

Modified artificial viscosity in Smooth Particle Hydrodynamics

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Abstract. Artificial viscosity is needed in Smooth Particle Hydrodynamics to prevent interparticle penetration, to allow shocks to form and to damp post shock oscillations. Artificial viscosity may, however, lead to problems such as unwanted heating and unphysical solutions. A modification of the standard artificial viscosity recipe is proposed which reduces these problems. Some test cases discussed.

Key words: accretion, accretion disks – hydrodynamics – shock waves – stars: formation – galaxies: formation

1. Introduction

Smooth particle hydrodynamics (SPH) has become an increasingly popular method in simulations of different astrophysical phenomena. Its Lagrangian formulation allows the study of large density differences. The particle formulation makes grids unnecessary and allows for easy implementation in three dimensions. A recent review of the method can be found in Monaghan (1992), and a technical description in for example Hernquist & Katz (1989).

Two forms of artificial viscosity are needed in SPH, the bulk and the von Neumann-Richtmeyer artificial viscosity respectively. They prevent interparticle penetration, allow shocks to form and damp the post shock oscillations. They do, however, induce transfer of kinetic energy in fluid motions to thermal energy. Many simulations using SPH involve the compression of a gas, often in a gravitational collapse of an initially cool gas cloud. The velocities in these simulations can be highly supersonic, which implies that the unphysically large artificial viscosity may dominate the heating. This heating and deceleration of the gas prevent further collapse.

Martel et. al. (1995) have introduced a new formalism which they call Adaptive SPH. They use an ellipsoid kernel that adapts itself to the surrounding medium, and therefore avoid unnecessary use of artificial viscosity. The authors claim good results in cosmological collapse simulations.

In the present study a different approach is suggested. The von Neumann-Richtmeyer artificial viscosity is restricted to supersonic velocities. It is therefore used only to form shocks and

to prevent interparticle penetration for supersonic particles. Its adverse effect at subsonic velocities is avoided. A region under compression is described by a limited number of particles. The relative velocities among neighbouring particles in a region under compression can be supersonic due to the limited resolution, but despite that the gas is not expected to form shocks. Therefore the von Neumann-Richtmeyer viscosity is restricted to regions that are not under compression. To avoid spurious heating in the subsonic regions, but prevent interparticle penetration and maintain the damping of post shock oscillations, the bulk artificial viscosity is modified. Instead of integrating the effect from the neighbouring particles individually, their collective effect at one point is considered.

The SPH method is briefly described, and the suggested changes in artificial viscosity are presented. These modifications of artificial viscosity have been tested in the simulation of a shock tube, the homologous compression of a gas sphere and the gravitational collapse of an initially cool gas sphere at rest. The results are compared with results from a standard formulation of artificial viscosity.

2. The SPH methodology

In SPH one models a number of particles that carry the physical quantities, where the particles' distribution in space describes the density distribution. To simulate a fluid each particle's mass is smoothed over a radius r . Following for example Hernquist & Katz (1989), such a smoothed quantity at \mathbf{r} be written for N particles as

$$f(\mathbf{r}) = \oint d\mathbf{r}' f(\mathbf{r}') w(\mathbf{r}' - \mathbf{r}, h) = \sum_j^N \frac{\bar{m}_{ij}}{\bar{\rho}_{ij}} f(\mathbf{r}_j) w(\mathbf{r}_{ij}, h_{ij}), \quad (1)$$

where $\bar{m}_{ij} = (m_i + m_j)/2$, $\bar{\rho}_{ij} = (\rho_i + \rho_j)/2$, $\bar{h}_{ij} = (h_i + h_j)/2$ and $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. The smoothing kernel, w , has the property

