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Research Note

A remark on the inversion of the magnification bias in the quasar-galaxy associations

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Abstract. We present an alternative but intuitive method of obtaining the true or intrinsic quasar luminosity function through the observed one in the framework of gravitational lensing. Such a so-called inversion of the magnification bias can be straightforwardly performed by means of the Mellin transformation instead of other complex methods. Application of our approach to the well-known form of the observed quasar luminosity function identifies the previous results in literature.

Key words: galaxies: general – quasars: general

It has been argued for a decade whether the magnification bias due to gravitational lensing by the matter clumps in the universe affects our quasar number counts $N_o(S)$, or equivalently the determination of quasar luminosity function $\Phi_o(L, z)$, and therefore, accounts at least partially for the apparent evolution of quasars (Turner 1980;1981; Canizares 1981;1982; Avni 1981; Vietri & Ostriker 1983; Schneider 1986;1992; Kayer & Refsdal 1988; Pei 1995). In particular, an overdensity of background quasars near foreground galaxies, clusters and even quasars would occur as a result of the magnification bias [see Wu (1996) for a recent summary]. All these issues can be simplified as a convolution of the true quasar number count $N_t(S)$ or luminosity function $\Phi_t(L, z)$ around redshift z with a magnification probability function $p(\mu)$ or $p(\mu|z)$:

$$N_o(S) = \int_0^\infty d\mu p(\mu)\mu^{-1} N_t(\mu^{-1}S), \qquad (1)$$

or

$$\Phi_o(L, z) = \int_0^\infty d\mu p(\mu|z)\mu^{-1}\Phi_t(\mu^{-1}L, z), \qquad (2)$$

where S is the flux threshold of the quasar sample, L, the absolute luminosity and, μ, the magnification factor. It is evident that the magnification probability distribution should satisfy the following constraints

$$\int_0^\infty d\mu p(\mu|z) = 1, \qquad (3)$$

$$\int_0^\infty d\mu\mu p(\mu|z) = 1, \qquad (4)$$

which correspond to, respectively, the normalization and the flux conservation. The question now reduces to how to find the true quasar number count $N_t(S)$ or luminosity function $\Phi_t(L, z)$ using the observed quantities for a given magnification probability distribution $p(\mu)$ or $p(\mu|z)$, namely, the inversion of Eq. (1) or Eq. (2). To solve Eq. (1), Schneider (1992) utilized the Volterra equation of the second kind with a kernel function $K(S, x)$ combined with other mathematical techniques and a specific boundary value, whereas for Eq. (2) Pei (1995) performed an expansion of the true luminosity function $\Phi_t(L, z)$ into the Taylor series coupled with a symbolic operator method to separate the variable μ from $\Phi_t(L, z)$. These sophisticated methods should be in principle applicable to various matter distributions, allowing us to derive the true quasar number count or luminosity function. However, the actual application of these methods often turns to be complicated, aside from the unknown details of the magnification probability function. Motivated by the importance of Eq. (1) or Eq. (2) in the study of the associations of angular positions of distant quasars with foreground objects (galaxies, groups and clusters of galaxies and quasars), we present an alternative but intuitive approach to the inversion of Eq. (2). Similarly, this approach can be equivalently employed for the inversion of Eq. (1).

Our method is based on the Mellin transformation (Titchmarsh 1948). For a give function of $f(x)$, its Mellin transformation is an integral of

$$\tilde{f}(s) = \int_0^\infty x^{s-1} f(x)dx \qquad (5)$$

so that the source function itself reads

$$f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} x^{-s}\tilde{f}(s)ds, \qquad (6)$$

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where c should be properly chosen to ensure that the singularities of the function $x^{-s}\tilde{f}(s)$ are on the left of the routine. The Mellin transformation of Eq. (2) is thus

$$\tilde{\Phi}_o(s,z) =$$

$$\int_0^\infty \mu^{s-1}p(\mu|z)d\mu \int_0^\infty (\mu^{-1}L)^{s-1}\Phi_t(\mu^{-1}L,z)d(\mu^{-1}L), \quad (7)$$

in which we have adopted a physically reasonable boundary that the magnification probability function $p(\mu|z) \longrightarrow 0$ for a sufficiently large magnification μ. While the right-hand side of the above equation is fortunately a product of two Mellin transformations of functions $p(\mu)$ and $\Phi_t(\mu^{-1}L,z)$, we have

$$\tilde{\Phi}_o(s,z) = \tilde{p}(s|z)\cdot\tilde{\Phi}_t(s,z), \quad (8)$$

where $\tilde{p}(s|z)$ is the Mellin transformation of $p(\mu|z)$ and $\tilde{\Phi}_t(s,z)$ and $\tilde{\Phi}_o(s,z)$ are the corresponding quantities of $\Phi_t(L,z)$ and $\Phi_o(L,z)$, respectively. From Eqs. (6) and (8) the true quasar luminosity function can be expressed as

$$\Phi_t(L,z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} L^{-s}\tilde{\Phi}_t(s,z)ds. \quad (9)$$

As it has been shown, our procedure of inversion of the magnification bias is more straightforward and convenient in application than the previous methods.

