

# Understanding some moving groups in terms of a global spiral shock

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**Abstract.** We present a schematic model to interpret the kinematics of some major moving groups in the solar neighbourhood. The origin of these groups is assumed to lie in large scale spiral shocks which may be responsible for the spiral arms in the galactic disk, which provide the initial conditions in the velocities of stars formed as a consequence of them. These initial conditions provide a position-velocity relationship which is used to establish an alternative definition of membership in moving groups. Currently available samples suitable for detecting the existence of moving groups are restricted to a small distance from the Sun, where our definition is in practice equivalent to the classical one based on the similarity of the space velocity vectors. However, the difference between both definitions may become relevant when the high quality astrometric data provided by Hipparcos become available. We apply the formulation developed in this paper to a sample of B, A and F stars to derive some parameters of our model.

**Key words:** stars: kinematics – Galaxy: open clusters and associations; solar neighbourhood – galaxies: spiral

## 1. Introduction

The existence of *moving groups* or *superclusters* of stars in the solar neighbourhood has been known for over one century (Eggen 1989). Nevertheless, their study has been largely restricted to the descriptive aspects of their stellar contents, probably due at least in part to the fact that their very definition as swarms of stars moving with near parallel velocities is fundamentally phenomenological. On the other hand, increasing research in the recent years has been devoted to the study of large star complexes in our and other galaxies, and their relationship to the large scale processes of star formation in galactic disks (Efremov & Chernin 1994). An immediate question is therefore whether the moving groups in our solar neighbourhood may be related to these larger complexes, and how we may use the detailed knowledge about the kinematics of nearby stars to gain

some insight on the formation and evolution of larger structures in galactic disks or about the recent history of star formation in the solar neighbourhood. In this respect, attempts have been made to trace back the orbits of member stars of moving groups to determine their birthplaces (Grosbøl 1976, Yuan 1977, Yuan & Waxman 1977, Palous & Hauck 1986), or to interpret them in terms of bursts of star formation (Gómez et al. 1990), former clusters disrupted by the close encounter with a massive molecular cloud (Wielen 1985), or dispersing coronae of open clusters (Agekian & Belozerova 1979, Mülläri et al. 1994).

A property of some of the best studied moving groups in the solar neighbourhood, such as the Hyades and the Ursae Majoris groups, and, although less pronouncedly, the Pleiades group, is the deviation of the motion of their member stars from the purely circular velocity. Such deviations in fact make their recognition possible, as their members occupy regions of the space velocity diagram falling on the wings of the velocity distribution of field stars; see for instance Fig. 2 in Palous & Hauck 1986. The peculiar velocities of these moving groups are well above the velocity dispersion of field stars with comparable ages. It is interesting to wonder whether the large spatial velocities of the stars may be related to the initial conditions of their formation, as a consequence of some energetic mechanism involved both in triggering the formation of the member stars and in determining their further kinematical behavior.

One such possible mechanism, which we will consider in this paper, is related to the existence of large scale shocks in the interstellar gas associated to the spiral structure of the Galaxy. The aim of this paper is to suggest an interpretative framework for at least some of the moving groups based on the existence of spiral shocks. In doing this, we will suggest a redefinition of what may be understood by a moving group in the context of our model, which in practice is equivalent to the classical definition when one is restricted to study samples limited to a small volume around the Sun. We will also provide some predictions that can be tested in the near future, when higher precision velocities and distances for stars belonging to these moving groups are available thanks to the data obtained by the Hipparcos satellite.

## 2. Effects of a spiral shock on stellar kinematics

The possibility that the perturbation on the galactic potential which induces the spiral structure could trigger star formation via a large scale spiral shock in the interstellar medium was already studied by Roberts 1969 and Shu et al. 1972; see also the reviews of Wielen 1974, Rohlfs 1977, Toomre 1977, and Chapter 3 of Bertin & Lin 1995 for an overview of the process of shock-induced star formation in spiral arms. The actual existence of such a shock has been controversial, as it depends on the dominant phase of the galactic interstellar medium. As noted by Combes & Gerin 1985, a medium dominated in volume by coronal gas and containing most of the denser phases in the form of discrete molecular clouds would react very differently to a spiral perturbation as would do a more homogeneous medium dominated in volume by warm atomic hydrogen. Also, Elmegreen 1992 has discussed other ways in which spiral arms can trigger cloud collapse and star formation without the need for a spiral shock. However, the existence of a large scale spiral shock has been assumed in other galaxies in order to explain the azimuthal separation of tracers of different stages of star formation (see Bertin & Lin 1995 and references therein), radial trends in the efficiency of star formation (Roberts, Roberts, & Shu 1975; Cepa & Beckman 1990; Puerari & Dottori 1996), and color gradients across the spiral arms (Yuan & Grosbøl 1981). In our Galaxy, the kinematic properties associated to a spiral shock producing a phase transition in the diffuse interstellar medium have been applied to explain the overall dynamics of molecular clouds (Bash & Peters 1976; Bash et al. 1977) and the vertex deviation of O and B stars in the solar neighbourhood (Hilton & Bash 1982). The kinematical perturbations induced by spiral arms in the solar neighbourhood have been applied to address problems such as the vertex deviation (Yuan 1971), the local values of the Oort constants (Lin et al. 1978; Lindblad 1980) and the heating of the galactic disk (Binney & Lacey 1988).

Our main hypothesis throughout this paper is that the main moving groups presently observed in the solar neighbourhood were formed in such shocks, and that the jump in the velocity of the gas crossing the shocks is the main responsible of their deviation from the circular motion. As such deviations are observed to be of order  $\sim 20 \text{ km s}^{-1}$ , i.e., less than 10% of the circular velocity at the position of the Sun, and the induced radial excursions are therefore small as compared to the radius of the solar circle (also less than 10%), we will use the first order epicyclic approximation to find out the kinematic signature expected in the stars formed by this process. This enables us to obtain a simple formulation, which predicts observable correlations among the constants of the epicyclic orbits, and a criterion of membership in moving groups based on these correlations. In doing so, we will be forced to introduce some simplifications, such as neglecting the streaming motions in the velocity induced by the gravitational potential associated to the density waves; including these perturbations would require to link the local spiral-arm structure to a grand design spiral pattern in our Galaxy. However, such a connection between the solar neighbourhood and the overall spiral structure of the Galaxy has been conflictive

ever since the density wave theory was proposed. We will further discuss this point in Sect. 3.3, and will provide an estimate of the errors introduced by our simplifications by integrating stellar orbits, including the effects of the spiral perturbation of the gravitational field.

