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LETTER

Letter to the Editor

Stellar evolution of low and intermediate-mass stars

IV. Hydrodynamically-based overshoot and nucleosynthesis in AGB stars

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Abstract. The focus of this study is on the treatment of those stellar regions immediately adjacent to convective zones. The results of hydrodynamical simulations by Freytag et al. (1996) show that the motion of convective elements extends well beyond the boundary of the convectively unstable region. We have applied their parametrized description of the corresponding velocities to the treatment of overshoot in stellar evolution calculations up to the AGB (Pop. I, $M_{\rm ZAMS}$ = $3M_\odot$).

Our calculations show the 3[rd] dredge-up already at the 7[th] thermal pulse (TP), and the dredge-up parameter reaches λ = 0.6 during the next five pulses. Accordingly, the amount of dredged up ^{12}C is up to $10^{-3}M_\odot$. Our models develope a small so-called ^{13}C-pocket consisting of a few $10^{-7}M_\odot$. Finally, this treatment of boundaries of convective regions leads to intershell abundances of typically (^{4}He/^{12}C/^{16}O)=(23/50/25) (compared to (70/26/1) in the standard treatment).

Key words: Stars: evolution – Stars: AGB, post-AGB – Stars: abundances

1. Introduction

The question of burning and mixing along the AGB is intimately linked to the so-called *carbon star mystery* (Iben 1981): Observations of the LMC and SMC prove that most of the carbon (and s-process element) enriched AGB stars are found at rather low luminosities indicating that they also have rather *low* masses (1...$3M_\odot$, cf. Smith et al. 1987, Frogel et al. 1990). On the other hand, AGB models predict the 3[rd] dredge-up, i.e. the mixing of interior carbon to the surface during the recurrent He shell flashes on the AGB, mostly for much higher luminosities, i.e. larger (core) masses. Thus, we meet two problems: (i) Why do we not observe luminous carbon stars?; and (ii) Why do we not find dredge up in low mass AGB models?

The first one belongs most likely to the occurrence of hot bottom burning and mass loss (Iben 1975, Blöcker & Schönberner 1991, Boothroyd et al. 1993), see also D'Antona & Mazzitelli (1996) for a recent discussion.

The second problem will be subject of this *Letter*. Synthetic AGB calculations utilize the stellar parameters known from evolutionary AGB model sequences and take the minimum core mass for dredge up, $M_{\rm H}^{\rm min}$, and the dredge-up parameter λ (ratio of dredged-up mass to growth of the hydrogen exhausted core per flash cycle) as adjustable parameters (among others) in order to match the observations, i.e. the luminosity function of LMC carbon stars. For example, van den Hoek & Groenewegen (1997) find $M_{\rm H}^{\rm min}$ = 0.58M$_\odot$ and λ = 0.75 as best fit in contrast to the results of evolutionary calculations ($M_{\rm H}^{\rm min} \gtrsim 0.65M_\odot$ and $\lambda \approx 0.25$, see Wood 1997). The amount of dredge-up found in evolutionary calculations depends sensitively on metallicities, core and envelope masses of the models (Wood 1981). Additionally, as pointed out by Frost & Lattanzio (1996), numerical details may play an important role. Finally, it is in particular the treatment of convection which bears large uncertainties concerning the dregde-up efficiency.

Another problem related to this subject concerns the s-process nucleosynthesis. The required neutrons are probably not released via the ^{22}Ne$(\alpha, n)^{25}$Mg reaction. Instead, the s-process is most likely driven by the ^{13}C$(\alpha, n)^{16}$O neutron source, raising the question how to mix protons into carbon-rich layers in order to produce sufficient amounts of ^{13}C . Iben & Renzini (1982) found that the protons can diffuse from the bottom of the convective envelope into the intershell zone due to semiconvection. However, this scenario is restricted to low metallicities (Iben 1983). Often, "standard calculations" artificially ingest a given amount of ^{13}C at the onset of the pulse into the convec-

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tively unstable He shell. However, Straniero et al. (1995) have shown that ^{13}C formed during the dredge-up phase is burnt under radiative conditions during the interpulse phase (for a recent review see Lattanzio 1995).