To demonstrate how efficiently the present method works, we take the exponential $L^{1/4}$ form of the observed quasar luminosity function used by Pei (1995)

$$\Phi_o(L,z) = \frac{\Phi_*}{L_z}\left(\frac{L}{L_z}\right)^{-\beta}\exp\left[-\left(\frac{L}{L_z}\right)^{1/4}\right] \quad (10)$$

with the luminosity evolution

$$L_z = L_*(1+z)^{-(1+\alpha)}\exp[-(z-z_*)^2/2\sigma_*^2], \quad (11)$$

here $(\Phi_*,\beta,\alpha,z_*,\sigma_*)$ are the parameters fitted by observations. Applying the Mellin transformation of Eq. (5) to Eq. (10) yields

$$\tilde{\Phi}_o(s,z) = 4\frac{\Phi_*}{L_z}{L_z}^s\Gamma\left[4(s-\beta)\right], \quad (12)$$

where Γ is the usual Γ-function with a variable of $4(s-\beta)$. If we adopt the same notation as Pei (1995) by defining

$$Z(s-1|z) \equiv \ln\langle\mu^{s-1}\rangle = \ln\left[\int_0^\infty d\mu\mu^{s-1}p(\mu|z)\right] \quad (13)$$

then the Mellin transformation of $p(\mu|z)$ reads

$$\tilde{p}(s|z) = \exp\left[Z(s-1|z)\right] \quad (14)$$

A combination of Eq. (12) and Eq. (14) gives rise to the Mellin transformation of the true quasar luminosity function $\tilde{\Phi}_t(s,z) = \tilde{\Phi}_o(s,z)[\tilde{p}(s|z)]^{-1}$, and the inverse Mellin transformation of $\tilde{\Phi}_t(s,z)$ results in the true quasar luminosity function

$$\Phi_t(L,z) = \frac{1}{2\pi i}\int_{(\beta+1)-i\infty}^{(\beta+1)+i\infty} 4\frac{\Phi_*}{L_z}\left(\frac{L}{L_z}\right)^{-s}$$

$$\Gamma\left[4(s-\beta)\right]\cdot\exp\left[-Z(s-1|z)\right]ds, \quad (15)$$

in which we have chosen $c = \beta+1$. Finally, the ratio of the true quasar luminosity function to the observed one is simply

$$\frac{\Phi_t}{\Phi_o} = \exp(-4m+e^m)\times\int_{-\infty}^{+\infty} ds\ \Gamma(4+2\pi is)$$

$$\cdot\exp\left[-2\pi ism - Z(\beta_r+2\pi is\beta_i|z)\right], \quad (16)$$

i.e., the result of Pei (1995) [Eq. (39)], where $m \equiv \beta_i\ln(L/L_z)$, $\beta_r \equiv \beta$, and $\beta_i \equiv 1/4$.

Nevertheless, we point out that a quantitative analysis of Φ_t/Φ_o depends on the observed quasar luminosity function $\Phi_o(L,z)$ and the magnification probability function $p(\mu|z)$. Except for some specific forms of $\Phi_o(L,z)$ [e.g. Eq. (10)] and $p(\mu|z)$, numerical computations should be often employed in finding the Mellin transformations $\tilde{\Phi}_o(s,z)$ and $\tilde{p}(s|z)$, and hence, the true quasar luminosity function $\Phi_o(L,z)$. In particular, it is relatively hard to get a simple form of $p(\mu|z)$ when the lens exhibits a complicated matter structure (see, for example, Schneider 1992). A detailed investigation for various objects as lenses is beyond the scope of this short note. We emphasize that the present method may be useful in the study of the associations of background quasars with foreground objects. Recall that the association problems, if real, have not been well account for to date in terms of gravitational lensing (Zhu et al. 1997). One of the possibilities is to abandon the unaffected background hypothesis, namely, the observed quasar number-magnitude relation or luminosity function has probably been contaminated by gravitational lensing according to Eq. (1) or Eq. (2). A further study based on more realistic lensing models, incorporating with the cosmological simulations of formation and evolution of large-scale structures, would provide a helpful insight into the problem.

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