

# The de-censoring of the faint stars in Tycho photometry<sup>\*</sup>

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**Abstract.** The Tycho catalogue provides astrometric and photometric data for more than 1 million stars. The derivation of the mean magnitudes of stars fainter than about 8 mag required a dedicated treatment, in order to take into account the observations that were too faint for providing a measurable signal. This treatment, called “de-censoring”, is based on a statistical model permitting to substitute mean values to the missing data. Faint photometric standard stars are used to demonstrate that the de-censoring produces reliable magnitudes and uncertainties.

**Key words:** methods: data analysis – methods: statistical – techniques: photometric – catalogs

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## 1. Introduction

Tycho is a project using the star mapper from the ESA Hipparcos satellite for deriving the positions, magnitudes and color indices of about 1 050 000 stars. The photometric passbands, called  $B_T$  and  $V_T$ , are close to the  $B$  and  $V$  passbands of the Johnson photometry. The scientific mission of the satellite began in November 1989, and ended in March 1993. The reduction of the Tycho data was completed in mid-1996, and the Tycho Catalogue (ESA 1997) will be published in June 1997. The instrument and the general organisation of the data analysis have been described in Høg et al. (1992a); the most important steps of the data reduction were the preparation of the Tycho Input Catalogue (Egret et al. 1992), the prediction of transits (performed at Astronomisches Rechen-Institut in Heidelberg), the detection of transits (Bässgen et al. 1992), the background determination (Wicenc and Bässgen 1992), the revision of the input catalogue (Halbwachs et al. 1992, Halbwachs et al. 1994), the astrometric reduction (Høg et al. 1992b, Høg et al. 1995), and the photometric reduction (Scales et al. 1992, Großmann et al. 1995). The

main points related to the Tycho photometry are summarized hereafter:

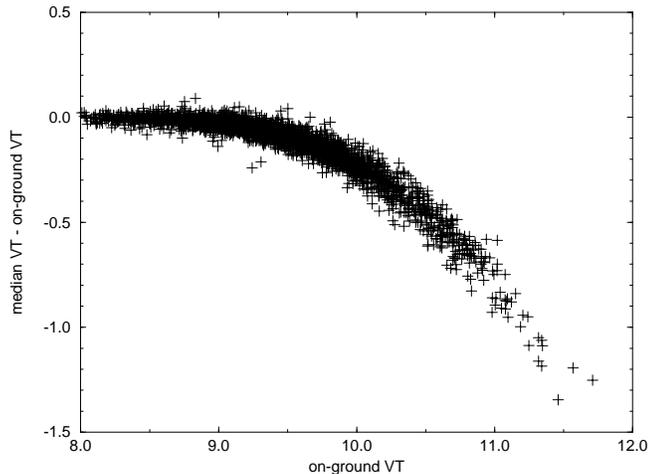
- The star mapper consists of two groups of slits in the focal plane of the telescope. One of them is called the “vertical” slit system, and the other one is the “inclined” slit system. When a star was crossing a slit, its light was simultaneously recorded by two photometers, one for each colour. The raw Tycho data are the photon counts that were constantly recorded with a sampling rate of 600 Hz.
- The data analysis is based on predictions of crossings of the slits systems by the stars of an input catalogue. A detection algorithm is used to search star signals in the photon counts of the  $B_T$  and of the  $V_T$  channels, considered together (the sum of the  $B_T$  and  $V_T$  channels is called the  $T$  channel hereafter). This search is performed within small intervals centered on the predicted epochs of the transits. A detection is recorded when its signal-to-noise ratio is larger than 1.5. The amplitudes of the signals are then estimated in the  $B_T$  and in the  $V_T$  channels. In a few cases however, the estimation algorithm fails to produce a signal amplitude.
- The Tycho photometry is calibrated on the basis of a set of photometric standard stars from the Geneva database (Großmann et al. 1995).  $B_T$  and  $V_T$  magnitudes are derived every times the signal amplitudes were obtained.
- The final selection of stars is performed after the final astrometric reduction. Among other criteria, one condition is that each star must have received at least 30 transit detections considered as acceptable for astrometry. Since the selection of detections was more restrictive for photometry than for astrometry, the Tycho catalogue contains a few stars with even less than 30 photometric measurements, although the mean number of transits per star is more than 100. As a consequence, the vast majority of the stars are too faint for being measured every time they were crossing a slit system.