$$\oint d\mathbf{r} w(\mathbf{r}, h) = 1. \quad (2)$$

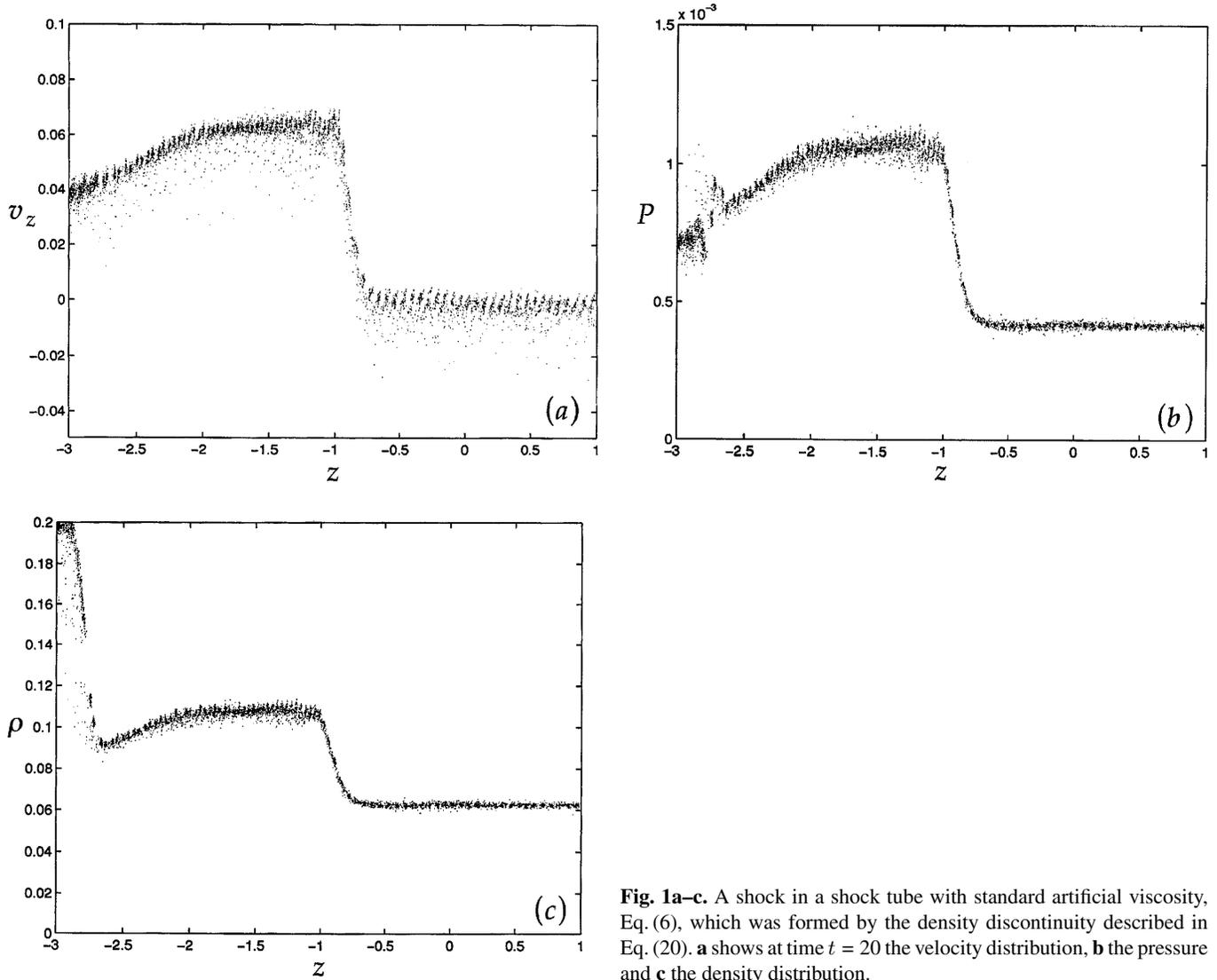


Fig. 1a–c. A shock in a shock tube with standard artificial viscosity, Eq. (6), which was formed by the density discontinuity described in Eq. (20). **a** shows at time $t = 20$ the velocity distribution, **b** the pressure and **c** the density distribution.

The kernel used here is the spline kernel from Monaghan and Lattanzio (1985):

$$w(\mathbf{r}, h) = \frac{1}{\pi h^3} \begin{cases} 1 - \frac{3}{2} \left(\frac{r}{h}\right)^2 + \frac{3}{4} \left(\frac{r}{h}\right)^3 & 0 \leq \frac{r}{h} \leq 1 \\ \frac{1}{4} \left(2 - \frac{r}{h}\right)^3 & 1 \leq \frac{r}{h} \leq 2 \\ 0 & \frac{r}{h} \geq 2 \end{cases} \quad (3)$$

This is a smooth kernel with compact support over the radius $2h$ around the particle. The smoothing length h is varying in time and updated every iteration to keep the interactions with other particles to a specific number. They are called the particle's neighbours, and the number of neighbours for each particle in the tests in this paper is 64.

In the SPH formulation there are different forms of the discretization of the Navier Stokes equations. Here the expressions of Hernquist & Katz (1989) are used, i.e.

$$\begin{cases} \frac{dv_i}{dt} = - \sum_j^N \tilde{m}_{ij} \left(\frac{\bar{P}_{ij}}{\bar{\rho}_{ij}^2} + \Pi_{ij} \right) \nabla_i w_{ij} \\ \frac{du_i}{dt} = - \frac{1}{2} \sum_j^N \tilde{m}_{ij} \left(\frac{\bar{P}_{ij}}{\bar{\rho}_{ij}^2} + \Pi_{ij} \right) \mathbf{v}_{ij} \cdot \nabla_i w_{ij} \end{cases}, \quad (4)$$

where $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$ and $\bar{P}_{ij} = (P_i + P_j)/2$. The continuity equation is automatically satisfied due to the Lagrangian formulation, and the density calculated from Eq. (1) becomes