Let us consider a rotating reference frame whose center, momentarily occupied by the Sun, rotates in a circular orbit around the galactic center with a velocity  $\Omega$ . The  $x$  axis is directed towards the galactic center and the  $y$  axis points in the sense of galactic rotation. Orbits deviating little from circularity can be described by the simple epicyclic expressions (e.g. King 1989)

$$x = X_1 + C \cos(\kappa t + \phi) \quad (1a)$$

$$y = Y_1 + 2AX_1t + \frac{2\Omega C}{\kappa} \sin(\kappa t + \phi) \quad (1b)$$

$$\dot{x} = -C\kappa \sin(\kappa t + \phi) \quad (1c)$$

$$\dot{y} = 2AX_1 + 2\Omega C \cos(\kappa t + \phi) \quad (1d)$$

where  $A$  is the corresponding Oort constant,  $\kappa$  is the epicyclic frequency, and  $X_1$ ,  $Y_1$ ,  $C$ ,  $\phi$  are constants describing the size and position of the orbit and that of the star in it. They can be easily determined from observations by fixing  $t = 0$  at the present time.

The spiral pattern and its associated shock rotate with an angular velocity  $\Omega_p$ . As long as the Sun is far from the corotation circle, the spiral shock can be pictured as a ridge rapidly crossing this reference system at periodical intervals, leaving star formation behind it. The velocity vector of the gas entering the shock, together with the shock jump conditions, determine the initial conditions in the motions of the stars.

The spiral shock forms an angle  $i$  (the pitch angle of the spiral pattern) with the direction of the galactic rotation, that we will assume to be small. As the interstellar gas crosses the shock, its velocity conserves the component parallel to the shock front, while, assuming the shock to be strong and rapidly dissipative, the velocity component perpendicular to it is greatly reduced. The shocked gas thus moves in a direction nearly tangent to the spiral arm; we will henceforth assume that this is also the initial velocity of the stars formed as a consequence of the compression associated to the passage of the gas by the spiral arm. In doing so, we implicitly assume that stars are formed out of clouds resulting from a phase transition in the diffuse interstellar medium or, alternatively, that the drag exerted by the shocked diffuse medium on pre-existing clouds can trigger star formation in them and slow them down to the velocity of the shocked gas in a time which is short as compared to the epicyclic period. This is also the approximation used by Bash & Peters 1976. Yuan & Grosbøl 1981 have discussed the case of longer drag timescales, based on the results of Woodward 1976. In that case, the period over which star formation proceeds in the clouds is shorter than the timescale for drag of the clouds by the diffuse medium. If this were so, then the initial velocities of the stars should be of order of the streaming motions induced by shockless spiral arms (as they would form before the kinematic effects of the

shock could be transmitted to the star forming clouds), rather than reflecting the post-shock velocity of the diffuse gas. Given that such streaming motions have smaller amplitudes than the deviations from circularity observed in the moving groups discussed here (see Sect. 3.3), we have chosen the first scenario to proceed in our study.

The circular velocity in the reference frame moving with the spiral shock is

$$v = R(\Omega - \Omega_p) \quad (2)$$

where we will use for  $R$  (the distance to the galactic center) and  $\Omega$  the values of 8.5 kpc and  $25.9 \text{ km s}^{-1} \text{ kpc}^{-1}$ , respectively, appropriate for the solar neighbourhood (Kerr & Lynden-Bell 1986). The velocity deviations of the gas from the circular motion in the galactocentric direction and in the direction of galactic rotation are respectively  $\xi$  and  $\eta$ . The components tangential and perpendicular to the shock are:

$$v_{t0} = R(\Omega - \Omega_p) \cos i + \xi \sin i + \eta \cos i \quad (3a)$$

$$v_{p0} = R(\Omega - \Omega_p) \sin i - \xi \cos i + \eta \sin i \quad (3b)$$

For a strong shock to exist, the condition  $v_{p0} \gg c_s$  must be fulfilled, with  $c_s$  being the effective sound speed in the interstellar gas. In a tightly wound spiral, this condition can be fulfilled far from the corotation circle, where  $(\Omega - \Omega_p)$  is large. Moreover, the response of the gas to the spiral gravitational potential provides a positive contribution to  $v_{p0}$  from the term  $-\xi \cos i + \eta \sin i$  at the position of the shock fronts (Shu et al. 1972). On the other hand, the velocity dispersion in the unshocked gas adds random terms in (3a) and (3b), expected to be of order of  $c_s$ ; therefore, this condition also ensures that the velocity dispersion of the gas does not introduce any essential changes in our treatment. Such velocity dispersion in the gas entering the shock, implying a range of values in the initial values for the jump conditions, may be expected to be reflected in the internal velocity dispersion of the moving groups formed out of it.

After the shock,  $v_t = v_{t0}$ ,  $v_p \simeq 0$ . In the reference frame used to define the epicyclic approximation, where  $\dot{x} = v_t \sin i$ ,  $\dot{y} = 2AX_1 + v_t \cos i + R(\Omega - \Omega_p)$ , we obtain

$$\dot{x} = (R(\Omega - \Omega_p) \cos i + \xi \sin i + \eta \cos i) \sin i \quad (4a)$$

$$\dot{y} = 2AX_1 - R(\Omega - \Omega_p) \sin^2 i + \eta \cos^2 i + \xi \cos i \sin i \quad (4b)$$

In writing Eqs. (4) in this form, we are assuming a trailing spiral and define the pitch angle  $i$  to be positive. Under the conditions of validity of our approximation, the term  $\xi \sin i + \eta \cos i$  in (4a) is much smaller than  $R(\Omega - \Omega_p) \cos i$ , so that

$$\dot{x} \simeq R(\Omega - \Omega_p) \cos i \sin i \quad (4c)$$

On the other hand, the term  $-R(\Omega - \Omega_p) \sin^2 i - \xi \cos i \sin i$  in (4b) is much smaller than the term  $R(\Omega - \Omega_p) \cos i \sin i$  in the right hand side of (4a). By comparison to (4a), Eq. (4b) can then be simply approximated by

$$\dot{y} \simeq 2AX_1 + \eta \cos^2 i \quad (4d)$$

The Eqs. (4c) and (4d) thus determine the approximate initial velocity at the time  $\tau$  of the passage of the spiral shock by a given position  $x_0, y_0$ . If the term  $R(\Omega - \Omega_p) \sin i$  is still larger than  $\eta$ , the impulse is mostly in the radial direction, and the resulting expressions are essentially the same as if we had assumed a purely circular motion for the gas at the start. Eqs. (4c) and (4d), together with Eqs. (1), give the following relations for the epicyclic elements:

$$C \cos(\kappa\tau + \phi) = \frac{\eta \cos^2 i}{2\Omega} \quad (5a)$$

$$C \sin(\kappa\tau + \phi) = -\frac{R(\Omega - \Omega_p) \cos i \sin i}{\kappa} \quad (5b)$$

$$C^2 = \frac{R^2(\Omega - \Omega_p)^2}{\kappa^2} \cos^2 i \sin^2 i + \frac{\eta^2 \cos^4 i}{4\Omega^2} \quad (5c)$$

$$x_0 = X_1 + \frac{\eta \cos^2 i}{2\Omega} \quad (5d)$$

$$y_0 = Y_1 + 2AX_1\tau - \frac{\Omega R(\Omega - \Omega_p) \sin 2i}{\kappa^2} \quad (5e)$$

Let us consider now the evolution of a star forming region along a spiral shock. Initially, the region is elongated along the spiral arm and forming with the direction of the galactic rotation an angle  $i$  (the pitch angle of the spiral arm). The shocked region then moves independently of the spiral pattern, and the galactic differential rotation tends to align its axis with the direction of the galactic rotation in a characteristic timescale given by shearing due to galactic differential rotation,  $t \sim (2A)^{-1} \simeq 4 \cdot 10^7$  years. Eventually, this region may approach the solar neighbourhood, appearing as a ridge of stars perpendicular to the direction of the galactic center. If the distance from the Sun until which we can observe these stars is small enough, the fragment of this ridge accessible to observations will actually correspond to a small portion of the spiral arm which we can characterize by a single value of  $x_0, y_0$  or, by Eqs. (5d), (5e), of  $X_1, Y_1$ .

The solar neighbourhood can be expected to contain some such fragments of ancient star forming ridges, left behind by the spiral arm as it moved across the local region of the galactic disk. Each such fragment will be characterized by the position of the guiding center of the epicyclic orbits of the stars belonging to it,  $X_1, Y_1$ . We can easily relate  $X_1, Y_1$ , and  $\tau$ : the position of the spiral shock can be represented locally by a straight line moving across our reference system:

$$y_0 = a + bx_0 + c\tau \quad (6)$$

where  $a$  is related to the present position of the spiral arm, and

$$b = \frac{1}{\tan i} \quad ; \quad c = -R(\Omega - \Omega_p) \quad (7)$$

Moreover, by Eqs. (5a), (5b),

$$\kappa\tau + \phi = \arctan\left[-\frac{2\Omega(\Omega - \Omega_p)R}{\kappa\eta} \tan i\right] + 2\pi n \quad (8)$$

and relating  $x_0, y_0$  to the positions of the epicyclic orbits guiding centers by means of (5d), (5e),

$$a + \frac{x_0}{\tan i} - R(\Omega - \Omega_p)\tau = Y_1 + 2AX_1\tau - \frac{\Omega R(\Omega - \Omega_p) \sin 2i}{\kappa^2} \quad (9)$$

whence

$$\tau = \left(a + \frac{1}{\tan i}(X_1 + \frac{\eta \cos^2 i}{2\Omega}) - Y_1 + \frac{\Omega(\Omega - \Omega_p)R \sin 2i}{\kappa^2}\right) \times \frac{1}{2AX_1 + R(\Omega - \Omega_p)} \quad (10)$$

This expression can be somewhat simplified if the Sun is far from the corotation and if the observational horizon is restricted to no more than a few hundred parsecs; in that case, the maximum value of  $X_1$  can be estimated from (1a) and (5c), making  $x \sim 0$ :

$$X_1 \simeq \frac{1}{\kappa} R(\Omega - \Omega_p) \cos i \sin i$$

and the ratio of the two terms in the denominator of Eq. (10) is

$$\frac{2AX_1}{R(\Omega - \Omega_p)} < \frac{A}{\kappa} \sin 2i$$

For values of  $A, \kappa$  typical of a flat rotation curve and small  $i$ , we have that  $\frac{2AX_1}{R(\Omega - \Omega_p)} \ll 1$ , and therefore we can write (10) as

$$\tau \sim \frac{1}{R(\Omega - \Omega_p)} \left(1 - \frac{2AX_1}{R(\Omega - \Omega_p)}\right) \left[a + \frac{1}{\tan i} \left(X_1 + \frac{\eta \cos^2 i}{2\Omega}\right) - Y_1 + \frac{\Omega(\Omega - \Omega_p)R \sin 2i}{\kappa^2}\right] \quad (11a)$$

Developing the products, and retaining only the first-order terms,

$$\tau \sim \frac{1}{R(\Omega - \Omega_p)} \left[ \left(\frac{1}{\tan i} - \frac{2Aa}{R(\Omega - \Omega_p)}\right) X_1 - Y_1 \right] + \left[ \frac{a}{R(\Omega - \Omega_p)} + \frac{\eta \cos^2 i}{2\Omega R(\Omega - \Omega_p) \tan i} + \frac{\Omega \sin 2i}{\kappa^2} \right] \quad (11b)$$

We note that, from Eq. (6),  $a = R(\Omega - \Omega_p)\bar{\tau}$ , where  $\bar{\tau}$  is the average age of the group defined as the time when the star-forming spiral shock passed by the local standard of rest; therefore,  $a$  can be very large for old moving groups.

