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ASTRONOMY AND ASTROPHYSICS

On the radiative transfer in atmospheres with randomly distributed inhomogeneities

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Abstract. We consider the problem of determining the statistical properties (intensity distribution, mean value and relative mean square deviation) of radiation which emerges from a medium with randomly distributed inhomogeneities when scattering is taken into account. The analytical formalism we propose is based on Ambartsumian's method of addition of layers. This can be applied to both local thermodynamic equilibrium (LTE) and Non-LTE cases for a wide variety of situations. A more detailed treatment of the special case of pure scattering, considered as the prototype problem for the transfer of the L_α-line radiation, is given. The present investigation is assumed to apply for interpreting the spectra of various inhomogeneous structures on the Sun. It is shown in particular that some specific features of the extreme ultraviolet (EUV) spectra of solar prominences may be accounted for within the framework of the theory we propose.

Key words: radiative transfer – Sun: prominences

1. Introduction

The consistent interpretation of emission by a variety of inhomogeneous structures on the Sun such as chromospheric spicules, fibrils, coronal plumes, prominences, etc., requires a suitable theory for the transfer of radiation through a gas which contains randomly distributed inhomogeneities. A procedure for calculating the statistical properties of the radiation emerging from a multicomponent LTE-atmosphere is presented by Lindsey (1987) and Jefferies and Lindsey (1988; hereafter JL88) for the interpretation of total solar eclipse observations in the far-infrared. These authors derived relations describing the evolution behaviour of the mean value of the intensity and its variance along the radiation path. It should be noted that the assumption of LTE is crucial for the approach proposed in this work. Because of the absence of scattering, and hence reflection from structural elements, the mean value of intensity emerging from any part of the multicomponent atmosphere remains unaltered when new components are added. This fact is essential in the sense that it simplifies the problem and allows one to establish immediately the recurrence relationship for both the mean intensity and variance so that the problem has a simple closed-form solution.

The main purpose of the present paper is to demonstrate an important situation that allows for scattering, in an inhomogeneous atmosphere with statistically varying properties, and admits a relatively simple representation. The case of conservative scattering ($\varepsilon = 0$, here ε is the probability of photon destruction per scattering) is meant, which is opposite to that of LTE ($\varepsilon = 1$) handled by the above-mentioned authors. The results we obtain are of primary concern in analysing the spatial and temporal variations of the L_α-line intensity for chromospheric and coronal structures. The approach developed in this article is based on Ambartsumian's method of addition of layers, which, applied to the LTE-atmosphere, allows a simple derivation of the equations obtained in JL88. This approach may become applicable to various astrophysical problems, as the theoretical interpretation of the EUV-spectra of quiescent prominences observed with the Harvard College Observatory spectrometer aboard ATM-Skylab, shown in the present study. We are interested also in optically thin lines such as $\lambda\lambda$ CII 1336Å, CIII 977Å, OVI 1032Å, the consistent interpretation of which may be also given by using the results of JL88. A concise treatment of the present problem was given by Nikoghossian and Pojoga (1997).

2. Statement of the problem

For the sake of simplicity we limit ourselves to the one-dimensional model atmosphere adopted in JL88. Let us consider a plane-parallel multilayer medium containing energy sources. Each of layers is characterised by the optical thickness τ, and the power of energy sources B. We shall assume that (τ, B) takes randomly one of two possible sets of values (τ_1, B_1) and (τ_2, B_2). The probabilities associated with these two events are p_1 and p_2 (= $1 - p_1$). No radiation incident on the medium is

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supposed, although it can easily be included if necessary. We are interested in the statistical properties of the intensity of the emerging radiation such as the expected value and the relative mean square deviation (RMSD), depending on the number N of layers. Formulated in this way, the problem generally enables one to obtain some information upon the statistical features of the observed intensity in the presence of different types of structures, one of which may be identified in some cases with the interposing ambium. Note also that the formulation of the problem can be extended to more general distributions of the occupancy and optical properties of structural elements.

