

The effect of increasing helium abundance on the NLTE model atmospheres of hot white dwarfs

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Abstract. Plane parallel and spherically symmetric model atmospheres for hot ($T_{\text{eff}} = 100000\text{K}$) high gravity ($\log g = 7.5$) stars are calculated and compared for different hydrogen/helium abundance ratios. The effect of the abundance changes on the emergent radiation is studied. Strong dependence of the profile of the He II 4686Å line on the abundance is found for low helium abundances. Also, pronounced dependence on the abundance ratio is found for the hydrogen H_{α} line for high helium abundances. Combination of these two lines may serve as a powerful diagnostic tool for the determination of hydrogen/helium abundance ratio in hot white dwarfs.

Key words: stars: atmospheres – line: profiles – white dwarfs

1. Introduction

First NLTE plane-parallel model atmospheres for high-gravity stars were calculated by Kudritzki (1976) and Auer & Shipman (1977). Wesemael et al. (1980) calculated a grid of LTE model atmospheres for high-gravity stars and compared them with NLTE models with lines in detailed radiative balance. They found the NLTE effects without treatment of lines to be unimportant and correctly concluded that “more definite predictions about the influence of NLTE effects . . . must await the inclusion of line radiative transitions”. Wesemael (1981) obtained similar results for pure helium models. However, in the course of years these results became tacitly assumed to be valid also for NLTE models with lines and lead to the total rejection of NLTE models for the analysis of atmospheres of white dwarfs. However, recent results of NLTE atmospheric modelling showed its importance. For a brief critical review of previous model atmosphere calculations (both LTE and NLTE) see Hubeny & Lanz (1995) and Lanz & Hubeny (1995). Recently, white dwarf model atmospheres calculations were brought to a very high degree of sophistication by including NLTE line blanketing effects by Lanz & Hubeny (1995) and Dreizler & Werner (1993, 1994). One interesting feature in white dwarf atmospheres is the abundance ratio of helium to hydrogen. Abundances of these two

elements in the atmospheres of known white dwarfs change significantly from almost pure hydrogen atmospheres to helium rich atmospheres with almost no hydrogen.

The abundances of hydrogen and helium at the hottest end of the white dwarf sequence (the DO and hot DA white dwarfs) were analysed by a number of authors. Koester et al. (1979) studied the helium content in white dwarf atmospheres systematically by means of LTE atmospheres. Wesemael et al. (1984, 1985) analyzed DO and DAO white dwarfs with a help of a grid of LTE model atmospheres calculated for different hydrogen/helium ratio. Barstow et al. (1995) analyzed the helium content of the DA white dwarf HZ43 by means of stratified and homogeneous LTE model atmospheres. However, use of LTE model atmospheres for the determination of the hydrogen/helium ratio is subject to a systematic error (see Napiwotzki 1993, 1995). It is much better to use NLTE model atmospheres, though some people favor LTE models since they allow easy inclusion of vertical stratification in the atmosphere (Jordan & Koester 1986, see also Koester 1996).

NLTE analyses were performed mostly with the help of the plane-parallel code of Werner (1986, 1987a,b, 1988, 1989, see also Dreizler & Werner 1993). Napiwotzki et al. (1995) analyzed the cool DO white dwarf HD149499B and determined the hydrogen/helium ratio using LTE model atmospheres. However, they checked for NLTE effects and found no difference. Three DO white dwarfs were analysed to determine the trace amount of hydrogen using NLTE profiles of H_{α} line by Werner (1996c). Dreizler & Werner (1996b) analysed a large number of DO white dwarfs and determined also their hydrogen content with the help of NLTE model atmospheres. Dreizler et al. (1996) discovered a hydrogen rich PG1159 type star HS2324+3944. Accurate knowledge of atmospheric content of hydrogen and helium is of a great importance not only for understanding the physics of stellar atmospheres, but also for the studying of the final stages of stellar evolution. Here we would like to present concise view of effects caused by increasing helium abundance on a very hot white dwarf atmosphere, and, consequently, on emergent spectra.