$$\rho_i = \sum_j^N \tilde{m}_{ij} w_{ij}. \quad (5)$$

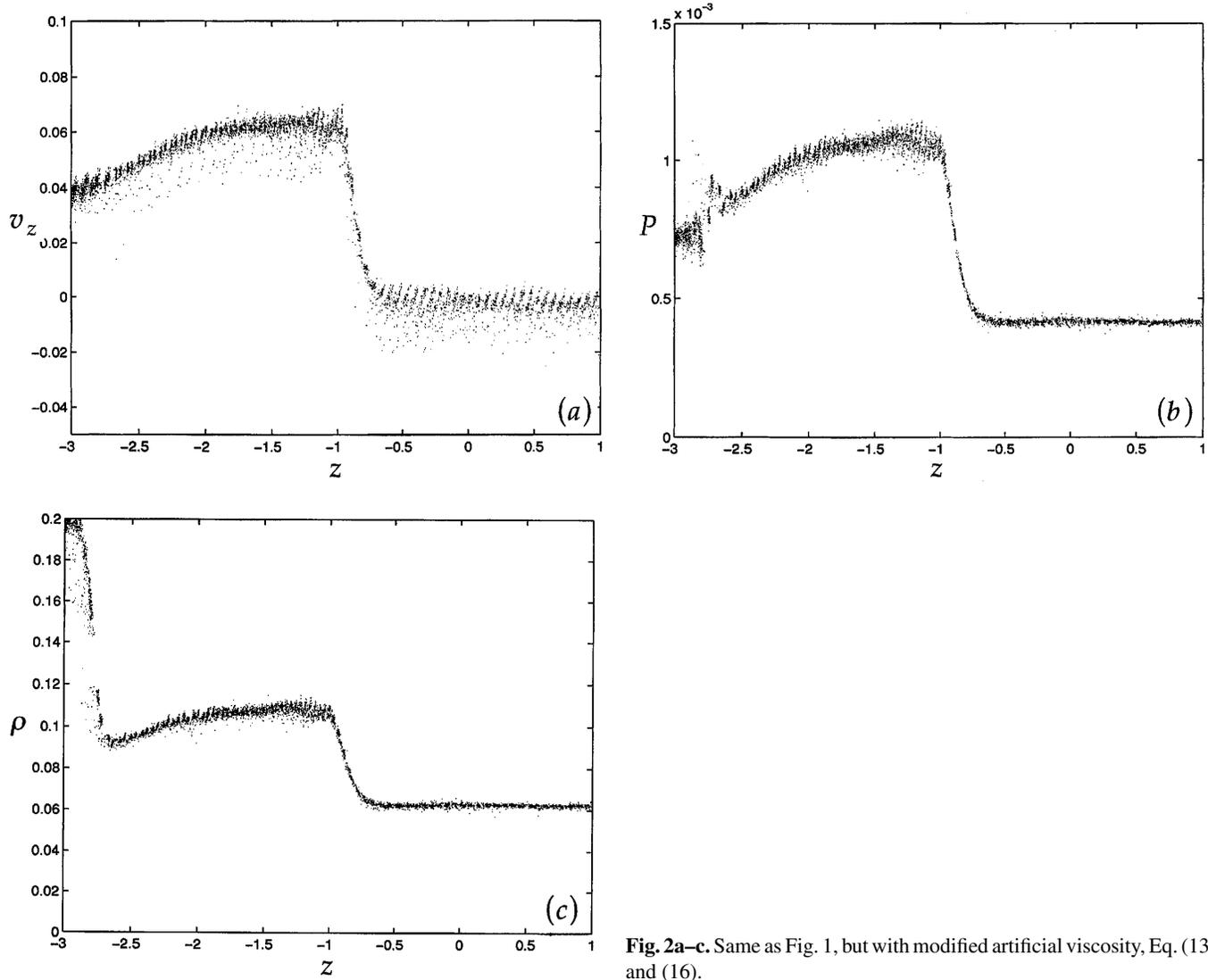


Fig. 2a–c. Same as Fig. 1, but with modified artificial viscosity, Eq. (13) and (16).

The standard artificial viscosity term, Π_{ij} , is defined

$$\begin{cases} \Pi_{ij} = \frac{1}{\bar{\rho}_{ij}}(-\alpha\mu_{ij}\bar{c}_{ij} + \beta\mu_{ij}^2) \\ \mu_{ij} = h_{ij} \frac{\mathbf{r}_{ij} \cdot \mathbf{v}_{ij}}{r_{ij}^2 + \nu^2} \delta_{ij} \\ \delta_{ij} = \begin{cases} 1 & \text{if } \mathbf{r}_{ij} \cdot \mathbf{v}_{ij} < 0 \\ 0 & \text{if } \mathbf{r}_{ij} \cdot \mathbf{v}_{ij} > 0 \end{cases} \end{cases}, \quad (6)$$

where $\bar{c}_{ij} = (c_i + c_j)/2$. The first and second term in the expression of Π_{ij} in Eq. (6) represents the bulk and the von Neumann Richtmeyer artificial viscosity respectively. The constant $\nu = 0.01h$ is a fudge parameter to prevent the artificial viscosity to become too large. The artificial viscosity is used only when $\mathbf{r}_{ij} \cdot \mathbf{v}_{ij} < 0$, that is when two particles are approaching each other. To close the system, the pressure is defined as $P + (\gamma - 1)\rho u$, where u is the thermal energy density and γ the adiabatic index. The particles' quantities are updated using a standard leapfrog integrator with the time step

$$\delta t = \text{Min}[\delta t_i] = \frac{\varepsilon h_i}{v_i + c_i + 1.2(\alpha c_i + \beta \mu_{i,max})}, \quad (7)$$

where $\varepsilon = 0.3$ is the Courant factor to stabilize the integration, and μ_{max} the maximum μ from the interactions with the other particles. Since all particles are integrated with the same time step, the smallest time step from all particles is used in the integration. In the leapfrog integrator the velocity and internal energy density are integrated at half time steps, $t_{n-1/2}, t_{n+1/2}, \dots$, while the position is integrated at whole time steps, t_n, t_{n+1}, \dots , as

$$\begin{cases} \mathbf{v}_{i,n+1/2} = \mathbf{v}_{i,n-1/2} + \frac{d\mathbf{v}_i}{dt} \delta t \\ u_{i,n+1/2} = u_{i,n-1/2} + \frac{du_i}{dt} \delta t \\ \mathbf{r}_{i,n+1} = \mathbf{r}_{i,n} + \mathbf{v}_{i,n+1/2} \delta t \end{cases}. \quad (8)$$

The viscous acceleration terms in Eq. (4) scales with $\alpha = \beta = 1$ as

$$\begin{cases} \frac{dv}{dt} \alpha \propto v c \\ \frac{dv}{dt} \beta \propto v^2 \end{cases}. \quad (9)$$

The artificial viscosity can therefore lead to undesirable effects, because the velocity differences are smoothed on a time scale of