Considering the scattering process, we make the transfer problem more difficult so that the state of a gas at a given point is no longer completely specified by the local values of the thermodynamic parameters as in LTE, but also depends on the radiation field throughout the atmosphere. Another difficulty stems from the fact that physical conditions at the same point may be altered during the random walk of a photon through the medium. In this case the possibility of application of our model problem, depends on the time-scales characterising the variation of structural elements and the scattering process.

Let us examine this point in more details for the scattering of a L_α-line photon in a typical quiescent prominence. Being opaque in this line ($\tau \sim 10^5 \div 10^6$; see e.g., Tandberg-Hanssen, 1995), the prominence may be considered as a semi-infinite atmosphere. In this case, the average time taken by a photon in diffusing is limited due to the destruction in the continuous spectrum and it is of the order of $(nkc)^{-1}$ s (Nikoghossian, 1986), (here n is the number density of the hydrogen atoms, c is the velocity of light and k is the absorption coefficient per atom in the Lyman continuum). The commonly accepted values for density are $n \sim 10^{10} \div 10^{11}$ cm^{-3} (Tandberg-Hanssen, 1995), which lead to an upper limit of the order of $10^{-4} \div 10^{-5}$ s for an average time of diffusion. On the other hand, it is well known that prominences generally show minor changes over periods of hours or days. Even during the activation of quiescent prominences, typical values of velocities are of the order of 30 to 50 km s^{-1}. Thus, we may safely assume in modelling the transfer problem that a given structural element (or thread) will be in the same state while being encountered several times by a travelling photon. It follows from these arguments that in observing a given distribution of intensity of the line-radiation over the set of instrument pixels we are concerned with an ensemble of various realisations of inhomogeneities, each of which corresponding to a well-defined deterministic problem.

3. The method of addition of layers

The above considerations imply that the stochastic problem we are concerned of assumes the knowledge of the solution of the transfer problem for an inhomogeneous multilayer atmosphere with properties (B_i, τ_i; $i = 1, 2$), being arbitrarily distributed over the layers. In its general Non-LTE formulation, this last problem is deeply connected with the procedure of addition of layers developed by Ambartsumian (1960) (see also Sobolev, 1963) as an alternative representation of the principle of invariance.

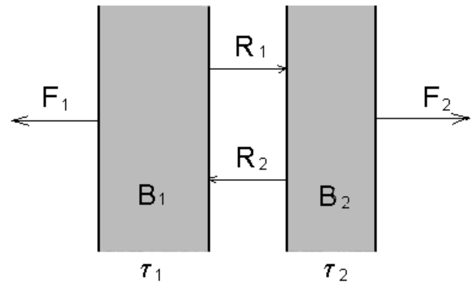

Fig. 1. Sketch of the transfer of radiation trough two slabs.

We consider the simplest situation when a medium is composed of two layers (slabs) of optical thicknesses τ_1 and τ_2 (see Fig. 1). The slabs contain energy sources of power B_1 and B_2, respectively, which produce respective fluxes f_1 and f_2 in each direction. Denoting by $\rho(\tau_i)$ and $q(\tau_i)$ the reflectance and transmittance of each slab, we may write a set of relations for the emerging fluxes

$$F_1 = R_2 q(\tau_1) + f_1, \qquad R_1 = R_2 \rho(\tau_1) + f_1, \tag{1}$$

$$F_2 = R_1 q(\tau_2) + f_2, \qquad R_2 = R_1 \rho(\tau_2) + f_2, \tag{2}$$

which imply

$$F_1 = f_1 \left[1 + a(\tau_1, \tau_2) q(\tau_1) \rho(\tau_2)\right] + f_2 a(\tau_1, \tau_2) q(\tau_1), \tag{3}$$

$$F_2 = f_1 a(\tau_1, \tau_2) q(\tau_2) + f_2 \left[1 + a(\tau_1, \tau_2) q(\tau_2) \rho(\tau_1)\right] \tag{4}$$

where $a(\tau_1, \tau_2) = [1 - \rho(\tau_1) \rho(\tau_2)]^{-1}$.

