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Do we see free precessing pulsars?

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Abstract. The evidence for free precessing neutron stars is discussed. Her X-1 and the Crab pulsar show periodic modulations in their light curves which can be ascribed to free precession. We show that both measured periods can be interpreted in terms of a unique model. It is interesting to note that the same model also explains well the eccentricities of rotating planets.

Key words: pulsars: individual: Her X-1 – stars: neutron – stars: rotation

1. Introduction

Precession or free precession is an often asked question regarding neutron stars, and many periodic phenomena observed with pulsars have been tentatively connected to it (Ögelman 1989, Schwartzenberg-Czerny 1992). The free precession of a rigid spinning top rotating about an axis, which is not one of its proper axis, reflects the deviation of the top from spherical symmetry. With neutron stars the question of free (or forced) precession may be more complicated by the possibility that they are not entirely rigid. Therefore, the angular momentum exchange between different parts of the star is possible. A dissipative exchange between the core superfluid and the neutron star crust eventually aligns the angular velocity with the preferred principal axis of the star, rendering the free precession unobservable, or damped out in some 10^3 years (Alpar and Ögelman 1987), while elastic nondissipative angular momentum exchange may lower the free precession period (Leins 1992) so much, that the astrophysical identification of the phenomenon becomes difficult. It is mainly due to these issues, as well as to different attempts to explain the phenomenon, that authors have reported on free precessional periods of neutron stars on wildly different time scales ranging from seconds to years (Carlini and Treves 1989, Huguenin et al. 1973, Helfland and Fowler 1977, Nolan et al. 1993, Ulmer et al. 1994).

Still, there is yet no obvious observed candidate for the free precession of a pulsar. The system Her X-1 – HZ Her may be an exception, where Trümper et al. (1986) (and references therein) find the free precession of the pulsar as a plausible explanation for the 35-day period. We observed a 60-second periodic modulation in the visible light of the Crab pulsar and we find that both Crab and Her X-1 pulsar fit well a simple free precession model.

2. Observations of the Crab pulsar and the 60 seconds period

Our optical photometric observations of the Crab pulsar were carried out at the $1.82m$ telescope of Asiago and Padua Observatories, Italy. Our aim was to search for short time–scale (from about 40-seconds to several minutes) small–amplitude periodic modulation in the Crab pulsar's optical light–curve. The observing apparatus, which was designed to search for amplitude and phase modulation, is described in Čadež and Galičič (1996a). In the same article, the observational run from December 1994 is presented along with the data reduction and analysis methods. Our latest (December 1995) Asiago Crab pulsar data, which were obtained by the same observational system as a year before (Čadež and Galičič, 1996c), strenghten previous results. In addition to our observational data, we have also analysed the Hubble Space Telescope (HST) Crab pulsar data which were obtained in 1991 by the HST instrument *High Speed Photometer* (Percival et al. , 1993). The analysis is described in Čadež and Galičič (1996a, 1996b). All three data sets give modulation frequencies which are in the same error–box (Čadež and Galičič, 1996b). This is illustrated by the average power spectrum (Fig. 1), obtained by adding normalised power spectra of Asiago 94, Asiago 95 and HST data. It shows a 5 sigma peak at 1/60 s = 0.0167 Hz.[1]

3. The Her X-1 35-day cycle

Her X-1 is known as a powerful X-ray source, and a member of a binary system (Smith 1977). The binary occultation occurs periodically at 1.7-day intervals and the pulsar is pulsing every 1.24s. The system also shows a regular 35-day signal variability (Tananbaum et al. 1972) which still lacks a conclusive

[1] The very low frequency rise of the spectrum is the $\approx 1/f$ noise characteristic of atmospheric scintillation and is observed with the same strength also with other stars.

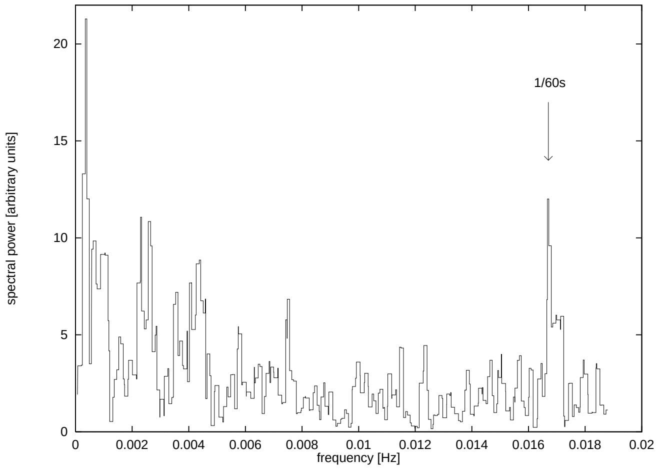

Fig. 1. Average power spectrum obtained as a sum of normalized power spectra for Asiago 94, Asiago 95 and HST data.