We can obtain a relationship among the epicyclic elements of stars with their origin in the same spiral arm by using Eq. (8). In doing so, we should include the dependence of  $\kappa$  with

galactocentric distance; to simplify, we will use that, for a flat rotation curve,

$$\frac{1}{\kappa} \simeq \frac{1}{\kappa_0} \left(1 - \frac{X_1}{R}\right) \quad (12)$$

where  $\kappa_0$  is the value at the position of the Sun. Replacing (11b) in (8), using the approximation (12), and neglecting the term containing  $Y_1$ , which is in general much smaller than those containing  $X_1$ , we finally obtain

$$\phi \simeq \mathcal{A} + \mathcal{B}X_1 + \mathcal{C}X_1^2 \quad (13)$$

where

$$\mathcal{A} = S + 2\pi n - \kappa_0\bar{\tau} - \frac{\eta\kappa_0 \cos^2 i}{2R\Omega(\Omega - \Omega_p) \tan i} - \frac{\Omega \sin 2i}{\kappa_0} \quad (14a)$$

$$\mathcal{B} = \frac{\kappa_0}{R(\Omega - \Omega_p)} \left[\bar{\tau}(2A - \Omega + \Omega_p) - \frac{1}{\tan i}\right] \quad (14b)$$

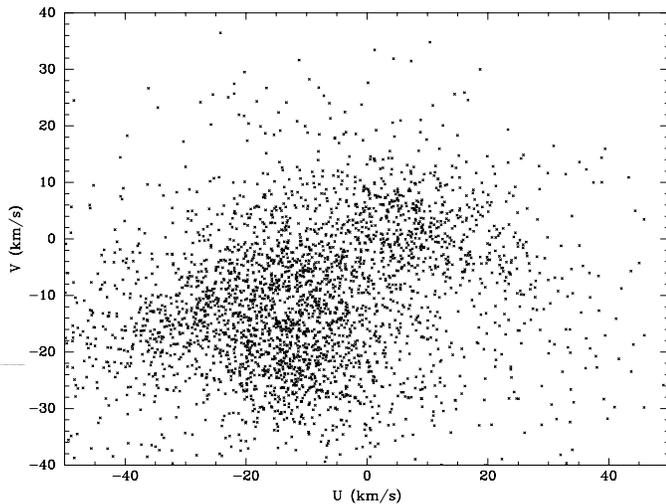
$$\mathcal{C} = \frac{\kappa_0}{R^2(\Omega - \Omega_p)} \left(2A\bar{\tau} - \frac{1}{\tan i}\right) \quad (14c)$$

with

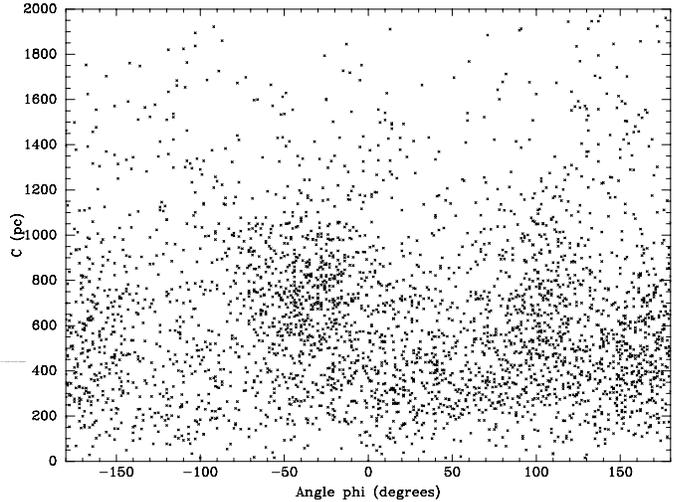
$$S = \arctan\left[-\frac{2\Omega(\Omega - \Omega_p)R}{\kappa_0\eta} \tan i\right] \quad (14d)$$

Eq. (13), plus the condition  $C = \text{constant}$ , may thus be used to provide an alternative definition of moving group, by which member stars are identified by having been formed in the same passage of a spiral arm by the solar neighbourhood. Notice that this definition, unlike classical ones, does not imply that the members of a moving group have isoperiodic orbits around the galactic center, that their space velocity vectors are similar, or that the stars are coeval. However, the restriction in heliocentric distance of the samples of stars whose motions we can analyze implies strong correlations among their orbital elements, resulting in kinematical similarities among the stars belonging to a group and observed in the solar neighbourhood. On the other hand, Eq. (10), together with the restriction in accessible values of  $X_1, Y_1$ , and correlations among these and other orbital elements limits the range of ages among the stars belonging to the so-defined moving groups.

It is in practice convenient to use an alternative form of Eq. (13), due to the fact that if  $x$  is of order of or smaller than  $C$ , then  $\phi$  and  $X_1$  are always strongly correlated, regardless of the existence of a relationship like (13). Eq. (13), however, clearly shows the dependence of velocity on position implied by our definition of moving group, taking into account that  $X_1 = x - C \cos \phi$ . Since, by Eqs. (1c), (1d),  $\phi$  essentially determines the direction of the velocity, our definition implies a gradient in both  $\dot{x}$  and  $\dot{y}$  with the  $x$  coordinate, independent of the gradient in  $V$  derived from the condition of isoperiodicity (see e.g. Eggen 1992a), noticeable only when  $x$  becomes of the same order as  $C$ . Moreover, Eq. (13) can have several possible solutions for



**Fig. 1.**  $U$ ,  $V$  components of the velocity of the B, A and F stars in our catalog



**Fig. 2.** Distribution of the epicyclic elements  $\phi$ ,  $C$  of the stars in our catalog.

$\phi$  in the proximities of  $x = 0$  for some values of  $\bar{\tau}$ ; it is therefore possible in principle that stars of the solar neighbourhood having a common origin in the same spiral arm could appear as kinematically distinct groups. On the other hand, in some regions of the  $\phi - x$  diagram the gradient of velocity with position can be expected to be difficult to appreciate, given the scatter arising from uncertainties in the velocity determinations. High precision spatial velocities extending to more distant stars (up to a few hundred parsecs) should permit the detection of this effect and thus provide a justification of our proposed definition of moving groups on physical grounds.

### 3. An application to nearby B, A and F stars

To study the properties of the spiral structure in the solar neighbourhood as derived from some moving groups, we used a sample of 3373 B-, A-, and F-type main sequence stars with known proper motions, radial velocities, and Strömgren photometry. The data are mostly from the Hipparcos Input Catalogue (Turon et al. 1992), complemented with Strömgren photometry and new radial velocities from several sources; see details in Jordi et al. 1996 and Chen et al. 1997. The existence of moving groups in this sample can be appreciated in the diagram shown in Fig. 1. In this diagram,  $U$ ,  $V$  are the components of the velocity in a non rotating reference frame (while  $\dot{x}$ ,  $\dot{y}$  are defined in a rotating reference frame):  $U$  is directed towards the galactic center, and  $V$  towards the direction of the galactic rotation. The relation between  $U$ ,  $V$  and  $\dot{x}$ ,  $\dot{y}$  is

$$\dot{x} = (U + U_{\odot}) - \Omega y \quad (15a)$$

$$\dot{y} = (V + V_{\odot}) + \Omega x \quad (15b)$$

where  $U_{\odot}$ ,  $V_{\odot}$  are the components of the solar motion with respect to the circular velocity. Fig. 1 essentially reproduces the

features in Fig. 1 of Palous & Hauck 1986, but with a considerably larger sample of stars. There are three major clusterings of stars visible in Fig. 1: the Hyades moving group near  $(U, V) = (-35, -16)$  km s<sup>-1</sup>; the Ursae Majoris moving group (also called the Sirius moving group) near  $(U, V) = (5, 2)$  km s<sup>-1</sup>; and the Pleiades moving group near  $(U, V) = (-10, -20)$  km s<sup>-1</sup>. For the components of the solar motion, we adopt  $(U_{\odot}, V_{\odot}) = (9, 12)$  km s<sup>-1</sup>.