These difficulties of standard stellar evolution calculations to confront the observations often led to the conclusion that mixing outside the formally convective boundaries may take place (e.g. Hollowell & Iben 1988, D'Antona & Mazzitelli 1996, Wood 1997), or even that only a hydrodynamic approach of modelling the H/He interface will overcome the drawbacks of the local treatment of convection (Arlandini et al. 1995).

In brief the situation can be summarized as follows: The mixing length theory (MLT) (Böhm-Vitense, 1958) is usually applied to stellar evolution calculations. But it does not describe additional (possibly partial) mixing beyond the Schwarzschild boundary. To meet the observations for main sequence stars the *instantaneous* mixing of the convective core is extended beyond the classical Schwarzschild boundary by some fraction of the pressure scale height. This treatment is commonly referred to as overshoot. It has been discussed for example by Alongi et al. (1993, and ref. therein). The method we present in the following has in some aspects similar consequences as former overshoot treatments but it is based on hydrodynamic calculations.

Freytag et al. (1996) have carried out two-dimensional numerical radiation hydrodynamics simulations to study the structure and dynamics of a variety of shallow surface convection zones. Their work reveals an independent theoretical evidence for the actual existence of extra mixing beyond the boundary of convectively unstable regions. The parametrisation of the exponentially declining velocities of convective elements beyond the classical convective border can be applied to one-dimensional stellar evolution calculations. This treatment leads then to some extra partial mixing (Blöcker et al. 1997). In the following chapters we confine ourselves to the description of the method and the most important consequences. Further implications and details will be given elsewhere.

2. The stellar evolution code and the method of mixing

The computations for this study are based on the code described by Blöcker (1993, 1995) with major modifications. Nuclear burning is accounted for via a nucleosynthesis network including 31 isotopes and 74 reactions up to carbon burning. We use the most recent opacities (Iglesias et al. 1992, Alexander & Ferguson 1994). The initial composition is $(Y, Z) = (0.28, 0.02)$, the mixing length parameter of the MLT is $\alpha = 1.7$. For the $3M_\odot$ model sequence presented in this paper a Reimers-type mass loss with $\eta = 1$ has been applied.

2.1. Equation for abundance changes

The main modification relevant for this study is the introduction of a time-dependent model for mixing in the formulation adopted by Langer et al. (1985) for the study of semiconvection in massive stars:

$$\frac{\mathrm{d}X_i}{\mathrm{d}t} = \left(\frac{\partial X_i}{\partial t}\right)_{\mathrm{nuc}} + \frac{\partial}{\partial M_\mathrm{r}}\left[\left(4\pi r^2\rho\right)^2 D\frac{\partial X_i}{\partial M_\mathrm{r}}\right] \quad (1)$$

with X_i being the mass fraction of the respective isotope. The first term on the right-hand side considers the abundance changes due to nuclear burning and the corresponding rates are determined by the nucleosynthesis network. The second part describes the mixing of the elements by means of a diffusion term given here with respect to the mass coordinate M_r. The diffusion coefficient D depends on the assumed mixing model. For instance, in convective unstable regions within the Schwarzschild boundaries D is derived from the MLT (see below).

In our present calculations we make the assumption that the burning rates, i.e. $(\partial X_i/\partial t)_{\mathrm{nuc}}$, do not change during one time step. The nuclear network is invoked just once for each time step and the resulting burning rates enter Eq. 1.

2.2. The choice of D

Eq. 1 is integrated over the whole star for each isotope. For the radiative zones D is obviously $D_r = 0$. Within the convectively unstable regions we follow the prescription of Langer et al. (1985): $D_\mathrm{c} = 1/3 v_\mathrm{c} l$ with l being the mixing length and v_c the average velocity of the convective elements according to the MLT (Böhm-Vitense, 1958). Then the diffusion coefficient can be written as

$$D_\mathrm{c} = \frac{1}{3}\alpha^{2/3}H_\mathrm{p}\left[\frac{c}{\kappa\rho}g\beta(1-\beta)\nabla_{\mathrm{ad}}(\nabla_{\mathrm{rad}}-\nabla_{\mathrm{ad}})\right]^{1/3}, \quad (2)$$

where H_p is the pressure scale height, κ the opacity, α the mixing length parameter, c the velocity of light, ρ the density, g the gravitational acceleration, ∇_{rad} and ∇_{ad} the radiative and adiabatic temperature gradient respectively, and β the gas pressure fraction.