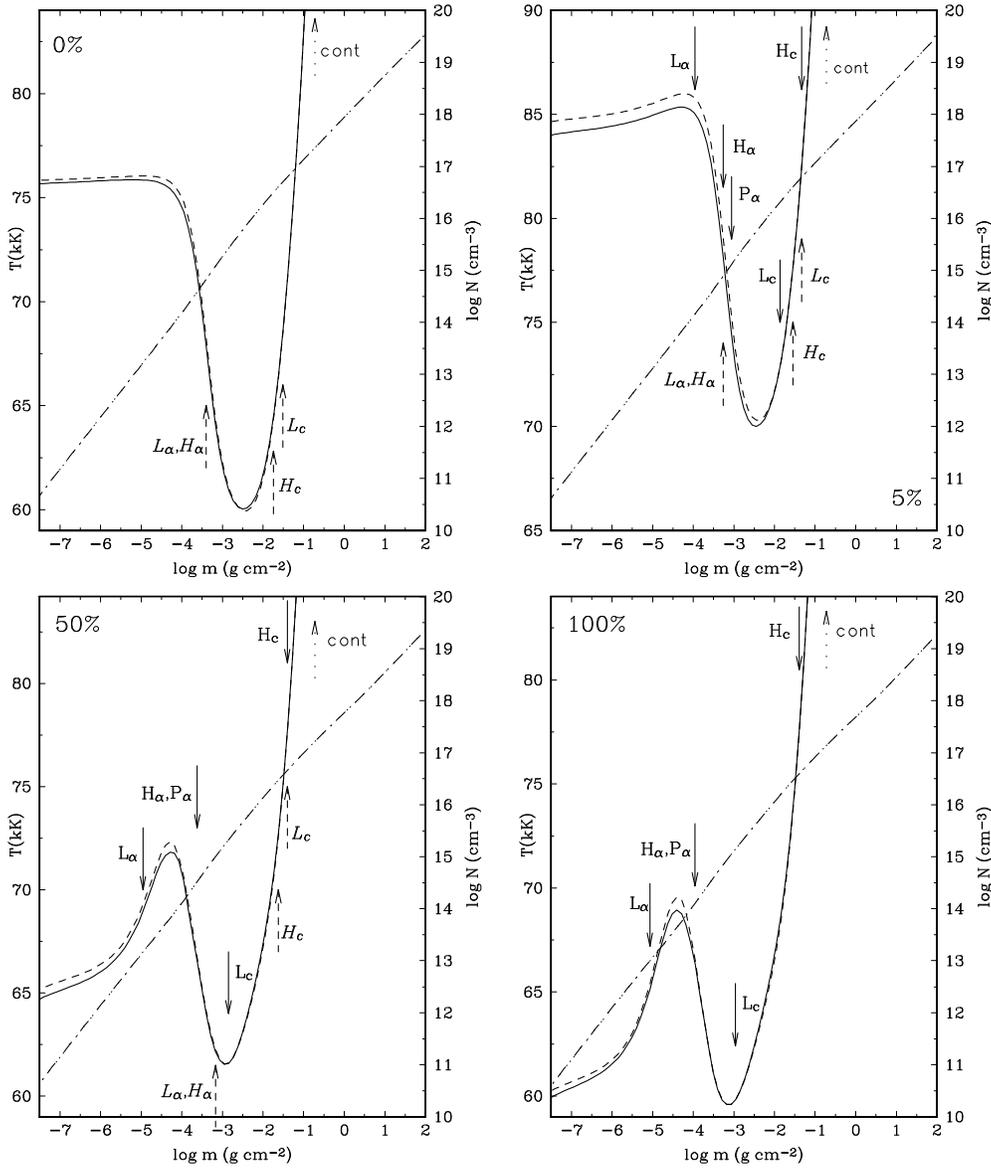


Fig. 1. The temperature and density structures of some selected models of our grid. The spherically symmetric models are plotted with the solid line, the plane-parallel ones with the dashed line. The density structure is plotted with the dot-dashed line for both models. Each figure is labeled with a helium abundance (in %) with respect to all elements by particle number. Here m is the column mass depth, N the total number particle density, and T the temperature. In addition, the depths of formation of selected important transitions are marked with arrows. Dashed arrows denote hydrogen transitions, full line arrows stand for He II transitions. The arrow labeled *cont* points to the depth where the Rosseland optical depth is of the order of unity.

