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LETTER

Letter to the Editor

Inhibition of turbulent transfer in self-gravitating flows below the Jeans length

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Abstract. Turbulent flows within clouds with Jeans numbers of order unity are observed in the interstellar medium. In the absence of self-gravitation, compressible turbulent flows steepen, form shocks, and hence dissipate rapidly. We show here that, for wavelengths immediately below the gravitational instability threshold, this nonlinear steepening does not occur, and dissipation is drastically reduced. This result might alleviate somewhat the problem of feeding the turbulence in marginally stable clouds (with Jeans number slightly below unity).

Key words: Gravitation – Hydrodynamics – Turbulence – Interstellar matter: kinematics and dynamics – Methods: numerical

1. Introduction

The interstellar matter in molecular clouds shows a quasi-steady turbulent state (Larson, 1981, Falgarone and Pérault 1987). Feeding this turbulence is a problem, in view of the rapid energy dissipation of compressible flows in shocks (Shu et al 1987). Magnetic fields could possibly reduce the energy requirement by reducing the dissipation rate (Kraichnan 1967). In this letter, we indicate another possible solution valid in the framework of hydrodynamics, by showing that spectral transfer and dissipation of waves with wavelength near the instability threshold are reduced or even stopped by the dispersive effect of self-gravitation. We show this by phenomenological arguments as well as by numerical simulations. We finally discuss the applicability of this result to interstellar clouds.

2. Phenomenology of dispersive effects and nonlinearities

A well-known example of inhibition of nonlinear transfer by dispersive terms is given by the Korteweg-DeVries (KdV) equation, $\partial u/\partial t + u\partial u/\partial x + \partial^3 u/\partial x^3 = 0$. In that case, initial fluctuations begin to steepen but as soon as modes with large enough wavenumbers k are generated, they escape from the shock forming region, because the difference in the phase velocities v_ϕ of two neighbouring interacting modes, say, $v_\phi(2k) - v_\phi(k)$, becomes larger than the velocity fluctuation u itself, so that finally the velocity jump does not become thinner than a characteristic length $1/k*$, and the shock disperses itself. Since the phase velocity difference increases with wavenumber as k^2, the dispersion inhibits the growth of all modes with k larger than $k*$. In the Navier-Stokes equations with self-gravitation, the gravity also introduces dispersion in the equations; the (Jeans) dispersion relation is $\omega^2 = k^2c^2 - \Omega^2$ (c being the sound speed, ρ_0 the density and $\Omega = (4\pi G\rho_0)^{1/2}$ the inverse of the free-fall time), and the phase velocity is (Fig. 1a):

$$v_\phi = c(1 - \kappa^{-2})^{1/2} \quad (1)$$

where κ is the wave number expressed in units of the Jeans wave number $k_J = \Omega/c$. One sees that the dispersion is maximal not at small scales as in the KdV case, but just below the Jeans length. Hence, if gravitation is to play a role comparable to that due to the dispersive term in the KdV equations, it is in a wave band just below the Jeans length, and instead of inhibiting the end of the shock formation process, it will prevent the onset of steepening of fluctuations with $k \leq k*$. The critical wavenumber $k*$ bounding this “frozen” range is estimated as above by $v_\phi(2k) - v_\phi(k) = u$ (where u is the characteristic wave amplitude) or:

$$M = (1 - 1/(2\kappa)^2)^{1/2} - (1 - 1/(\kappa)^2)^{1/2} \quad (2)$$

where $M = u/c$ is the Mach number. One sees from Fig. 1b that $\kappa*$ decreases rapidly with increasing Mach number. For instance, when $M = 0.1$, Eq. (2) predicts that $\kappa* = 2$, i.e., the frozen wave band fills just one octave $(k_J, 2k_J)$.

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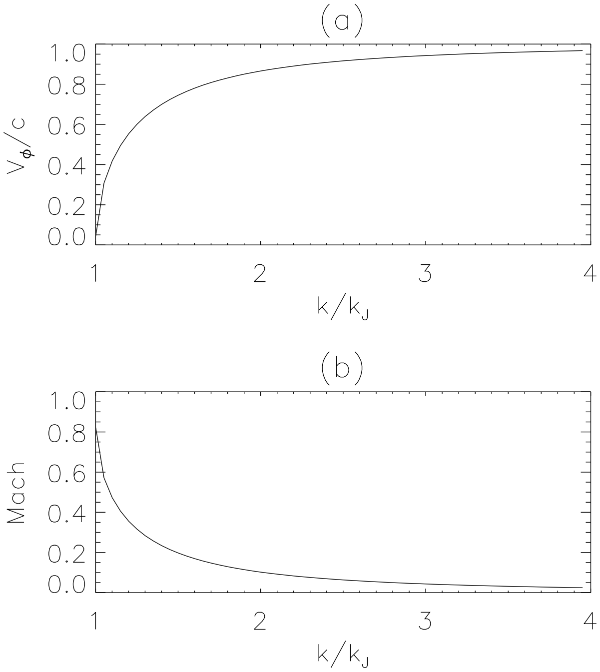