Eqs. (3), (4) constitute a wealth of information on the radiation field in a composite atmosphere and enables one to infer specific features of this model problem and its possibility to apply in more complicated multilayer problems. The complex nature of these equations consists in their non-linear form with respect to reflection and transmission coefficients. This fact makes in practice impossible their multiple application in attempting to derive an analytical closed-form solution for the multi-layer problem.

4. The statistical characteristics of the emerging radiation

Let us consider two special situations which are, however, of great interest in astrophysics.

4.1. The LTE-atmosphere ($\varepsilon = 1$).

In this case we have $\rho(\tau_i) = 0$, $a(\tau_1, \tau_2) = 1$, $q(\tau_i) = \exp(-\tau_i)$, and $f_i = B_i \left[1 - \exp(-\tau_i)\right]$ so that Eqs. (3) and (4) take the following form

$$F_1 = B_1 \left[1 - \exp(-\tau_1)\right] + B_2 \left[1 - \exp(-\tau_2)\right] \exp(-\tau_1), \tag{5}$$

$$F_2 = B_1 \left[1 - \exp(-\tau_1)\right] \exp(-\tau_2) + B_2 \left[1 - \exp(-\tau_2)\right]. \tag{6}$$

It is noteworthy that the observed value of intensity depends, as it follows from relations (5) and (6), not only on the number of realised layers having various physical properties, but also on their order within the medium. Now assuming that the pair of quantities (B, τ) is random and takes values (B_i, τ_i) with probability p_i, we are led to the results obtained in JL88.

The mean value of the emerging intensity in this simplest case may be written immediately by examining all of the four possible events. Two of them occur with probability $p_1 p_2$ and correspond to differing slabs; in this case the required intensities are given by Eqs. (5) and (6). Two other possibilities (the case of similar slabs) are realised with probabilities p_i^2 and yield for the emerging intensity $F_i = B_i \left[1 - \exp(-2\tau_i)\right]$. Consequently we have

$$\langle I_{(2)} \rangle = (1 + \alpha) \langle I_{(1)} \rangle \tag{7}$$

where $\alpha = \langle \exp(-\tau) \rangle$, $\langle I_{(1)} \rangle = \left\langle B \left[1 - \exp(-\tau)\right] \right\rangle$; the index specify the number of layers.

The generalisation of this result over the multilayer problem is straightforward. Because of the absence of the reflected fluxes, the expected value of intensity of radiation emerging from any part of the medium does not depend on whether the following layers are concerned or not. This fact implies that the process of averaging can be performed by parts. In particular, we may regard the front slab, shown in Fig.1, as an ensemble of $(N - 1)$ layers to obtain

$$\begin{aligned} \langle I_{(N)} \rangle &= p_1 \left\{ \langle I_{(N-1)} \rangle \exp(-\tau_1) + B_1 \left[1 - \exp(-\tau_1)\right] \right\} \\ &\quad + p_2 \left\{ \langle I_{(N-1)} \rangle \exp(-\tau_2) + B_2 \left[1 - \exp(-\tau_2)\right] \right\} \\ &= \alpha \langle I_{(N-1)} \rangle + \langle I_{(1)} \rangle . \end{aligned} \tag{8}$$

This is the recurrence formula derived in JL88 on the base of other arguments. Our processing allows to write more general formulae for N slabs

$$\begin{aligned} \langle I_{(N)} \rangle &= \alpha^k \langle I_{(N-k)} \rangle + \langle I_{(k)} \rangle \\ (k &= 0, 1, 2..N; I_{(0)} = 0). \end{aligned} \tag{9}$$

Eq. (8) admits a closed-form solution

$$\langle I_{(N)} \rangle = L \langle I_{(1)} \rangle \tag{10}$$

where $L = \left(1 - \alpha^N\right) / (1 - \alpha)$.