2. The sphericity effects

We would also like to draw the attention to the systematic error that is introduced into all (NLTE, line blanketed, ...) *plane parallel* model atmospheres of white dwarfs by neglecting the curvature of the atmosphere. Unlike NLTE and line blanketing, the geometry of the atmosphere has generally been overlooked and assumed to be plane parallel without any discussion. However, an attempt to test our spherically symmetric code also for the case of hot high gravity stars brought new results. To the best of our knowledge, the very first spherically symmetric NLTE model atmospheres of white dwarfs were calculated by Kubát (1995a) for the case of $T_{\text{eff}} = 100000\text{K}$ and $\log g = 8.0$. Initially, we expected a totally negligible difference between plane-parallel and spherically symmetric model atmospheres. The result was surprising since small differences were found in the emergent spectrum in the core of H_{α} line. Further calcula-

tions (Kubát 1995b) confirmed the preceding results and found slightly more pronounced differences for stars with lower gravity of $\log g = 6.0$. The sphericity effects in the model atmospheres of white dwarfs are summarized in Kubát (1997a). The differences found were large enough to be surprising, still they were too small to solve the Balmer line problem (Napiwotzki 1992, 1993). The Balmer line problem was the fact that for some particular stars the observed Balmer line profiles were deeper than those predicted by plane-parallel NLTE calculations. The largest part of the Balmer line problem was recently successfully solved by Werner (1996a,b) by more accurate treatment of line opacities of C, N, and O. Nevertheless, small differences between computed and observed line profiles are still present. The possibility of explaining these small differences by sphericity effects should be tested by repeating the calculations of Werner in spherical geometry. This remains to be done in the future. In addition, similar differences between computed

Table 1. The outer temperature for plane-parallel (PP) and spherically symmetric (S) models and the ratio of the thickness of the atmosphere to the stellar radius R_* . The thickness is calculated as a geometrical distance between depth points with $\log \tau_r = -7.5$ or $\log \tau_r = -5$, and $\log \tau_r = 2/3$; τ_r is the Rosseland optical depth.

He (%)	PP	S	$\log \tau_r = -7.5$	$\log \tau_r = -5$
	T(K)		r/R_*	
0	75830	75670	3.5 -3	2.0 -3
0.01	76030	75830	3.5 -3	2.0 -3
0.1	77240	76900	3.6 -3	2.1 -3
1	82020	81500	3.8 -3	2.2 -3
5	84640	84000	3.5 -3	2.0 -3
10	80240	79880	3.1 -3	1.8 -3
15	76360	75790	2.7 -3	1.6 -3
20	73530	73030	2.4 -3	1.5 -3
30	69270	68760	2.1 -3	1.2 -3
40	66710	66120	1.8 -3	1.1 -3
50	65170	64970	1.6 -3	9.6 -4
60	63570	63180	1.5 -3	9.2 -4
70	62730	62350	1.4 -3	8.3 -4
80	61770	61430	1.3 -3	7.8 -4
90	60990	60670	1.2 -3	7.6 -4
99	60390	60050	1.2 -3	7.4 -4
99.9	60330	60010	1.2 -3	7.1 -4
100	60230	59910	1.1 -3	7.0 -4

and observed line profiles are present in model atmospheres of DO white dwarfs (hot helium rich white dwarfs with effective temperatures $T_{\text{eff}} > 45000\text{K}$ – for a recent review see Dreizler & Werner 1996a). According to the analysis of Dreizler & Werner (1996b) they may also be removed with the help of C, N, and O opacity, but the abundances of these elements must be enhanced.

In this paper we extend our calculations of pure hydrogen model atmospheres of white dwarfs and our estimates of sphericity effects to slightly more complicated case of atmospheres consisting of hydrogen and helium. In order to obtain better insight into the effects caused by helium on the atmospheric structure, we calculated a grid of model atmospheres for $T_{\text{eff}} = 100000\text{K}$ and $\log g = 7.5$ for different abundances of helium from a pure hydrogen to a pure helium atmosphere. Preliminary results of our calculations have already been presented (Kubát 1996b).

