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On dynamical scattering of Kuiper Belt Objects in 2:3 resonance with Neptune into short-period comets

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Abstract. A large number of trans-Neptunian comets might have been detected by the Hubble Space Telescope. If a significant fraction of these Halley-sized objects are stored in the 2:3 orbital resonance with Neptune, random gravitational scattering by the Kuiper belt objects (KBOs) with diameters in the hundred km range is found to be of potential importance in destabilizing the resonant orbits and hence acts as an injection mechanism of short-period comets. The relevant feeding rate of short-period comets from the 2:3 resonant population is estimated to be comparable to the injection rate as a result of dynamical sculpting by long-term planetary perturbations. Similar dynamical interaction could also lead to the injection of KBOs of a few hundred km diameter from the 2:3 resonant zone into the region inside the orbits of Neptune and Uranus. The comet-like object, 2060 Chiron might be such an example. The number of similar objects is estimated to be 5–10, while the total number of KBOs of 100 km size or larger leaking across the orbit or Neptune, e.g. Centaurs, may be on the order of 50.

Key words: comets: general – planets and satellites: general

1. Introduction

Comets are the most primitive objects in the solar system containing key information on the early history of planetary formation. It is therefore important to know where they first originated. The long-period comets with orbital periods > 200 years have isotropic inclination distribution and are most likely injected from the distant cometary Oort cloud via stellar perturbations, Galactic tide and perturbation effect of the Giant Molecular Clouds. According to theoretical models (Safronov, 1972; Fernandez & Ip, 1981) these icy bodies with radii of a few km represent the remnants of gravitational accumulation and scattering of icy planetesimals in the accretion of Uranus and Neptune. Earlier dynamical studies suggested that upon injection to elliptical orbits with perihelion distances < 30 AU (the orbital distance of Neptune), planetary perturbations will become effective in capturing them into short-period orbits with periods < 20 years (Everhart, 1972). There is therefore a continuous transition between the long-period and the short-period comet populations. However, such a scenario has been shown to be untenable on two counts. First, the average value of the capture efficiency of long-period comets is in fact too small to maintain a steady-state population of short-period comets (e.g. Joss, 1973). This consideration led Fernandez (1980) to postulate the existence of a belt of comet- and lunar-sized objects just beyond the orbit of Neptune as the feeding ground for the short-period comets. Second, numerical calculations by Duncan et al. (1988) demonstrated that the low-inclination orbits of the observed short-period comets could only have come from the orbital capture of a population of parent bodies initially in similar orbital inclinations. The same result has since been obtained by Ip & Fernandez (1991) using a statistical approach. These theoretical studies hence have reinforced the previous proposals by Edgeworth (1949) and Kuiper (1951) on the possible existence of trans-Neptunian objects (or Kuiper belt objects) as an extension of the outer edge of the Solar System.

The observational search of the Kuiper belt objects (KBOs) by several dedicated groups has born important results. More than twenty objects with diameters between 100 and 400 km (assuming an albedo of 0.04) have been discovered by now (Jewitt & Luu, 1995; Williams et al., 1993). From observational statistics, Jewitt & Luu (1995) estimated that beyond Neptune's orbit there should be in total about 38 000 objects of this size range. On the basis of recent detections of more KBOs, D. Jewitt (review paper in ACM meeting, Versailles, France, July 7.–11., 1996) updated the total number of KBOs to be 70 000. Anticipating further discoveries, we will use 10^5 as the working number for the total population of KBOs. The probable detection of a population of "comets" with sizes comparable to that of comet Halley (diameter ≈ 10 km) was recently reported by Cochran et al. (1995) from Hubble Space Telescope observations. These authors suggested that the total number of these Halley-sized objects could be as many as 2×10^8. Because of the lack of orbital data, Cochran et al. (1995) tacitly assumed that these objects detected by HST are all moving in orbits in 2:3 commensurability with the motion of Neptune. This hypothesis is based on the interesting finding that a significant fraction of the KBOs follows this relation (Jewitt & Luu, 1995; Duncan et al., 1995). Maholtra (1993, 1995) proposed that about 50% of the KBOs could be trapped in resonant orbits (i.e. 2:3, 3:4, and 1:2, etc.) on the basis of her theory of the resonant capture of Pluto by Neptune during the accretion of the outer planets (see Fernandez and Ip, 1984).