explanation. Among the different models proposed to explain it, Trümper et al. (1986) and Shaham (1986) have presented the free precessional model as a strong candidate. The correspondence of the 35 day period with a model free precession period has been left rather loose by most authors, since the effective inertial eccentricity $((J_{zz}/J_{xx}-1))$ to go with the Euler free precession formula:

$$\omega_{fp} = (J_{zz}/J_{xx}-1)\omega_{rot} \tag{1}$$

has been a matter of many discussions involving the question of superfluidity of neutron star interior and the question of rigidity of the crust (Shaham 1977, Link, Epstein and Baym 1993). Another concern expressed by many authors is the effective viscosity due to the interaction of superfluid vortices with the solid crust which is expected to effectively damp the free precession (Alpar and Ögelman 1987) if there is no mechanism driving it (according to our knowledge none was suggested). In the case of Her X-1 the problem seems even more severe, since it is accreting matter and angular momentum with it. Therefore, Peterson, Rotschild and Gruber (1991) considered the slaved precession of the accretion disk responsible for the 35 day cycle. Their kinematic model works well in describing relative intensity variations of the main pulse with the 35 day period. However, it is very difficult (if not impossible) to realize it dynamically, since it requires an unusually large torque to drive such a fast precession (Sharp, Calvani and Turolla 1984). The measured changes of the orbital period of Her X-1 also seem to be inconsistent with this model (Deeter et al. 1991).

4. On oblatness of rotating gravitating bodies and how well do planets obey the theory

The question of the shape of a rotating gravitating body goes all the way back to Newton. Already in 1742 Maclaurin wrote an expression relating the eccentricity of a rotating liquid drop to the angular velocity of rotation (Lang 1978, p278). The theory of self gravitating homogeneous liquids has been extensively treated by Chandrasekhar (1969) and extended to rotating polytropic stars by Tassoul (1978, p244). It was further extended to rotating relativistic compact stars by Komatsu et al. (1989), Cook, Shapiro and Teukolsky (1994), Salgado, Bonazzola and Gourgoulhon (1994). Slowly rotating (compact) stars were considered in particular (Cohen 1971 and references therein). The free precession of slowly rotating neutron stars in the framework of general relativity is considered by Thorne and Gürsel (1983), who prove that rigidly rotating, fully relativistic bodies undergo precessional motion equivalent to that described by classical Euler equations for rigid body free precession (Goldstein 1981).

In addition to the above works, treating mostly the simpler problem of a rotating self gravitating gas, there was considerable discussion in the literature about the importance of superfluidity and about the amount of elastic energy stored in a star (Pandharipande, Pines and Smith 1976, Shaham 1986, Schwartzenberg-Czerny 1992). It was shown that these issues make a firm prediction about the inertial eccentricity of a realistic neutron star very difficult, since many parameters are poorly known. Therefore, it seems reasonable to apply the simplest theory to the astrophysical problem of which we know most answers, and compare the results with observations. Planets of the solar system offer reliable data which can be compared with theory.

If one assumes that planets can be modeled as rotating polytropes (with polytropic index n), then the theory of slowly rotating non relativistic polytropes gives the following relations for the flatness and the mass quadrupole moment

$$f \;=\; k_f(n)\Omega^2 \tag{2}$$

and

$$J_2(n) = \frac{J_{zz}-J_{xx}}{Ma_{eq}^2} \;=\; k_J(n)\Omega^2 \tag{3}$$

where

$$\Omega^2 = \frac{\omega_{rot}^2}{2\pi G\langle\rho\rangle} \quad , \tag{4}$$

and flatness is:

$$f = \frac{a_{eq}-a_{pol}}{a_{eq}}, \tag{5}$$

ω_{rot} is the angular frequency of rotation, a_{eq} and a_{pol} are the equatorial and polar radius of the star, respectively, G is the gravitational constant and $\langle\rho\rangle$ the average density of the star. The moment J_2 is the source of the quadrupole term in the gravitational potential of the star ($V_2 = [GMa_{eq}^2 J_2/r^3]\,P_2(\cos\theta)$). The proportionality factors can be obtained numerically. They and the inertial eccentricity coefficient (ε_I) defined by

$$\frac{J_{zz}-J_{xx}}{J_{zz}} \;=\; \varepsilon_I(n)\Omega^2 \tag{6}$$

Table 1. Coefficients relating the flatness, the gravitational quadrupole and the inertial eccentricity to the rotational velocity of a polytropic star (with polytropic index n).