3. Model atmospheres

Our model atmosphere code was described in Kubát (1993, 1994, 1996a). Recently, we included the NLTE occupation probability formalism of Hubeny et al. (1994). This improvement is described in detail in Kubát (1997c). Using our code we have calculated a grid of hydrogen-helium model atmospheres for $\log g = 7.5$, and $T_{\text{eff}} = 100000\text{K}$ for several abundances of helium with respect to the total abundance of hydrogen and helium, namely for 0%, 0.01%, 0.1%, 1%, 5%, 10%, 15%, 20%, 30%, 40%, 50%, 80%, 90%, 99%, 99.9%, and 100% of helium (by particle number).

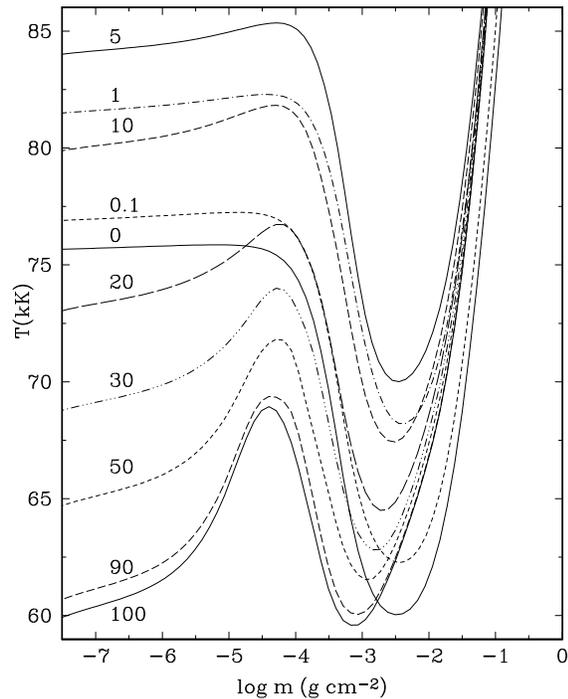


Fig. 2. The temperature structure of some spherically symmetric model atmospheres from our grid. Each model is labeled with a helium abundance (in %) with respect to the total number of particles (by particle number).

We considered an 11 level hydrogen atom (10 levels of H I + H II) and a 16 level helium atom (15 levels of He II + He III). Neglecting He I is acceptable, since its total population is at least six orders of magnitude lower than the total population of He II, and, consequently, has no significant effect on atmospheric structure. Oscillator strengths for both elements were taken from Wiese et al. (1966), lines were assumed to have a Doppler profile. Photoionization cross sections for hydrogen and He II were calculated using a standard hydrogenic formula (see e.g. Mihalas 1978). However, the effect of level dissolution of the uppermost levels on the photoionization cross section was taken into account after Hubeny et al. (1994). The collisional excitation and ionization rates for hydrogen and He II were calculated using the polynomial fit of Napiwotzki (1993) to experimental data. As in our previous paper (Kubát 1995b), the higher, non-explicit levels were assumed to be in LTE with respect to the ground level of the next higher ion. However, their population numbers were calculated using the occupation probability formalism of Hummer & Mihalas (1988). The ionization from these levels is taken into account by means of the so-called modified free-free cross-section (Auer & Mihalas 1969). The collisional transitions between explicit and non-explicit levels are taken into account by means of a modified collisional ionization rate (see Hubeny 1988).