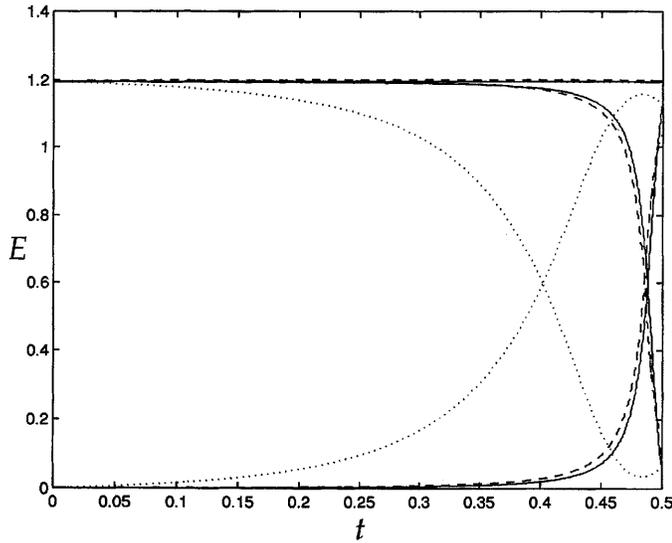


Fig. 3. The energy curves for the homologous compression of a gas with different forms of artificial viscosity. The solid line represents the compression without any artificial viscosity at all, the dashed the modified artificial viscosity, Eq. (13) and (16), and the dotted the standard artificial viscosity, Eq. (6). The curves that decline in the beginning represent the kinetic energies for the three models, those that rise represent the internal and the uppermost straight curves represent the total energy.

roughly $h/(c+v)$. The velocities in the model will therefore be smoothed out unless it expands or if there is some driving mechanism such as gravitation. To conserve the energy the particles are heated, which may be unphysical. The heated gas may reach an equilibrium state earlier than expected. A way of preventing interparticle penetration without unnecessary heating of the gas could therefore be useful, and this implies that there is a need to restrict the artificial viscosity to the shocks as much as possible.

3. Restrictive use of von Neumann-Richtmeyer artificial viscosity

In SPH the formation of shocks is mostly an effect of the von Neumann-Richtmeyer viscosity. To avoid the undesirable effects in the subsonic region, I propose to restrict the use of it to those regions. This modification will allow shocks to form, and prevent interparticle penetration at supersonic velocities.

Consider a gas cloud in a spherical symmetric gravitational collapse. Suppose the physical viscosity is small, so that the compression can be assumed to be adiabatic. When the model has reached an equilibrium, the pressure force that prevents further gravitational compression balance the gravitational force. If the cloud is warm, the forces may balance each other even in its initial state. But if on the other hand the cloud is initially cool, it must be compressed to gain the required thermal energy density, perhaps by orders of magnitude. In many cases this is not possible with standard artificial viscosity, Eq. (6), due to poor resolution. If the cloud in the example above has a radius of unit length and is modelled with 10^3 particles, the mean in-

terparticle distances are about 0.1. In the spherical compression the particles at different radii therefore have supersonic relative velocities if the gas is cool enough. Standard artificial viscosity, Eq. (6), will decelerate and heat the gas, and prevent further compression.

Since $\rho \propto h^{-3}$ the time derivative for any particle can be written

$$\dot{\rho} = -\frac{3\rho\dot{h}}{h}. \quad (10)$$

This relation can be used to decide whether a particle follows the fluid or if artificial viscosity is necessary. Consider two particles with a separation of r moving towards each other with a speed of \dot{r} they follow the fluid in the neighbourhood, the neighbourhood is under compression and the particles' relative velocity will approximately satisfy

$$\frac{\dot{r}}{r} \approx \frac{\dot{h}}{h}. \quad (11)$$

If two particles are approaching each other at a velocity exceeding the sound speed, that is if

$$\mathbf{r}_{ij} \cdot \mathbf{v}_{ij} < 0 \quad \text{and} \quad v_{ij} > \bar{c}_{ij}, \quad (12)$$

artificial viscosity is necessary to prevent interparticle penetration. I therefore propose to restrict the von Neumann-Richtmeyer artificial viscosity as

$$\begin{cases} \Pi_{vNR,ij} = \frac{\beta\mu_{ij}^2}{\bar{\rho}_{ij}} \\ \mu_{ij} = \bar{h}_{ij} \frac{\mathbf{r}_{ij} \cdot \mathbf{v}_{ij}}{r_{ij}^2 + \mu_{ij}^2} \delta_{ij} \\ \delta_{ij} = \begin{cases} 1 & \text{if } \begin{cases} \mathbf{r}_{ij} \cdot \mathbf{v}_{ij} < 0 \\ v_{ij} > c_{ij} \\ \frac{\bar{p}_{ij}}{3\bar{\rho}_{ij}} r_{ij} > v_{ij} - \bar{c}_{ij} \end{cases} \\ 0 & \text{otherwise} \end{cases} \end{cases} \quad (13)$$

4. Modified bulk viscosity

A smoothed quantity can be calculated at any point in the fluid by Eq. (1). The smoothed velocity at \mathbf{r}_i is

$$\Delta \mathbf{v}'_i = \sum_j \Delta \mathbf{v}'_{ij} = \sum_j \mathbf{v}_j \frac{\bar{m}_{ij}}{\bar{\rho}_{ij}} w_{ij}. \quad (14)$$

If this point coincides with a position for a particle, the smoothed and the particle's individual velocity will be different in general. This difference is used to construct a modified bulk viscosity. Benz (1990) suggests that this quantity could be used when integrating the position to prevent interparticle penetration. It is true that it prevents interparticle penetration, but it unfortunately introduces conservation problems when the particles are not moved at their individual velocity. If the contribution from particle i subtracted, it does however say something about the