Of the observed population, there are eleven objects probably trapped in 2:3 resonance with Neptune. This means that the proportion of the resonant component is on the order of 35% (Maholtra, 1993). It is, however, important to note that the identification of the 2:3 resonant objects of large eccentricities is subject to strong selection effect. This is because the small values of their perihelion distances ($\approx$ 30 AU) permit a higher detection probability of this group of "eccentric" KBOs. Also, the precise computation of orbital elements for individual objects is hampered by the limitation in measurements (i.e., small numbers of oppositions, short arcs, etc.). The large proportion of KBOs in 2:3 resonance is therefore still an assumption to be verified by further observations. More comprehensive survey with better statistical results of the orbital distribution of the KBOs may modify this estimate. With this said, we would like to explore in this work the possibility that these resonant bodies – if indeed they are so numerous – could be a significant source of the short-period comets.

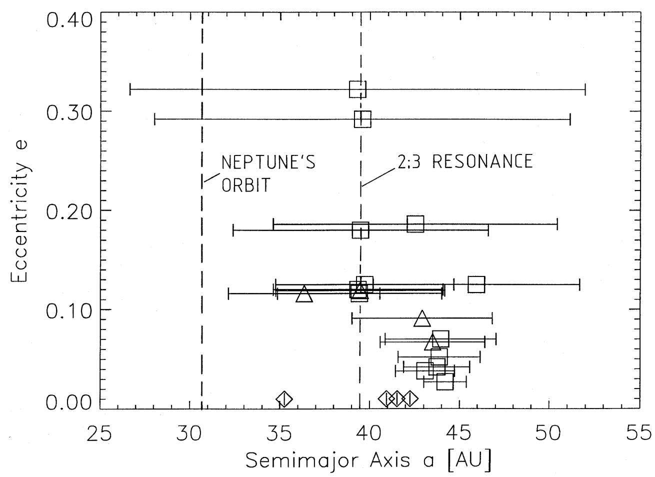

Fig. 1. A scatter plot of the radial distribution and orbital ranges of the KBOs. The squares (□) are for objects with orbital parameters listed in Duncan et al. (1995). The triangles (△) are for objects with orbital parameters listed in Jewitt & Luu (1995). The diamonds (◊) are for objects assigned with zero eccentricity. Gravitational scattering effect is facilitated by the overlapping of the orbital rayers of the objects trapped in resonant motion with those of the outer population

2. Scattering effects

The dynamical injection of short-period comets from the Kuiper belt is closely related to the long-term stability of the orbits of KBOs under the gravitational perturbations of the major planets. Torbett (1989) and Torbett & Smoluchowski (1990) are the first to investigate this effect by numerical integration and they found that orbits of particles initially placed at radial distances smaller than 45 AU are most likely of chaotic nature with a Lyapunov time of about a million years. Subsequently, a number of authors have performed simulations with longer dynamical times and larger samples of test particles (Levison & Duncan, 1993; Holman & Wisdom, 1993; Duncan et al., 1995). The general result is that the original population of the trans-Neptunian objects is subject to gradual erosion continuing until today. In the dynamical calculations of Duncan et al. (1995) it was shown that the inner edge of the Kuiper belt should have been sculpted away such that objects with semimajor axes smaller than 35 AU would have been completely eliminated over the age of the solar system. The average loss probability of the KBOs orbiting within 50 AU is on the order of 3–5$\times 10^{-11}$ per year depending on the assumed eccentricity ($e \approx$ 0.05–0.15).

