.823 d). Moreover, the progressive weakening of the amplitude of the $f_1 + f_2$ term is clearly visible in Fig. 3 (lower panel); the same trend can be evidenced by considering different types of normalized amplitudes and mode energies and it must be considered as a well established fact. On the other hand, if we look at the $f_2 - f_1$ term we observed a smoother behaviour considering both the $G_{-1,1}$ progression (Fig. 2, upper panel) and its amplitude (Fig. 3, upper panel). All these facts strongly support the action of a resonance effect involving the $f_1 + f_2$ term.

The third order progressions ($G_{2,1}, G_{1,2}, G_{-1,2}$) do not show any particular feature except a slight tendency of increasing values for longer periods. The fourth order term $G_{3,1}$ progression mimics, at a different level, the $G_{3,0}$ one.

3. Double-mode, classical and s-Cepheids

Fig. 4 represents the actual scenario of single- and double-mode Cepheids with $P <8$ d in our galaxy. The ϕ_{21}-P plane provides a quantitative representation of the Hertzsprung progression, formed by the Classical Cepheids (open circles); the $\phi_{2,1}$ points obtained from the F mode of DMCs (filled circles) are perfectly

superimposed. The "Z" progression is defined by the single-mode Cepheids which do not follow the Hertzsprung progression; in our opinion, these stars are to be considered the true s-Cepheids and their $1O$ nature is now definitely established by the perfect superimposition of the ϕ_{21} values obtained from the $1O$ mode of DMCs.

There is again a some underlying confusion about the nomenclature of these variables. The general assumption is that "s-Cepheids have a quasi-sinusoidal light curve", but to prove it it is necessary to perform a quantitative analysis: in such a case it is easy to establish the quasi-sinusoidal shape. Other criteria (first look evaluation, full amplitude values, ...) are quite arbitrary and not as useful as the amplitude ratios R_{21} for f_1(i.e. the ratio between the amplitude of f_1 and its harmonic $2f_1$) and f_2 (between the amplitude of f_2 and $2f_2$). As the lower panel of Fig. 3 shows, the stars forming the "Z" sequence also show a small R_{21} value and hence the light curve deviates very slightly from a sine-wave shape. However, it should be noted that the R_{21} values for the F-mode of AX Vel (particularly), GZ Car, BQ Ser are smaller than the expected ones; also the R_{21} value of the F-mode of VX Pup is small and it is located in the gap between the two sequences, very close to that of the single-mode AV Cir. Hence, the Fourier parameters have to be considered globally to perform a reliable mode identification.
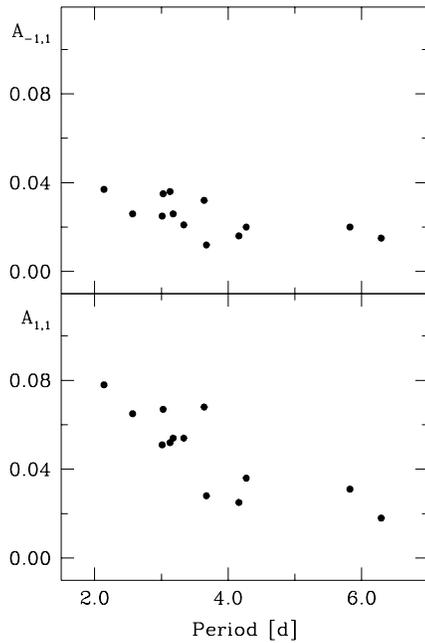
Fig. 3. The amplitude of the cross-coupling terms $f_1 + f_2$ decreases towards longer periods and it reaches its minimum at P∼6.0 d (lower panel); the amplitude of the $f_2 - f_1$ term is small over the whole range of period values

The $G_{i,j}$ values of the fit of the CO Aur data were not used in the previous discussion because they strongly deviate from the progressions described by the others; this is undoubtedly due to the $1O/2O$ pulsating nature of this star. It should be noted that the $G_{i,j}$ values are deviating only when considering the period, but they are in the range of those of $F/1O$ pulsators (see Fig. 2 for the $f_1 + f_2$ and $f_2 - f_1$ cases). In Paper I we already discussed its perfectly sine-shaped $2O$ light curve.

4. Conclusions

In the previous sections we supplied a quantitative description of the DMC light curves by means of their Fourier parameters and their progressions as function of the period. We detected and determined very precise ranges of generalized phase differences as function of the order fit, also establishing the uniformity of the Fourier parameter distribution. In such a context, these uniformities make the DMC light curves very similar to the single-mode Cepheid ones. Hence, the physical properties of the pulsation should be the same and, at least to a first approximation, the adiabatic approximation provides a satisfactory understanding of the phenomenon (Antonello 1994b)

It will be very interesting now to verify the properties of the generalized phase differences in the more numerous sample constituted by the DMCs of the Large Magellanic Clouds (Alcock et al. 1996, Welch et al. 1996); to do this, we consider it an essential step to apply the frequency analysis described in Paper I in order to avoid spurious or unreliable results.
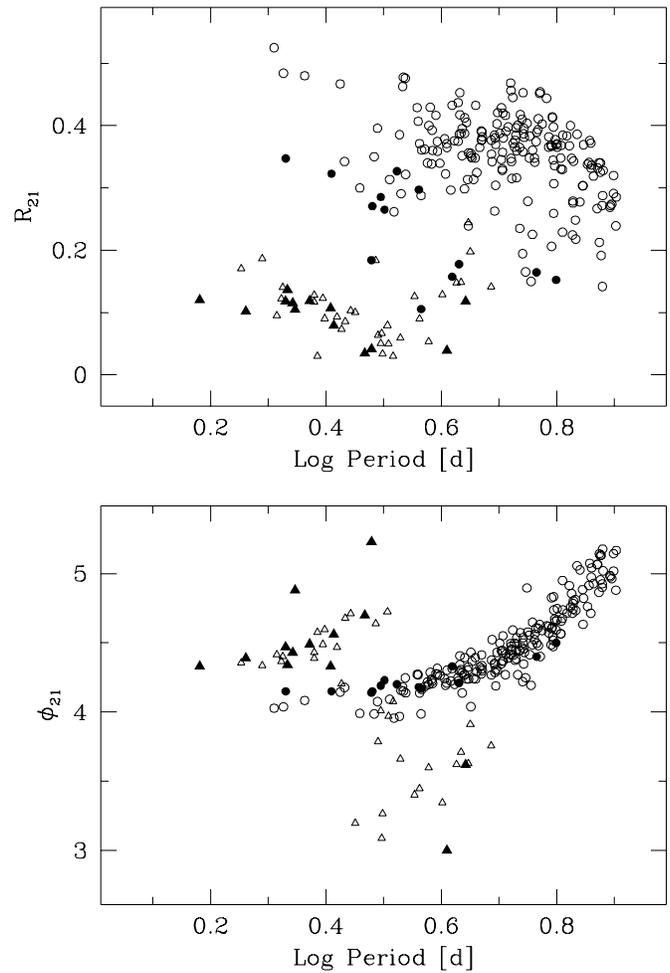
Fig. 4. The Fourier parameters ϕ_{21} and R_{21} provide a powerful discrimination between F and $1O$ pulsators when plotted versus Period. Dots: single-mode Classical Cepheids. Triangles: s-Cepheids. Filled dots: Fundamental radial mode of DMCs. Filled triangles: $1O$ radial mode of DMCs

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