Fig. 1. Dispersive properties of gravito-acoustic waves in the stable range ($k \geq k_J$). (a) phase velocity versus wavenumber. (b) Maximum Mach number for which the dispersive term should inhibit wave steepening, versus wavenumber.

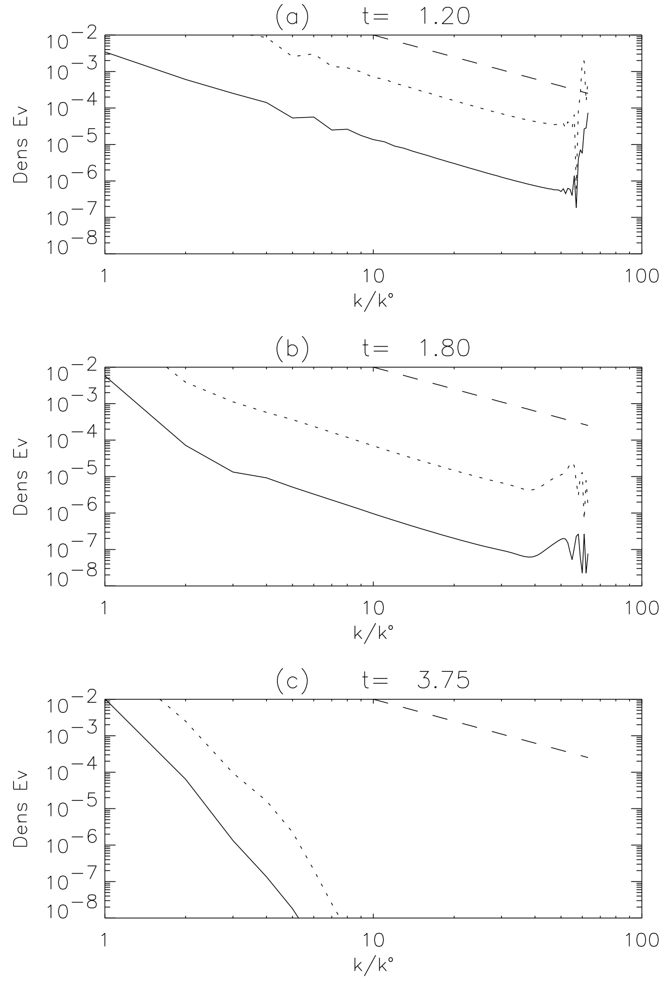

Fig. 2. Energy spectra resulting from the evolution of a plane acoustic wave, at three Jeans number, M_0 = 0.1. Continuous: density; dotted: kinetic energy; dashed line: reference k^{-2} spectrum. (a) J = 0, t = 1.2 (b) $J = \sqrt{0}.5$, t = 1.8 (c) $J = \sqrt{0}.8$, t = 3.75.

3. Numerical results: one-dimensional acoustic waves

To check numerically these phenomenological ideas, we consider one- and two-dimensional gravitoacoustic waves. We integrate the compressible Navier-Stokes equations in the isothermal case with periodic boundary conditions, the gravitational acceleration $\boldsymbol{g}$ being computed from $div\boldsymbol{g} = -4\pi G(\rho - \rho_0)$, ρ_0 being the average density of the medium. We use a pseudo-spectral method, as in (Léorat Passot and Pouquet 1990). Varying the Mach number based on the initial wave amplitude and the Jeans number $J = k_J/k_0$ associated with some wave number k_0, one should be able to determine numerically the parameter region in which transfer is inhibited. We restrict here ourselves to three values of the Jeans number, J = 0 (no gravitation), $J = \sqrt{0}.5$, and $\sqrt{0}.8$. Consider first the inviscid evolution of a one-dimensional plane progressive wave with velocity u_x and density ρ:

$$u_x = U_0 sin(k_0 x), \rho = 1 + M_0 sin(k_0 x) \quad (3)$$

where $M_0 = U_0/c$ = 0.1 is the initial Mach number. The time unit is the nonlinear time $1/(k_0 U_0)$. Fig. 2a presents for reference the case with no gravitation (J = 0): a k^{-2} kinetic energy spectrum forms after about one nonlinear time, $t \simeq 1$. After that time, the calculation looses physical meaning, as the scales near the grid size become excited significantly. With $J = \sqrt{0}.5$, the -2 slope forms later on, around $t \simeq 1.8$ (Fig. 2b). Notice that the k^{-2} spectrum has a much lower level than previously and starts only at $k = 3k_0$. Thus, the critical wave number is $\kappa* = 3/J \simeq 4$, i.e., the frozen wave band is twice as large as predicted by Eq. (2). With a still larger Jeans number, $J = \sqrt{0}.8$, no excitation larger than 10^{-8} appears at $k \geq 8$, in spite of the absence of diffusion, even after about four turnover times (Fig. 2c). Fig. 3 shows the corresponding oscillations of density and velocity at a given point for $J = \sqrt{0}.5$ and $\sqrt{0}.8$. The relative amplitude increase of the density oscillations is due to the decrease in phase speed with increasing J (since for linear eigenmodes $\delta\rho/\rho = \pm\delta u/v_\phi$). For $J = \sqrt{0}.8$ the density oscillations are seen to remain quasi-monochromatic.

4. Numerical results: two-dimensional waves

In two- or three-dimensional flows, the phenomenology presented above might no longer apply, due in particular to the presence of a solenoidal component. We shall consider the following initial configuration:

$$U_x = U_0 sin(k_0 x), U_y = U_0 sin(k_0 y), \rho = 1 + M_0 sin(k_0 x) \quad (4)$$

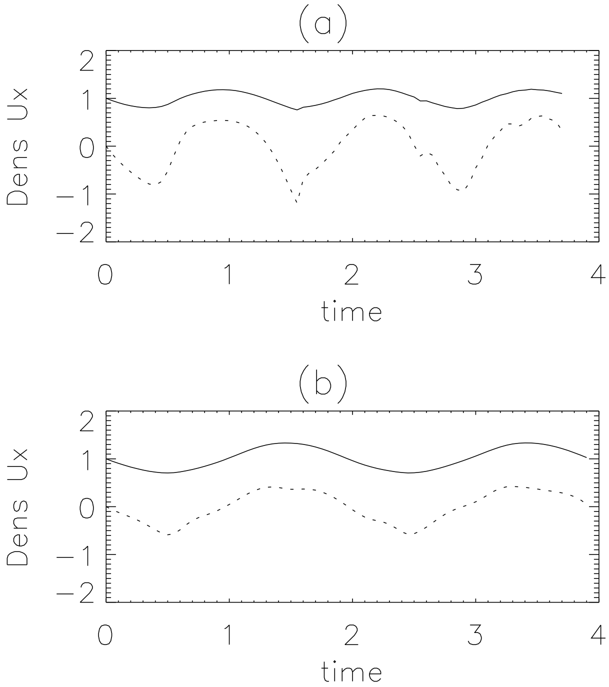

Fig. 3. Temporal evolution of the density (continuous) and velocity (dotted) at a given point, starting with a plane wave, $Mach$ = 0.1 (a) $J = \sqrt{0}.5$ (b) $J = \sqrt{0}.8$.