The clearance of the inner population of the KBOs within 5 AU of the orbit of Neptune is consistent with the earlier finding by Torbett & Smoluchowski (1990) that Neptune's gravitational perturbation is able to drive objects in its near vicinity into chaotic orbits. The large eccentricities of these chaotic orbits will mean that the corresponding KBOs would be injected into the inner solar system or be ejected into escape orbits via random planetary encounters. The only exception, of course, is the objects in 2:3 resonance with Neptune which avoid close encounters with Neptune as a result of their librational motion. However, objects in 2:3 resonant orbits could be de-captured either because of long-term gravitational perturbations with a time scale of billions of years (Duncan et al., 1995) or because of mutual gravitational scattering. Given the present observed orbital distribution of the KBOs which is characterized by a cluster of 2:3 resonant objects with semimajor axes $a \approx$ 39.5 AU well separated from another population of objects orbiting at larger radial distances (see Fig. 1), the gravitational interaction among the KBOs might be important in determining their dynamical evolution. To evaluate this effect, we will first examine what are the minimum orbital changes required to perturb a KBO out of the stable 2:3 resonance and then we will estimate whether such orbital perturbations are of any significance.

According to the analytical treatment of Dermott et al. (1988), the periodicity of the libration angle of two bodies with masses, m_1 (Neptune) and m_2 (KBOs), and semimajor axes, a_1 and a_2 ($a_2 > a_1$) in a $p : p+q$ resonance can be expressed as

$$T_l = T_2 \left[3(p+q)^2 \frac{m_2}{M_\odot} (1+g) \alpha f(\alpha) e_2^q \right]^{-\frac{1}{2}} \tag{1}$$

where $M_\odot$ is the solar mass, $g = (m_1/m_2)/\alpha$, $\alpha = a_1/a_2$, e_2 and T_2 are the eccentricity and orbital period of m_2, respectively, and $f(\alpha)$ = 2.484 for the 2:3 resonance (Fernandez & Ip, 1996). The ratio α of semimajor axes will oscillate during the libration with an amplitude given by

$$\left(\frac{\Delta\alpha}{\alpha} \right)_r = 8(p+q) \frac{m_2}{M_\odot} (1+g) \alpha f(\alpha) e_2^q \left(\frac{T_e}{T_z} \right) \tag{2}$$

Combining Eqs. (1) and (2) and using the assumption that $m_2 \ll m_1$ and p = 2 and q = 1 (for 2:3 resonance) we find (Dermott et al. 1988; Malhotra, 1993)

$$\left(\frac{\Delta\alpha}{\alpha}\right)_r = 8\left[\frac{1}{3}\left(\frac{m_1}{M_\odot}\right)\alpha f(\alpha)e_2\right]^{\frac{1}{2}} \tag{3}$$

For α = 0.763, $m_1/M_\odot$ = 5.17 × 10^{-5}, and e_2 = 0.25, we have $(\Delta\alpha/\alpha) \approx 0.026$. Thus a change of the $alpha$ value of comparable magnitude will probably be sufficient to destabilize the resonant motion such that the perturbed KBO will no longer be protected from close encounters with Neptune.

Investigations by Morbidelli et al. (1995), Malhotra (1995) and Levison & Stern (1995) showed that the resonant structure of the Kuiper belt is very complex. Because of the overlapping of secular resonances and the Kosai resonance at nearby orbital regions (36 and 41 AU). the 2:3 resonance is stable only at small libration amplitude. This means the above-mentioned value of $\Delta\alpha/\alpha \approx 0.026$ for de-capture of resonant orbit might be an overestimate. As a matter of fact, using numerical methods, Duncan et al. (1995) have examined the positions of stable orbits as functions of a and e with a step size of Δa = 0.05 AU. From their graphical chart of the "islands of stability" for the 2:3 resonance and other commensurabilities, it can be seen that the width of Δa for long term stable libration is always very narrow with a value of $\Delta a \approx$ 0.1–0.2 AU for $e \approx$ 0.1–0.3. This means the empirical value of $\Delta\alpha/\alpha$ may be taken to be $\approx 2.5 \times 10^{-3}$–$5.0 \times 10^{-3}$ instead of the value of $\Delta\alpha/\alpha \approx 0.026$ as given in Eq. (3). Detailed long-term numerical integrations tracing the dynamical evolution of the orbits of the scattered KBOs would be required to provide a more accurate picture. Before this step, a simplified analytical model will be used to explore this new effect of potential importance to the interrelation between the KBOs and short-period comets.

