

Research Note

The residence time of dust grains in turbulent molecular clouds

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Abstract. The residence time of dust grains inside turbulent molecular clouds is calculated. Earlier estimates of this time by Boland & de Jong (1982) have been criticized by Prasad et al. (1987) for not taking into account properly the turbulent character of the velocity field in molecular clouds which implies that the trajectory of a dust grain should be treated as a random walk. Taking some minimal assumptions regarding the turbulent velocity field a time scale is derived that depends on the Reynolds number of the flow. The near proportionality with Reynolds number of this time scale results in a much longer time scale over which dust grains will remain inside a molecular cloud.

Key words: ISM: dust – ISM: molecular clouds – turbulence

to be test particles of the turbulent flow, it is possible to derive a circulation time scale in terms of the Reynolds number of the flow. From the study of Weidenschilling and Ruzmaikina (1994) it is clear that in particular sonic turbulence strongly affects grain growth and destruction since it affects the rate of collisions and the energy of such collisions with other grains and with gas phase material. The point of focus of this paper is not the growth and destruction of grains but rather the stochastic migration of an ensemble of grains throughout a dense cloud. It is demonstrated that for quite general properties of the turbulence the random walk through a cloud indeed considerably increases the time grains stay in a region of high optical depth, in accordance with the argument of Prasad et al. (1987) although the time scale derived here is different from theirs.

1. Introduction

Chemical surface reactions on dust grains form an essential pathway in the formation of complex molecules, and even some quite simple ones, in interstellar molecular clouds. The time that a grain resides inside clouds with a high optical depth determines its life time and therefore the time over which it can play a role in the chemical processing of its host molecular cloud. This residence time scale has been estimated previously by Boland & de Jong (1982) to be L/σ_v from the observables L which is the size of the cloud and σ_v which is its velocity dispersion. Prasad et al. (1987) argued that this essentially implies a linear path through the cloud for any dust grain. This does not properly account for the fact that the velocity field in the cloud is turbulent and that the grains, dragged along by the gas motions, perform a random walk. Using a constant step size random walk Prasad et al. (1987) derive a time scale $L^2/\sigma_v l_s$ where l_s is the (typical) step size in the random walk, which they estimate to be the scale of the largest eddies in the cloud. However using a single scale eddy size to represent the large range of eddy sizes present in highly turbulent flow is also inappropriate. Taking dust grains

2. Power law turbulence

The coupling between gas and dust grains in stellar winds and in the interstellar medium can be shown to be quite strong, because of the strong drag that dust grains experience when drifting with respect to the gas. The dust therefore is treated here as completely momentum coupled to the gas: effectively it is a test particle for the turbulent flow.

The length scale of dissipation (internal scale) of the turbulence l_d is set by the viscosity in the gas of the molecular cloud or the magnetic diffusivity (the inverse of the conductivity) if the cloud contains a (tangled) magnetic field. Unless this length scale of dissipation of the turbulent velocity field is nearly equal to the size of the molecular cloud the energy density E in the velocity field per length scale interval dl , or equivalently per wave vector interval dk , will have an ‘inertial range’ over which it behaves as a power law as a function of length scale. Outside of this range the energy density or power in the turbulent velocity field decreases rapidly, either due to dissipation or because the cloud is not large enough to contain the length scales larger than its own size. For stationary hydrodynamical turbulence in the absence of magnetic fields, and in the inertial range, the energy density has a slope of this power law which conforms to the well known Kolmogorov-Obukhov scaling: $E(k)dk \propto k^{-5/3}dk$

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(cf. Landau & Lifshitz, 1987). As was pointed out by Scalo (1987) molecular clouds are highly magnetized and subject to self-gravity. Also energy is injected into the turbulent velocity field from a variety of sources on a variety of spatial scales, for instance shear due to galactic rotation, so the slope of the energy spectrum can differ from the value of $-5/3$. However even for magnetohydrodynamic (MHD) turbulence the velocity field behaves as a power law over a large range of length scales (cf. Stanišić, 1985). Gravity does not impose a characteristic length scale nor do effects of compressibility. The energy injection mechanisms can be subject to one or more characteristic length scales which could introduce humps or roll-overs in the turbulent spectrum, where locally the energy spectrum is not a power law. However the requirement of stationarity of the turbulent spectrum imposes strong constraints on the transfer of energy between scales. Only if the turbulent cascading of energy is the least effective (slowest) of the energy transfer mechanisms operating, which is unlikely, will the spectrum be severely affected by the other effects, because then energy can build up at the injection length scale before the energy cascading from larger scales and to smaller scales can compensate for this. Summarizing, the energy density probably does not follow exactly a power law with a slope equal to $-5/3$ over the entire range of scales possible, but the departures from this behaviour are not likely to be large.