For each helium abundance we have calculated a plane parallel model and a spherically symmetric model for a typical white dwarf mass of $0.6M_{\odot}$. We stopped our calculations when

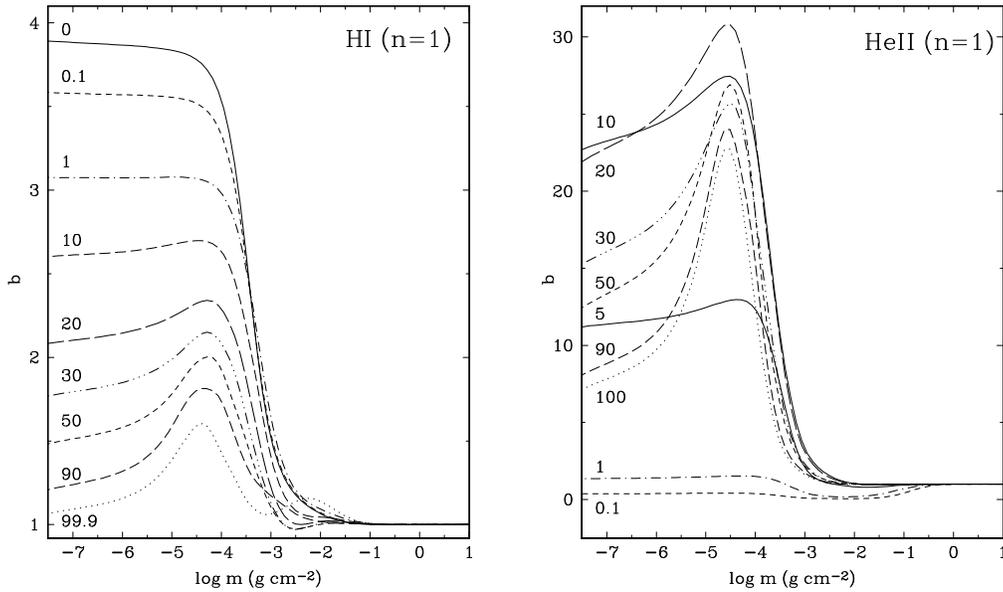


Fig. 3. The departure coefficients for the ground levels of H I and He II. Each curve is labeled with a helium abundance (in %) with respect to all elements by particle number.

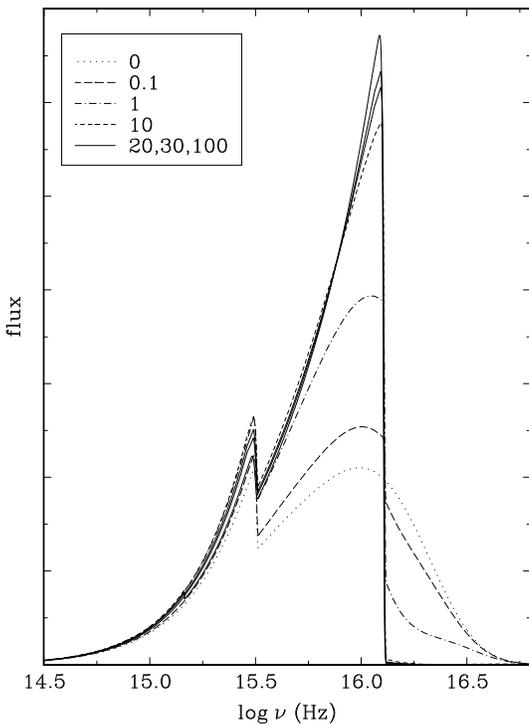


Fig. 4. The continuum emergent flux F_ν from some spherically symmetric model atmospheres from our grid. The numbers indicate the abundance of helium atoms with respect to the total number of particles (in %).

the maximum relative error between two successive iterations was lower than 10^{-3} . In order to speed up the convergence, we successfully implemented the acceleration technique of Ng (1974) and the Kantorovich acceleration (Hubeny & Lanz 1992) into the inner iteration cycle (linearization for given ΔJ_ν).

The temperature structures of some of our resulting models are plotted in Fig. 1. The differences between plane-parallel and spherically symmetric models are of a similar magnitude as in Kubát (1995b). Again, the spherically symmetric models are slightly cooler than the plane-parallel ones.

