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ASTRONOMY AND ASTROPHYSICS

Semiclassical collisional functions in a non ideal plasma

N. Ben Nessib[1], **S. Sahal-Bréchot**[2], **and Z. Ben Lakhdar**[1]

[1] Laboratoire de Physique Atomique et Moléculaire, Département de Physique, Faculté des Sciences de Tunis, 1060 Campus Universitaire, Tunis, Tunisia

[2] Laboratoire " Atomes et Molécules en Astrophysique", Unité associée au C.N.R.S. No 812, Département Atomes et Molécules en Astrophysique, Observatoire de Paris-Meudon, F-92195 Meudon Cedex, France

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Abstract. Collisions between atoms (or ions) and electrons play an important role in Astrophysics for spectroscopic diagnostics and modellisation of stellar interiors and atmospheres. Plasma shielding effects due to electron and ion correlations are not negligible in the physical conditions of white dwarfs, owing to their high density. They can also play a role in the case of rather cool stars and for hydrogen lines, because their excited levels are quasi-degenerated.

The model which usually describes collective effects is the Debye-Hückel potential: the two-particle Coulomb field is shielded by the ensemble of surrounding electrons. However, for low plasma temperatures, when the electron density increases, the plasma becomes non ideal. Consequently this potential is no longer valid and the Coulomb cutoff potential is more appropriate. The Coulomb cutoff potential is in fact especially suitable for the case of a strongly non ideal plasma.

In the present paper, we calculate inelastic cross-sections by describing the electron atom interaction with a Coulomb cutoff potential and using a semiclassical perturbation approach. These cross-sections enter the expressions of the collisional linewidths in the impact approximation, and the statistical equilibrium equations leading to atomic populations for non-LTE studies. The case of cross-sections which are needed for impact polarization studies are also treated.

Key words: atomic processes – line: formation plasmas – stars: interiors – stars: dwarfs – stars: atmospheres

1. Introduction

Collisions between atoms (or ions) and electrons play an important role in Astrophysics for the interpretation of line spectra and for the modellisation of stellar interiors. Collisional profiles enter the opacities and the spectral profiles. Collisional excitation and deexcitation cross-sections averaged over appropriate velocities distributions enter balance equations (the so-called statistical equilibrium equations).These equations lead to atomic populations and then to line intensities and atomic line polarization when anisotropies are present in the plasma. Atomic coherences can also play a role in certain cases but this is outside the scope of this paper.

Correlation effects and thus plasma shielding effects are not negligible in the physical conditions of white dwarfs atmospheres, owing to their high density. They also play a role in the case of rather cool stars and for atomic transitions which are quasi-degenerated: Stark profiles of hydrogen lines for instance, or electron and proton collisional depolarization of hydrogen lines (Sahal-Bréchot et al. 1996).

In the standard formalism of Stark impact broadening of spectral lines and of cross sections, the electrostatic Coulomb potential is used for describing the interaction between the perturbing electrons and the emitting atom. Electronic correlations (screening effects) are usually taken into account by introducing a cutoff in the interaction when the electron-atom distance exceeds the Debye radius R_D. A more consistent treatment was proposed by Cooper et al. (1971) by using the Debye-Hückel potential. It is often a good approximation for high temperature and low density plasmas (weakly non ideal plasmas), while for the opposite case of low temperature and high density (strongly non ideal plasmas) the Coulomb cutoff potential or the ionsphere potential are more appropriate (Stewart & Kedar 1966). These potentials which can be written as the Coulomb term with correcting terms, are widely used in the literature. With the first one correcting term, we get the continuum lowering potential (Suchy 1964 and Kraeft et al. 1983). With two correcting terms, this gives rise also to spectral line shifts (Skupsky 1980, Nguyen et al. 1986, Salzmann & Szichman 1987).

Dimitrijević et al. (1989) used the potential with the first one correcting term, to calculate semiclassical collisional line widths and shifts in the adiabatic limit.

In the present paper we obtain new analytic expressions of the non adiabatic semiclassical collisional functions which generalize the standard ones to a strongly non ideal plasma by

Send offprint requests to: S. Sahal-Bréchot (sahal@obspm.fr)

using the Coulomb cutoff potential. We will take account the first correcting term only.