Observationally it is relatively straightforward to determine a power law spectrum for the mass M contained in clumps within molecular clouds, so the starting point is this mass spectrum, where $f(M)$ is the fraction of cloud clumps with a mass between M and $M + dM$:

$$f(M)dM \propto M^{-\alpha}dM \quad (1)$$

Recent measurements of α by Williams et al. (1995) yield a value for $\alpha = 1.27 \pm 0.09$ for the Rosette Nebula. Values of up to 1.7 for this and other cloud complexes have also been reported (cf. Blitz, 1991). It appears that α is roughly constant over a range of scales from 100 pc down to 10^{-4} pc in at least one cloud (cf. Rouan et al. 1997).

2.1. In the absence of density-velocity correlation

The mass is related to the gas density ρ and l by:

$$M \propto \rho l^3 \quad (2)$$

In principle the gas density could be a function of the length scale l , which is discussed in Sect. 2.2. Assuming a ρ independent of l the mass spectrum can be converted to a spectrum in terms of sizes between l and $l + dl$:

$$f(l)dl = C_L l^{2-3\alpha} dl \quad (3)$$

Outside of this range the amount of material is assumed to be negligible and the value of the power-law index α is assumed constant over the entire inertial range. Even though the observed

clumps should probably not be individually identified with turbulent eddies it is likely that the turbulent motions follow a similar distribution of energy density versus size, since the clumps are a consequence of random convergence of the turbulent flow. In this paper it is assumed that the distributions are identical. The constant C_L is determined by demanding that the integral of $f(l)$ over l between the dissipation length scale l_d and the size of the molecular cloud L be equal to unity.

$$\int_{l_d}^L C_L l^{2-3\alpha} dl = 1 \quad (4)$$

implies:

$$C_L = (3\alpha - 3) \frac{L^{3\alpha-3}}{R_e^{\frac{9\alpha-9}{4}} - 1} \quad \alpha \neq 1$$

$$C_L = \frac{4}{3 \ln R_e} \quad \alpha = 1 \quad (5)$$

where the Reynolds number R_e is related to L and l_d by:

$$R_e = \left(\frac{L}{l_d} \right)^{4/3} \quad (6)$$

If a cloud is optically thin in the spectral line in which the velocity dispersion is measured the observed velocity dispersion of a cloud must be the expectation value of the square of the velocity. If the cloud is optically thick the line only measures the turbulence in the surface layers. Since the turbulent velocity field is probably not homogeneous a velocity dispersion determined with an optically thick line is then not representative of the interior of the cloud. In the following it will be assumed that a measured velocity dispersion is representative however, so that either the method with which it is measured is sensitive to velocity fields throughout the clouds or the turbulence is assumed to be homogeneous and isotropic. In the usual turbulent cascade (cf. Landau & Lifshitz, 1987) of fully developed free turbulence the velocity v_l of a turbulent eddy with size l is:

$$v_l = v_L \left(\frac{l}{L} \right)^{1/3} \quad (7)$$

where v_L is the overturning velocity of the largest eddies supported by the molecular cloud which have a size of the order of the cloud size L . As argued at the start of this section a number of effects could change the power 1/3 in relation (7), but the departures are not likely to be very large. The velocity dispersion σ_v^2 is the expectation value of the square of the turbulent velocity:

$$\sigma_v^2 = \int_{l_d}^L C_L v_l^2 l^{2-3\alpha} dl$$

$$= C_L v_L^2 L^{3-3\alpha} \frac{3}{11-9\alpha} \left(1 - R_e^{\frac{9\alpha-11}{4}} \right) \quad \alpha \neq \frac{11}{9}$$

$$\approx \left| \frac{9\alpha-9}{9\alpha-11} \right| v_L^2 R_e^\gamma \quad \alpha \neq 1, \frac{11}{9} \quad (8)$$

In the approximate equality it has been assumed that the Reynolds number $R_e \gg 1$. The exponent γ satisfies:

$$\gamma = \begin{cases} -1/2 & \alpha > \frac{11}{9} \\ \frac{9-9\alpha}{4} & 1 < \alpha < \frac{11}{9} \\ 0 & \alpha < 1 \end{cases} \quad (9)$$

Note that the intrinsic thermal width of the line is assumed to be much smaller than the width of the velocity distribution, which implies that the turbulent velocities are supersonic. If this is not the case a proper convolution should be done of the line profile and the velocity distribution.