3. Time scale

We have applied the statistical method developed by Öpik (1950) and Arnold (1965) to compute the dynamical time scales of gravitational scattering between a small object in 2:3 resonance with Neptune and a population of KBOs of different sizes. In the original theory of Öpik (1951), the probability of collision between a test body of semimajor axis a, eccentricity e, and inclination i with a planet in circular orbit with semimajor axis a_0 was formulated in the context of two-body problem as follows. With $A = a/a_0$, the three components of the encounter velocity at orbital intersection (U_x in the radial direction, U_y in the azimuthal direction, and U_z in the vertical direction) can be expressed as

$$U_x^2 = 2 - \frac{1}{A} - A(1-e^2)\,, \tag{4}$$

$$U_y = \sqrt{A(1-e^2)}\cos i - 1\,, \tag{5}$$

$$U_z = \sqrt{A(1-e^2)}\,\sin\, i\,. \tag{6}$$

For direct motion, cos $i > 0$ and U_y is small; for retrograde motion, cos $i < 0$ and U_y has a large negative value. The total encounter velocity is

$$U^2 = 3 - \frac{1}{A} - 2\sqrt{A(1-e^2)}\cos i\,. \tag{7}$$

With these velocity components (which are all normalized to the orbital velocity of the planet, U_0) the classical formula for encounter probability per orbital period between a stray body and a planet can be written as

$$p = \frac{S^2U}{\pi\sin i|U_x|} \tag{8}$$

where πS^2 is the effective collisional cross seetion with S in unit of a_0. As an example, for a = 40 AU, a_0 = 45 AU, e = 0.25, e_0 = 0.05 and sin i = 0.30, we have U_x = 0.204, U = 0.375, and the surface impact probability $p = 1.12 \times 10^{-22}D^2$ per years where D is the diameter of the KBO according to Eq. (8). Note that in case the orbital eccentricity (e_0) of the planet is non-zero, collision or close encounter between the two objects is possible only for a fraction of the time. This fraction is given by (Arnold, 1965)

$$f = \frac{1}{\pi}\cos^{-1}\left[\pm\frac{(1-e_0^2) - A(1\pm e)}{e_0A(1\pm e)}\right] \tag{9}$$

for the two possible cases. The encounter probability per orbital period can thus be rewritten as

$$p = \frac{fS^2U}{\pi\sin i|U_x|} \tag{10}$$

In the Monte Carlo simulations of the random gravitational scattering process the numerical statistics can be improved by assigning the effective cross section πS^2 to be a multiple value ($K^2 > 1$) of the gravitaional capture cross section,

$$\pi R_e^2 = \pi R^2\left(1 + \frac{U_e^2}{U^2}\right) \tag{11}$$

where R is the geometrical cross section of the interacting planet or KBO and U_e is the surface escape speed in unit of U_0. The actual value of impact parameter (the closest distance between the KBO and the comet trapped in 2:3 resonance) is chosen at random within the effective cross section given by $\pi S^2 = \pi K^2 R_e^2$. This means a small value of K (= 3, say) will lead to a large fraction of surface impact collision events while a large value of k (> 20) will favor mostly distant gravitational encounters. For non-collisional interactions, changes in the orbital elements (Δa, Δe, and Δi) can be computed from the two-body approximation for gravitational scattering. Thus from the choice of the k-parameter, we can in principle derive the relation between variations in $\Delta\alpha/\alpha$ and the scattering time scale (T_s).

Following the basic computational scheme described in Ip (1989), we have considered the orbital scattering effect of a population of KBOs of diameter D on the cometary object in 2:3