The next important statistical parameter is the relative mean square deviation defined as

$$\delta_{(N)} = \left(\left\langle I_{(N)}^2 \right\rangle / \langle I_{(N)} \rangle^2 \right) - 1. \tag{11}$$

The expression for $\left\langle I_{(N)}^2 \right\rangle$ can be readily derived (see Appendix) as

$$\begin{aligned} \left\langle I_{(N)}^2 \right\rangle &= M \left\langle I_{(1)}^2 \right\rangle + K \langle I_{(1)} \rangle \\ &\quad \times \langle B \exp(-\tau) (1 - \exp(-\tau)) \rangle \end{aligned} \tag{12}$$

where $M = \left(1 - \beta^N\right) / (1 - \beta)$, $\beta = \langle \exp(-2\tau) \rangle$ and $K = 2 (M - L) / (\beta - \alpha)$. We finally obtain for the RMSD value

$$\begin{aligned} \delta_{(N)} &= M \left\langle I_{(1)}^2 \right\rangle / L^2 \langle I_{(1)} \rangle^2 \\ &\quad + K \langle B \exp(-\tau) (1 - \exp(-\tau)) \rangle / L^2 \langle I_{(1)} \rangle - 1. \end{aligned} \tag{13}$$

Bearing in mind the optically thin lines, consider separately the case when the total optical thickness of an atmosphere is small. Now equations (3) and (4) take the form

$$F_1 = F_2 = B_1 \tau_1 + B_2 \tau_2. \tag{14}$$

It is obvious from (14) that the arrangement of the layers (structural elements) with distinct physical properties is of no importance in observing one or another value of the intensity. This fact makes easier the problem at hand, enabling one to write down immediately the statistical characteristics of the emerging radiation.

Consider a random event consisting in that a medium is composed of k layers characterised by the pair of values (B_1, τ_1), and $(N - k)$ layers having the property (B_2, τ_2). It is obvious that the probability of such an event is $p_k = C_N^k p_1^k p_2^{N-k}$ (where $C_N^k = N! / k! (N - k)!$ are the binomial coefficients), while the associated value of intensity is $[k B_1 \tau_1 + (N - k) B_2 \tau_2]$. Hence we obtain the value of the observed intensity for such event

$$I_{(N)}^{(k)} = C_N^k p_1^k p_2^{N-k} \left[k B_1 \tau_1 + (N - k) B_2 \tau_2\right]. \tag{15}$$

Summing-up over all values of k we arrive - in accordance with the formula (10) - at the following expression for the mean value of intensity

$$\langle I_{(N)} \rangle = N \langle I_{(1)} \rangle . \tag{16}$$

The corresponding value for the mean square deviation is then

$$\delta_{(N)} = \frac{r}{N} \left(\frac{1 - \gamma}{1 + r\gamma} \right)^2 \tag{17}$$

where $\gamma = B_2 \tau_2 / B_1 \tau_1$ and $r = p_2 / p_1$.

4.2. The NLTE atmosphere ($\varepsilon = 0$).

This case differs significantly from that of LTE because the layers are mutually coupled throughout the medium due to the scattering process. This fact is displayed by the specific nonlinear form of Eqs. (3) and (4). Hence we conclude that the mean value of the intensity of emerging radiation cannot be found successively by such a simple way as in LTE.

To visualize the basic features of the problem, we shall discuss largely the special case of pure scattering ($\varepsilon = 0$) which is of importance when interpreting a resonance line as L_α, for instance. The essential simplification comes from the fact that the overall amount of radiative energy released in the atmosphere escapes it. This means that $\rho(\tau) + q(\tau) = 1$, which leads the Eqs. (3) and (4) to

$$\begin{aligned} F_1 &= f_1 + f_2 \\ &\quad + a(\tau_1, \tau_2) \left[f_1 \rho(\tau_2) q(\tau_1) - f_2 \rho(\tau_1) q(\tau_2) \right], \end{aligned} \tag{18}$$

$$\begin{aligned} F_2 &= f_1 + f_2 \\ &\quad - a(\tau_1, \tau_2) \left[f_1 \rho(\tau_2) q(\tau_1) - f_2 \rho(\tau_1) q(\tau_2) \right]. \end{aligned} \tag{19}$$