2. Theory

We will study two cases where electron impact (or ion-impact) semiclassical collisional functions are needed:

1. Total inelastic cross-sections entering the expressions of the impact width of isolated (non hydrogenic) lines and the coefficients of the statistical equilibrium equations (balance equations) leading to atomic populations for non-LTE studies.

2. Inelastic cross-sections for impact polarization studies (cross-sections for creation of alignment for instance).

2.1. Total inelastic cross-sections and collisional impact broadening of isolated lines

Within the impact approximation the profile is lorentzian for isolated lines. Overlapping lines are outside the present study. For the line corresponding to the transition between the initial level i and the final level f, the half half-width w and the shift d are given by the Baranger's formula (1958):

$$w + i\,d = N_{\mathrm{P}} \int_0^{\infty} v f(v) \mathrm{d}v \int_0^{\infty} 2\pi\,\rho\,\mathrm{d}\rho \times \left\{1- < i \mid S \mid i >< f \mid S^{-1} \mid f >\right\}_{\mathrm{AV}} \tag{1}$$

N_P is the density of the perturbers, S is the scattering matrix obtained for the atom-perturber interaction corresponding to the impact parameter ρ and the relative velocity v, $f(v)$ is the relative atom-perturber Maxwell distribution of velocities and $\{..\}_{\mathrm{AV}}$ is the angular average over the magnetic quantum numbers which will not be detailed here.

For the transition between the levels i $(n_i l_i L_i S J_i)$ and f $(n_f l_f L_f S J_f)$, the total width at half intensity $W = 2w$ can be put under the form (Sahal-Bréchot 1969a, Sahal-Bréchot 1969b, Sahal-Bréchot 1974):

$$W = 2w = N \int_0^{\infty} v f(v) \mathrm{d}v \left(\sum_{j \neq i} \sigma_{ij}(v) + \sum_{j' \neq f} \sigma_{fj'}(v) + \sigma_{el} \right) \tag{2}$$

j, j' are the perturbing levels.

The elastic contribution to the width σ_{el} does not concern the present paper. The inelastic cross-sections $\sigma_{ij}(v)$ (resp. $\sigma_{fj'}(v)$) are given by an integration over the impact parameter ρ of the transition probabilities $P_{ij}(v,\rho)$ (resp. $P_{fj'}(v,\rho)$) as:

$$\sum_{j \neq i} \sigma_{ij}(v) = \pi R_1^2 \sum_{j \neq i} P_{ij}(v, R_1) + \int_{R_1}^{R_D} 2\pi\rho \mathrm{d}\rho \sum_{j \neq i} P_{ij}(v, \rho) \tag{3}$$

The perturbation theory used for the derivation of the S-matrix leads to a divergence in the integration over the impact parameter: a lower cutoff R_1 is required. In fact, for high densities or for very small energy differences, an upper cutoff R_D (the Debye radius $R_{\mathrm{D}} = \left(\frac{kT}{4\pi N_{\mathrm{e}} e^2}\right)^{\frac{1}{2}}$) is also introduced in order to take into account the Debye shielding.

The expression for P_{ij} (resp. $P_{fj'}$) is given within the second order time dependent perturbation theory by an average over the initial Zeeman states M_i and a sum over the final states M_j (Seaton 1962):

$$P_{ij}(v,\rho) = \frac{1}{2J_i + 1} \sum_{M_i, M_j} \frac{1}{\hbar^2} \times \left| \int_{-\infty}^{+\infty} \langle n_i l_i J_i M_i |\, V(t)\, | n_j l_j J_j M_j \rangle\ \mathrm{e}^{\frac{i(E_j - E_i)t}{\hbar}}\ \mathrm{d}t \right|^2 \tag{4}$$

where $V(t)$ the interaction potential between the atom and the charged perturber moving along a classical path at time t. E_i (resp. E_j) is the energy of the i (resp. j) level.

This semi-classical description has been extensively used for Stark broadening calculations and for collisional transition probabilities entering the statistical equilibrium equations for non LTE studies.