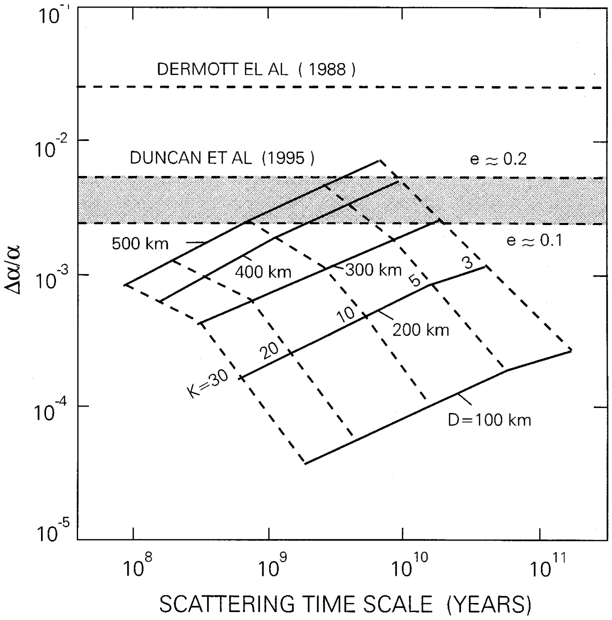

Fig. 2. The relation between the change in semimajor axis and the corresponding dynamical time scale as a result of gravitational scattering by KBOs with diameters between 100 km and 500 km. The threshold values of resonance instability given in Dermott et al. (1988) and Duncan et al. (1995) are indicated. The grid points of $\Delta\alpha/\alpha$ and the dynamical times obtained by using different k-parameters (k = 3, 5, 10, 20, 30) in the Monte Carlo computations are linked up by dashed lines.

resonance with Neptune. The resonant orbit is characterized by a_r = 39.44 AU, e_r = 0.25 and sin (i) = 0.30 while the scattering KBOs are distributed between 40 and 50 AU with an eccentricity $e_k \approx 0.05$ and sin $(i_k) \approx 0.05$. In other words, we place all non-resonant KBOs in the orbital region which might interact with the 2:3 resonant objects. The total number of the hypothetical KBOs is $N_k = 100$ and in each Monte Carlo run our numerical algorithm seeks out the object with the highest scattering probability and then computes the post-encounter changes in the semi-major axis (Δa), eccentricity (Δe) and inclination (Δi) of the resonant body. In order to accumulate enough statistics we have repeated this computation 1000 times. In the numerical calculations, the cases of mutual collision are excluded from the statistics. [For D = 200 km and the value of the impact parameter set to be k = 10, the collisional probability (p_c) is about 0.2%, and $p_c \approx 1\%$ if k = 5.] As mentioned before the k value is adjusted so that the relation between Δa and the scatterin time scale can be derived. The results relating the probable changes in $|\Delta\alpha|/\alpha$ as functions of the corresponding time scale are summarized in Fig. 2.

Fig. 2 shows the absolute values of the expected changes (which could be positive or negative) in $\Delta\alpha/\alpha$ after random encounters of a small object with KBOs with D ranging from 100 km to 500 km. The runs obtained by adopting particular values of the k parameters (controlling the interacting cross sections) are also indicated. In this figure the total number of KBOs (or scatters) with semimajor axes between 40 AU and 50 AU is assumed to be $N_s = 10^5$. The scattering time scale (T_s) can thus be rescaled by noting that $T_s\alpha 1/N_s$.

If all KBOs have $D \approx 400$ km, it is expected that the HST objects will suffer gravitational scattering into the strip of orbital instability (i.e., the shaded region) in 3 billion years after an encounter with a KBO of such large size. On the other hand, for smaller-sized KBOs with $D \approx 100$ km, say, the HST objects in 2:3 resonance would be extremely stable over a time scale $\approx 10^{11}$ years. This consideration hence indicates that the size distribution of the KBOs is important in determining the effective dynamical time scales. If the differential size distribution follows the expression $dn/dD = A \cdot D^{-q}$ where A is a normalization factor the total number of KBOs with diameters between D_1 and D_2 will be $n(D_1, D_2) = A \int_{D_2}^{D_1} (D)^{-q} dD$.

Therefore, the actual scattering time scale can be estimated by using the following formula

$$T_s = \left[A \int_{D_2}^{D_1} t(D)^{-1} \cdot D^{-q} dD \right]^{-1} \quad (12)$$

where $t(D)$ is the scattering time of an object with diameter D.