As it is known, in the conservative case (see, e.g. Sobolev, 1963, Chapt. 6)

$$\rho(\tau_i) = \tau_i/(\tau_i+2);\; q(\tau_i) = 2/(\tau_i+2);\; f_i = B_i\tau_i. \tag{20}$$

The emerging flux f_i is equal the total amount of radiative energy released by the internal sources in each direction so that Eqs. (18) and (19) can be rewritten as

$$F_1 = f_1 + f_2 + \Delta_{(2)}, \qquad F_2 = f_1 + f_2 - \Delta_{(2)} \tag{21}$$

where

$$\Delta_{(2)} = (B_1 - B_2)\left[\tau_1\tau_2/(\tau_1+\tau_2+2)\right]. \tag{22}$$

The fact that $F_1 \neq F_2$ stated by Eqs. (21) (or Eqs. (18) and (19)), implies the non-commutativity of layers with respect to the observed value of intensity of emerging radiation. Moreover as follows from Eq. (22), this effect is due to the difference in power of the energy sources. When this difference is negligible, the observed value of the intensity is additive with respect to the contribution of each layer, so that these are interchangeable. We shall refer to this point just below.

Returning to the stochastic problem under discussion, we note that values of F_1 and F_2 given by Eqs. (18) and (19), are observed with a probability (p_1p_2)whereas two other possible events, yielding to fluxes $2f_i$, occur with probabilities p_i^2. Hence, as expected on physical grounds, we find

$$\langle I_{(2)}\rangle = 2\langle f\rangle = 2\langle I_{(1)}\rangle. \tag{23}$$

This result may be generalised inductively over the multilayer problem to yield

$$\langle I_{(N)}\rangle = N\langle I_{(1)}\rangle. \tag{24}$$

It is obvious that this result remains valid for more general probability distributions of inhomogeneities. We see that the expression of the mean intensity in the case of conservative scattering (Eq. 24) is equivalent with that obtained in the case LTE, when the total optical thickness of an atmosphere is small (Eq. 16).

The derivation of the closed-form solution for the RMSD is a significantly more difficult problem to address here. A direct derivation of δ for some low values of N gives

$$\delta_{(2)} = \frac{p_1p_2}{2\langle f\rangle^2}\left[(f_1-f_2)^2 + \Delta_{(2)}^2\right], \tag{25}$$

$$\delta_{(3)} = \frac{p_1p_2}{3\langle f\rangle^2}\left[(f_1-f_2)^2 + \frac{2}{3}\Delta_{(3)}^2\right] \tag{26}$$

where $\Delta_{(2)}$ is given by the Eq. (22), and

$$\Delta_{(3)}^2 = p_1\Delta'^2 + p_2\Delta''^2; \tag{27}$$

where

$$\Delta' = (B_1-B_2)\left[2\tau_1\tau_2/(2\tau_1+\tau_2+2)\right]$$

$$\Delta'' = (B_1-B_2)\left[2\tau_1\tau_2/(\tau_1+2\tau_2+2)\right].$$

In general case, $\delta_{(N)}$ may be represented in the form

$$\delta_{(N)} = \frac{p_1p_2}{N\langle f\rangle^2}\left[(f_1-f_2)^2 + \omega(\tau_1,\tau_2)(B_1-B_2)^2\right], \tag{28}$$

where $\omega(\tau_1,\tau_2)$ is a certain weighting factor depending on τ_i and the realized number N of layers.