Finally, for the regions which are immediately adjacent to convectively unstable zones we adopt the depth dependent diffusion coefficient derived by Freytag et al. (1996: Eq. 9) from their numerical simulations of two-dimensional radiation hydrodynamics of time-dependent compressible convection :

$$D_{\mathrm{os}} = D_0 \exp\frac{-2z}{H_\mathrm{v}}, \qquad D_0 = v_0\cdot H_\mathrm{p}, \qquad H_\mathrm{v} = f\cdot H_\mathrm{p}, \quad (3)$$

where z denotes the distance from the edge of the convective zone ($z = |r_{\mathrm{edge}} - r|$ with r: radius), and H_v is the velocity scale height of the overshooting convective elements at r_{edge} being proportional to the pressure scale height H_p. According to Freytag et al. (1996), D_0 has to be taken near the edge of the convective zone. Here the velocity field varies only slighty. We take the corresponding typical velocity of the convective elements, v_0, obtained from the MLT in the unstable layers immediately before the Schwarzschild border.

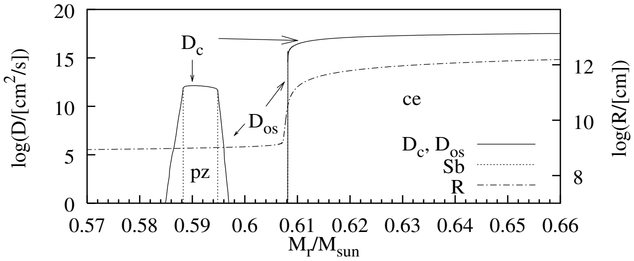

Fig. 1. Diffusion coefficient ($D_{\rm c}$, $D_{\rm os}$) versus mass coordinate for an TP-AGB model with f = 0.02, 49yr before the largest ^{4}He-luminosity (beginning of 12$^{\rm th}$ pulse cycle). The pulse-driven convective zone (pz) occupies the lower half of the intershell zone. The latter extends from $0.587M_\odot$ (location of the ^{4}He-shell) to $0.607M_\odot$ (where H-burning takes place). The vertical dotted lines indicate the Schwarzschild boundaries (Sb). For the base ($M_{\rm r} = 0.608M_\odot$) of the convective envelope (ce) it can not be distinguished from $D_{\rm os}$ since the H-shell is very extended. This can be seen from the dashed-dotted line which shows the radius (r). However, this overshoot region comprises $5.8*10^{-5}M_\odot$ accounted for in 16 mass shells.

We summarize the choice of D (Fig. 1):

$$D = \begin{cases} D_{\rm c} & \text{instantaneous mixing, ordinary convection} \\ D_{\rm os} & \text{mixing efficiency declining, overshoot region} \\ D_{\rm r} & \text{no mixing, radiative layers} \end{cases}$$

2.3. The determination of f

The parameter f is a measure of the efficiency of the extra partial mixing. It defines the velocity scale height of the convective elements beyond the boundary of the convectively unstable zone. As can be seen from Eq. 3 the decline of $D_{\rm os}$ is steeper for smaller values of f, thus the larger f the further extends the extra partial mixing beyond the convective edge.

Freytag et al. (1996) find from their simulations of overadiabatic convective envelopes of A-stars and DA white dwarfs $f = 0.25 \pm 0.05$ and 1.0 ± 0.1 respectively. These results already reveal that f depends on the stellar parameters. For deep envelope (or core) convection f can be expected to be considerably smaller since the ratio of the Brunt-Väisälä timescales of the stable to the instable layers decrease with increasing depth, i.e. adiabacity. However, hydrodynamical simulations of such deep convection zones are not available yet and we have to rely on indirect methods. For example, $f \approx 0.02$ results in the same main-sequence width as calculations by Schaller et al. (1992) who applied instantaneous overshoot with a parameter of $d_{\rm over}/H_{\rm p} = 0.2$. We took this value of f for our calculations assuming that it holds also for the deep convection zones of AGB stars. Although the width of the calculated main sequence depends very sensitively on f ($\Delta f = 0.005$ corresponds to a displacement of the terminating age main sequence of $\Delta \log g \approx 0.08$) our experiments with AGB evolution calculations have shown that the character of our findings remains qualitatively valid even if f is changed by a factor of two.