The next step of the data reduction in photometry is the derivation of the mean magnitudes of the stars. The treatment of bright stars is presented in Großmann et al.; it consists in taking the median of the measurements obtained for each channel. This

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<sup>\*</sup> Based on observations made with the ESA Hipparcos satellite



**Fig. 1.** The distribution of the differences between the median  $V_T$  magnitudes of the photometric standard, and the magnitudes coming from on-ground measurements.

method cannot be applied to the faint stars, however: only a part of their transits were detected, and the faintest measurements coming from the statistical variations of the photon counts were censored. The median of the measurements is then biased toward bright magnitudes. This effect was already pointed out by Großmann et al., and it is shown again in Fig. 1: the difference between the observed Tycho  $B_T$  magnitudes and the actual magnitudes of the photometric standards is increasing almost linearly for stars fainter than  $B_T=11$  mag, so that the median of the measurements is always around  $B_T=10.5$ . It is then impossible to evaluate the bias for about 50 % of the Tycho catalogue when only the detections are taken into account.

It was contemplated to derive the median magnitudes by considering the missing measurements as faint measurements. Unfortunately, the validity range of this simple method is severely limited: for the bright stars, the median of the actual measurements is better than the median of all transits, since some missing measurements are “spurious non-detections”, as explained in Sect. 3.2. On the other side, the faintest magnitudes that could be derived from the median of all transits depend on the maximum value permitted for the background; as a consequence, it comes from the model presented in this paper that the  $V_T$  magnitudes of stars with  $B_T - V_T = 1.5$  would have been properly computed only until  $V_T = 10.2$  mag. Therefore, since another method was necessary anyway, the median of all transits was ignored.

The method used to derive the magnitudes of the faint stars is explained hereafter. It is based on the “de-censoring” of the transits that did not lead to measurements. The principles of de-censoring are presented in Sect. 2. The model describing the data acquisition is in Sect. 3, and the complete procedure is described in Sect. 4. The results are presented in Sect. 5.

## 2. The basic principles of Tycho de-censoring

The mean  $B_T$  and  $V_T$  magnitudes must be derived by taking the missing measurements into account. Various techniques for solving statistical problems involving missing data were developed during the last decades (Little & Rubin 1987). Some of them were successfully applied in astronomy (Feigelson & Nelson 1985, Isobe et al. 1986). According to the definition given by Kendall & Stuart (1961), the data in Tycho photometry are affected by a “Type I censoring”, since the measurements smaller than a threshold were lost. However, their treatment is rather complicated, because two different censorings must be considered: the censoring coming from detection, that depends on the photon counts in the  $T$  channel, and, when a transit is detected, the censoring coming from the possible failure to evaluate the signal amplitude in  $B_T$  or in  $V_T$ . Moreover, the censoring threshold is not constant, but it depends on the circumstances of the transits: the most important ones are the slit group, and the background in the photon counts. Therefore, an iteration technique is preferred to the sophisticated methods of “survival analysis” presented in the papers mentioned above.

The algorithm is based on a model describing the data acquisition. The model is used to compute the mean luminosity corresponding to each censored measurement, assuming the  $B_T$  and  $V_T$  magnitudes of the star are known. The mean luminosities are imputed to the censored measurements. The next step is the calculation of the mean  $B_T$  and  $V_T$  magnitude of the star, taking all transits into account. The values thus obtained are input for another iteration, until the calculation has converged.

## 3. The model of data acquisition

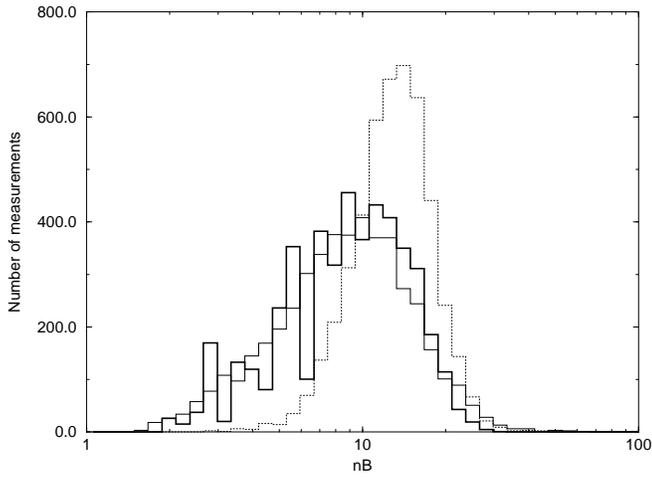
The model hereafter is a simplified summary of the part of the Tycho data reduction that is related to photometry. It consists of a mathematical description of the treatment of the transit observations obtained for a star with magnitudes  $B_T$  and  $V_T$ .