The inspection of the right-hand side of Eq. (28) reveals the difference in the physical nature of the two terms. While the first term depends on the difference (f_1-f_2) $(= B_1\tau_1 - B_2\tau_2)$, and is stipulated by fluctuations in the realised numbers of layers with different physical properties, the second term depends on (B_1-B_2) and stems from the variations in the arrangement of layers for a fixed number of various structures. The mathematical difficulty in deriving $\delta_{(N)}$ arise when searching a closed-form expression for the function ω which determines the relative role of each of two mentioned effects. However, some conclusions on its behaviour can be drawn immediately by writing the detailed expression for this function. For small values of $\tau = \tau_1 \approx \tau_2$, then $\omega \sim \tau^4$, while $(f_1-f_2)^2 \sim \tau^2$, and hence the contribution in $\delta_{(N)}$ of variations due to the arrangement of layers is negligible and may be ignored. In this case Eq. (28) converts into Eq. (17). In the opposite situation when τ is large, $\omega \sim \tau^2$, so that both the two mentioned effects are equally important unless (B_1-B_2) is too small. It is reasonable to expect that the product $B\tau$ undergoes greater random variations than the power of energy sources B alone, so that the first term in the brackets of Eq. (28) may be assumed not to be smaller than the second one. In the limiting case when $B_1 \approx B_2$, the dispersion of the values of intensity is due to variations in the optical thickness of the atmosphere and we are led again, as in LTE thin case, to Eq. (17).

As pointed out at the beginning of the article, the results obtained are widen out to describe more general and realistic model patterns for interpreting the radiation emerging from media with randomly distributed inhomogeneities. Thus the total number of layers N may be also seen as a random variable being distributed according to a certain law (e.g. a Poisson law). The Poisson distribution seems to be suitable in applying it to structures with a relatively small number of elements and large fluctuations, as it is the case especially at the top regions of prominences. In this case, Eq. (17) must be replaced by

$$\delta_{(\bar{N})} = \frac{1}{\bar{N}}\left[1 + r\left(\frac{1-\gamma}{1+r\gamma}\right)^2\right] \tag{29}$$

where $\bar{N}$ is the Poisson mean of N. Other expressions for δ would be modified accordingly. It is readily seen that the quantity N entering Eq. (24) must be replaced now by $\bar{N}$.

An important extension of our results is connected with an increase in the number of possible values taken by the quantities B and τ. The resulting polynomial distribution for the intensity is remembering the observational intensity distributions and reproduces a variety of their specific features.

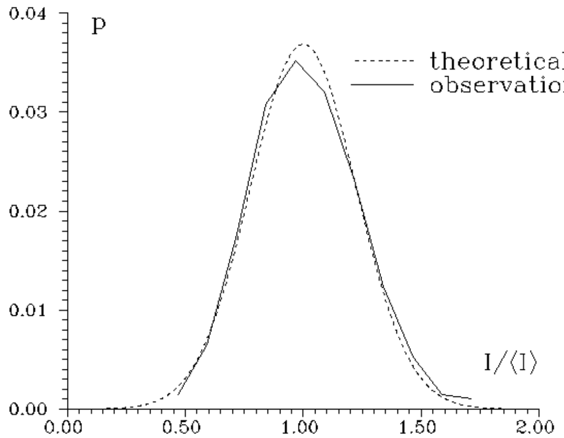

Fig. 2. The typical intensity distribution of the L_α-line. The solid curve represents the observed probability p; the dotted one is the Gaussian law for the same value of δ (=0.05). According to Kolmogorov's λ-criterion, the degree of confidence is 99%.

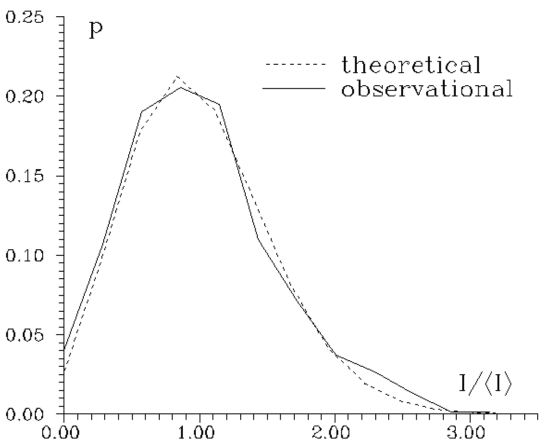

Fig. 3. The typical intensity distribution of the CII 1336Å-line. The solid curve is the observed probability p; the dotted one is the Poisson law for the same value of δ (=0.32).

5. The results of numerical calculations

Models discussed above are certainly too idealised to apply for a quantitative interpretation of the observed images of solar inhomogeneous structures. Even though, the theoretical distributions derived for the emergent intensity allow to account, at least quantitatively, for some specific features of observational data.