As shown by Eq. (13), stars from a moving group with a deviation from the circular motion (characterized by the value of  $C$ ) considerably larger than their average distance to the Sun can be expected to have similar values of  $\phi$ . A diagram of  $\phi$  vs.  $C$  can thus be used too to identify moving groups, in a similar way as is done in a  $U$ , vs.  $V$ . diagram. The  $(\phi, C)$  diagram corresponding to our sample of stars appears in Fig. 2.

The clusters in the  $(U, V)$  diagram translate into clusters in the  $(\phi, C)$  diagram. The stars of the Hyades, Pleiades and Ursae Majoris moving groups now cluster around  $\phi \simeq 100^{\circ}$ ,  $\phi \simeq 170^{\circ}$ , and  $\phi \simeq -40^{\circ}$ , respectively. Although with a rather large scatter, due in a large part to observational errors, the average values of  $C$  are similar for these three groups, lying around 600 pc. This result is in an acceptable agreement with the value  $C = 450$  pc obtained by using the best fitting parameters to the HI kinematics in our Galaxy, obtained by Lin et al. 1969 ( $i = 8^{\circ}$ ,  $\Omega_p = 13$  km s<sup>-1</sup>).

#### 3.1. Ages of the moving groups

Eq. (8) predicts a relationship between  $\phi$  and the age. In principle, this could be used as a consistency test for our initial hypothesis if the ages of the groups were known with precision. However, some of the simplifications we have made in our treatment, such as the neglect of the gravitational perturbation associated to the density wave, can be expected to influence the real evolution of  $\phi$  with time. We will evaluate this influ-

ence in Sect. 3.3; however, to do it, and to use Eq. (13) as a first approximation to the recognition of moving groups with our alternative criterion, it is convenient to have at least an estimate of the ages of the moving groups observed in the solar neighbourhood.

The age of groups may be in principle better determined when there is an open cluster associated to them, as is the case of the Hyades and the Pleiades. The situation is somewhat worse for the Ursae Majoris group, whose age determination relies on stars selected only by their velocity, regardless of their position in the sky. An independent approach, used by Chen et al. 1997, is based on the joint use of proper motions, radial velocities, and Strömgren photometry to detect moving groups and simultaneously derive the kinematical properties and mean age of each one.

Unfortunately, even a short review of the literature shows that uncertainties in the determination of the age of even thoroughly studied open clusters are comparable or even larger than the epicyclic period. Discussions of this problem and its relation to the features of the adopted stellar models can be found in Bertelli et al. 1992, and it is illustrated with ages derived for single clusters by different authors in Janes & Phelps 1994. Therefore, the published values are of use only to the extent of allowing an estimate of the number of epicyclic orbits elapsed since the formation of the group, related to  $n$  in Eq. (8). This equation can then be used to derive a kinematic age for the group.

Several kinematic ages are in principle possible for each moving group. As the differences among them are an entire number of epicyclic periods ( $\simeq 1.7 \cdot 10^8$  yr), it is usually possible to choose only one or two values of  $n$  best fitting the actual observations. To estimate the possible ages of moving groups, we use Eq. (8), with the approximation (12) and the condition that, near the Sun,  $X_1 \simeq -C \cos \phi$ . In this way, we will adopt the following ages:

- \* Hyades group:  $\tau = -7.6 \cdot 10^8$  yr,  $n = -5$
- \* Ursae Majoris group:  $\tau = -3.8 \cdot 10^8$  yr,  $n = -3$
- \* Pleiades group:  $\tau = -1.1 \cdot 10^8$  yr,  $n = -1$

These ages are found assuming  $\eta = 0$  in Eq. (8); however, using  $\eta = 10 \text{ km s}^{-1}$  as a representative value of the amplitude of the streaming motions induced by the spiral arms would change the ages by only  $10^7$  years.

It should be noted that these ages are those of the group members which are passing by the solar neighbourhood at present, and are in general different from the average age  $\bar{\tau}$  defined before and used in Eqs. (13). The ages  $\bar{\tau}$  are found from Eq. (13), using the fact that the coefficients  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\mathcal{C}$  are linear in  $\bar{\tau}$ , and replacing  $\phi$  by its value in the solar vicinity and  $X_1$  by  $-C \cos \phi$ . In this way, and assuming as before  $\eta = 0$ , we obtain:

- \* Hyades group:  $\bar{\tau} = -7.7 \cdot 10^8$  yr
- \* Ursae Majoris group:  $\bar{\tau} = -3.1 \cdot 10^8$  yr
- \* Pleiades group:  $\bar{\tau} = -1.7 \cdot 10^8$  yr

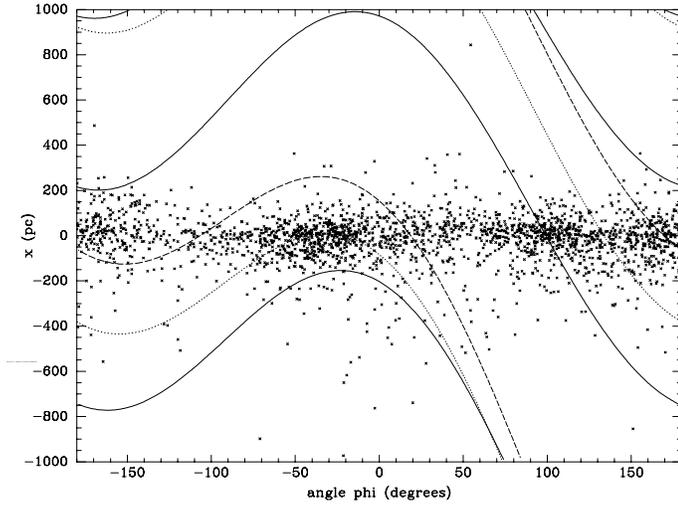
These are the values to be used in Eqs. (13); however, the former set of  $\tau$ - values should be used when comparing to inde-

pendent age estimates of each moving group in the solar neighbourhood. The distinction is not very important for the two older groups, but can be relevant for the Pleiades, as the difference between both values amounts to over 50 % of the present age of the group. On the other hand, it will be shown in Sect. 3.3 that best-fitting ages based only on stars with  $x \simeq 0$  are not reliable, due to our neglect of the spiral arm gravitational field which can cause group displacements several hundred parsecs in amplitude.