2.2. Impact polarization of Zeeman sublevels

The collisional transition probability between the sublevels i $(n_i l_i L_i S J_i M_i)$ and j $(n_j l_j L_j S J_j M_j)$, taking into account the exact energy separation in zero-magnetic field $E_j - E_i$ is given by Sahal-Bréchot et al. (1996). The transition probability between Zeeman sublevels is expressed as follows:

$$P(n_i l_i J_i M_i \to n_j l_j J_j M_j,\ v,\ \theta,\ \rho) = \frac{1}{\hbar^2} \times \left| \int_{-\infty}^{+\infty} \langle n_i l_i J_i M_i |\, V(t)\, | n_j l_j J_j M_j \rangle\ \mathrm{e}^{\frac{i(E_j - E_i)t}{\hbar}}\ \mathrm{d}t \right|^2 \tag{5}$$

θ is the angle between the direction of the relative atom-perturber velocity $\boldsymbol{v}$ and the quantization axis in the laboratory frame centered on the radiating atom (cf. Fig. 1 of Sahal-Bréchot et al. 1996)

The cross-section follows by an integration over the impact parameter ρ:

$$\sigma(n_i l_i J_i M_i \to n_j l_j J_j M_j, v, \theta) = 2\pi \int_0^{\infty} P(n_i l_i J_i M_i \to n_j l_j J_j M_j, v, \theta, \rho) \rho \mathrm{d}\rho \tag{6}$$

Indeed, appropriate cutoffs are also required.

3. Not correlated semiclassical collisional functions

3.1. Not correlated functions for the total cross-sections

In the standard treatment, the atom-perturber interaction V is the electrostatic Coulomb potential between the N atomic electrons, (with $N = 1$ for hydrogen), the nucleus of charge ($Z+N$)

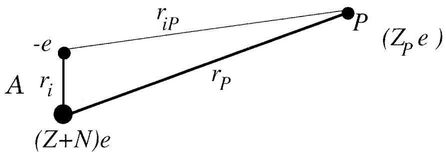

Fig. 1. The system atom A + perturber P

(with $Z = 0$ for a neutral) and the perturber of charge Z_P (+1 for a proton and -1 for an electron)

$$V = \frac{Z_P\, e^2 (Z+N)}{r_P} - Z_P\, e^2 \sum_{i=1}^{N} \frac{1}{r_{iP}} \tag{7}$$

The coordinates are defined on Fig. 1.

$\frac{1}{r_{iP}}$ is expanded in multipolar components and only the long range part is retained in the perturbation theory:

$$V = \frac{Z Z_P e^2}{r_P} - \sum_{\lambda=1}^{\infty} \frac{4\pi\, Z_P\, e^2}{2\lambda+1} \frac{1}{r_P^{\lambda+1}} \times \sum_{\mu=-\lambda}^{+\lambda} \sum_{i=1}^{N} r_i^{\lambda}\, Y_{\lambda\mu}(\hat{r}_P)\, Y^*_{\lambda\mu}(\hat{r}_i) \tag{8}$$

The first term of that expansion is the Coulomb term (it is zero for a neutral atom) and does not play any role in the calculation of the cross-section due to its spherical symmetry. The following ones have only to be retained. And in addition we only retain the dipole term ($\lambda = 1$) for the calculation of the cross-sections between the levels that are dipolar electric transitions:

$$V_{\mathrm{dipolar}} = -\frac{4\pi\, Z_P\, e^2}{3} \frac{1}{r_P^2} \sum_{\mu=-1}^{+1} Y_{1\mu}(\hat{r}_P) \sum_{i=1}^{N} r_i\, Y^*_{1\mu}(\hat{r}_i) \tag{9}$$

We use the parametrization of the straight path trajectory in the collision frame. The quantization axis is along the direction of the relative velocity. (cf. Fig. 2 of Sahal-Bréchot et al. 1996):

$$\begin{cases} r_P = \sqrt{\rho^2 + v^2 t^2} = \dfrac{\rho}{\sin\theta_P} \\ x_P = \rho\cos\phi_P \\ y_P = \rho\sin\phi_P \\ z_P = vt = r_P\cos\theta_P \end{cases} \tag{10}$$