In this section, we shall limit ourselves to considering several points connected with the interpretation of EUV-images of quiescent prominences. As is known, along with the opaque lines such as L_α, the EUV-spectrum contains a number of lines ($\lambda\lambda$ CII 1336Å, CIII 977Å, OVI 1032Å), for which prominences seem to be approximately transparent. Depending on the optical thickness of the lines, the spatial distributions of the observed values of the intensity possess distinct features. While the L_α-line obeys most commonly a Gaussian-like distribution with a relatively small dispersion ($\delta \sim 0.04 \div 0.07$) (Fig.2), optically

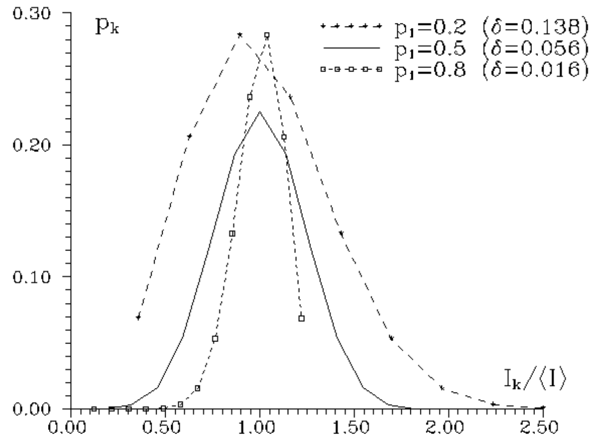

Fig. 4. Theoretical distributions of the line-radiation for $N = 10$, $\gamma = B_2\tau_2/B_1\tau_1 = 0.1$ for various values of p_1, with the corresponding values of δ . The value p_k represents the probability to have k layers characterised by the pair of values (B_1, τ_1), and $(N - k)$ layers having the property (B_2, τ_2).

thin lines exhibit an asymmetrical distribution profile with a larger dispersion ($\delta \sim 0.07 \div 0.3$), tailed to the range of higher intensities, and are strongly likeness to the Poisson distribution (see Fig.3). It is easily seen, however, that this kind of distributions are derivable from the binomial law we already discussed in section 4 (see Fig.4). When $p_1 \approx p_2$, the binomial distribution differs only slightly from a Gaussian one even for relatively small values of N. In the case that the atmosphere contains a small portion of strongly radiating elements (i.e. p_1 is small), we are led to a Poisson-like asymmetrical distribution which fits the observational profile with a high degree of confidence. It should be emphasised that this distribution may be obtained by assuming that the total number of structural elements varies according to a Poisson law. The latter possibility leads, however, to somewhat low values of N as compared to the other estimates (see e. g. Fontenla et al., 1996) and, on the contrary, yields greater values of N, while being applied to the optically thick lines. Actually both of the above-mentioned reasons act together, and each of them may become essential in forming one or another intensity distribution for the observed line-radiation. In any case, the asymmetry of the observed intensity distribution for optically thin lines is an evidence of the smallness of the line-of-sight number of structural elements ($N < 10$). These numbers were found by fitting together theoretical and observational intensity distributions and they typically range between 3 and 8 structural elements. The degree of asymmetry makes possible to infer, in principle, the scale and the probability distribution of inhomogeneities.

Important information is contained in the observational values of the RMSD. Having in mind the high opacity of prominences in the L_α line, one may expect even lower values of δ than usually observed. Now we see from Eq. (17) that depending on γ (i.e. on the scale of the inhomogeneities), values of δ are widely ranged. Knowing δ allows one to estimate the dispersion in the amount of energy emitted by a gas, or alternatively in the emission measure. The symmetrical distribution profile of the L_α line implies a uniform distribution of the structural elements with various physical properties.

