

Abundance and clustering of QSOs in cosmic structure formation models

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Abstract. When combining the observations of spatial abundance and two-point correlation function of QSOs, we can effectively set constraints on models of cosmic structure formation. Both the abundance of gravitationally confined halos and their two-point correlation functions can be calculated in the conventional Press-Schechter formalism. We apply this method to examine the properties of possible host halos of QSOs in three popular models: the standard cold dark matter (SCDM) model, the low density flat cold dark matter (LCDM) model and the cold-plus-hot dark matter (CHDM) model. The LCDM and CHDM models are normalized to the COBE-DMR observations, and the SCDM is normalized to $\sigma = 0.58$. We find that the SCDM and LCDM models can pass the abundance-plus-correlation test for QSOs. However, the CHDM are difficult to produce host halos to fit with the number density of high redshift QSOs and their clustering on large scales ($10 h^{-1}$ Mpc) simultaneously. We studied various mechanisms, originated both gravitationally and non-gravitationally, which may lead to a biasing of the halo clustering. We conclude that these effects are too weak in order to release the trouble of the CHDM models.

Key words: quasar: general – cosmology: large-scale structure of universe – dark matter

1. Introduction

Recently, we have developed a method to test popular dark-matter models of structure formation using the number density and the two-point correlation function of high redshift objects (Bi & Fang 1996). We found an approximate expression of two-point correlation function of mass and collapsed halos in the Press-Schechter formalism (Press & Schechter 1974, hereafter PS). In this approximation, the nonlinear gravitational interaction was treated as the sum of various individual spherical

top-hat clustering. These top-hat spheres consist of both collapsed PS halos and uncollapsed regions. Moreover, the bias that massive PS halos have stronger correlation than the background mass can naturally be introduced by considering that no collapsed halo of mass M exists in initial regions (or top-hat spheres) of mass less than M .

This method was applied to CIV absorption systems in QSO spectra. Because CIV systems should be hosted by collapsed halos, one can obtain an lower limit to the spatial number density of these host halos from the observed CIV number density. This requires that the hosts of CIV should consist of halos with mass as low as M_{th} . On the other hand, the two-point correlation of halos is stronger when the mass of the halos is larger. The observed two-point correlation functions set a lower limit, M_{co} , to the mass of host halos. Obviously, a reasonable model has to give $M_{th} > M_{co}$. The standard cold dark matter (SCDM) model normalized to $\sigma = 0.58$ and the low density flat cold dark matter (LCDM) model can pass this test. However, for the cold-plus-hot dark matter (CHDM) models with parameters $\Omega_c = 0.7$ and $\Omega_b = 0.3$, or $\Omega_c = 0.8$ and $\Omega_b = 0.2$, the two-point correlation functions of halos with mass M_{th} are too small to explain the observed correlation functions. In order to have enough number of collapsed halos to host CIV systems, M_{th} should not be larger than $10^{11} M_{\odot}$. But the observed two-point correlation function on the scales of $\Delta v \sim 300 - 1,000$ km/s indicates that M_{co} should not be less than $10^{12} M_{\odot}$.

In this paper, we study the same problem, but using QSOs as the discriminator. The topic of finding constraints on dark matter models from QSOs is not new. As early as 1980s, the highest redshift of QSOs was used to rule out the model of hot dark matter (HDM), because the HDM predicted that collapsed halos cannot form at redshift larger than 3, which is much less than the redshifts of many existing QSOs. Consequently, the formation of high redshift QSOs cannot be explained in the HDM model (e.g. Efstathiou & Rees 1988). This redshift test is passed for current dark matter models. For instance, the tilted CDM, scaled CDM and CHDM models all are able to produce with the observed abundance of QSOs (Nusser & Silk 1993).