Since the scattering effect is dominated by the large objects, some qualitative idea can pe obtained by examining the time scales determined by those KBOs with $D \approx$ 300–400 km. If q = 2, about 11% of the KBO population will have D between 300 km and 400 km, and the maximum value of the scattering time scale will be $T_s(q = 2) \approx 3 \times 10^{10}$ years with $N_s \approx 10^5$. If q = 3.5 which is the expected value from equilibrium evolution of collisional fragmentation, about 3.5% of the KBOs have $D \approx$ 300–400 km and the corresponding value of $T_s(q = 3.5)$ will be $\approx 9.4 \times 10^{10}$ years. As we discussed in the Introduction section, it is possible that about 35–50% of the KBOs are trapped in 2:3 (Malholtra, 1993). In this scenario $N_s \approx 5 \times 10^4$, and we have $T_s(q = 2) \approx 6 \times 10^{10}$ years and $T_s(q = 3.5) \approx 1.91 \times 10^{11}$ years.

Because the orbital evolution driven by random gravitational encounters is itself an accumulative process determined by the time history of successive encounters. We have performed Monte Carlo computations tracing the orbital changes of the test particles under the scattering effect of an ensemble of KBOs with prescribed size distributions. A run is terminated when the cumulative change in $\Delta\alpha/\alpha$ reaches a certain threshold value. From these computations we can produce histograms of the distributions of the sacttering time. Fig. 3 shows the case for a threshold value of $\Delta\alpha/\alpha$ = 0.5% in de-capture from the 2:3 resonance.

While the average scattering times are estimated to be between 5.7×10^{10} years (q = 2.0) and 7.1×10^{10} years (q = 3.5), half of the scattered bodies will actually be lost within a time scale of about 10^{10} years. This is because the average dynamical time scale is biased by the long tail of large survival ages in the histogram. We have also computed cases with the threshold value of $\Delta\alpha/\alpha$ set to be 1.0×10^{-2}. The basic results are shown in Fig. 4. The average lifetimes are about three times longer as in the previous case (cf. Fig. 3). In the example with q = 3.5, 50% of the KBOs in 2:3 resonance will be lsot within 4×10^{10}

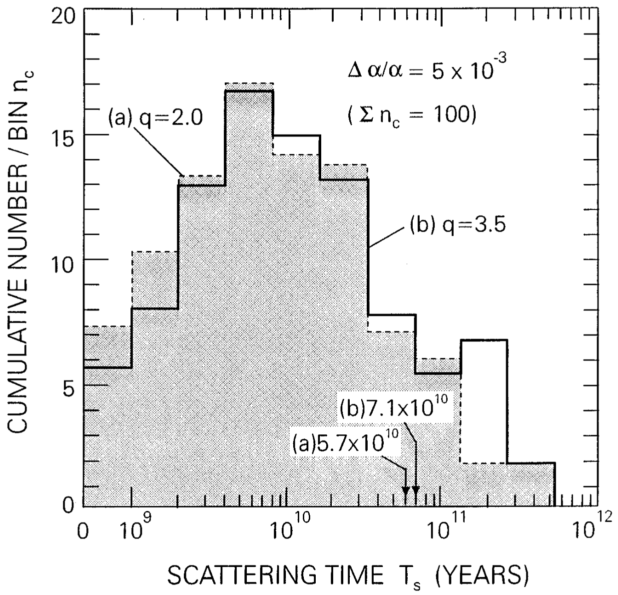

Fig. 3. Histogram of the cumulative numbers of KBOs in 2:3 resonance with Neptune de-captured by other non-resonant objects. Total number of test particles is 100 in the simulation runs obtained by setting the k-parameter to be 20 and a total population of scattering KBOs normalized to be 5×10^4. The threshold value of $\Delta\alpha/\alpha$ is assumed to be 0.5% and two cases of the size distributions are illustrated (a) q = 2.0 and (b) q = 3.5.