On the other hand, the effect of increasing helium abundance is much more dramatic. Even a ratio of helium to hydrogen 10^{-4} causes changes in the temperature of the outer regions. In order to see how increasing helium abundance affects the temperature structure, we plotted selected spherically symmetric models in one graph (see Fig. 2). We find that the outer temperature changes even when helium is only a minor constituent. First, the outer temperature rises with increasing helium abundance reaching a maximum around the value of 5% of He. Then the outer temperature decreases down to the value about 60000K for a pure helium atmosphere. The strong effect of helium at its low abundances is caused by the fact that for stars with $T_{\text{eff}} = 100000\text{K}$ the maximum flux falls on the He II Lyman continuum edge. On the other hand, trace abundances of hydrogen in helium atmospheres do not change the temperature structure significantly since its absorption coefficient is relatively low at the maximum flux. In Table 1 we list temperatures of the spherical models at the depth of $\log m = -7.5$ where all lines are optically thin. In addition, we found that increasing helium abundance affects the extension of the atmosphere in the same manner as the temperature. The maximum extension is for one of the "hottest" atmospheres with 1% of helium.

In order to emphasize the NLTE effects we plotted the depth dependence of the departure coefficients $b_i = n_i^{\text{NLTE}}/n_i^{\text{LTE}}$ (Menzel 1937, n_i is the population number of the level i) of the ground levels of both H I and He II in Fig. 3. The departure coefficients start to differ from 1 at depth about $m \sim 0.1$, which corresponds to the Rosseland optical depth of $2/3$ for our models. The hydrogen ground level departure coefficient has its highest values for pure hydrogen atmosphere, then it decreases with

decreasing hydrogen abundance. We found the maximum value of the departure coefficient of the ground state of He II for the model with 20% of helium. The maximum value is almost 10 times higher than for hydrogen ground state. For higher helium abundances, the departure coefficients decrease. The departure coefficients for low helium abundances are about 0.4 for 0.1% of helium and 1.4 for 1% of helium. Notice that for the abundance of 10% they reach the value of 26. Thus we have found large NLTE effects on population numbers. Consequently, we confirm the result of Napiwotzki (1995) that LTE atmospheres are not acceptable for the analysis of very hot white dwarfs.

4. Emergent radiation

4.1. Continuum flux

The net continuum flux without lines is plotted in Fig. 4 for some models of our set. The effect of increasing helium abundance is clearly seen. The absorption due to the opacity of the ionization from the ground state of He II is so effective that even an abundance of 1% causes a significant drop of emergent flux and an abundance of 10% causes the difference of an order of magnitude. The absorbed energy is reemitted at $\lambda > 227\text{\AA}$ and produces a sharp peak around $\nu \sim 10^{16}$. The flux for energies below the He II resonance ionization edge rises with increasing helium abundance. This rise is very large for relatively low helium abundances (tenths of percent). When helium starts to be a dominant element, the rise is much weaker. The behaviour of hydrogen Lyman ionization edge is more complicated, since the flux is affected by both H and He absorption.

4.2. Line profiles

We have calculated detailed He II Paschen and Pickering line profiles for some representative model atmospheres from our grid to show their dependence on the hydrogen/helium abundance ratio. The line profiles were calculated using the approximate formula for Stark broadening presented in the Appendix of Hubeny et al. (1994). They were then normalized to the continuum.

The resulting profiles are plotted in Fig. 5. We see the strong dependence of the intensities of He II P_α and Pi_α lines on the helium abundance for the models for which the helium abundance is the lowest. However, due to the fact that the influence of helium on the atmospheric structure is strong even for an abundance of 0.01%, helium with such low abundance can not be considered as a trace element. On the other hand, the shape of the P_α and Pi_α lines for the helium abundance of 10% is almost the same as for the pure helium atmosphere and changes only marginally for intermediate values of helium abundances. Thus these helium lines (especially the α lines) may serve as an excellent tool for determination of the value of its abundance for low abundances of this element. However, their usage as a diagnostic tool is limited to low abundances of He. Higher members of the Pickering series show similar behavior, however on a more moderate scale.

In addition, even members of the Pickering series (β, δ, \dots) are blended with the hydrogen Balmer lines. Their dependence on the hydrogen/helium abundance ratio is even more interesting. The intensity of the central emission in H_α line rises with decreasing hydrogen abundance reaching the maximum for an abundance of 30%. For lower abundances of hydrogen, the intensity of the central emission decreases, as expected. The magnitude of the dependence of the H_α line profiles for low hydrogen abundances confirms results found by Werner (1996c) in the analysis of the white dwarf PG1034+001 which has the same values of the effective temperature and gravity as our models. The H_β line behaves similarly, however, it is much weaker.