This parametrization of the trajectory yields to:

$$Y_{10}(\hat{r}_P) = \sqrt{\frac{3}{4\pi}}\cos\theta_P = \sqrt{\frac{3}{4\pi}} \frac{vt}{\sqrt{\rho^2+v^2t^2}} \tag{11}$$

and

$$Y_{1\pm1}(\hat{r}_P) = \mp\sqrt{\frac{3}{8\pi}}\sin\theta_P\ \mathrm{e}^{\pm i\phi_{\mathrm{P}}} = \mp\sqrt{\frac{3}{8\pi}} \frac{\rho}{\sqrt{\rho^2+v^2t^2}}\ \mathrm{e}^{\pm i\phi_{\mathrm{P}}} \tag{12}$$

Using Eq. (9), the expression (4) becomes

$$P_{ij}(v,\rho) = \frac{1}{3}\frac{Z_P^2 e^4}{\hbar^2}\frac{4\pi}{3} R_{line}^2 R_{mult}^2 l_> I^2 \sum_{\mu=0,\pm1} |J_{1\mu}|^2 \tag{13}$$

where

$$J_{1\mu} = \int_{-\infty}^{+\infty} \mathrm{e}^{i\omega_{ij}t} \frac{Y_{1\mu}(\hat{r}_P)}{r_P^2}\mathrm{d}t \tag{14}$$

R_{line} and R_{mult} are defined in Shore & Menzel (1968).

$l_> = max(l_i, l_j)$, $\omega_{ij} = \dfrac{E_j - E_i}{\hbar}$ and

$I = \displaystyle\int_0^{\infty} R_{n_i l_i}(r) R_{n_j l_j}(r) r \mathrm{d}r$ is the radial integral.

Analytical calculations which have be done elsewhere (Seaton 1962; Griem 1962 and Sahal-Bréchot 1969a) give:

$$J_{10} = \sqrt{\frac{3}{4\pi}} \frac{2}{\rho v} i z K_0(z) \tag{15}$$

and

$$J_{1\pm1} = \mp\mathrm{e}^{\pm i\phi_{\mathrm{P}}} \sqrt{\frac{3}{8\pi}} \frac{2}{\rho v} z K_1(z) \tag{16}$$

where $z = \dfrac{\rho\omega_{ij}}{v}$, K_0 and K_1 are Bessel modified functions.

So that

$$\sum_{\mu=0,\pm1} |J_{1\mu}|^2 = \frac{3}{\pi\rho v} A(z) \tag{17}$$

We obtain the so called $A(z)$ function:

$$A(z) = z^2[K_o^2(|\ z\ |) + K_1^2(|\ z\ |)] \tag{18}$$

and

$$P(n_i l_i J_i \rightarrow n_j l_j J_j, v, \rho) = P_{ij}(v,\rho) = \frac{4 I_H^2}{E(E_j - E_i)} \frac{m}{m_e} f(n_i l_i J_i \rightarrow n_j l_j J_j) \frac{a_0^2}{\rho^2} A(z) \tag{19}$$

where $f(n_i l_i J_i \rightarrow n_j l_j J_j)$ is the oscillator strength, I_H the ionization energy of hydrogen, a_0 the Bohr radius, m_{e} the electron mass and E the energy of the perturber of reduced mass m. After integration of the transition probability over the impact parameter, the total cross-section is given by the expression

$$\sigma(n_i l_i J_i \rightarrow n_j l_j J_j, v) = \pi R_1^2 P(n_i l_i J_i \rightarrow n_j l_j J_j, v, R_1) + \pi a_0^2 Z_P^2\ \frac{8 I_H^2}{E(E_j - E_i)} \frac{m}{m_e} \times f(n_i l_i J_i \rightarrow n_j l_j J_j)[a(z_1) - a(z_D)] \tag{20}$$

where $z_1 = \dfrac{R_1\omega_{ij}}{v}$ and $z_{\mathrm{D}} = \dfrac{R_{\mathrm{D}}\omega_{ij}}{v}$.