As a grain is swept along in the turbulent eddies it performs a random walk through the cloud. If all the steps were of equal length l then it would take $N_l = (L/l)^2$ steps to traverse a distance L , each step taking a time $T_l = l/v_l$. The step size obeys the distribution (3) and so the total residence time for a dust grain must be the expectation value of $N_l \times T_l$:

$$\begin{aligned} T_{tot} &= \int_{l_d}^L C_L N_l T_l l^{2-3\alpha} dl \\ &= C_L \frac{L^{4-3\alpha}}{v_L} \frac{3}{|5-9\alpha|} \left(1 - R_e^{\frac{9\alpha-5}{4}}\right) \quad \alpha \neq \frac{5}{9} \\ &\approx \left| \frac{(9\alpha-9)^3}{(9\alpha-11)(9\alpha-5)^2} \right|^{1/2} \frac{L}{\sigma_v} R_e^\delta \equiv C_T \frac{L}{\sigma_v} R_e^\delta \\ &\quad \alpha \neq \frac{5}{9}, 1, \frac{11}{9} \end{aligned} \quad (10)$$

where the exponent δ satisfies:

$$\delta = \begin{cases} 3/4 & \alpha > \frac{11}{9} \\ (17-9\alpha)/8 & 1 < \alpha < \frac{11}{9} \\ (9\alpha-5)/4 & \frac{5}{9} < \alpha < 1 \\ 0 & \alpha < \frac{5}{9} \end{cases} \quad (11)$$

The slope α of the mass spectrum of the clouds appears in the exponent δ only for $\frac{5}{9} < \alpha < \frac{11}{9}$. For $\alpha = 1.27$ the exponent $\delta = 3/4$ and $C_T \approx 0.9$. For steeper ($\alpha > 1.27$) spectra C_T approaches unity asymptotically. It is only for mass spectra that are flat or rising ($\alpha < 0$) that the estimate of Boland & de Jong (1982) is appropriate, because only then is $\delta = 0$.

Of interest is the dependence of the residence time T_{tot} on the Reynolds number R_e for $\alpha > 5/9$. It seems perhaps counter-intuitive that for more vigorous turbulence (larger R_e) dust grains take longer to traverse a cloud. However one should realize that if the turbulence is more vigorous the velocity dispersion σ_v also increases. For a given (measured) velocity dispersion an increased Reynolds number implies that l_d is smaller so that there is a larger contribution to the integrals (8) and (10) from small scales. Thus a grain is likely to make many more small steps in a random walk than large ones.

2.2. Fully coupled density-velocity fluctuations

In Sect. 2.1 the density ρ is assumed to be constant, so that it can be ignored in the integral for the velocity dispersion. In view of

the Larson relations (Larson, 1981) which show that the density of clumps follows a power law as a function of clump size, it is instructive to consider also the effect of a gas density which fluctuates in a manner that is spatially correlated with the turbulent velocity and which has a power law spectrum of l : $\rho \propto l^{-\beta}$. The measured value of $\beta = 1.3 \pm 0.2$ (cf. Cernicharo, 1991 and references therein). The combined distribution function is assumed to be the product of the individual distribution functions for the density and the velocity, which implies that (3) becomes:

$$f(l)dl \propto l^{2-3\alpha+\beta(\alpha-1)} dl \quad (12)$$

In this paper the clouds are not assumed to be in virial equilibrium, thus no a-priori relation between ρ_l and v_l is postulated, and β is a free parameter. For a cloud that is in virial equilibrium on every scale one expects that the kinetic and potential energies of turbulent eddies are identical up to a constant factor. This implies that $v_l^2 \propto M_l/l \propto \rho l^2 \propto l^{2-\beta}$. Combining this with (7) shows that then one would expect $\beta = 4/3$, which is equal to the value actually observed to within 1σ .