Table 1. Parameters of the cosmological models

	SCDM	LCDM	CHDM
h	0.5	0.75	0.5
Ω_0	1	0.3	1 ($\Omega_\nu = 0.3$)
λ_0	0	0.7	0
σ_8	0.58	1	0.55

However, the QSO abundance test alone is not free from the uncertainties of discriminating among the models like SCDM, LCDM and CHDM. While some works claimed that the CHDM model is consistent with the QSO abundance, other works found oppositely (Ma & Bertschinger 1994). We will show that adding a clustering test to the abundance fitting will reduce the number of free parameters in the discrimination, and gives much better results.

1.1. Abundance of QSOs

The spatial number density of collapsed halos can be calculated from the standard PS theory. We define $\delta(\mathbf{x}, z)$ to be the 3-D density fluctuation field of dark matter extrapolated to redshift z assuming linear evolution. A density field $\delta_R(\mathbf{x}, z)$, representing the smoothed fluctuation on scale R , can be derived from $\delta(\mathbf{x}, z)$ by

$$\delta_R(\mathbf{x}, z) = \frac{1}{V_R} \int \delta(\mathbf{x}', z) W(R; \mathbf{x}' - \mathbf{x}) d\mathbf{x}', \quad (1)$$

where the function $W(R; \mathbf{x}_1 - \mathbf{x})$ is the top-hat window for the comoving volume $V_R = 4\pi R^3/3$. The total mass within V_R on average is $M = V_R \rho_0$, where ρ_0 is the mean density at the present if the scaling factor of the universe is set to be unity at $z = 0$.

For Gaussian perturbations, the mass fluctuation within a top-hat window of radius R is described by variance $\sigma^2(R, z)$, which is determined by the initial density spectrum $P(k)$ and normalization factor σ_8 , i.e. $\sigma(R = 8 \text{ h}^{-1} \text{ Mpc})$. In the case of $\Omega = 1$ Einstein-de Sitter universe, the linear evolution of the variance is $\sigma^2(R, z) \propto (1+z)^{-2}$. Thus, the fraction of the total mass $\rho_0 \Delta \mathbf{x}$ having fluctuations larger than a given δ_c in an arbitrary spatial domain $\Delta \mathbf{x}$ is

$$F_R = \int_{\delta_c(z)}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(R, z)} \exp\left(-\frac{\delta_R^2}{2\sigma^2(R, z)}\right) d\delta_R. \quad (2)$$

If we take $\delta_c(z)$ to be the critical overdensity for the collapse of the spherical mass M at z , $F_R \cdot \rho_0 \Delta \mathbf{x}$ should be identified as the sum of masses of all collapsed halos, each of which is massive greater than M . For the Einstein-de Sitter universe, we have $\delta_c(z) = (1+z)1.69$. For the flat Λ universe, function $\delta_c(z)$ is calculated by Bi & Fang (1996). The differential $-\frac{\partial}{\partial M}(F_R \rho_0 \Delta \mathbf{x}) dM$ gives the total mass of collapsed halos in the mass range M to $M+dM$. Hence, if $n_c(M, z) dM$ is defined as the spatial number density of halos between M and $M+dM$ at z , we have

$$-\frac{\partial}{\partial M}(F_R \rho_0 \Delta \mathbf{x}) dM = n_c(M, z) dM \cdot M \Delta \mathbf{x}, \quad (3)$$

where we use the subscript c in n_c to emphasize that it is for collapsed halos. Considering the cloud-in-cloud problem, the above defined n_c should be multiplied by a factor of 2. The normalization $\int_0^{\infty} n_c(M, z) M dM = \rho_0$ can then be fulfilled. Therefore, the spatial number density of halos with mass between M and $M+dM$ at redshift z is given by

$$n_c(M, z) dM = -\frac{\rho_0}{M} \frac{\partial}{\partial M} \text{erfc}\left(\frac{\delta_c(z)}{\sqrt{2}\sigma(R, z)}\right) dM, \quad (4)$$

where $\text{erfc}(x)$ is the complementary error function.