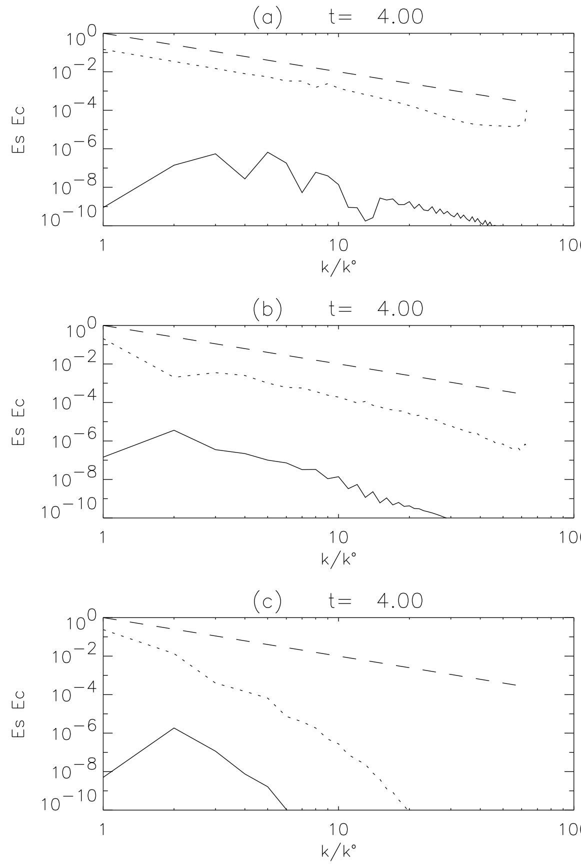

Fig. 4. Energy spectra resulting from the evolution of two-dimensional acoustic waves with modified dissipation. Time $t = 4$. Continuous: solenoidal velocity spectrum; dotted: potential velocity spectrum; dashed: reference k_2 spectrum. (a) $J = 0$ (b) $J = \sqrt{0}.5$ (c) $J = \sqrt{0}.8$.

Such perturbations generate plane waves in the x and y directions, without vorticity. Note however that there is always some vorticity generated by the numerical code, due to numerical errors: this provides for an initial noise which may be amplified by nonlinear interactions with the mother wave (4), as we will see. We integrate the two-dimensional Navier-Stokes isothermal equations with again successively $J = 0$, $\sqrt{0}.5$ and $\sqrt{0}.8$, and $M_0 = 0.1$, which (see Eq. (4)) corresponds to a mean Mach number $\simeq 0.14$ (thus larger than in the one-dimensional case). One observes that with the initial conditions (4) it is no longer possible to integrate the equations without viscous damping, as the excitation at the grid scale reaches rapidly a level much higher than in the one-dimensional case. To measure the eventual transfer reduction in good conditions, we clearly have to eliminate the grid scale excitation. A standard diffusive term would do the job, but would at the same time damp a large fraction of the wave energy even with reduced spectral transfer, which would hide the phenomenon which we are interested at. Hence we use a modified dissipation term, which is nonzero only for wavenumbers larger than $k = 10k_0$. The equations with modified diffusion mimic a high Reynolds number flow where large-scale dissipation is negligible; we have checked that with $J = 0$, and 128*128 grid points, the spectral shape is not basically different from that obtained with standard dissipation. One observes that, as expected, the solenoidal component is amplified starting from a numerical noise via an inverse cascade to the larger scales, and finally one ends up at four turnover times with solenoidal and compressive spectra having comparable slopes, both showing a correct damping of grid scale excitation (Fig. 4). When the Jeans number is large enough, after 4 turnover times the spectral transfer is significantly reduced, which resembles the one-dimensional result (compare Fig. 4 and Fig. 2), even though the average Mach number is larger in the present runs.

A direct measurement of the reduction of turbulent energy dissipation is provided by plotting the energies versus time: kinetic energy $E_K = <\rho u^2>/2$, internal energy $E_i = \rho ln\rho c^2$, and total energy $\epsilon = E_K + E_i + E_G$, where E_G is the gravitational energy. In the absence of dissipation, the total energy is an invariant; in presence of modified dissipation, it is conserved up to the time where substantial spectral transfer has set in. One sees in Fig. 5a that with $J = 0$ the total energy is conserved as expected up to time $t \simeq 1$; subsequently, it decreases due to nonlinear transfer in the range where viscous dissipation is nonzero. In the two cases with $J > 0$ and $M_0 = 0.1$ (Fig. 5b and c), the damping rate is considerably reduced in the time range shown, compared to the gravitationless case (Fig. 5a). The large oscillations of internal energy correspond to exchanges with the gravitational energy, with the period growing with Jeans number as in linear gravito-acoustic waves.

We have partially explored the limits of the dispersive domain. Increasing the Mach parameter up to $M_0 = 0.2$, with