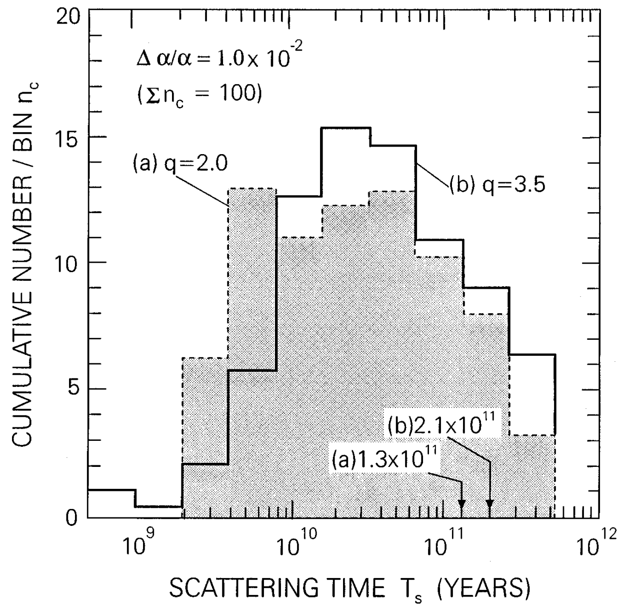

Fig. 4. Same as in Fig. 3 but with a threshold value of $\Delta\alpha/\alpha = 10^{-2}$.

years. This shows that the gravitational scattering mechanism described here is indeed an efficient process. Our results thus suggest that – in comparison with the dynamical sculpting time (T_d) of 2–3 $\times 10^{10}$ years as estimated by Duncan et al. (1995) – the scattering mechanism described here is significant in ejecting the trans-Neptunian comets into short-period orbits. This interesting new pathway must be confirmed by more elaborate numerical integration calculations, however. Finally, in our calculations, it was found that the long-term orbital behaviour of the resonant KBOs tends to be characterized by an inward drift in the semimajor axes as a result of angular momentum exchange with the objects exterior to their orbital zone. This effect further indicates the importance of mutual gravitational interaction of the KBOs in shaping the general structure of the Kuiper belt.

4. Discussion

One advantage of the present consideration is that our treatment is based on the observed orbital resonant structure of the KBOs without invoking theoretical extrapolation of their original radial distribution in the past. Following the assumption by Cochran et al. (1995) that the number of Halley-size objects trapped in 2:3 resonance with Neptune is on the order of 2×10^8, the corresponding injection rate of such bodies into chaotic orbits would be 3×10^{-3}–10^{-2} per year. The injection rate of smaller bodies with D between 1 and 10 km will be larger depending on the size distribution in this diameter range.

It is interesting to note that the gravitational scattering mechanism investigated here is of the Fernandez (1980) scenario of orbital diffusion of trans-Neptunian comets in near-circular orbits into short-period orbits due to the action of a system of lunar-sized bodies (see also Ip & Fernandez, 1991). Because of the libration motion of the resonant objects, KBOs with diameters of 200–300 km will be effective in destroying their 2:3 resonance with Neptune and transfer them immediately into short-lived orbits of chaotic nature. With the emerging picture of the orbital configuration of the KBOs this is clearly a potentially important souce mechanism of short-period comets. We hope that more comprehensive detections of the KBO population will permit a comparison of this scattering process with other dynamical effects.

Following the preset scenario, the mutual gravitational interactions of the KBOs observed by Jewitt & Luu (1995) and Williams et al. (1993) should send some of these large objects into the inner solar system. The comet-like object 2060 Chiron – with a diameter $\approx$ 150–200 km – might be just such an example.

How many of them should there be? In a separate study, we have examined the orbital evolution of KBOs and trans-neptunian objects once scattered into non-resonant orbits. In summary, it is found that the average dynamical lifetime of such objects before loss by planetary impact or ejection into interstellar orbit is on the order of 100 million years. The average dynamical lifetime (T_u) for objects with aphelia inside the orbit of Uranus (i.e. $Q < 20$ AU) is about 12 million years. The expected number of KBOs in Chiron-like orbits therefore is $N^* \approx 5 \times 10^4 \times (T_u/T_s) \approx$ 5–10 for $T_s \approx$ 3.7–7.4$-(T_u/T_s) \approx$ 5–10 for $T_s \approx$ 3.7–7.4 $\times 10^{10}$ years. The dynamical lifetime of KBOs with Q inside the orbit of Neptune is estimated to be $T_n \approx 5.6 \times 10^7$ years. The number of such transient objects leaking inward of the Kuiper belt is hence expected to be $N^{**} \approx$ 50 resulting from the resonant decapuring mechanism alone.