We will study models of SCDM, LCDM and CHDM, for which the density parameter Ω_0 , the cosmological constant λ_0 , and σ_8 , are listed in Table 1 (the Hubble constant is taken to be $H_0 = 100 \text{ h km s}^{-1} \text{ Mpc}^{-1}$.) It is worth to point out that the σ_8 of the CHDM and LCDM models are compatible with the COBE-DMR observation, but that of the SCDM model is not. The value $\sigma_8 = 0.58$ gives a good fit for the SCDM model to all data except for the COBE result. We assumed that the initial primordial power spectrum is the Harrison-Zel'dovich type. The linear transfer functions of the SCDM and the LCDM models are taken from Bardeen et al. (1986) and that of the CHDM model from Klypin et al. (1995).

The cumulative number density, $N_c(M)$, of all halos with mass greater than M should be

$$N_c(M, z) = \int_M^{\infty} n_c(M_1, z) dM_1. \quad (5)$$

The abundance of halos calculated from Eq. (5) has been verified by a number of N-body simulations (e.g. Lacey & Cole 1994).

The redshift evolution of the number density of collapsed halos, $N_c(M, z)$, are shown in Figs. 1a, 1b and 1c for the SCDM, LCDM and CHDM models, respectively. The eight curves in Figs. 1a and 1b correspond to $M = 10^{13.5+n0.25} M_\odot$, with $n = 0, 1, \dots, 7$ from top to bottom. In Fig. 1c, it is $M = 10^{11.5+n0.25} M_\odot$, with $n = 0, 1, \dots, 7$.

Pairs and multiples of QSOs are not common to see. Therefore, it is reasonable to assume that each collapsed halo hosts only one, or at most a few QSOs. The number density of QSOs with magnitude $M_B < -26$ is plotted as crosses in Fig. 1. The data points are taken from Pei (1995) whose result is based on the observations of Hewett et al. (1993) and Schmidt et al. (1992). In Pei's paper, $n(z)$ is measured in the Einstein-de-Sitter cosmological model ($\Omega_0 = 1, \lambda_0 = 0$ and $h = 0.5$). When comparing the observations to the LCDM model, we have made corrections of the cosmological effect on $n(z)$ due to the non-zero λ_0 .

Fig. 1 shows that for all the models, the lower mass limits are mainly determined by the QSO abundance at $z = 4$. The possible hosts of QSOs in the SCDM and LCDM models can be provided by halos with masses above $10^{13.75}$, and $10^{14} M_\odot$, respectively. In the CHDM model, the number densities of collapsed halos at these mass range are much fewer. Therefore, for the CHDM, we have to include lower mass halos to host QSOs. The mass of possible QSO halos will be as low as about $M = 10^{11.75} M_\odot$, which is more than two orders of magnitude below those in the models of SCDM and LCDM.

2. Two-point correlation functions of QSOs

The spatial number density of uncollapsed spherical regions can also be calculated in PS formalism. We define $m(M_0, r, z)dM_0dr$ to be the number density of regions with mass $M_0 \rightarrow M_0 + dM_0$ and radius $r \rightarrow r + dr$ at z . Similar to Eq. (4), we have

$$m(M_0, r, z) = -\frac{\rho_0}{M_0} \frac{1}{2} \frac{\partial^2}{\partial M_0 \partial r} \operatorname{erfc} \left[\frac{\delta(r, z)}{\sqrt{2}\sigma(r_0, z)} \right]. \quad (6)$$

where r_0 is defined by $M_0 = V_0\rho_0 = 4\pi r_0^3\rho_0/3$. $\delta(r, z)$ is the overdensity, for which a region M_0 will evolve into a sphere with radius r at z . Function $\delta(r, z)$ is also calculated in Bi & Fang (1996). The total number of such spherical regions in an arbitrary volume $\Delta\mathbf{x}$ should on average be given by $m(M_0, r, z)dM_0dr\Delta\mathbf{x}$.