In the analysis presented here σ_v depends on ρ since the density plays a role in the determining the profile of the spectral line from which the velocity dispersion is measured. If one takes a density weighted average instead of (8) then:

$$\begin{aligned} \sigma_v^2 &= \frac{1}{\langle \rho \rangle} \int_{l_d}^L C_L \rho_l v_l^2 l^{2-3\alpha+\beta(\alpha-1)} dl \\ &= v_L^2 \frac{\rho_L}{\langle \rho \rangle} C_L L^{3-3\alpha+\beta(\alpha-1)} \frac{1 - R_e^{(9\alpha-9-3\beta(\alpha-2))/4}}{11-9\alpha+3\beta(\alpha-2)} \\ &\quad \alpha \neq \frac{6\beta-11}{3\beta-9} \end{aligned} \quad (13)$$

where $\langle \rho \rangle$ is the expectation value for the density.

$$\langle \rho \rangle = \int_{l_d}^L C_L \rho_l l^{2-3\alpha+\beta(\alpha-1)} dl \quad (14)$$

Note that there is a problem with the velocity dispersion that results from this analysis. Observations appear to indicate that $\sigma_v \propto L^{0.3}$ (cf. Scalo, 1987), whereas with the observed values of α and β one obtains $\sigma_v \propto v_L^2 R_e^{-1/2}$ which is independent of L because of relations (6) and (7). One way to explain this is that when observing at increasing resolution (smaller L) one concentrates on ‘clumps’, which are the regions with higher density and lower velocity dispersion. This ‘selection effect’ might introduce the observed trend, bringing it closer to relation (7).

The relation (10) becomes now:

$$\begin{aligned} T_{tot} &= \int_{l_d}^L C_L N_l T_l l^{2-3\alpha} dl \\ &= \frac{L}{\sigma_v} \frac{\sigma_v}{v_L} C_L L^{3-3\alpha+\beta(\alpha-1)} \frac{1 - R_e^{(9\alpha-5-3\beta(\alpha-1))/4}}{5-9\alpha+3\beta(\alpha-1)} \\ &\quad \alpha \neq \frac{3\beta-5}{3\beta-9} \end{aligned} \quad (15)$$

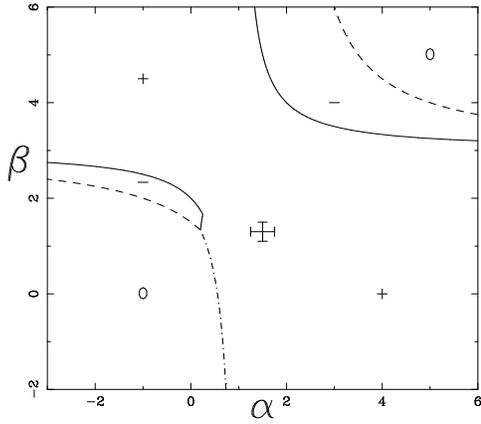


Fig. 1. T_{tot} is a power law of R_e for $R_e \gg 1$. The sign of the power law index is shown as a function of the power-law exponents α and β . It is positive in areas indicated by a +, negative where there is a – and zero on the solid curves and in areas indicated by 0. On the dashed curves $T_{tot} \propto (\ln R_e)^{-1/2}$. On the dash-dotted curve $T_{tot} \propto \ln R_e$. The error bars show the range of measured values for α and β for clumps in molecular clouds

It is not particularly illuminating to show the equations for all possible values of the exponents α and β , since this separates out into 73 separate cases. Instead in Fig. 1 is shown the behaviour of the exponent of the Reynolds number R_e as a function of α and β . Indicated by +, 0, and – are regions where this exponent is positive, zero, or negative respectively. For $2/3 < \beta < 3$ and $\alpha > 1$ and for $\beta < 2/3$ and $\alpha > 2$ the dependence of T_{tot} on the Reynolds number is always $T_{tot} \propto R_e^{3/4}$ for $R_e \gg 1$. From Fig. 1 it is clear that only for quite steep distributions for the density (large β) the exponent is negative and T_{tot} decreases as R_e increases. In such cases the velocity dispersion is weighted strongly towards the high density, low velocity clumps so that for more vigorous turbulence the estimate $T_{tot} = L/\sigma_v$ is increasingly an overestimate of the residence time. For the measured $\alpha = 1.27$, T_{tot} is a decreasing function of R_e only for $\beta > 6.7$. For the observed range of values α and β the relation is

$$T_{tot} = C_T \frac{L}{\sigma_v} R_e^{3/4} = C_T \frac{L^2}{l_d \sigma_v} \quad (16)$$

where the value of $C_T \approx 0.35$. This appears to be very similar to the expression of Prasad et al. (1987) who have l_s replacing l_d , where l_s is the typical eddy size. The dissipation length scale l_d is much smaller than the typical eddy size $l_d \ll l_s$ however, so that the time scale derived here is longer than that derived by Prasad et al. (1987).