### 3.1. The transit detection

When the star is crossing a slit group, the photon counts exhibits a peak within an interval of  $1/90$  second  $\simeq 6.7$  samples (this does not concern the raw photon counts, but the non-linearly folded photon counts presented in Bässgen et al. 1992). In the  $T$ -channel, the number of photons recorded during the transit within the interval of  $1/90$  second,  $N_T$ , obeys a Poisson distribution, with the parameter  $\langle N_T \rangle$ :

$$\langle N_T \rangle = \langle n_{back_B} \rangle + \langle n_{back_V} \rangle + \langle n_B \rangle + \langle n_V \rangle \quad (1)$$

where  $\langle n_{back_B} \rangle$  and  $\langle n_{back_V} \rangle$  are the average numbers of background photons recorded in the interval in the channels  $B_T$  and  $V_T$ ; they are estimated from the photon counts around the transits. In practice, the total background,  $\langle n_{back_B} \rangle + \langle n_{back_V} \rangle$ , is 50 counts on average, but it ranges from about 20 to 133 counts. The average numbers of photons received from the star,  $\langle n_B \rangle$



**Fig. 2.** The distribution of the signals in  $B_T$ , derived from measurements of red standard stars with  $B_T - V_T > 0.7$  fulfilling the following conditions: transit of the inclined slit system,  $\langle n_B \rangle$  between 3.3 and 13.3 photon counts (corresponding to amplitudes of 0.5 and 2 counts/sample respectively), and  $\langle back_B \rangle$  between 6.7 and 13.3 counts (ie a flux between 1 and 2 counts/sample). In order to take into account the magnitude censoring, the measurements with  $SNR_B < 0.5$  are discarded. The dotted line refers to the signals estimated in the photometric reduction, and the thin solid line to the corrected signal. The distribution derived from the model is plotted as a thick line for comparison; it contains gaps and peaks because the total photon count in  $B_T$ ,  $N_B$ , is assumed to be an integer number.

and  $\langle n_V \rangle$ , are related to the  $B_T$  and  $V_T$  magnitudes by the equations:

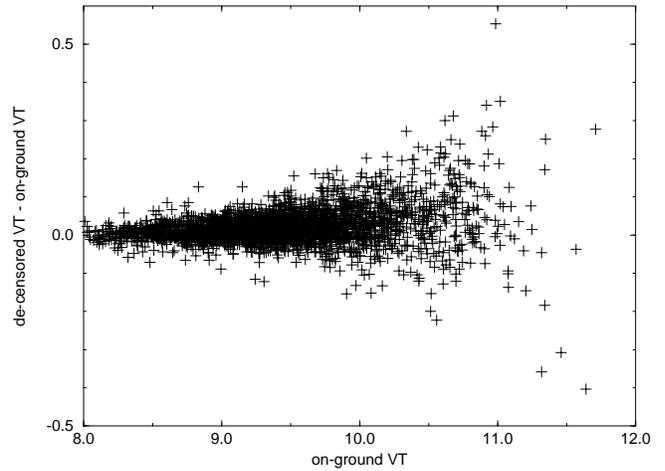
$$\langle n_B \rangle = 10^{\frac{B_{cal} - B_T}{2.5}} \quad \text{and} \quad \langle n_V \rangle = 10^{\frac{V_{cal} - V_T}{2.5}} \quad (2)$$

where  $B_{cal}$  and  $V_{cal}$  are calibration terms depending on the instrumental parameters of the transit (slit group, field, and part of the slit). These terms were derived in the photometric calibration;  $B_{cal}$  ranges from 13.4 to 13.9 mag and  $V_{cal}$  ranges from 13.1 to 13.5 mag. Therefore, the distribution function of  $N_T$  is determined. This is slightly unrealistic, however, since  $N_T$  is an integer number in the model, whereas the actual data reduction provided estimations of photon counts obeying a continuous distribution.