The mass correlation function $\xi(r)$ is determined by the relative enhancement of mass density in the spherical shell $r \rightarrow r + dr$ around area $\Delta\mathbf{x}$. Only the spheres with radius $r \rightarrow r + dr$ can contribute to this enhancement. The mean enhancement of each M_0 sphere is approximately described by its mass variance $M_0^2\sigma^2(r_0, z)$. Therefore, the mass correlation function can be estimated by

$$\begin{aligned} \xi(r, z) &= \frac{\int_0^\infty m(M_0, r, z)dM_0dr\Delta\mathbf{x}M_0^2\sigma^2(r_0, z)}{\rho_0 4\pi r^2 dr \cdot \rho_0 \Delta\mathbf{x}} \\ &= \int_0^\infty dM_0 m(M_0, r, z) M_0^2 \sigma^2(r_0, z) / 4\pi r^2 \rho_0^2. \end{aligned} \quad (7)$$

The factors $\rho_0 4\pi r^2 dr$ and $\rho_0 \Delta\mathbf{x}$ are the mean mass in the spherical shell $r \rightarrow r + dr$ and the volume $\Delta\mathbf{x}$, respectively. In deriving Eq. (7), we implicitly assumed that there is no correlation among the initial spheres, so the mass fluctuation in the shell is simply given by the sum of the individual components.

Because it is impossible that a collapsed halo with mass M formed in an uncollapsed region with $M_0 < M$, the number density of M collapsed halos in an uncollapsed sphere of mass M_0 should be zero if M is greater than M_0 . Thus, it is reasonable to assume that the number density of collapsed halos in each PS sphere r at z on average is

$$\mathcal{N}_c(M, M_0) = \begin{cases} AN_c(M, z) & \text{for } M \leq M_0, \\ 0 & \text{for } M > M_0, \end{cases} \quad (8)$$

where the constant $A \geq 1$ is introduced to maintain the normalization condition

$$\int_0^\infty \mathcal{N}_c V_0 n(M_0, z) dM_0 = N_c(M, z). \quad (9)$$

We have then $A = 2/\operatorname{erfc}[\delta(R, z)/\sigma(R, z)]$.

Like calculating Eq. (7), let's consider a typical spherical shell $r \rightarrow r + dr$. Only the spheres with radius $r \rightarrow r + dr$ can contribute to the mass enhancement in this shell. The total number of uncollapsed regions with radius $r \rightarrow r + dr$ and masses $M_0 \rightarrow M_0 + dM_0$ in a volume $\Delta\mathbf{x}$ is $m(M_0, r, z)dM_0dr\Delta\mathbf{x}$. The total number of collapsed halos M in each M_0 sphere is