Finally, we note that the large KBOs and the Halley-sized objects detected by Cochran et al. (1995) could also be coupled by collisional processes even though the corresponding time scale is relatively long (Stern, 1995). Suppose that a fraction (f_1) of the HST objects can interact with the large KBOs with $D \geq 100$ km and that a projectile of 10-km diameter is sufficient to cause its catastrophic breakup, the corresponding destruction rate will be $P_{KBO} = N_{HST} \cdot P_c$ where $N_{HST} = f_1 \cdot 2 \times 10^8$ and the collisional probability per year can be estimated to be $P_c = 1 \times 10^{-22}$. D^2 with D in km (see Sect. 3). Hence, for $D \approx 100$ km, we have $P_{KBO} \approx 2.2 \times 10^{-10}\ f_1$ year^{-1}. The time scale for the destruction loss of the KBOs detected by Jewitt & Luu (1995) and Williams et al. (1993) is therefore on the order of $T_c \approx 2.9 \times 10^9/f_1$ years. The KBOs with $D \geq 100$ km are therefore relatively long-lived against collision destruction from this point of view even with $f_1 \approx 1$. This estimate is in good agreement with Stern (1995) and Farinella & Davis (1996).

It is important to point out that a population of small fragments will be implanted into the Kuiper belt after each collisional breakup event. If the size distribution of the fragments follows a power law as given above with $q \approx 3.5$ and that the maximum fragment size is half that of the target body and the minimum fragment size is 1 km, the total number of fragments produced by the breakup of a KBO with $D = 100$ km will be $N_c = 7.6 \times 10^5$. This means that, if the total number of KBOs subject to collisional interaction with the smaller objects identified by HST is given to be $N^*_{KBO} \approx f_2\ 3.8 \times 10^4$ (with $f_2 \leq 1$), the total fragment production rate will be approximately $\dot{N}_c = P_{KBO} \cdot N^*_{KBO} N_c$. We find that $\dot{N}_c \approx 10 \cdot f_1 f_2$/year just by invoking the collisional interaction of the KBO population inferred from the ground-based observations and the Halley-size objects detected by HST. With $f_1 f_2 \approx 0.1$, $\dot{N}_c \approx 1$ new "comet" per year and this shows that collisional process can in principle play a very important role in supplying the short-period comets as well as determining the mass distribution of the trans-Neptunian population with $D \leq 100$ km. A detailed treatment, however, would require the incorporation of impact physics pertinent to the production and destruction of objects of different sizes as well as the velocity distribution of the impact fragments (cf. Marzari et al. 1994; 1995; Farinella & Davis, 1996). The latter effect is of particular relevance to the transition from the stable libration motion characterized by the 2:3 resonance and the short-lived chaotic orbits discussed earlier.

5. Conclusion

In this study a novel mechanism of transporting KBOs and their collisional fragments from the trans-neptunian region to the inner solar system is proposed. The key element of this orbital transfer process depends critically on the assumption that a large number of the the KBOs and the smaller trans-neptunian objects detected by HST are trapped in 2:3 (perhaps also 3:4 and 4:5) resonances. The dynamical time scale for gravitational scattering of these resonant bodies into chaotic orbits by other "field" objects is found to be comparable with the orbital sculpting time scale estimated before by Levison & Duncan (1993), Holman & Wisdom (1993) and others. Such gravitational mechanism is therefore expected to play an important role in determining the structure of the Kuiper belt and the injection rate of short-period comets. By the same token, KBOs of large size in 2:3 resonance could also be scattered into orbits like that of 2060 Chiron. As many as 50 of such objects could exist in region beyond Jupiter's orbit. Because the resonant structure in the Kuiper belt is very complex, some of these estimates need to be verified or improved by more sophisticated computations of which we intend to pursue in future.

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