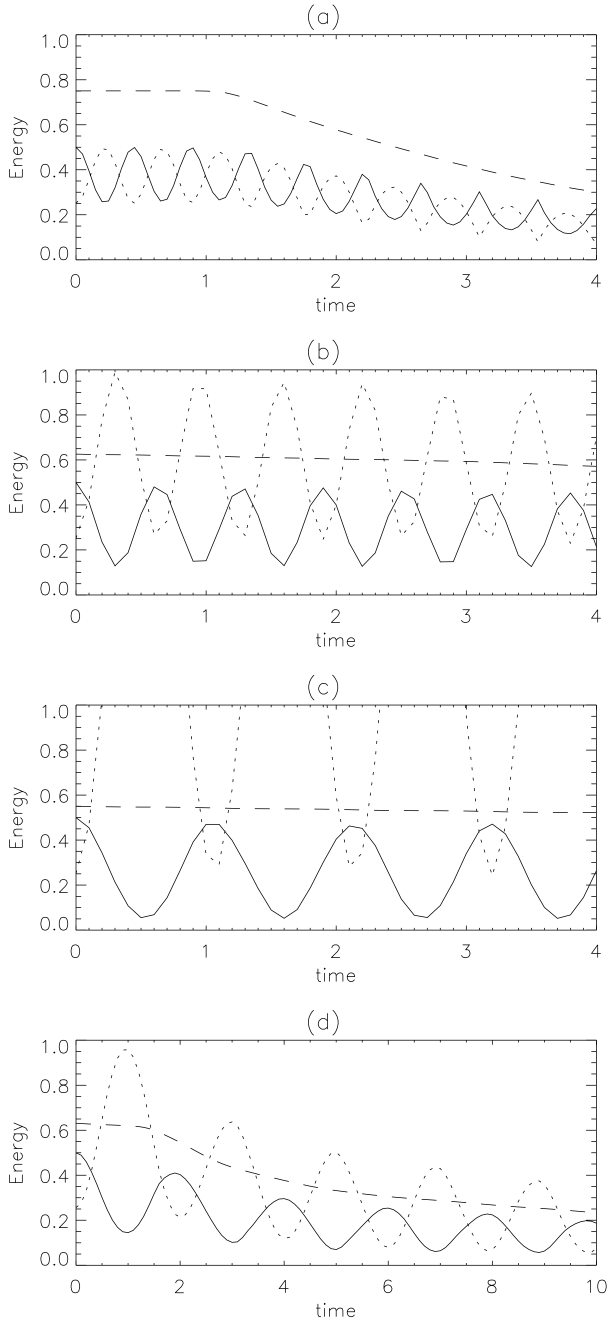

Fig. 5. Temporal evolution of energies in two-dimensional acoustic waves with modified dissipation ($\nu = 0$ for $k <= 10k_0$). Continuous: kinetic energy; dotted: internal energy; dashed: total energy ϵ. (a) $J = 0$, $M_0 = 0.1$ (b) $J = \sqrt{0}.5$, $M_0 = 0.1$ (c) $J = \sqrt{0}.8$, $M_0 = 0.1$ (d) $J = \sqrt{0}.5$, $M_0 = 0.3$.

$J = \sqrt{0}.8$ still exhibits reduced dissipation, as in the $M_0 = 0.1$ case. The system falls back into a strongly dissipative regime similar to the gravitationless case when $M_0 = 0.3$, $J = \sqrt{0}.5$ (Fig. 5d). If however, keeping the same Mach number $M_0 = 0.3$, we consider $J = \sqrt{0}.8$, the system is no longer stable; it collapses slowly, the density extrema reaching 472 and 0.08 in 2.6 turnover times. This slow collapse conserves the total energy; it corresponds mainly to a transfer of gravitational into internal energy, with no important increase in kinetic energy.

5. Conclusion

The initial conjecture that spectral transfer is reduced for waves with length smaller but comparable to the Jeans length is validated by our numerical experiments, which agree reasonably well with the phenomenological relation (2) between Mach number and wave number (or Jeans number). Such an inhibition of nonlinear interactions in fluids by dispersive effects is not unique; geostrophic turbulence also presents at large scales a similar phenomenon (Rhines 1979, Dubrulle and Valdettaro 1992). Non-dispersive (MHD) Alfvén waves in conducting fluids also provide an interesting case of decrease of nonlinear coupling, due to the specificity of the nonlinear coupling, actually leading to dispersive-like effects (Iroshnikov 1963, Kraichnan 1965, Grappin and Mangeney 1996).

For the present mechanism to play a role in sustaining the fully developed turbulence of interstellar matter, it is however necessary that dispersion effects be important in a spectral domain larger than considered above. For this to actually happen, it is sufficient to assume that the pressure gradient roughly balances the gravitational forces not only at a definite scale, but in the whole spectral range where the condensations are observed, since this is a sufficient condition for dispersive effects to be important. This assumption is compatible with observations, which indicate that the Jeans number is close to unity for a whole range of scales. This scenario does no longer require to invoke the so-called “turbulent pressure”, which effect has been demonstrated only in artificially forced cases, (Léorat et al., 1990). To test this hypothesis numerically requires considering a hierarchy of condensations all close to pressure equilibrium, a situation very different from the uniform case considered here (work in progress).

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