The detection of the transit depends on its signal-to-noise ratio ( $SNR$ ). Since the signal-to-noise ratio of the signal  $n$  is generally defined as  $n/\sigma_n$ , the signal-to-noise ratio of the transit in the  $T$  channel is:

$$SNR_T = \frac{N_T - \langle n_{back_B} \rangle - \langle n_{back_V} \rangle}{\sqrt{N_T}} \quad (3)$$

The transit is detected when  $SNR_T$  is larger than 1.5. In a conservative estimation, for stars with  $B_T - V_T = 0.7$ , and assuming a moderate total background of 50 counts, this threshold corresponds to  $V_T = 11$  or 11.4 mag, according to the calibration terms; these limits are 0.5 mag brighter when the maximum background is assumed. In reality, the cut-off at  $SNR_T = 1.5$



**Fig. 3.** Same as Fig. 1, but with the de-censored  $V_T$  magnitudes in place of the medians of the measurements.

is not sharp, because the signal is estimated with a fast routine providing only an approximate value. However, this is ignored in the model.

When a transit is detected, the next step is the estimation of the  $B_T$  and  $V_T$  magnitudes, that is presented in Sect. 3.3. When  $SNR_T$  is below 1.5, since the transit was predicted, a non-detection is recorded. The failure to detect transits with  $SNR_T$  less than 1.5 is called the “detection censoring” hereafter.

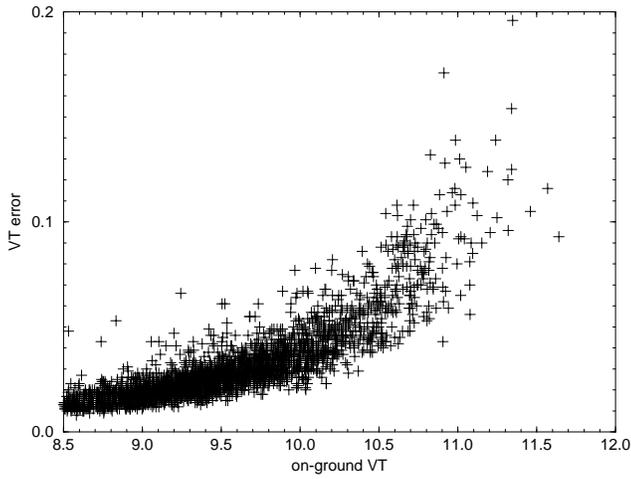
### 3.2. The spurious non-detections

The non-detections are not only the censored transits. Transits with  $SNR_T$  larger than 1.5 were sometimes omitted, for two reasons. The first one is that the predicted epoch used in the detection process may be wrong, since they were based on a preliminary determination of the attitude of the satellite. In such cases, the transit should be discarded, but, in practice, the information that the epoch was wrong was obtained only when a background transit was detected by chance; the transit was considered as valid otherwise (this concerns about half of the erroneous predictions). The second reason was that, in photometry, the detections were considered to be due to the stars only when the distance between the star and the position of the slit was less than 0.6 arcsec. As a consequence, even the stars bright enough for being detected each time they were crossing a slit group get some non-detections. These non-detections are not censored transits, since they are not due to signals below the detection threshold, but they are referred to as “spurious non-detections”.

It appears from the bright standard stars that 6 % of the transits are spurious non-detections. This rate is assumed to be constant.

### 3.3. The determination of the magnitude

When a transit is detected,  $N_B$  and  $N_V$  photons are counted in the channels  $B_T$  and  $V_T$  respectively. The numbers of photons



**Fig. 4.** The errors of the  $V_T$  magnitudes of the photometric standard stars, computed with the de-censoring software.