The age adopted for the Hyades group is consistent with the several age determinations listed by Janes & Phelps 1994 for both the Hyades and the Praesepe clusters, both having similar kinematics and age and possibly belonging to the same moving group. The average adopted by Janes & Phelps 1994 suggests that  $\tau = -7.6 \cdot 10^8$  yr is the most likely value; these authors take  $\tau = -9 \cdot 10^8$  yr as an average for Praesepe, a value closer to the estimate of Eggen 1993 and Chen et al. 1997 for members of the group at large, i.e., not only cluster members. Nevertheless, other studies (Soderblom et al. 1993, Williams et al. 1994, Balachandran 1995) support the coevality of both clusters. We will therefore adopt  $\tau = -7.6 \cdot 10^8$  yr,  $n = -5$  for the Hyades moving group, although adopting  $n = -6$ ,  $\tau = -9.3 \cdot 10^8$  yr could also have some observational support. On the other hand, we should notice that, in his study of the Hyades moving group members, Eggen 1992a finds indications of several age groups with ages ranging from  $3 \cdot 10^8$  yr to  $8 \cdot 10^8$  yr.

There are also large discrepancies as to the age of the Ursae Majoris group. A short review of derived ages is given by Boesgaard et al. 1988. The commonly accepted value is around  $\tau = -3 \cdot 10^8$  yr (e.g. Soderblom & Mayor 1993a), but Eggen 1992b finds indications of a large age spread and the existence of groups considerably older than that value. Younger ages, more concordant with the above quoted value, are found by Palous & Hauck 1986, as well as Eggen 1983. Chromospheric activities among G and K members of the group (Soderblom & Mayor 1993b) also support an age around  $3 \cdot 10^8$  yr. We will adopt the above estimate of  $n = -3$ ,  $\tau = -3.8 \cdot 10^8$  yr in this paper.

As for the Pleiades, the only value compatible with our hypothesis about the origin of its moving group and its estimated age is  $n = -1$ ,  $\tau = -1.1 \cdot 10^8$  yr, near the independent age estimate of Chen et al. 1997. Although the age commonly accepted for the Pleiades is  $\tau \simeq -8 \cdot 10^7$  yr (Lyngå 1987), this value has been a subject of controversy, given the apparent evidence for a significantly older age of the lower mass stars (Herbig 1962) or a large age spread among very low mass members (Steele et al. 1993). These problems have been recently reassessed: Mazzei & Pigatto 1989 have found an age in excess of  $1.0 \cdot 10^8$  yr for the more massive stars using improved evolutionary models, a result confirmed by Meynet et al. 1993. On the other hand, Stauffer et al. 1995 do not find indications of a significant non-coevality among very low mass members. Convincing evidence for a Pleiades age exceeding  $10^8$  yr has been recently presented by Basri et al. 1996 based on the lithium test applied to some of the lowest mass members. We conclude that current estimates are well consistent with the kinematic age inferred by us assum-



**Fig. 3.** Expected variations of the epicyclic element  $\phi$  along the direction to the galactic center  $x$ , superimposed on the distribution of stars of our sample with  $300 \text{ pc} < C < 900 \text{ pc}$ . The solid line corresponds to the Hyades moving group; the dashed line, to the Pleiades moving group; and the dotted line, to the Ursae Majoris moving group

ing the Pleiades moving cluster to have been formed in a spiral shock.

### 3.2. Model predictions

The fact that the relationship (13) implies that  $x \simeq 0$  at well defined values of  $\phi$  makes our definition of moving groups equivalent in practice to the classical ones, in the sense that membership criteria can be established on the basis of the similarity of the space velocities, whose direction is determined by the angle  $\phi$ . As discussed above, this is a consequence of the correlations among epicyclic elements which appear when the samples analyzed are restricted to a distance of few hundred parsecs from the Sun, such as the one used in this paper. However, our model makes some predictions about the spatial variations of some epicyclic elements that may be checked in the future, when precise velocities are available for more distant stars, thus possibly revealing distant members of the moving groups discussed here.

Fig. 3 shows the relation (13) for the different moving groups, indicating the variations to be expected with  $x$ . The lines in Fig. 3 show the average locus expected to be occupied by stars formed in the same passage of a spiral arm, whose vertical coordinate (depending on the adopted age  $\bar{\tau}$ ) has been adjusted according to the position of the actually observed moving groups, as explained in Sect. 3.1. The observational uncertainties may thus move these curves upwards or downwards, while the inflection centered near  $\phi = -90^\circ$  is a feature of the model, independent of the spiral structure parameters and common to all groups.

As can be seen, an extension of the observations out to a distance of 500 pc or more should clearly reveal our predicted spatial gradients in  $\phi$ , and consequently whether or not our hypothesis about the origin of the discussed moving groups is sup-

ported by them. The case of the Ursae Majoris moving group is especially tantalizing, as it is located near a local maximum in the  $\phi$ - $x$  curve; data extending out to a few hundred parsecs away from the galactic center may reveal velocity gradients consistent with this picture. It should be pointed out that, although the presence of a spiral gravitational field is expected to distort the  $\phi$ - $x$  relationship, as will be discussed in the next section, the qualitative behavior remains similar, still leaving Ursae Majoris as the best placed group to test our hypothesis. Interestingly, a possible indication of the local maximum in the  $\phi$ - $x$  diagram associated to the Ursae Majoris group near the Sun may be present in the results of Méndez et al. 1992, who found a smaller spatial extent for the Ursae Majoris group as compared to the Hyades group: if one bases the criterion for membership allocation on the similarity of velocities, then the Ursae Majoris group should have a small extent in the radial direction. The results of Méndez et al. 1992 may thus be reflecting the abrupt velocity gradient with  $x$  expected at the location of the Ursae Majoris group.