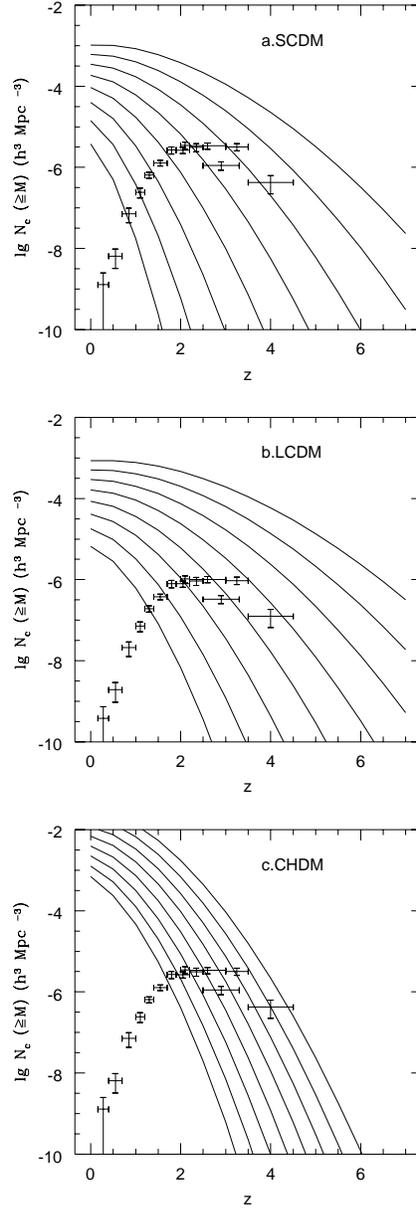


Fig. 1a–c. Evolutions of the comoving number density of halos with mass larger than a given number M . **a** for SCDM model, the eight curves correspond to $M = 10^{13.5+n0.25} M_\odot$, with $n = 0, 1, \dots, 7$ from top to bottom. **b** The same as **a** but for LCDM model. **c** for CHDM model, here $M = 10^{11.5+n0.25} M_\odot$, with $n = 0, 1, \dots, 7$.

$\mathcal{N}_c(M)V_0$, and the variance of the number is $(\mathcal{N}_c V_0)^2 \sigma^2(r_0, z)$. Therefore, the correlation function of collapsed halos with mass larger than M can be approximated as (Bi & Fang 1996)

$$\begin{aligned} \xi(r; \geq M, z) &= \frac{\int_M^\infty dM_0 m(M_0, r, z) dr \Delta\mathbf{x} \cdot (\mathcal{N}_c V_0)^2 \sigma^2(r_0, z)}{N_c 4\pi r^2 dr \cdot N_c \Delta\mathbf{x}} \\ &= A^2 \int_M^\infty dM_0 m(M_0, r, z) V_0^2 \sigma^2(r_0, z) / 4\pi r^2 \end{aligned} \quad (10)$$

Eq. (10) implicitly assumed that in uncollapsed regions, the collapsed halos have the same linear variance as the mass. The approximation given by Eq. (10) is found to be in good agreement with the linear approximation on scales larger than R , but higher than the empirical formalism of Hamilton et al. (1991) on scales less than R , here R is the scale within which the non-linear effects are significant. Therefore, Eq. (10) can be used at least as a upper limit to the correlation function.

Using Eq. (10), we have calculated the correlation function $\xi(r, \geq M)$ at redshift $z = 2.5$. The results are shown in Fig. 2, in which 2a and 2b are for the models of SCDM and LCDM, and 2c for CHDM. The eight curves in Figs. 2a and 2b are corresponding to masses $10^{13.5+n0.25} M_{\odot}$, $n = 0, \dots, 7$ from left to right, respectively. In Fig. 2c, it is $10^{11.5+n0.25} M_{\odot}$ and $n = 0, \dots, 7$.

The two-point correlation function of QSOs is found to be obey the same power law as galaxies $\xi_{qq}(r) = A_{qq} r^{-1.8} = (r/r_0)^{-1.8}$, and the amplitude $A_{qq} \sim 25$ (or the correlation length $r_0 \sim 6 h^{-1} \text{Mpc}$) at $z = 1.5$ when q_0 is taken to be 0.5 (Mo & Fang 1993). This gives $\xi_{qq}(r) > 0.1$ for $r = 5 - 10 h^{-1} \text{Mpc}$. The clustering of QSOs on scales of $r \sim 30 h^{-1} \text{Mpc}$ is also found to be significant (Deng et al. 1994). Some observations and statistics have even indicated the possible existence of groups of QSOs with comoving sizes as large as about $100 h^{-1} \text{Mpc}$ (e.g. Clowes & Camppusano 1991, Komberg et al. 1996.) Despite A_{qq} is decreasing for $z \geq 1.5$, the amplitude ξ_{qq} is still larger than 0.1 (Mo & Fang 1993; Komberg et al. 1994.)

Fig. 2a and 2b show that the correlation function of $M \geq 10^{14} M_{\odot}$ halos in the SCDM and LCDM models are consistent with the observational data. These halos have also proper spatial number density as the observed QSOs. The difference between the two models are very small. In fact, the top-hat evolutions of a spherical mass in SCDM and LCDM are indistinguishable, because it has almost the same dynamic trajectory in all flat universes with $\lambda_0 \leq 0.8$.