received from the star,  $n_B$  and  $n_V$  are estimated as the differences between the total numbers  $N_B$  and  $N_V$  and the mean backgrounds  $\langle n_{back_B} \rangle$  and  $\langle n_{back_V} \rangle$ . The measured magnitudes are then:

$$B_{meas} = B_{cal} - 2.5 \log(N_B - \langle n_{back_B} \rangle) \quad (4)$$

and

$$V_{meas} = V_{cal} - 2.5 \log(N_V - \langle n_{back_V} \rangle) \quad (5)$$

However, this calculation was actually not possible when the total numbers of photons were close to the mean background. In practice, a minimum signal-to-noise ratio of 0.5 is assumed in the model. When the  $SNR$  is below this limit in one of both channels, the corresponding magnitude cannot be derived; in reality, the cut-off is rather unsharp, but, when a magnitude was derived although the  $SNR$  is less than 0.5, the measurement is considered as doubtful and it is ignored. The failure to measure the magnitude when the  $SNR$  is less than 0.5 is called “the magnitude censoring” hereafter. In practice, since its threshold is much smaller than the detection threshold, the magnitude censoring only concerns faint stars with very small or very large color indices.

The calibration terms  $B_{cal}$  and  $V_{cal}$  were derived from the bright standard stars, as explained in Großmann et al (1995). Due to the censoring, it was then not verified if they were valid also for faint stars. This point is investigated hereafter. In order to avoid the bias due to the detection censoring, the  $B_T$  measurements of the faint red standard stars are considered: since these stars have  $B_T$  much fainter than  $V_T$ , their detections depend essentially on  $V_T$ , and the statistic of  $B_T$  is affected only by the magnitude censoring. Therefore, the theoretical distribution of the signals  $n_B$  is derived by assuming that  $N_B$  obeys a Poisson distribution with the parameter  $\langle N_B \rangle = \langle n_B \rangle + \langle n_{back_B} \rangle$ , and with a cut-off on the left side when  $SNR_B = 0.5$ .

The distribution of the measured  $n_B$  thus reveals an excess of bright magnitudes, as shown in Fig. 2 for the inclined slit

group, and for  $\langle n_B \rangle$  and  $\langle n_{back_B} \rangle$  within small intervals. In fact, this is due to a non-linearity of the relation between the number of photons coming from the star and the signal estimated in the data processing. The medians of the actual measurements and of the theoretical distributions are considered for estimating the correction to apply to the signal. For instance, it comes from Fig. 2 that, for the inclined slit group, the excess is 4 counts when the measured signal is 13 and when the average background is 10 counts; in that case, this corresponds to an excess of 0.4 mag for a measured  $B_T$  of 10.8 mag. All the measurements of the red photometric standard stars are used to determine the correction as a function of the slit group, of the background,  $\langle n_{back_B} \rangle$ , and of the measured signal, called  $n_{B_{meas}}$ . When  $n_{B_{meas}}$  is larger than 13.3 counts (ie when the amplitude of the signal is larger than 2 counts per sample), it appears the true signal,  $n_{B_{cor}}$ , is given by the equation:

$$n_{B_{cor}} = n_{B_{meas}} - C_s \times \frac{\langle n_{back_B} \rangle^2}{(n_{B_{meas}} + \langle n_{back_B} \rangle)^2} \quad (6)$$

with  $C_s = 20$  counts for the transits of the vertical slit group, and 13.3 counts for the inclined slits. Other relations exist for smaller values of  $n_{B_{meas}}$ . When these corrections are applied, the statistical properties of the magnitude measurements are fairly well fitting the model (see Fig. 2). The same relations are used to correct the estimations of  $n_V$ , since they were obtained with the same algorithm.

The corrections of  $n_B$  and  $n_V$  does not affect the detection threshold  $SNR_T = 1.5$ , since the detection was performed with another routine for estimating the signal.

## 4. The de-censoring procedure

### 4.1. The transit selection

Only the transits with background flux in the T channel smaller than 12 000 counts/second are used in photometry. Moreover, it is verified that no other star, among the half a million brightest ones, is crossing a slit system simultaneously; the transit is discarded otherwise.

Each transit is characterized by several parameters:

- Flags referring to the part of slit system that was crossed by the star. The calibration terms in Eq. 2,  $B_{cal}$  and  $V_{cal}$  are determined by these flags.
- The background fluxes in the  $B_T$  and in the  $V_T$  channels. These fluxes provide the mean numbers of background photons collected during the transit,  $\langle n_{back_B} \rangle$  and  $\langle n_{back_V} \rangle$ .