Table 1. Dredge-up data for selected thermal pulses of a $M = 3M_\odot$ sequence: mass of the hydrogen exhausted core ($M_{\rm H}$), dredge-up parameter ($\lambda = \Delta M_{\rm Dup}/\Delta M_{\rm H}$, $\Delta M_{\rm H}$: increase of the core mass by nuclear burning during the last pulse cycle), total amount of dredged-up material ($M_{\rm DUP}$), amount of dredged up ^{12}C ($M_{\rm DUP(^{12}C)}$), composition of the intershell zone, and content of the ^{13}C-pocket ($M_{(^{13}{\rm C})}$).

TP Nr.	7	8	12
$\frac{M_{\rm H}}{M_\odot}$	0.5779	0.5837	0.6073
λ	0.11	0.12	0.60
$\frac{M_{\rm DUP}}{M_\odot}$	$6.3 * 10^{-4}$	$7.0 * 10^{-4}$	$3.7 * 10^{-3}$
$\frac{M_{\rm DUP(^{12}C)}}{M_\odot}$	$1.4 * 10^{-4}$	$1.8 * 10^{-4}$	$1.5 * 10^{-3}$
(^{4}He/^{12}C/^{16}O)	(23/50/25)	(23/49/26)	(26/45/27)
$\frac{M_{(^{13}{\rm C})}}{M_\odot}$	$3.9 * 10^{-7}$	$3.7 * 10^{-7}$	$2.0 * 10^{-7}$

3. First results from the calculations

Applying this treatment of convection to AGB models we found the 3$^{\rm rd}$ dredge up for a $3M_\odot$ model. The onset was already at the 7$^{\rm th}$ thermal pulse at $M_{\rm H} = 0.58M_\odot$ reaching a dredge-up parameter of $\lambda = 0.6$ within the next five pulses (see Tab. 1). Note, that corresponding calculations without extra mixing (standard calculations) did not show any dredge up.

Due to the application of depth-dependent overshoot to all convective zones the abundances in the intershell zone change in comparison to standard calculations. The pulse-driven convective zone extends deeper into the underlying C/O-core already during the first TP and considerable amounts of ^{16}O and ^{12}C are mixed up. Compared to the values given in Tab. 1 the abundances of the intershell zone in standard calculations is typically (^{4}He/^{12}C/^{16}O)=(70/26/1). The comparison of these intershell abundances with observed surface abundances of Wolf-Rayet (WR) central stars and PG 1159 stars can be used in the future as an additional constraint for f.

This strong increase of ^{12}C in particular causes now a corresponding increase of opacity and thereby of the radiative gradient $\nabla_{\rm rad}$ and leads to semiconvection (Iben & Renzini 1982) immediately below the ^{4}He-rich layer. In our calculations this shows up as thin convective shells which may be connected by their overlaping diffusive tails. The effect is that the sharp border between the intershell zone and the ^{4}He-rich layer is wiped out (Fig. 2, $M \approx 0.607M_\odot$). In the following, the envelope convection continuously progresses downwards, and finally its base proceeds through the previously semiconvective zone (Fig. 3). Typically, 50% of the dredged-up material consist of ^{12}C, the rest is given by ^{4}He and ^{16}O, resp., with 25% each (Tab. 1).

Finally the convective envelope base remains at the position of deepest penetration for about fifty years. This is the phase where the adjacent H and ^{12}C profiles can built up a very thin overlap zone due to the diffusive tail of the envelope convection ($D_{\rm os}$, Eq. 3). At this location in the star the ^{13}C-pocket can form (Fig. 4). We confirm the findings of Straniero et al. (1995) that the built up ^{13}C- pocket is burnt under radiative conditions well before the next thermal pulse.