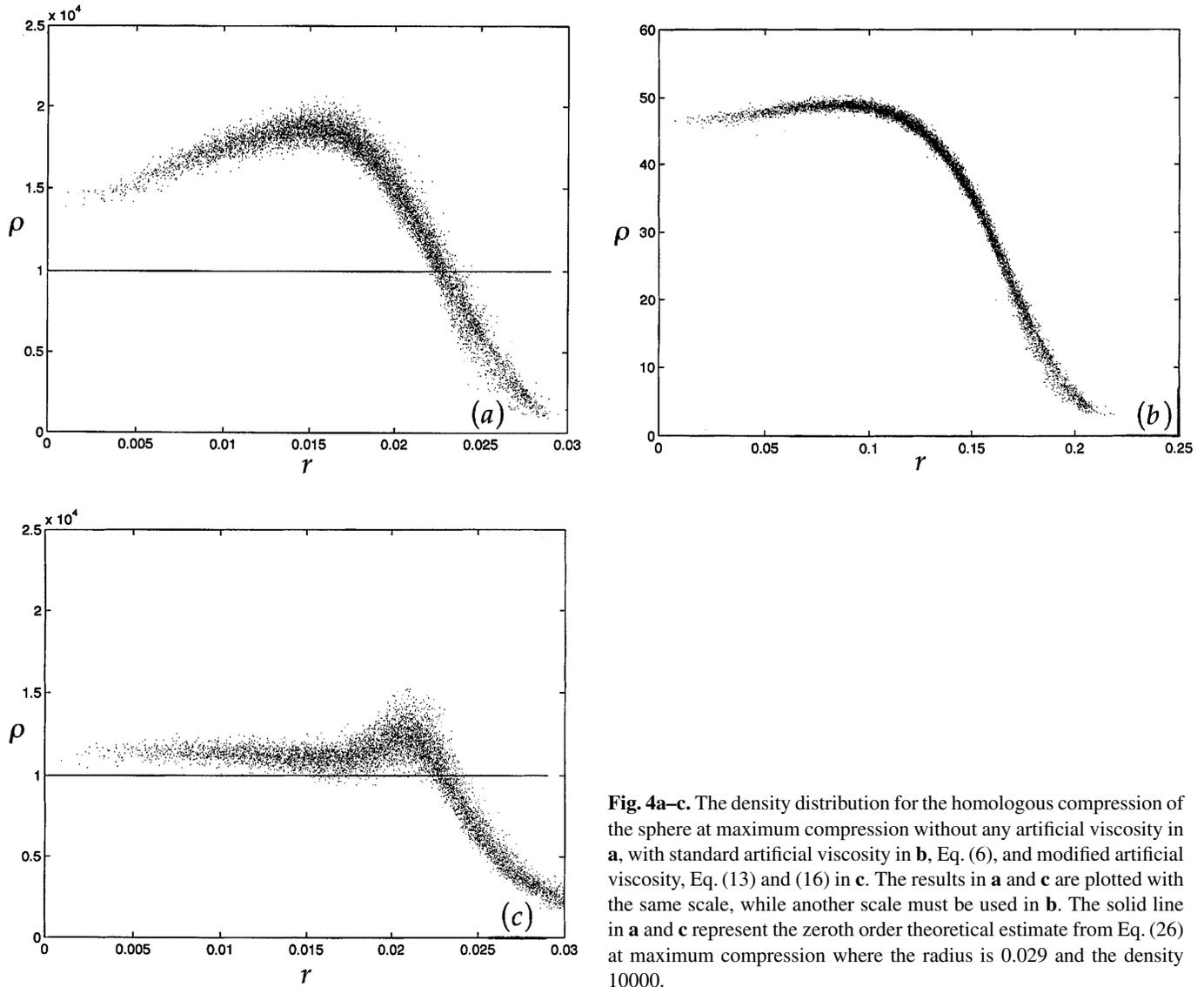


Fig. 4a–c. The density distribution for the homologous compression of the sphere at maximum compression without any artificial viscosity in **a**, with standard artificial viscosity in **b**, Eq. (6), and modified artificial viscosity, Eq. (13) and (16) in **c**. The results in **a** and **c** are plotted with the same scale, while another scale must be used in **b**. The solid line in **a** and **c** represent the zeroth order theoretical estimate from Eq. (26) at maximum compression where the radius is 0.029 and the density 10000.

fluid around the particle. The smoothed velocity at r can then be redefined as

$$\Delta \mathbf{r}'_i = \frac{1}{1 - w_i} \left(\sum_j^N \mathbf{v}_j \frac{\bar{m}_{ij}}{\bar{\rho}_{ij}} w_{ij} - \mathbf{v}_i \frac{\bar{m}_{ii}}{\bar{\rho}_{ii}} w_{ii} \right), \quad (15)$$

where $w_{ij} = w(r_{ij}/\bar{h}_{ij} = 0) = 1/\pi \bar{h}_{ij}^3$. The denominator scales the expression so that $\Delta \mathbf{v}_i = \mathbf{v}_i$, if for all j $\mathbf{v}_j = \mathbf{v}_i$.