On the other hand, Fig. 2c shows that the two-point correlation functions of the CHDM model are very small on large scales. For halos with mass $10^{13.25} M_{\odot}$, the amplitude of the correlation function is well less than 0.1 on scales of $r = 5 - 10 h^{-1} \text{Mpc}$. Therefore, the CHDM model seems to be unfavored under the abundance-plus-correlation test. According to the abundance, the mass of QSO hosts in the CHDM model should be as small as $10^{12} M_{\odot}$. On the other hand, the two-point correlation function of such halos is much less than what is observed.

3. Biasing problem

A possible way to save the CHDM model is to assume that there is a very high biasing factor to raise the correlation function of $10^{11.5} M_{\odot}$ halos in the CHDM model. From Fig. 2c, if this factor is as large as 5, the power of two-point correlation functions mass $10^{11.5} M_{\odot}$ would be able to fit with QSO's clustering. However, biasing is actually not a free parameter which can be arbitrarily chosen (Einasto et al. 1994).

Generally speaking, the correlation functions of objects identified from a mass field will be different from the mass

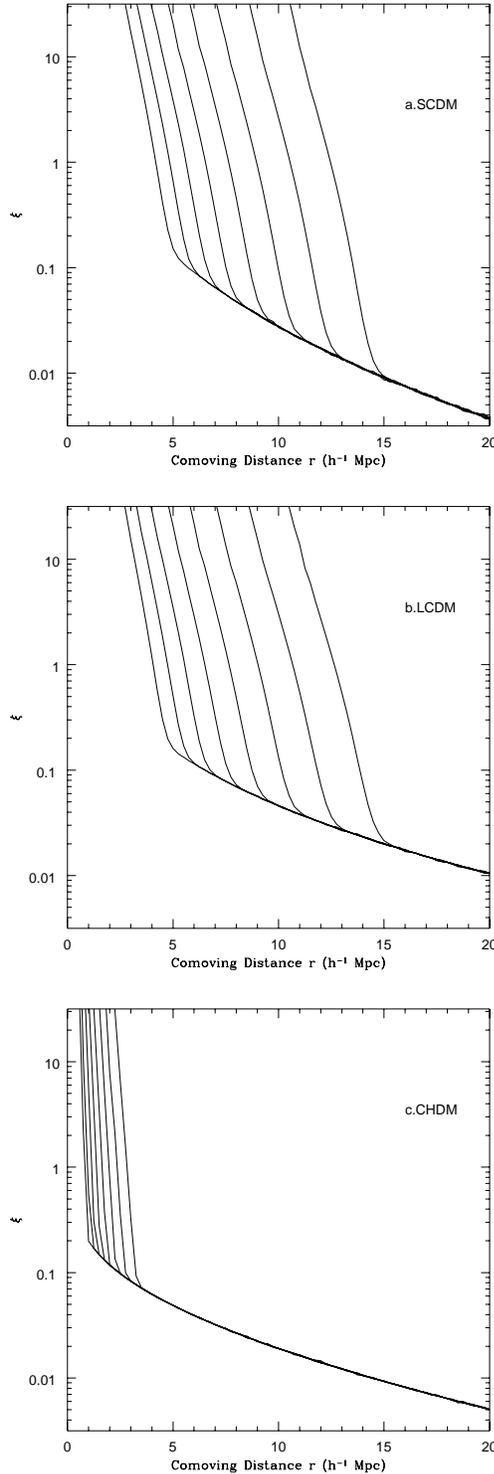


Fig. 2a–c. Two point correlation functions of M halos at $z = 2.5$. **a** For SCDM model, the eight curves from left to right are corresponding to masses $M = 10^{13.5+n0.25} M_{\odot}$ and $n = 0, \dots, 7$, respectively. **b** The same as **a** but for LCDM model. **c.**) For CHDM model, here $M = 10^{11.5+n0.25} M_{\odot}$ and $n = 0, \dots, 7$.

itself. A different method in the identification procedure can lead to a different biasing. The question now is: can we choose other QSO identifications to produce larger biasing, and then to give higher amplitude of the two-point correlation functions of QSOs in the model?