The selected transits all contribute to the determination of the  $B_T$  and  $V_T$  magnitudes of the star, but they have not the same quality: the background is variable from 1700 to 12 000 counts/second, and, moreover, the sensitivity of the vertical slit group is 0.3 mag better than that of the inclined one. In order to take into account the observation conditions, any transit receives two weights, one for each channel. Each weight is defined as the square of the average signal-to-noise ratio, which is derived from the background and from the average signal computed

with Eq. 2. Therefore, the weights do not depend on the result of the transit, but only on the conditions, and on the  $B_T$  and  $V_T$  magnitudes of the star that are assumed in the iteration step, as explained in Sect. 2.

#### 4.2. The detected transits

A transit is detected when a detection is found closer than 0.6 arcsec from the position of the slit. When a magnitude measurement is available, the number of photons received from the star is corrected with Eq. 6. Next, if the signal-to-noise ratio of the measurement is larger than 0.5 after the correction, the corrected signal is accepted. It is considered as affected by the magnitude censoring otherwise.

When a measurement is missing due to magnitude censoring, the mean total number of photons below the censoring limit is derived from the model. The mean number of photons received from the star is then imputed to the measurement.

#### 4.3. The non-detected transits

The mean numbers of photons corresponding to a transit not detected are simultaneously computed for both channels, taking into account two possible causes: the detection censoring, and the spurious non-detections.

The treatment of the detection censoring is not trivial, since this effect depends on the T-channel. Pairs of photon counts ( $n_B$ ,  $n_V$ ) are generated by simulation; those with  $n_B + n_V$  below the detection limit are taken into account to compute the mean numbers of photons received from the star when the transit occurred. This simulation is also used to derive the probability of getting a censored detection, when the prediction of the transit was reliable.

The mean numbers of photons corresponding to a spurious non-detection,  $\langle n_B \rangle$  and  $\langle n_V \rangle$ , are directly derived from Eq. 2, assuming the values of  $B_T$  and  $V_T$  used in the iteration step.

The mean numbers of stellar photons corresponding to a non-detected transit are then derived by combining these two origins, assuming the 6 % proportion of spurious non-detections.

#### 4.4. The mean $B_T$ and $V_T$ magnitudes of the star

In on-ground photometry, the mean magnitude of a star is computed as the average of the magnitude measurements. This method is used since the distribution of the logarithm of the photon counts obeys then a Gaussian law, due to scintillation (Sterken & Manfroid, 1992). This is not true for Tycho photometry, and the mean magnitudes are derived from the average intensities, as explained hereafter.

In the calculations above, all transits received a mean signal for each channel. These photon counts are transformed into intensities by the equation:

$$I_M = n_M \times 10^{-\frac{M_{cal}}{2.5}} \quad (7)$$

where  $M$  refers to the channel  $B_T$  or  $V_T$ . The average intensities  $\langle I_B \rangle$  and  $\langle I_V \rangle$  are then derived from all the selected transits, taking their weights into account. The mean  $B_T$  and  $V_T$  magnitudes of the star are finally:

$$M_T = -2.5 \log \langle I_M \rangle \quad (8)$$

The next iteration is then based on these values, until the calculation has converged.

#### 4.5. Estimation of the errors

The errors are derived from the variance of the average intensity of the star in each channel, ie:

$$\text{var}(\langle I_M \rangle) = \frac{\sum W_M^2 \cdot \text{var}(I_M)}{(\sum W_M)^2} \quad (9)$$

where  $W_M$  is the weight of a transit in the  $M$  channel (ie  $B_T$  or  $V_T$ ). The problem is to derive the variances of the intensities for each transit. When the intensity comes from an actual measurement, the calculation is simple. Since  $n_M = N_M - n_{backM}$ , it is derived from Eq. 7 that:

$$\text{var}(I_M) = I_M \cdot 10^{-\frac{-M_{cal}}{2.5}} + n_{backM} \cdot 10^{-\frac{-M_{cal}}{1.25}} \quad (10)$$

The variance of the mean intensities imputed to the censored data are much more difficult to estimate; they are related to the accuracy of the model, including that of the assumed magnitudes, but this is the very result that is searched. Moreover, a fast computation is required, since the de-censoring process must be applied to a very large number of stars. It appears finally that the simplest way to solve this problem is just to ignore the contribution of the censored data in the calculation of the errors. This approximation is quite acceptable in practice, as shown in the next section.