In the standard formulation of the bulk artificial viscosity particle i interacts with each neighbour separately, where the acceleration and time derivative of the internal energy is added to the particle as described in Sect. 2. This introduces problems described in Sect. 3. The individual velocity of particle i can then be seen as a deviation from the smoothed value at the particle's position and the artificial viscosity as a correction to the individual velocity. I propose a modified bulk viscosity to replace the standard bulk viscosity, where the fluid around the particle is considered from a collective contribution from

the neighbours in one single interaction. This modified bulk artificial viscosity is defined as

$$\begin{cases} \frac{d\mathbf{v}_i}{dt} = -N_n \eta \frac{c_i \hat{\mathbf{v}}_i}{h_i} \sqrt{\mathbf{v}_i \cdot \Delta \mathbf{v}_i} \delta_i \\ \delta_i = \begin{cases} 1 & \text{if } \mathbf{v} \cdot \Delta \mathbf{v}_i < 0 \\ 0 & \text{if } \mathbf{v} \cdot \Delta \mathbf{v}_i > 0 \end{cases} \end{cases}, \quad (16)$$

where N_n the number of neighbours. The constant η around unity, and used in the same way as the constants α and β in Eq. (6). This interaction can be seen as if the particle interacts with a virtual particle with the smoothed velocity $\Delta \mathbf{v}_i$, have a mass of $N_n m_i$ and lies at a distance of h the direction of \mathbf{v}_i . This expression thus becomes similar to Eq. (6) and affects the same velocity regime. The difference is that in Eq. (16) the collective contribution from all neighbours is considered in one single interaction.

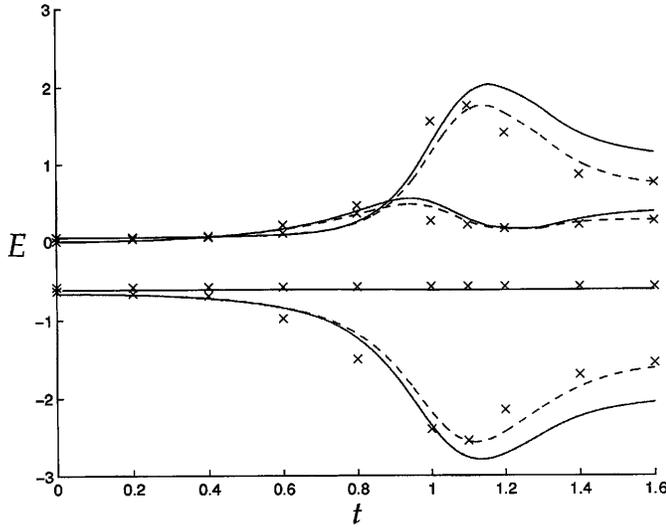


Fig. 5. The energy curves for the Evrard gravitational collapse using standard artificial viscosity, Eq. (6), represented by a dashed line, and modified artificial viscosity, Eq. (13) and (16), represented by a solid line. The uppermost curves represent the internal thermal energy, the next the kinetic, the straight line the total and the two curves below the others' the potential energy. The solid line is from an accurate one-dimensional PPM calculation at $t = 0.77$ from Steinmetz & Müller (1993).

Now consider the integration of the velocities and internal energy density from time $t_{n-1/2}$ to $t_{n+1/2}$ with the time step δt . From Eq. (8) the velocity for particle i at $t_{n+1/2}$ is

$$\mathbf{v}_{i,n+1/2} = \mathbf{v}_{i,n-1/2} + \frac{d\mathbf{v}_{i,n}}{dt} \delta t, \quad (17)$$

which gives the change in kinetic energy for particle i :

$$\Delta T_i = \frac{m_i}{2} (\mathbf{v}_{i,n+1/2} \cdot \mathbf{v}_{i,n+1/2} - \mathbf{v}_{i,n-1/2} \cdot \mathbf{v}_{i,n-1/2}) = \frac{m_i}{2} \mathbf{v}_{i,n-1/2} \cdot \frac{d\mathbf{v}_{i,n}}{dt} \delta t + \frac{m_i}{2} \frac{d\mathbf{v}_{i,n}}{dt} \cdot \frac{d\mathbf{v}_{i,n}}{dt} \delta t^2. \quad (18)$$

If $\Delta U_i = m_i (du_i/dt) \delta t$ is the change in internal energy for the particle it is possible to conserve the energy, that is require that $\Delta T_i + \Delta U_i = 0$. The time derivative of the internal energy for the particle is then consequently defined as

$$\frac{du_{i,n}}{dt} = \frac{\Delta U_i}{m_i \delta t} = -\mathbf{v}_{i,n-1/2} \cdot \frac{d\mathbf{v}_{i,n}}{dt} \delta t - \frac{1}{2} \frac{d\mathbf{v}_{i,n}}{dt} \cdot \frac{d\mathbf{v}_{i,n}}{dt} \delta t^2 \quad (19)$$

to conserve the total energy.

The artificial also must prevent particle penetration. The modified artificial viscosity is calculated with respect to an integrated mean of the neighbours. Therefore a particle will move less than ϵh from the definition of the time step, Eq. (7), regardless of the neighbours' individual sound speed and individual velocities. This should be compared with the distance to the closest which are approximately one h with 64 neighbours. Since the neighbours have no identity, this may lead to penetration with a few particles which have a sufficient deviation from the integrated mean.

If the particles i and j are each others neighbours and that their other neighbours give the same contribution to their respective velocities, one concludes from Eq. (15) and (16) that the impulse is conserved. Their other neighbours do, however, not give the same contribution, due to the limited resolution. Tests of self gravitating rotating disks show that the angular momentum and impulse are well conserved.

5. Tests

Any form of artificial viscosity must be able to form and propagate a shock. The modification of the artificial viscosity, Eq. (13) and (16), is tested in a shock forming test and compared with the standard artificial viscosity, Eq. (6), introduced by Monaghan and Gingold (1983). The ability of the modified artificial viscosity to compress the gas without viscous deceleration and heating has also been tested in a homologous compression of a gas sphere. The test constructed by Evrard (1988) is used to study the differences between the artificial viscosities in a gravitational collapse of an initially cool gas cloud.