As an example, let us first consider the QSO identification by velocity dispersion. It has been known for a decade that QSOs with low redshift are preferentially located in small groups of galaxies. This is in evidence from QSO-galaxy correlation function (Yee & Green 1987), CIV-associated absorption in high redshift radio-loud QSOs (Flotz et al. 1988), clustering analyses of QSO distribution (Bahcall & Chokshi 1991) and the galaxy environments around QSOs (Ellingson et al. 1991a). It has been also shown that the velocity dispersion of galaxies around QSOs is $\sim 400 \text{ km s}^{-1}$ (Ellingson et al. 1991b). Therefore the environment suitable of QSOs formation seems to be small groups of velocity dispersion $\sim 400 \text{ km s}^{-1}$. The strength of the QSO correlation function, is intermediate between galaxies and rich clusters, so once again it is similar to galaxy groups.

If we assume that high redshift QSOs formed in the same environment as that in the low redshift, the QSOs should be identified as collapsed halos with 3-dimensional velocity dispersion $\sigma_v \sim \sqrt{3} \times 400 = 700 \text{ km s}^{-1}$ between $z = 2$ and $z = 4$.

In the PS formalism, the comoving number density of halos with velocity dispersion σ_v can be calculated by

$$n(\sigma_v, z)d\sigma_v = -\frac{3}{(2\pi)^{3/2}R^3} \times \frac{d \ln \sigma(R, z)}{d \ln R} \frac{\delta_c(z)}{\sigma(R, z)} \exp\left(-\frac{\delta_c^2(z)}{\sigma^2(R, z)}\right) \frac{dR}{R} \quad (11)$$

As in Eq. (4), $\delta_c(z)$ in Eq.(11) is the critical overdensity for collapsed at redshift z . The meaning of R here is the same as in §2. The relationship between σ_v and R is given by (Narayan & White 1988)

$$\sigma_v = c_\sigma \sqrt{3} H_0 R (1+z)^{1/2} \quad (12)$$

for Einstein-de-Sitter universe, and

$$\sigma_v = c_\sigma \sqrt{3} \Omega_0^{1/2} H_0 R (1+z)^{1/2}. \quad (13)$$

for open universe or the LCDM universe. The coefficient c_σ has been determined by comparing Eqs. (11) and (12) with N-body simulation (Jing & Fang 1994). It found $c_\sigma = 1.1 - 1.3$. For most calculations, $c_\sigma = 1.2$ is a preferred value. Hence, one can safely use Eqs.(11)-(13) even when the 3-D velocity dispersion is as large as about 800 km s^{-1} .

The total number density of the collapsed halos with the velocity dispersion greater than a certain value, say σ^{lim} , is

$$N(> \sigma^{lim}, z) = \int_{\sigma^{lim}}^{\infty} n(\sigma_v, z) d\sigma_v. \quad (14)$$

Using Eq. (14), we found that the SCDM and LCDM models can produce sufficient number of $\sigma^{lim} = 700 \text{ km s}^{-1}$ halos to fit with the number density of QSOs. But the number of such halos

in the CHDM model is too small. To have enough number of halos, we should use $\sigma^{lim} = 300 \text{ km s}^{-1}$ or less in the CHDM model.

The mass of the σ_v -selected halos can be determined from Eqs.(12) or (13). One can then calculate the two-point correlation functions of them by Eq. (10). Here we get almost the same results as before because σ_v is one-to-one related to the mass or the radius of the top-hat windows. Similar to Fig. 2c, the $\sigma^{lim} = 300 \text{ km s}^{-1}$ halos still lack of correlation power on all scales larger than $2 \text{ h}^{-1} \text{ Mpc}$ in the CHDM model.