6. Summary and conclusions

In this article we treated the problem of the statistical characteristics of radiation which emerges from a gas which contains randomly distributed inhomogeneities, considering an approach based on Ambartsumian's method of addition of layers. In a plane-parallel medium composed by N layers (slabs), we considered that the optical thickness τ of a layer and the power of energy sources B are random variables that take only one of two possible values (τ_1 and τ_2 and respectively, B_1 and B_2) with the associated probabilities p_1 and p_2.

Equations for the mean value and relative mean square deviation of the intensity were derived in both LTE (Eqs. 10, 13) and NLTE-pure scattering (Eqs. 24, 28) cases.

These results were applied to the interpretation of the EUV-spectra of quiescent prominences. The observed Gaussian-like distribution of the L_α-line (Fig.2) can be derived from the binomial law when our two possible events occur with equal probabilities. On the other hand, the Poisson-like asymmetrical distribution showed by optically thin lines (Fig.3) can be obtained when the number of strongly radiating elements is small (i.e. p_1 is small). To give further insight to this point, note that the treated lines carry information on prominence regions with different physical properties (temperature, density). Now the symmetrical intensity distribution and small values of δ for the L_α-line, indicate the homogeneity or slab-like structure of relatively cool regions ($T \sim 10^4$K), whereas, for instance, the asymmetry of distribution and large dispersions in the intensity of the line CII λ 1336Å, may be regarded as an evidence of the small line-of-sight number of structural elements and of a patchy structure of relatively hot regions ($T \sim 2 \cdot 10^4$K). One may expect such structure in the case of a steep temperature gradient in the prominence-corona transition zone, for which the regions of the line formation may become too thin to be observable.

The main conclusions of this article remain to be valid for more complex situations concerning the probability distribution of structural elements in the atmosphere. A more detailed comparison of the present theoretical results with observational data will be addressed in a future article.

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Appendix A

Let us consider a set of N layers and assume for the moment that the Nth layer is characterised by the pair of fixed values $\left(\tilde{B}, \tilde{\tau}\right)$ so that

$$I_{(N)} = I_{(N-1)} \exp\left(-\tilde{\tau}\right) + \tilde{B}\left[1 - \exp\left(-\tilde{\tau}\right)\right]. \qquad \text{(A1)}$$

Then it is obvious that

$$\left\langle I^2_{(N)} \right\rangle = \left\langle I^2_{(N-1)} \right\rangle \exp\left(-2\tilde{\tau}\right) + 2\left\langle I_{(N-1)} \right\rangle \tilde{B} \exp\left(-\tilde{\tau}\right) \times \left[1 - \exp\left(-\tilde{\tau}\right)\right] + \tilde{B}^2 \left[1 - \exp\left(-\tilde{\tau}\right)\right]^2. \qquad \text{(A2)}$$

Now let us assume that the pair $\left(\tilde{B}, \tilde{\tau}\right)$ takes random values (B_i, τ_i) with probabilities p_i ($i = 1, 2$). According to our assertion in Sect.4, the averaging process may be carried-out by parts so that we are led to the recurrence relation

$$\left\langle I^2_{(N)} \right\rangle = \beta \left\langle I^2_{(N-1)} \right\rangle + 2\left\langle I_{(N-1)} \right\rangle \times \left\langle B\left[1 - \exp\left(-\tau\right)\right] \exp\left(-\tau\right) \right\rangle + \left\langle I^2_{(1)} \right\rangle \qquad \text{(A3)}$$

where we used the fact that $\left\langle I_{(1)} \right\rangle = \left\langle B\left[1 - \exp\left(-\tau\right)\right] \right\rangle$. A successive application of Eqs. (A3) and (10) yields

$$\left\langle I^2_{(N)} \right\rangle = M \left\langle I^2_{(1)} \right\rangle + 2\left\langle B\left[1 - \exp\left(-\tau\right)\right] \exp\left(-\tau\right) \right\rangle \times \left\langle I_{(1)} \right\rangle \sum_{k=0}^{N-2} \beta^k L_{N-k-1} \qquad \text{(A4)}$$

where $L_k = \left(1 - \alpha^k\right) / \left(1 - \alpha\right)$. After some simple algebra we finally arrive at Eq. (12).

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