The number of particles is varied between 8192 and 16384, and the number of neighbours for each particle is 64, so that h is varying in time and space. In the equation of state the adiabatic index is $\gamma = 5/3$ to model an ideal monoatomic gas. Dimensionless units are used to keep the quantities in the model around unity, where the gravitational constant $G = 1$. In a model the total mass $M = 1$, the typical length $L = 1$ and time $T = 1$, which relates to the gravitational constant as $G = L^3/MT^2$. The real quantities of the model can be calculated by inserting the corresponding quantities in the desired unit system.

5.1. Shock formation test

A box with dimensions $d_x \times d_y \times d_z = 1 \times 1 \times 9$ is used. Periodic boundary conditions are applied in the x - and y -directions. At the ends of the tube, $z = -5$ and $z = 4$, no boundary conditions were applied, so the particles are allowed to move away from the tube. Their velocities are however low compared with the velocity of the shock and do not affect the shock model. The 16384 particles are ordered in a cubic centered grid, such that 8192 particles are distributed at $-5 \leq z < -4$, and the rest at $-4 \leq z \leq 4$. With a total mass of 1.0 mass units the density distribution is

$$\rho = \begin{cases} 1/4 & \text{where } -5 \leq z < -4 \\ 1/16 & \text{where } -4 \leq z \leq 4 \end{cases} \quad (20)$$

The initial thermal energy density is set to 0.01, and the particles have no initial velocity. A shock is formed at the discontinuity at $z = -4$, which propagates to the right. This is a rather weak shock, which is a better test than a strong shock. The reason is that here the particle velocities are not much larger than the sound speed, because if they are the standard and modifies artificial viscosity become similar. Fig. 1 shows the shocks at $t = 20$ with the standard artificial viscosity, Eq. (6), and Fig. 2 using modified artificial viscosity, Eq. (13) and (16). A comparison between Fig. 1 and 2 shows that the shocks formed by the two

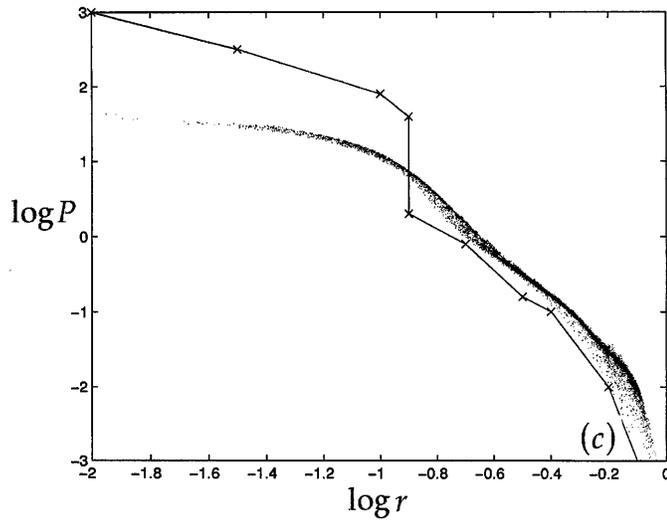
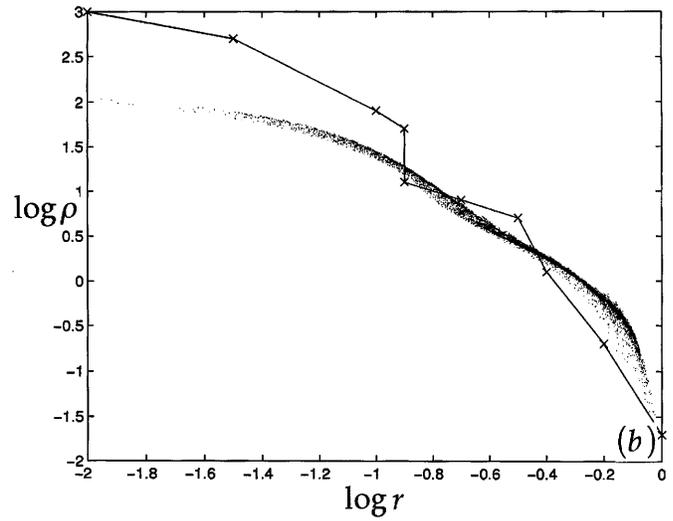
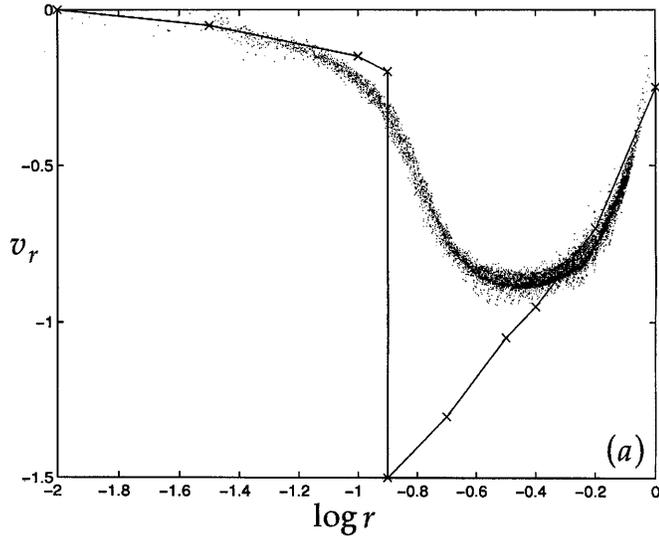


Fig. 6a–c. The velocity, density and pressure distributions for the Evrard gravitational collapse using standard artificial viscosity, Eq. (6). The radial velocity, density and pressure are plotted at $t=0.8$. The crosses are values at $t=0.77$ are results from an accurate one-dimensional PPM calculation from Steinmetz & Müller (1992).

versions of artificial viscosity are similar, so that the modified artificial viscosity is able to work in the same way as the standard artificial viscosity.