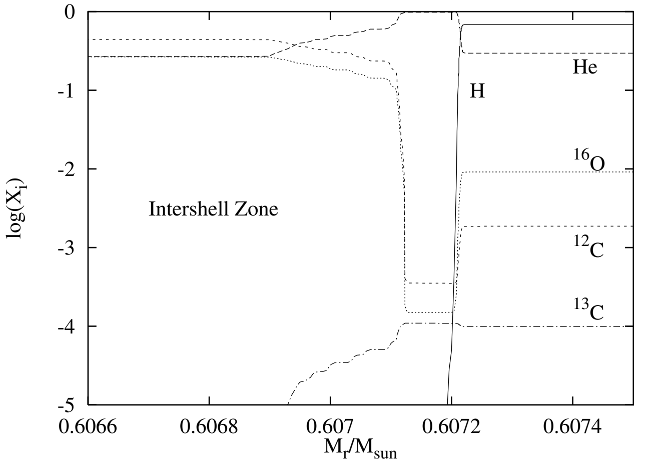

Fig. 2. Onset of the 3rd dredge-up during the 12th pulse cycle. At $0.6072 M_{\odot}$ is the bottom of the convective envelope which has already moved into the ^{4}He-dominant layer at $M_r/M_{\odot} \approx 0.6071 - 0.6072$. The latter is the product of the H-shell burning of the previous pulse. This situation is located 273yr after the beginning of 12th pulse cycle, i.e. 322yr later than shown in Fig. 1.

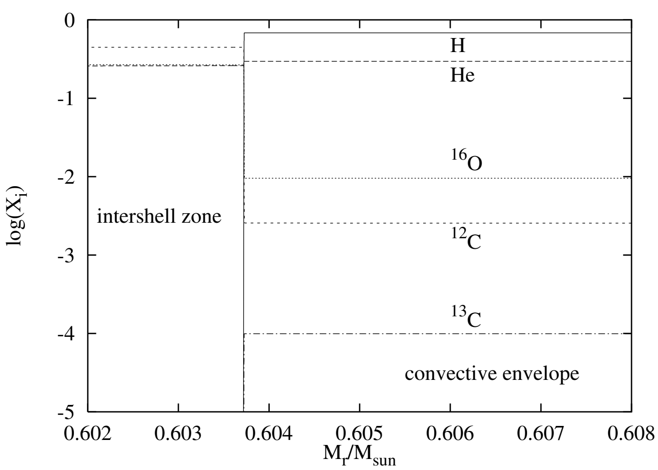

Fig. 3. The convective envelope continues to proceed downwards (210yr past Fig. 2) and will shortly hereafter stop. Note the different mass range compared to the previous figure.

4. Concluding remarks

We considered overshoot in evolutionary calculations based on results of 2-dimensional hydrodynamical simulations of convection. On the AGB, this leads to the formation of a ^{13}C-pocket in the intershell region in contrast to calculations based on standard overshoot considerations (i.e. on formal shifts of the Schwarzschild border). Additionally, the intershell abundances are considerably changed due to extra mixing. As opposed to calculations without any overshoot, we find now dregde-up even for models with masses as low as $3M_{\odot}$.

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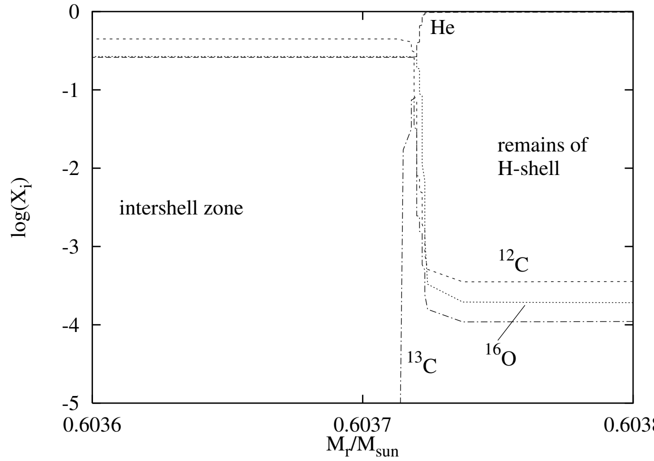

Fig. 4. The downward movement of the convective envelope has stopped, the dredge-up is over. A small ^{13}C-pocket has formed in the region of previously overlaping diffusion tails of th H- and ^{12}C-profile. The snapshot is 17660yr later than the Fig. 3.

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