The integration of $A(z)/z$ over the impact parameter introduces the so called $a(z)$ function:

$$a(z) = \int_z^{\infty} \frac{A(z')}{z'}\mathrm{d}z' = z K_0(z) K_1(z) \tag{21}$$

3.2. Not correlated impact polarization functions

The transition probability between Zeeman sublevels with $l_j = l_i \pm 1$ in the atomic frame can be expressed as follows (Sahal-Bréchot et al. 1996):

- For $M_i = M_j$

$$P(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j, v, \theta, \rho) = P_{oo}(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j) \times \left[A_o(z) cos^2\theta + A_\pm(z) sin^2\theta\right] \quad (22)$$

- For $M_i = M_j \pm 1$

$$P(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j, v, \theta, \rho) = P_{oo}(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j) \times \left[A_o(z) \frac{sin^2\theta}{2} + A_\pm(z) \frac{1 + cos^2\theta}{2}\right] \quad (23)$$

where $P_{oo}(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j)$ is the algebraic parameter

$$P_{oo}(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j) = Z_P^2 \frac{12 I_H^2}{E(E_j - E_i)} \frac{m}{m_e} \frac{a_0^2}{\rho^2} \times (2J_i + 1) f(n_i l_i J_i \rightarrow n_j l_j J_j) \begin{pmatrix} J_i & 1 & J_j \\ -M_i & \mu & M_j \end{pmatrix}^2 \quad (24)$$

In the above 3-j coefficient, $\mu = M_i - M_j$ due to the selection rules. The functions $A_o(z)$ and $A_\pm(z)$ are:

$$A_o(z) = z^2 \mid K_o(z) \mid^2 \quad (25)$$

and

$$A_\pm(z) = \frac{1}{2} z^2 \mid K_1(z) \mid^2 \quad (26)$$

We find again the total transition probability between the levels $(n_i l_i J_i)$ and $(n_j l_j J_j)$ by summing over the final states M_j and averaging over the initial states M_i:

$$P(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j, v, \rho) = P_{oo}(n_i l_i J_i M_i \rightarrow n_j l_j J_j M_j) A(z) \quad (27)$$

The semiclassical collisional function $A(z)$, can be written as:

$$A(z) = A_o(z) + 2A_\pm(z) \quad (28)$$

To obtain the cross-section between two Zeeman sublevels, we have to integrate the above expression over the impact parameter. We obtain functions which are specific to impact polarization:

$$a_o(z) = \frac{1}{2} z^2 \left[K_1^2(z) - K_o^2(z)\right] \quad (29)$$

and

$$a_\pm(z) = \frac{1}{4} \left\{z^2 \left[K_o^2(z) - K_1^2(z)\right] + 2z K_o(z) K_1(z)\right\} \quad (30)$$

As for $A(z)$, $a(z)$ can be put under the form:

$$a(z) = a_o(z) + 2a_\pm(z) \quad (31)$$

The function for the transition probability for creation of alignment is denoted by $A_2(z)$ (Sahal-Bréchot et al. 1996):

$$A_2(z) = 2\left[A_\pm(z) - A_o(z)\right] = z^2 \left[K_1^2(z) - 2K_o^2(z)\right] \quad (32)$$

whereas the function for the cross-section for creation of alignment is denoted by $a_2(z)$:

$$a_2(z) = 2[a_\pm(z) - a_o(z)] = z K_o(z) K_1(z) - \frac{3}{2} z^2 \left[K_1^2(z) - K_o^2(z)\right] \quad (33)$$

4. Correlated semiclassical collisional functions

4.1. Interaction potential for non ideal plasmas

The non-ideality factor γ is defined as the ratio of the mean interaction potential energy between charged particles to their kinetic energy

$$\gamma = Ze^2 \frac{(2N_e)^{1/3}}{kT} = 2.1\ 10^{-3} Z \frac{\left[N_e(\mathrm{cm}^{-3})\right]^{1/3}}{T(\mathrm{K})} \quad (34)$$

The number of particles in the Debye sphere N_D is defined by

$$N_D = \left(\frac{4}{3}\right) \pi R_D^3 2N_e \quad (35)$$

where N_e is the electronic density. For a weakly non ideal plasma, $0.1 \lesssim \gamma \lesssim 0.2$ and $1 \lesssim N_D \lesssim 10$. For a strongly non ideal plasma $\gamma \gtrsim 0.5$ and $N_D \lesssim 1$.