The errors of the magnitudes are derived from the variances of the intensities. The probability distribution of the derived magnitudes is not symmetrical, however, and the ‘‘error on the bright side’’ must be distinguished from ‘‘the error on the faint side’’. These errors are respectively:

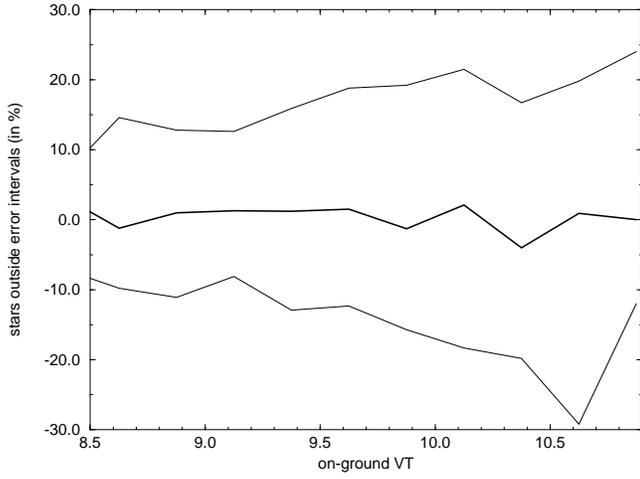
$$\sigma_M^- = 2.5 \log \left( 1 + \frac{\sqrt{\text{var} \langle I_M \rangle}}{\langle I_M \rangle} \right) \quad (11)$$

and

$$\sigma_M^+ = -2.5 \log \left( 1 - \frac{\sqrt{\text{var} \langle I_M \rangle}}{\langle I_M \rangle} \right) \quad (12)$$

The exact calculation of the errors of the colour index  $B_T - V_T$  is also not trivial, since the colour index is a function of the ratio of the average intensities  $\langle I_B \rangle$  and  $\langle I_V \rangle$ . When the errors are defined as corresponding to the percentiles 16 % and 84 %, they are calculated by the equations:

$$\sigma_{B_T - V_T}^- = 2.5 \log \frac{c + t}{c - \frac{1}{t}} \quad (13)$$



**Fig. 5.** The proportions of stars outside the error intervals for  $V_T$ . The upper line is the proportion of photometric standard stars with Tycho magnitudes too faint at the  $1 \sigma$  level when compared to the on-ground magnitudes. The thick line is the excess of stars with Tycho magnitudes fainter than the on-ground magnitudes (it is zero when the median of the Tycho magnitudes is exactly equal to the on-ground magnitude). The lower line is the proportion of photometric standard stars with Tycho magnitudes too bright at the  $1 \sigma$  level; Note that the figure contains the faint stars at the top.

and

$$\sigma_{B_T - V_T}^+ = 2.5 \log \frac{c + \frac{1}{t}}{c - t} \quad (14)$$

with  $c$  and  $t$  coming from

$$c = \sqrt{\left(\frac{\langle I_B \rangle}{\sigma_{\langle I_B \rangle}}\right)^2 + \left(\frac{\langle I_V \rangle}{\sigma_{\langle I_V \rangle}}\right)^2} - 1 \quad (15)$$

and

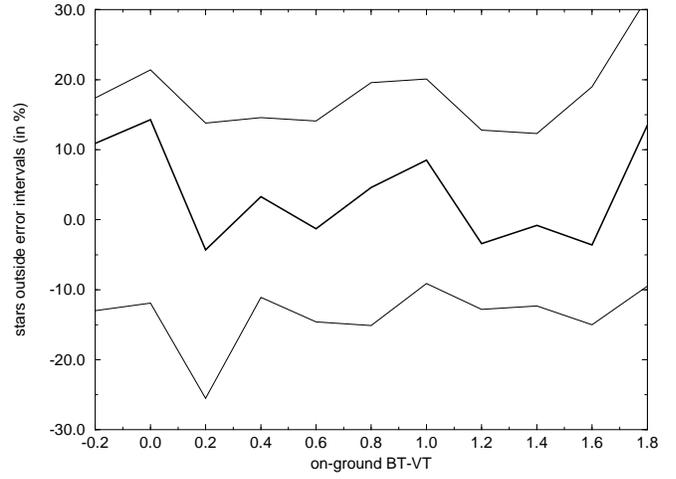
$$t = \frac{\sigma_{\langle I_B \rangle} \cdot \langle I_V \rangle}{\langle I_B \rangle \cdot \sigma_{\langle I_V \rangle}} \quad (16)$$