This result should be expected. An identification of objects from a density field is a sampling. As one knows, a sampled field will not be different from the original field on scales much larger than the characteristic scale, R_c , of the sampling (Vanmarke 1983). This means that, in principally, no biasing on scales larger than the characteristic scale can be introduced by the identification. For our question here, it is not easy to given the characteristic scale R_c of the identification. However, it is probably reasonable to choose $R_c = [1 + \xi(R, z)]^{1/3} R$, where $\xi(R, z)$ is the average of $\xi(r, z)$ inside R (Hamilton et al. 1991). So the scale R_c is actually the coherence length of the density field smoothed by window R . The distribution of halos should not be biased from mass distribution on scales larger than R_c . The correlation function can effectively be amplified by linear geometrical biasing only for large halos (Kaiser, 1984). Fig. 2 show a break in the correlation functions at r_b , which is roughly equal to R_c . When $r > r_b$, the correlation functions of the collapsed halos should approach to the mass correlations, while when $r < r_b$, the biasing leads to larger correlation functions.

All identifications based on gravitational parameters, such as circular velocity, virial temperature etc. are essentially equal to mass identifications, because for a gravitational collapsed systems these parameters are one-to-one related. Therefore, they can be expressed as an equivalence to M . Biases in these parameters cannot give significant different results from those using σ_v or M .

Obviously, any non-gravitational identifications are not constrained by the gravitational characteristic scale. For instance, if gas processes in the formation of QSOs play the role of biasing, the characteristic scale will not equal to R_c . However, the characteristic scale of gas processes should be much less than R_c , because the velocity of gaseous component is too small to segregate the QSO halos on scales of $5 \text{ h}^{-1} \text{ Mpc}$.

4. Conclusions

We showed that the abundance-plus-correlation test can serve as a promising discrimination among models of cosmic structure formation using the high redshift ($z > 2$) QSO number density and their clustering on large scales ($10 \text{ h}^{-1} \text{ Mpc}$). We found that the SCDM and LCDM model are consistent with the observations of abundance and two-point correlation, while the CHDM models is difficult to fit with the observed numbers.

The contributions of variously biasing mechanisms have been investigated. A bias, in fact, is a model of the environment suitable to form objects, or a phenomenological relation-

ship between the cosmic density field and QSOs. We have studied possible biasing processes, including both gravitational and non-gravitational origins. None of them seems to give large characteristic scale and clustering amplitude required to make the CHDM model success.

One should also consider the possibility that each halo may host more than one QSOs. In this case, the correlation function can be stronger than that of one-QSO/one-halo model. In order to fit with observed abundance of QSOs in the CHDM model, one can assume that each halo of $M = 10^{14} M_{\odot}$ host more than 10 QSOs. In this case, halos of $M = 10^{14} M_{\odot}$ in the CHDM can also fit with the observed two-point correlation function of QSOs (Fig. 2c). However, this apparently leads to the paradox that we did not find so many bright QSOs concentrating in galaxy groups. A possible way to explain it is that the lifetime of QSOs is just longer than the cosmic time at $z = 3$ but shorter than the cosmic time at the present. (If QSOs have a life time shorter than the cosmic time at $z = 3$, we require even more low mass halos, this makes the situation even worse.) However, the difference between the cosmic time at $z = 3$ and $z = 0$ is only a factor of 8 ($t \propto (1+z)^{3/2}$ in the $\Omega = 1$ universe). Therefore, we may predict that on average, each of 8 galaxy group will have a bright QSO. This is not true. Otherwise, we should require a very particular mechanism to make the QSO formation at $z = 3$.

It is theoretically possible to explain any correlations if we are allowed to introduce *unknown* inhomogeneity into the density field, and assume that the correlation of QSOs absorbers is given by these inhomogeneities. However, to plan these inhomogeneities is equal to put desired structures in the initial perturbations. Such models will, however, no longer be the SCDM, LCDM or CHDM models, which are based on initial fluctuations produced at the inflationary era. However, even in this case the CHDM will still be difficult to produce enough halos with reasonable velocity dispersions to fit with the abundance of high redshift QSOs.

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