In the case of a weakly coupled plasma, the atom-perturber Debye-Hückel potential is appropriate:

$$V_D(t) = \left(\frac{Z_P e^2 (Z + N)}{r_P} - Z_P e^2 \sum_{i=1}^{N} \frac{1}{r_{iP}}\right) \exp\left(-\frac{r_P}{R_D}\right) \quad (36)$$

where R_D is the electronic Debye radius, Cooper et al.(1971) found that the A and a functions become in the dipolar long range approximation:

$$A^D(z) = z^2 K_o^2(\beta) + \beta^2 K_1^2(\beta) \quad (37)$$

and

$$a^D(z) = \beta K_o(\beta) K_1(\beta) - \frac{q^2}{2} \left[K_1^2(\beta) - K_o^2(\beta)\right] \quad (38)$$

where $q = \frac{z}{z_D}$ with $z_D = \frac{R_D \omega_{ij}}{v}$ and $\beta = \sqrt{z^2 + q^2}$. In the non shielding limit ($q \rightarrow 0$), these functions are also reduced to the well know expressions of $A(z)$ and $a(z)$. These correlated functions are of greater utility, especially if q should gets large ($q \geq 0.1$).

However, in the case of strongly coupled plasmas (or strongly non ideal plasma), the Debye-Hückel potential is not valid. We now consider the so-called "Coulomb cutoff potential" which is defined for the atom-perturber interaction as follows:

$$\begin{cases} V_c(t) = \dfrac{Z_P e^2 (Z+N)}{r_P} - \\ \qquad Z_P e^2 \sum\limits_{i=1}^{N} \frac{1}{r_{iP}} \left[1 - \frac{r_P}{R_c}\right] \quad \text{for } r_P < R_c \\ V_c(t) = 0 \quad \text{for } r_P > R_c \end{cases} \tag{39}$$

And if only the dipolar long range part is retained in the perturbation theory:

$$\begin{cases} V_c(t) = -\dfrac{4\pi Z_p e^2}{3} \sum\limits_{\mu=0,\pm1} \left[\dfrac{Y_{1\mu}(\hat{r}_P)}{r_P^2} - \dfrac{Y_{1\mu}(\hat{r}_P)}{R_c r_P}\right] \times \\ \qquad \sum\limits_{i=1}^{N} r_i Y_{1\mu}^*(\hat{r}_i) \quad \text{for } r_P < R_c \\ V_c(t) = 0 \quad \text{for } r_P > R_c \end{cases} \tag{40}$$

where R_c is the cutoff parameter assumed to be equal to the ion sphere radius $R_c = \left(\dfrac{3Z}{4\pi N_e}\right)^{1/3}$ (Scheibner et al. 1987).

The semiclassical collisional functions for isolated neutral lines have to be revised using the cutoff potential instead of the Coulomb or the Debye ones. This is the principal aim of this work and for that new functions are obtained.

4.2. Validity conditions

The condition of validity of the impact approximation is (Baranger 1958):

$$\frac{4}{3}\pi\rho_{typ}^3 << \frac{1}{N} \tag{41}$$

This means that the collision volume must be very small compared to the inverse of the density of the perturber. If the plasma is strongly non ideal (high density and low temperature) the ρ_{typ} is of the order of the thermal de Broglie length $\lambda\!\!\!^{-} = \dfrac{\hbar}{m_e \bar{v}}$ where $\bar{v} = \sqrt{\dfrac{8kT}{\pi m_e}}$ is the mean thermal electron velocity and T is the electron temperature.