Thus a combination of He II P_α and hydrogen H_α lines may serve as a powerful diagnostic tool for the determination of the hydrogen/helium abundance ratio.

On the other hand, the sphericity effects found in Kubát (1995a,b) are present only for hydrogen lines. The helium lines are affected only marginally. This is mostly due to the fact that He II lines form at depths where the differences in temperature structure are very tiny. Despite the fact that the sphericity effects in the hydrogen H_α line are only about 1%, they are extremely important. The emission core of this line is often used for determination of trace amounts of hydrogen in helium atmospheres. Thus neglecting the sphericity effects may introduce systematic errors into the abundance analysis.

5. Conclusions

We have studied the effect of increasing helium abundance on the temperature structure of the NLTE model atmospheres of hot white dwarfs with the effective temperature $T_{\text{eff}} = 100000\text{K}$ and $\log g = 7.5$. We have found quite dramatic changes in the temperature structure even for very low abundances of helium. They are caused by the fact that the position of the He II ground state ionization edge falls into the region where the emergent radiation reaches a maximum. As a consequence, the absorption by the helium ions is extremely effective. These dramatic changes of the temperature structure and the emergent flux translate to the changes of the line profiles. In this paper we studied the effect on visual helium lines, namely on those from Paschen and Pickering series. The α lines of the above mentioned series are very sensitive to these changes.

Conversely, the difference between temperature profiles for 90% and 100% of helium is much less than between 0% and 1% of helium. For higher helium abundances the temperature structure of the atmosphere does not change as much as for lower helium abundances. The changes in the emergent flux as well as in the He II line profiles are much smaller. However, for high helium abundances (and consequently low hydrogen abundances) the hydrogen/helium ratio is sensitively reflected in the hydrogen Balmer line profiles. Very interesting is the fact that for the abundance of helium 30% (i.e. 70% of hydrogen) the H_α profile has stronger emission than for pure hydrogen atmosphere. A similar effect was observed for hydrogen lines in the NLTE abundance analysis of the much cooler star β Lyr by Dimitrov