$\sigma_{B_T - V_T}^-$  is the error on the “blue side”, ie so that the probability that the actual  $B_T - V_T$  is less than the computed one minus  $\sigma_{B_T - V_T}^-$  is 16 %. On the other side,  $\sigma_{B_T - V_T}^+$  refers to the “red side” (in practice, it appears that the formulae:  $\sigma_{B_T - V_T}^- = \sqrt{\sigma_{B_T}^{-2} + \sigma_{V_T}^{+2}}$  and  $\sigma_{B_T - V_T}^+ = \sqrt{\sigma_{B_T}^{+2} + \sigma_{V_T}^{-2}}$  are good approximations of Eq. 13 and 14, although they are much simpler).

## 5. Application of the de-censoring software to the photometric standard stars

### 5.1. The mean magnitudes

The mean  $B_T$  and  $V_T$  magnitudes of the photometric standards are derived from the transits collected during all the Tycho mission, using the de-censoring procedure. The differences between



**Fig. 6.** Same as Fig. 5, but for  $B_T - V_T$ . The upper line refers to the stars with Tycho colour indexes too large, and the lower to the stars with Tycho colour indexes too small when compared to the uncertainty intervals. Only the standard stars with  $B_T > 9.5$  mag are taken into account.

the derived  $V_T$  magnitudes and the on-ground measurements are plotted on Fig. 3. The large bias affecting the actual measurements has vanished, although a close examination still reveals that the de-censored magnitudes are about 0.06 mag too faint when  $V_T > 10.6$  mag. In the  $B_T$  channel, a bias of 0.05 mag appears also when  $B_T > 11.4$  mag. Simple functions are used to fit the biases, and the de-censored magnitudes and then corrected. The corrected de-censored magnitudes are called the “Tycho magnitudes” hereafter.

### 5.2. The uncertainties

As long as the measurements are considered individually, the uncertainties of the Tycho are rather large; for instance, it comes from the equations above that the error of a measurement of  $V_T = 9.5$  mag is 0.21 mag, when  $V_{cal} = 13.4$  mag and when a moderate background  $\langle n_{backv} \rangle = 25$  counts is assumed. Fortunately, a lot of measurement are used to derive the mean magnitudes: for the photometric standard stars, the median number of photometric transits is 109. Therefore, the error of the mean magnitude  $V_T = 9.5$  mag is only 0.023 mag when 100 measurements as the one considered in the example above are taken into account. The errors of the mean  $V_T$  magnitudes of the photometric standard stars are plotted in Fig. 4. They are around 0.013 mag when  $V_T = 8.5$  mag, but they increase dramatically for stars fainter than 10.5 mag, when the number of detections is falling off (for stars with  $V_T = 10.5$  mag, about 20 % of transits are censored).

For checking the validity of the errors, they are compared to the differences between the Tycho magnitudes and the on-ground magnitudes. The sizes of the error intervals are calculated as the quadratic sums of the errors of the de-censored magnitudes and of the on-ground magnitudes. The proportion of stars outside the error interval is plotted for  $V_T$  in Fig. 5.

It appears from this figure that the proportion is smaller than 16 % when  $V_T < 9.5$  mag, suggesting that the errors are overestimated. However, the Tycho error is usually smaller than the error of the on-ground magnitude when  $V_T < 9.5$  mag, and the overestimation is obviously due to the former. For the faint magnitudes, the proportion is a little larger than 16 %, but it remains on average less than 20 %.

A similar statistic is given in Fig. 6 for the Tycho colour index  $B_T - V_T$ . Only the stars fainter than  $B_T = 9.5$  mag are taken into account in this plot, since the de-censoring is not very frequent for the bright stars. The large deviations at the extremities are not statistically significant, but are due to the small numbers of blue and red standard stars.

In conclusion, the de-censoring software provides the mean magnitudes of faint stars with small biases that are easily corrected. Moreover, the estimated errors look fairly reliable. In the Tycho catalogue, the mean magnitudes were derived from the median magnitudes for 30 000 stars brighter than  $V_T = 8$  mag and  $B_T = 8.5$  mag, but the de-censored magnitudes were preferred for the 1 018 000 other stars.

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