Thus, for strongly non ideal plasmas, the impact approximation can be written as follows:

$$T N_e > \left(\frac{4\pi}{3}\right)^{23} \frac{\pi\hbar^2}{8 m_e k} \tag{42}$$

If the temperature is expressed in Kelvin and the density in cm^{-3}, this condition becomes $T N_e > 9 \quad 10^{-12}$

The validity condition for strongly non ideal plasma ($\gamma >$ 0.5) can be written as a function of the electron density and temperature as $T N_e^{-13} < 4.2 \quad 10^{-3}$ with the same units as above.

This permits us to construct a diagram which represents different plasma conditions (Fig. 2). The impact approximation is valid on the left of curve (1) (dotted region). this shows that there is a region where the impact approximation is valid for a strongly non ideal plasma.

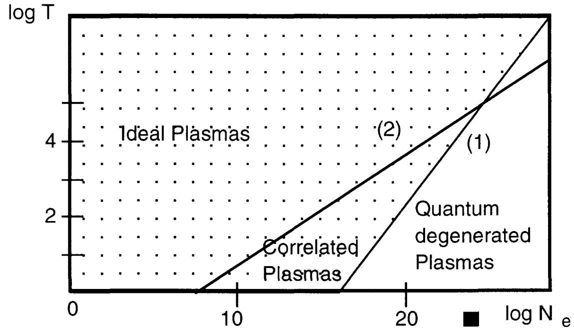

Fig. 2. Different plasmas according to temperature and density

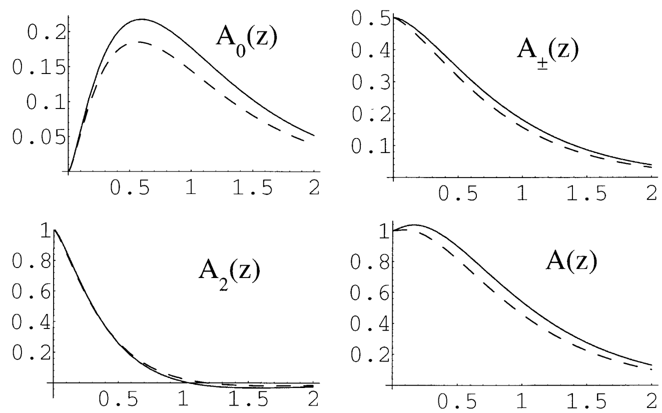

Fig. 3. Collision functions for the transition probability. Full lines: not correlated functions $A_0(z)$, $A_\pm(z)$, $A_2(z)$ and $A(z)$. Dashed lines: correlated functions $A_0^c(z)$, $A_\pm^c(z)$, $A_2^c(z)$ and $A^c(z)$ for $z_c = 20$.

4.3. Correlated functions for total cross-sections

For obtaining the transition probability, the $J_{1\mu}$ function in Eq. (14) has to be changed by the correlated function $J_{1\mu}^c$ given by:

$$J_{1\mu}^c = \int_{-\infty}^{+\infty} e^{i\omega_{if} t} \left[\frac{Y_{1\mu}(\hat{r}_P)}{r_P^2} - \frac{Y_{1\mu}(\hat{r}_P)}{R_c r_P}\right] \mathrm{d}t \tag{43}$$

To be rigourous, the above expression should be integrated from $-t_c$ to $+t_c$, because the potential is equal to zero for $r_P > R_c$. Nevertheless, we have decided to integrate from $-\infty$ to $+\infty$ as in the standard treatment. This will be compensated by introducing an upper cutoff at ρ_c in the integration over the impact parameter.

After calculations, the preceding $A(z)$ function becomes the correlated function $A^c(z)$:

$$A^c(z) = A(z) - \sqrt{2}\pi \frac{z^2}{z_c} \mathrm{e}^{-z} \left[K_0(z) + K_1(z)\right] + \pi^2 \frac{z^2}{z_c^2} \mathrm{e}^{-2z} \tag{44}$$

where $z_c = \dfrac{R_c \omega_{ij}}{v}$.