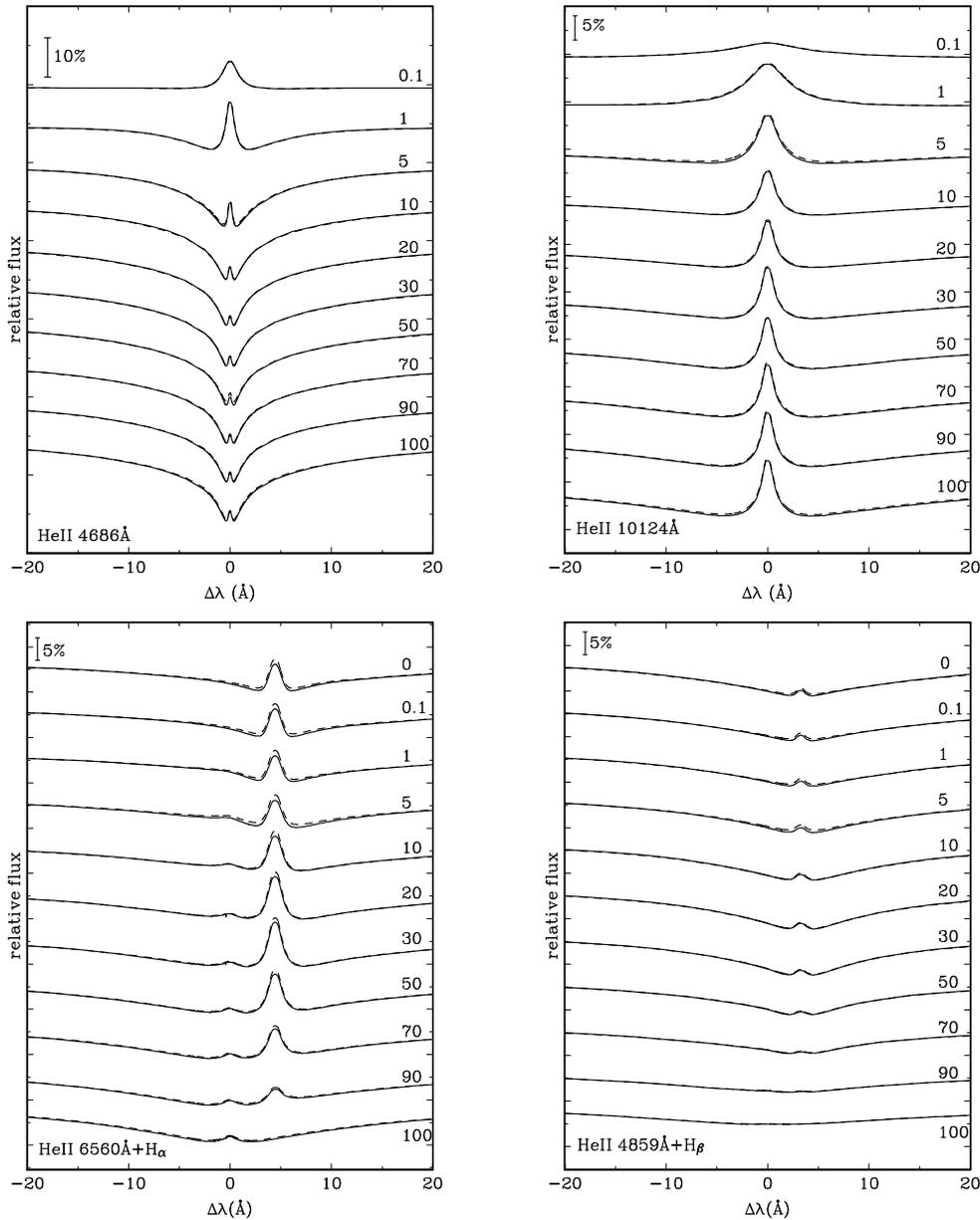


Fig. 5. The profiles for He II P_{α} , Pi_{α} , Pi_{β} , and Pi_{δ} lines. The profiles emerging from spherically symmetric models are plotted with a solid line, the profiles from the plane-parallel ones with the dashed line. For lines without a contribution of hydrogen the line profiles are practically the same for both geometries. Each figure is labeled with a helium abundance (in %) with respect to all elements by particle number.

& Kubát (1988). They found stronger Balmer absorption lines for models with higher abundance of helium.

With the help of our results, the combination of the He II P_{α} line (4686 Å) and the blend He II Pi_{β} + H_{α} may serve as a powerful diagnostic tool for the determination of the hydrogen/helium abundance ratio. For lower abundances of helium the line sensitive to the hydrogen/helium ratio is the He II 4686 Å line accompanied by the weaker dependence of the H_{α} line. For higher abundances of helium where the He II 4686 Å line is almost insensitive to the abundance changes we may use the H_{α} line for the determination of the hydrogen/helium abundance ratio.

The sphericity effects that are surprisingly large for the Balmer lines in the pure hydrogen atmospheres are also present in the Balmer lines in the hydrogen-helium atmospheres. The

He II lines are not affected for the $T_{\text{eff}} = 100000\text{K}$, $\log g = 7.5$ atmosphere. However, the missing effects of sphericity for He II lines for our stellar parameters do not mean that the plane-parallel atmospheres are completely safe also for other values of the effective temperature and surface gravity like e.g. hot central stars of planetary nebulae (see Kubát 1997b).

Our model atmosphere abundance analysis is restricted to models including only hydrogen and helium. The effect of “metals” like carbon, nitrogen, and oxygen should also be studied. However, we do not expect dramatic changes of these conclusions for atmospheres where either hydrogen or helium is a dominant element and metal (i.e. Fe, Ni) line blanketing is neglected. On the other hand, for fully blanketed model atmospheres, the results may change since the effect of blanketing may change physical parameters also in the regions of formation of hydro-

gen and helium lines of interest. Nevertheless, any conclusion about line blanketed model atmospheres must await detailed calculations.

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