Another consequence of our definition of moving group is that coevality is not strictly required, although in practice the age spread of stars of a given group within a radius of a few hundred parsecs should be generally small and hardly detectable. The age spread for a given group can be estimated from (11b), from which, neglecting the spread in  $Y_1$  (whose effect is much smaller than that of the spread in  $X_1$ ), we can write

$$\Delta\tau \simeq \frac{1}{R(\Omega - \Omega_p)} \left( \frac{1}{\tan i} - 2A\bar{\tau} \right) \Delta X_1 \quad (16)$$

Using spiral structure parameters from Lin et al. 1969, we obtain in the limits of very young ( $\bar{\tau} \rightarrow 0$ ) and very old ( $\bar{\tau} \rightarrow +\infty$ ) groups that  $\Delta\tau[\text{yr}] \simeq 6.3 \cdot 10^7 \text{ yr} \Delta X_1[\text{kpc}]$  and  $\Delta\tau/\bar{\tau} \simeq 0.24 \Delta X_1[\text{kpc}]$ , respectively. Given the large uncertainties reviewed in Sect. 3.1, age gradients in the galactocentric direction may be actually difficult to detect even extending the observational horizon out to 1 kpc from the Sun.

### 3.3. Relation to the spiral structure and validity of the approximations introduced

As was already noted in Sect. 2, the treatment of the orbits of the stars subsequent to their formation in a spiral shock under the epicyclic approximation implies a number of simplifications which can be expected to distort the relation (13). A major neglected effect is due to the gravitational potential associated to the spiral density wave, while only the axisymmetric, slowly varying overall gravitational potential is used for the epicyclic approximation.

To estimate the effect of our simplifications, we have numerically integrated the individual orbits of stars, which have been evolved in time under the influence of a radial force  $F_r = \Omega_0^2 R_0^2 / R$  (where the subindex "0" denotes values at the Sun's position), corresponding to a flat rotation curve, and perturbed by the force components due to the density wave:

$$F_{sr} = \alpha \Omega_0^2 R_0 \sin \psi \quad (17a)$$

$$F_{st} = \alpha \Omega_0^2 R_0 \tan i \sin \psi \quad (17b)$$

with

$$\psi = m\Omega_p(t - \tau) - m(\theta - \Omega\tau) - \frac{m}{\tan i} \ln \frac{R}{R_0} \quad (17c)$$

In these expressions,  $R$ ,  $\theta$  are galactocentric polar coordinates, having the Sun at the present time at ( $R = R_0$ ,  $\theta = 0$ );  $m$  is the number of arms, and  $\alpha$  is an adimensional factor representing the relative importance of the spiral perturbation as compared to the dominant axisymmetric galactic field.

For the estimate of the effects induced by the perturbing potential, we have used the best fitting parameters to the 21 cm observations of HI in our Galaxy derived by Lin et al. 1969. It is a well known fact that this set of parameters can fit rather successfully the overall HI structure of our Galaxy, but fails in explaining the details of the distribution of spiral arm tracers in the solar neighbourhood (Lin & Yuan 1978; Elmegreen 1985). Lin & Yuan 1978 consider that several spiral structures coexist in the Galaxy, and that the description of our Galaxy in terms of a single spiral pattern is an oversimplification; the latter objection would therefore apply to the perturbations described by Eqs. (17), and would leave the problem of integrating the orbits indetermined. However, we may expect that orbit integration with the mentioned set of parameters should still provide a valid estimate of the influence of the effects neglected in our treatment. The integration of the orbits has thus been performed using  $\Omega_p = 13 \text{ km s}^{-1}$ ,  $i = 8^\circ$ , and  $m = 2$ . The value of  $\alpha$  estimated by Lin et al. 1969 is  $\alpha = 0.05$ , which has been adopted in most subsequent works; however, the streaming motions of young stars and molecular clouds in our Galaxy derived from more recent observations suggest a smaller value of  $\alpha$ : for a population of objects with a small velocity dispersion  $\sigma$ , the amplitude of the perturbation in the tangential component of the velocity,  $\hat{v}$ , is