By integrating $A^c(z)/z$ over z, we obtain the correlated function $a^c(z)$ which replaces $a(z)$:

$$a^c(z) = a(z) - \sqrt{2}\pi \frac{z}{z_c} \mathrm{e}^{-z} K_1(z) + \frac{\pi^2}{4 z_c^2}(1+2z)\mathrm{e}^{-2z} \tag{45}$$

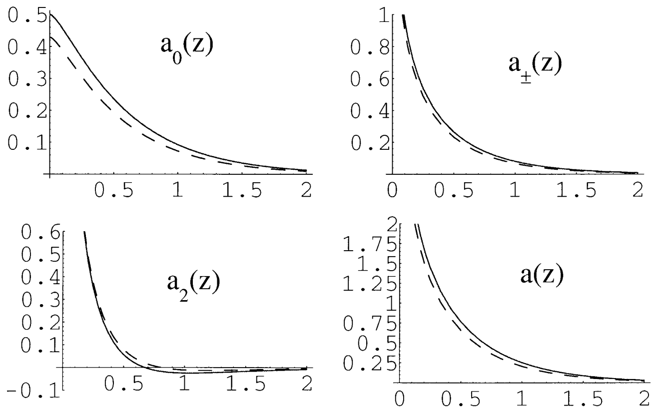

Fig. 4. Collision functions for the cross section. Full lines: not correlated functions $a_0(z)$, $a_\pm(z)$, $a_2(z)$ and $a(z)$. Dashed lines: correlated functions $a_0^c(z)$, $a_\pm^c(z)$, $a_2^c(z)$ and $a^c(z)$ for $z_c = 20$.

4.4. Correlated impact polarization functions

By using the potential V_c and after calculations, we obtain that $A_o(z)$, $A_\pm(z)$ and $A_2(z)$ have to be replaced by the corresponding correlated functions:

$$A_o^c(z) = \left[zK_o(z) - \frac{\pi z}{\sqrt{2} z_c} \mathrm{e}^{-z} \right]^2 \tag{46}$$

$$A_\pm^c(z) = \frac{1}{2} \left[zK_1(z) - \frac{\pi z}{\sqrt{2} z_c} \mathrm{e}^{-z} \right]^2 \tag{47}$$

and

$$A_2^c(z) = A_2(z) - \frac{\pi^2 z^2}{2 z_c^2} \mathrm{e}^{-2z} - \frac{\sqrt{2} \pi z^2}{z_c} \mathrm{e}^{-z} \left[K_1(z) - 2K_0(z) \right] \tag{48}$$

Integration over z gives the new functions:

$$a_0^c(z) = a_0(z) + \frac{\pi^2}{8 z_c^2} (1 + 2z) \mathrm{e}^{-2z} - \frac{\sqrt{2} \pi z \mathrm{e}^{-z}}{z_c} \left\{ K_1(z) - \frac{1}{3} z \left[K_2(z) - K_1(z) \right] \right\} \tag{49}$$

$$a_\pm^c(z) = a_\pm(z) + \frac{\pi^2}{16 z_c^2} (1 + 2z) \mathrm{e}^{-2z} - \frac{\sqrt{2} \pi z^2 \mathrm{e}^{-z}}{6 z_c} \left[K_2(z) - K_1(z) \right] \tag{50}$$

and

$$a_2^c(z) = a_2(z) - \frac{\pi^2}{16 z_c^2} (1 + 2z) \mathrm{e}^{-2z} + \frac{2\sqrt{2} \pi z \mathrm{e}^{-z}}{z_c} K_1(z) \tag{51}$$

The Figs. 3 and 4, show an example of the behaviour of these correlated functions.

When z_c becomes infinite (no correlations), these correlated collision functions $A_0^c(z)$, $A_\pm^c(z)$, $A_2^c(z)$, $A^c(z)$, $a_0^c(z)$, $a_\pm^c(z)$, $a_2^c(z)$ and $a^c(z)$ are reduced to the not correlated collision functions, which is expected.

5. Conclusion

By using a Coulomb cutoff potential, we have calculated new semiclassical functions. When comparing them to the standard ones, we find that the screening leads to lower the semiclassical collisional functions (cf. Fig. 2 and Fig. 3). These functions can now be used for calculating collisional widths and cross-sections for populations and polarization in the place of the standard ones when the plasma is strongly non ideal.

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