n	0.0	0.5	1.0	1.5	2.0	2.5	3
$k_f(n)$	1.875	1.426	1.140	0.965	0.861	0.802	0.772
$k_J(n)$	0.7500	0.4506	0.2599	0.1433	0.0739	0.0349	0.0144
$\epsilon_I(n)$	1.875	1.383	0.995	0.701	0.478	0.312	0.192

Table 2. The measured data for planets.

planet	f	J_2	$\Omega^2 = \frac{\omega^2}{2\pi\rho}$	f/Ω^2	J_2/Ω^2
Mercury	0	0.00008 ± 0.00006	0.000000677	0.0	118
Venus	0	0.000006 ± 0.000004	0.0000000407	0.0	147
Earth	0.003528	0.0010826	0.00230	1.459	0.471
Mars	0.0030428	0.001959	0.00304	1.704	0.644
Jupiter	0.06481	0.014736	0.0571	1.135	0.258
Saturn	0.10762	0.016479	0.101	1.0698	0.164
Uranus	0.03	0.0033434	0.0254	1.180	0.132
Neptun	0.022	0.0034105	0.0175	1.254	0.194

are listed in Table 1 (our calculations, compare with Tassoul 1978). The measured values for planets (Lang 1992) are listed in Table 2.

With the exception of Mercury and Venus the measured ratios f/Ω^2 and J_2/Ω^2 are close to theoretically predicted for polytropes with n between 0 (homogeneous fluid) and 1.5 (adiabatic ideal gas). The second smallest planet Mars is best modelled with the hard $n = 0$ polytrope, the Earth requires $n \approx 0.5$, which is a rather hard polytrope exhibiting some but not very pronounced central concentration of mass. The giant planets, however, are best fitted with $n = 1$ or even $n = 1.5$ polytropes, which fits well with the idea that they are much more differentiated (due to their large size and therefore higher internal temperatures, as well as by their ability to retain the volatile elements in their atmospheres) than the smaller planets. We understand these results as an indication that one cannot attribute significant elastic stress to any planet including Mercury and Venus that rotate so slowly that their flatness cannot be measured. The data indicate, for example, that Venuses abnormal quadrupole moment carries only 5×10^{-4} the amount of energy carried by the quadrupole moment of the Earth. Thus, all the planets whose dimensionless angular velocity (Ω) is larger than $\approx 1/30$ seem to be well relaxed about the ground energy shape determined by rotating polytropes.

The question of planetary free precession is more complicated. Only the Earth is known to free–precess with an amplitude no larger then a few tenths of an arc second. Two periods connected to this motion have been measured: 12 months and 14.2 months (Audouze and Israel 1994). On the other hand, the calculations, discussed above, would predict free precession with the period of 10.3 months. The actually observed free precession is thus more complicated and slower, but not much slower then expected. The reason for the discrepancy is understood to be due to the elasticity and viscosity of the molten interior of the Earth, which cannot quite follow the motion of the crust. In this regard it seems to be difficult to understand why the free precession persists at all in spite of the damping effect of viscosity. Mars would most likely be a much cleaner example of a free precessing planet because of its smaller mass and, therefore, probably much smaller portion of molten interior. However, its free precession has not yet been measured, although such measurements have already been proposed (Bills 1996).

Table 3. The expected free-precession period $P_{prec} = [\nu_{rot}\epsilon_I\Omega^2]^{-1}$ for the Crab pulsar ($\nu_{rot} = 30s^{-1}$) if the pulsar is a $1.3M_\odot$ neutron star according to Pion condensate (pi), Reid's (R), Three-nucleon interaction approximation (TNI), Bethe-Johnson (BJ), Mean field approximation (MF), Tensor interaction (TI) equation of state and assuming that the effective polytropic index (n) is a free parameter. The expected free precession period is tabulated in seconds.The window in the shading is centered on the best effective polytropic index for each equation of state for the given mass.