$$\hat{v} \simeq \frac{\hat{F}_{sr}}{2\Omega(1 - \nu^2 + x)} \quad (18)$$

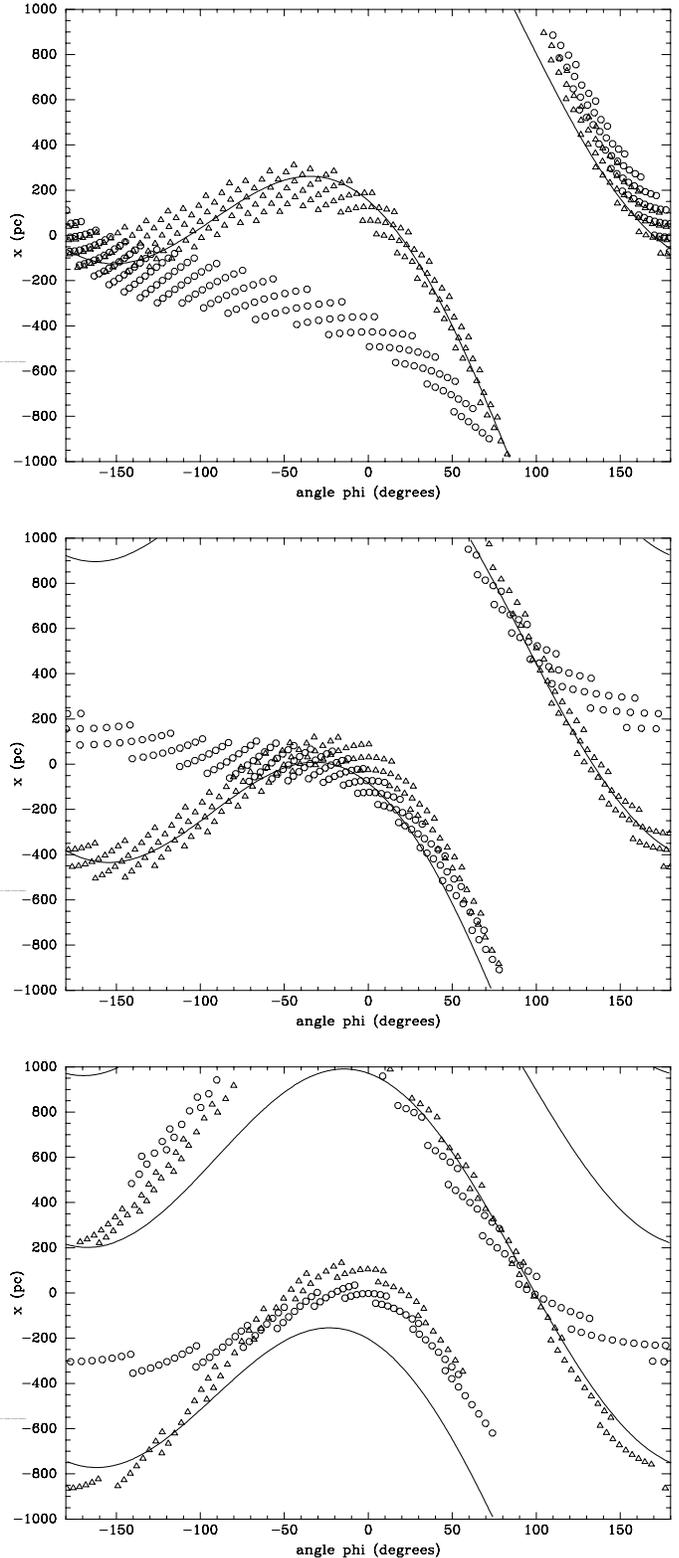
where

$$\hat{F}_{sr} = \alpha\Omega^2 R, \quad \nu = \frac{m(\Omega - \Omega_p)}{\kappa}, \quad x = \frac{m^2\sigma^2}{R^2\kappa^2 \tan^2 i}$$

Therefore,

$$\alpha \simeq \frac{2\hat{v}}{\Omega R}(1 - \nu^2 + x) \quad (19)$$

Using the values of  $\hat{v}$  and  $\sigma$  found by Clemens 1985 for galactic molecular clouds ( $\hat{v} \simeq 5 \text{ km s}^{-1}$ ,  $\sigma = 3 \text{ km s}^{-1}$ ), one finds  $\alpha \simeq 0.02$ , a value still sufficient to trigger galactic shocks (Shu, Milione, and Roberts 1973). Streaming motions in other low velocity dispersion populations such as OB stars also show small streaming amplitudes (Burton & Bania 1974, Comerón & Torra 1991), consistent with a value of  $\alpha$  below 0.05. We have adopted  $\alpha = 0.03$  for the calculations presented here.



**Fig. 4.** The locus occupied by stars in the  $\phi - x$  diagram as predicted by Eq. (13) (solid line) is compared to the results of the orbit integration of a set of probe stars. Triangles represent probe stars moving under the influence of the  $F_r$  force alone, while circles also include the spiral gravitational field. From top to bottom, the panels correspond to the adopted ages of the Pleiades, Ursa Majoris, and Hyades moving groups.

In our numerical integrations, we assume a number of probe stars to be formed at each position of the spiral shock, with initial motions given by the initial conditions (5). The  $\eta$  component of the pre-shock streaming motion is set to zero; as already discussed in Sect. 3.1, a non-zero value of  $\eta$  can be compensated for by a relatively small change in  $\bar{\tau}$ , and it is not expected to have any noticeable observational consequences. The spiral shock is assumed to be forming stars during an interval of time centered at the time  $\tau$  of its passage by the local standard of rest; the interval is determined by the condition that stars formed out of it do not have the possibility of migrating to the solar neighbourhood during the time span of our simulations. The orbit of each individual star is tracked until the present moment, and the epicyclic elements are calculated at the end of the integration.

The results are presented in Fig. 4, whose axes are the same as those of Fig. 3. The three values of  $\tau$  correspond to the three moving groups discussed in Sect. 3.1. For the sake of completeness, each panel shows three sets of data: the filled circles delineate the loci defined by Eq. (13); the open triangles are the result of numerical integrations without spiral arms ( $\alpha = 0$ ); and the open circles show the result of numerical integrations with spiral arms. The results for  $\alpha = 0$  clearly indicate the validity of the epicyclic approximation in our treatment. Only probe stars lying within 1 kpc from the Sun at the end of the integrations were considered in Fig. 4. The width of the bands of open triangles and circles are mostly due to the scatter in values of  $Y_1$ , which was neglected in deriving Eq. (13). Further scatter is expected in real observations as a consequence of the scatter in  $\eta$  resulting from the velocity dispersion in the pre-shocked medium.

The use and shortcomings of Eq. (13) can be appreciated from Fig. 4. The qualitative behaviour of the integrated orbits with and without spiral arms is similar, and over a large part of the  $\phi - x$  diagram, the solid line representing Eq. (13) is near the average locus of the open circles representing the stars moving under the spiral arm potential. The position of the inflection near  $\phi = -90^\circ$  is qualitatively reproduced in the orbits integrated with spiral arms, appearing also like an inflection or like a plateau, although this is generally the region where the difference between the cases with and without arms is larger.

Given the amplitude of the displacement between the armed and armless cases, the fit of the curves derived from Eq. (13) to the data in Fig. 3 (which depends in the single parameter  $\bar{\tau}$ ) is not meaningful as far as their intersections with the solar circle ( $x = 0$ ) are concerned; put in other words, the intersection of those curves with the  $x = 0$  line does not allow a precise dating of moving groups, as was noted in Sect. 3.1. However, Eq. (13) is still useful to estimate the gradients to be expected in the  $\phi - x$  plane, and the distance to the Sun where a sample of stars should extend in order to test the real existence of such gradients and the plausibility of the scenario proposed here. Moreover, if the observational data are extended out to a distance from the Sun larger than the size of the radial excursions induced by the spiral structure, more accurate dating of moving clusters based on the overall behaviour of  $\phi$  vs.  $x$  as given by Eq. (13) may become possible, as the cyclic nature of the perturbing force

only produces oscillations around the mean given by the non-armed case.

#### 4. Conclusions

We have proposed a framework for interpreting those moving groups in the solar neighbourhood whose members show relatively large deviations from the circular velocity around the galactic center. Our interpretation is based on the possible existence of a large scale shock associated with the density wave responsible for the spiral structure of our Galaxy, which determines the initial velocity of the stars formed by this process. New conditions for moving group membership have been derived, which happen to be equivalent to those based on the similarity of spatial velocities when they are applied to samples of very nearby stars. These conditions imply the existence of velocity gradients, independent of the isoperiodicity condition sometimes assumed for moving groups. An approximation to these gradients is derived, and the distortion due to the gravitational field associated to the density wave (not considered in our treatment) is estimated from numerical integrations. The predictions of our proposed definition of moving group membership may be tested as high precision spatial velocities for more distant stars are available, such as those provided by the Hipparcos satellite.

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