2.3. The influence of fractal structures

It is important to note that the residence time in the cloud complex derived here may be an over-estimate of the residence time of the grains. There is growing evidence that the gas density distribution in molecular cloud complexes is highly inhomogeneous or ‘fractal’ in structure (cf. Scalo, 1987, 1990; Falgarone

et al., 1991). Even though the grain can take a long time to traverse a cloud complex it will encounter regions within that complex that may well be sufficiently warm or exposed to ultraviolet (UV) radiation to be quite destructive for grains or at least their icy mantles. This affects the statistics of the distributions and the various expectation values evaluated in this paper. For instance the velocity dispersion (Eq. 8) is independent of the external scale and the density weighted velocity dispersion (13) is as well for any value $\beta \geq 3$, whereas it generally will not be for a fractal molecular cloud. In this case one should properly calculate the characteristic time it takes for grains to circulate into exposed regions from dense cold parts of the cloud core where the grain is shielded. The time of survival should then depend on the fractal dimensions of the hospitable and inhospitable parts of the cloud. One could think of this as considerably reducing the Reynolds number because in definition (6), the value for L should be the size of the high density (hospitable) regions. This means that for a fractal cloud L and the Reynolds number itself become stochastic quantities, for which expectation values must be calculated. If the regions with sufficiently high density occupy relatively small regions of space, e.g. if these regions are filamentary in structure, this reduces L on average and therefore also R_e and thus T_{tot} . However the Reynolds number cannot be as low as to be of order unity on average over a molecular cloud because that would imply that on average the flow in a cloud complex is laminar which is clearly not the case. Therefore the estimate of Boland & de Jong (1982) of $T_{tot} = L/\sigma_v$ for the residence time of grains in molecular clouds is too low.

3. Conclusions

It is shown in this paper that $t = L/\sigma_v$ severely under-estimates the actual time spent by grains in turbulent molecular clouds. In the limit of large Reynolds number for the turbulence, in clouds in which the velocity dispersion deduced from line widths represents well the velocity distribution throughout the cloud, the residence time is $T_{tot} \approx 0.35 R_e^{3/4} L/\sigma_v$. Unfortunately the Reynolds number is not a directly observable quantity. Using the definition (6) one can make a crude estimate however. The dissipation scale l_d is determined by the microphysics of the mechanism responsible for the dissipation of the turbulence such as viscosity or magnetic diffusivity. Such mechanisms usually become effective on length scales comparable to the mean free path of gas atoms or molecules. On larger scales the turbulence is essentially free and nearly dissipationless. Even at the low densities and low ionization fractions of cold interstellar matter this mean free path is not likely to be larger than 0.1 pc, at which scale ambipolar diffusion starts affecting the propagation of hydromagnetic waves (Mouschovias, 1991). The size L of a typical molecular cloud is more than 10 pc. Realistic values of the Reynolds number for molecular clouds can thus be in excess of 10^3 , and have even been estimated to be of the order of 10^6 (Scalo, 1987), which implies that the residence time of grains can be more than 10^2 or even 10^4 times longer than previously estimated.

Taking as an example the clumps in the Rosetta molecular cloud which have a velocity dispersion of typically ~ 1.5 km/s and a size of typically ~ 1.5 pc (cf. Williams et al., 1995) the time scale, for a Reynolds number of 10^4 , becomes $\sim 3 \cdot 10^8$ yr. This is already of the order of the life time of a cloud, so grains formed in the centre of clouds like this will barely manage to migrate out of the cloud, let alone out of clouds that have a larger Reynolds number. One might conclude that therefore molecules caught or formed on grains will not be returned to the ISM at all, since grains will not reach regions where photodesorption can play a role. However other effects might intervene. Vigorous turbulence is likely to lead to a higher frequency of grain-grain collisions, and also leads to heating because of dissipation of the turbulent energy. Inhomogeneity of a cloud produces a higher UV radiation field within the cloud than the cloud would have if it were homogeneous. What should be clear from this paper is that all physical and chemical processes in which grains are involved may well be modified by the presence of turbulence, and most certainly will have a much longer time to operate than estimated previously.

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