EOS	n=0.5	1.0	1.5	2.0
pi	264.3	367.3	521.4	764.6
R	191.1	265.6	377.0	552.9
TNI	125.7	174.8	248.1	363.8
BJ	105.6	146.8	208.3	305.5
MF	55.2	76.5	108.6	159.3
TI	42.5	59.1	83.9	123.1

5. Existence of free precession in pulsars?

Since gravity is the strongest force shaping neutron stars, it is reasonable to assume, that their shape and inertial eccentricity is also determined by the hydrostatic equilibrium. Furthermore, if pulsars are relatively as rigid as planets, one may expect that they could free precess. The notion of their relatively high rigidity is supported by the observation that no earthquake has ever

Table 4. Same as Table 3 but for the pulsar in Her X-1 (ν_{rot} = $0.808s^{-1}$); the expected free precessional period is tabulated in days.

EOS	n=0.5	1.0	1.5	2.0
pi	156.6	217.6	308.9	453.0
R	113.2	157.4	223.3	327.5
TNI	74.5	103.5	147.0	215.5
BJ	62.6	86.9	123.4	181.0
MF	32.6	45.3	64.3	94.4
TI	25.2	35.0	49.7	72.9

produced an innertial change comparable to that observed in a (giant) glitch.

As a test of these ideas, we estimate the inertial eccentricity of a rotating neutron star, just as the inertial eccentricity of a planet, and examine if neutron stars can be found showing modulation at expected free precession frequencies. We calculate $\langle\rho\rangle$ in eq. 1 from the mass radius relation ($M(R)$) obtained from neutron star models based on Pion condensate (pi), Reid's (R), Three–nucleon interaction approximation (TNI), Bethe-Johnson (BJ), Mean field approximation (MF), and Tensor interaction (TI) equation of state (after Lipunov 1992). Since the theory of relativistic free precession has not been fully implemented with the above equations of state, we use the polytropic index n as a somewhat free model parameter, whose best value $n_{eff} = 1 + 2/(1 - \frac{R}{M}\frac{dM}{dR})$ (Bowers and Deeming 1984) can be estimated from the slope of the mass-radius relation at the given mass, at least in those cases where relativistic effects are not too important. We choose a $1.3M_{\odot}$ neutron star (the measured mass of Her - X1), calculate it's dimensionless angular velocity (Ω) if it were rotating with $\omega_{rot} = 2\pi \times 30s^{-1}$ (Crab) and with $\omega_{rot} = 2\pi \times 0.808s^{-1}$ (Her X-1) and calculate the free precession period for different average densities that a polytropic model with given n would predict. The results are given in Fig. 3 and 4 and the best effective polytropic index for each model is indicated by the position of the unshaded window in each row.

It is surprising to find that one equation of state - the TI equation of state - can describe the two periods remarkably well!

6. Conclusions

The observed flatness and inertial eccentricity of planets can be explained quite well with the rotating polytropic model. The polytropic index making the best fit is consistent with what we know about the distribution of matter inside planets. This suggests that planets are well relaxed about their gravitational equilibrium configuration. The observed free precession period of the Earth disagrees only by a few ten percent from the prediction of the Euler theory of rigid rotators. The relatively small discrepancy can be understood (in qualitative terms) to be due to elasticity of the Earth.

The rotating polytropic model seems to do an equally good job in accounting for the fast 60-second periodicity that we observed in the Crab pulsar, as well as for the 35-day cycle of Her X-1. It is quite interesting to note that both the Crab and the Her X-1 pulsar require a model with almost identical neutron star parameters (the model predicts their average densities to be in the ratio 25:24). It is also worth mentioning that the same model can plausibly explain the 163-day precession period of the SS433 jets (even if the nature of the compact object in SS433 is not yet clear, since it has not been observed as a pulsar, see e.g. Zwitter and Calvani 1989, 1990). All other models have difficulty in coping with the shortness of this 163 day period (Sharp, Calvani & Turolla 1984).

A powerful test of the model would be the observation of the slow–down of the free precession. Its prediction that $P_{fp} \propto P_{rot}^3$ could be observed in Crab's light–curve in a few years. An observational agreement with theoretical prediction would further strengthen the case for the free precession explanation. The slow and interacting binary pulsar Her X-1 does not offer a chance for a similar clean comparison, however the fact that it is accreting may help elucidating the free precession driving mechanism.

Should the free precession be accepted as the correct explanation of the above phenomena, it will give an important tool to study neutron star equations of state.

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