5.2. Homologous compression of a gas sphere

The ability of the modified artificial viscosity, Eq. (13) and (16), to handle compression of a gas realistically is tested. Initially 8192 particles are distributed uniformly in a sphere with radius $R_{init} = 1$ on a slightly disturbed cubic centered grid. There are no boundary conditions applied, but there is an initial velocity distribution directed towards the origin according to

$$\mathbf{v}_{init}(\mathbf{r}) = -V_0 \frac{\mathbf{r}}{R_{init}}, \quad (21)$$

where $V_0 = 2$ is a constant and \mathbf{r} the position in the sphere. The particles are initially isothermal with a thermal energy density of $u_{init} = 0.001$, and with a total mass of $M = 1$, which gives

an initial density of $\rho_{init} = 3/4\pi$. The sum of the total kinetic, E_{kin} , and thermal, E_{th} , energies is

$$E_{tot} = E_{kin} + E_{th} = \int_0^M \frac{v^2}{2} dm + uM = \int_0^R \frac{4r^2}{2} 4\pi r^2 \rho dr + \mu M = \frac{6}{5} + 0.001 = 1.201 \quad (22)$$

Assume that the compression is adiabatic, so that Poisson's equation,

$$P = K\rho^\gamma, \quad (23)$$

is valid. From the equation of state, $P = (\gamma - 1)u\rho$, and the adiabatic index, $\gamma = 5/3$, it follows that

$$K = (\gamma - 1)\mu_{init}\rho_{init}^{\gamma-1} = 1.73 \cdot 10^{-3} \quad (24)$$

If it is assumed that all kinetic energy is converted to thermal energy at maximum compression, the thermal energy density at this point is

$$u' = \frac{E_{tot}}{M} = 1.201. \quad (25)$$

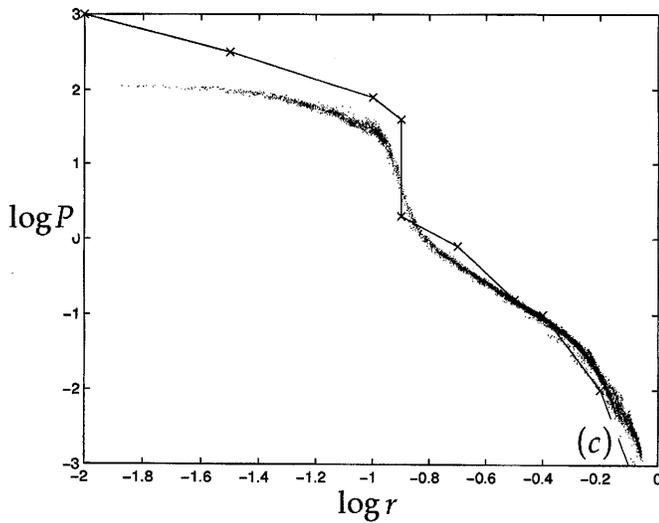
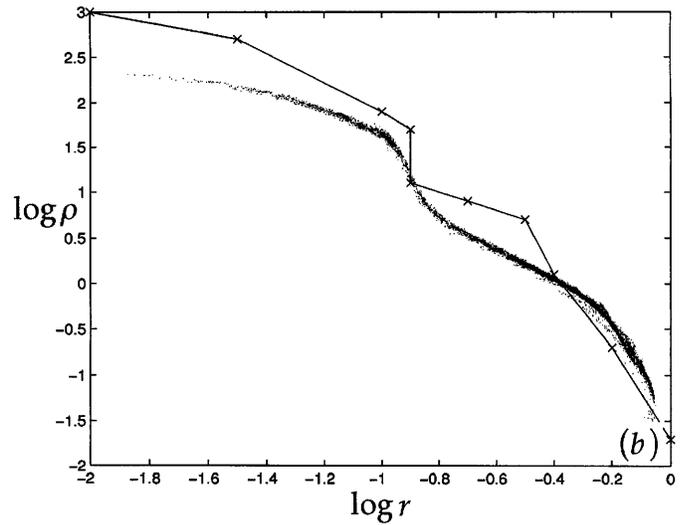
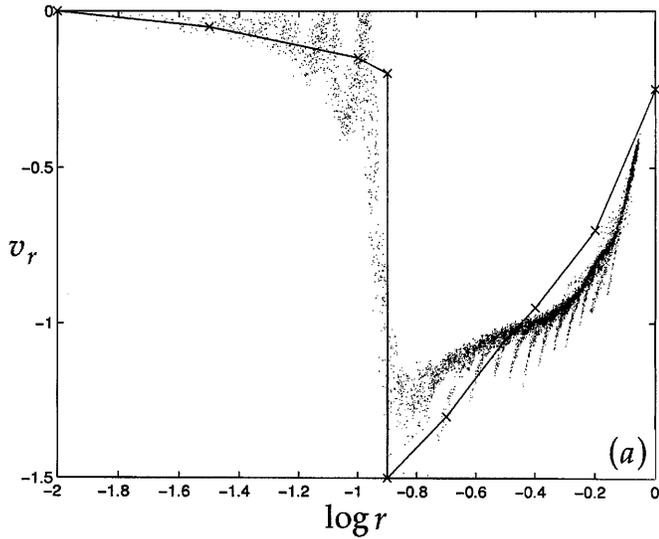


Fig. 7a and b. Same as Fig. 6, but with modified artificial viscosity, Eq. (13) and (16).

This gives a density and radius of the compressed gas sphere as

$$\rho' = \left[\frac{K}{(\gamma-1)u'} \right]^{\frac{1}{1-\gamma}} \tag{26}$$

$$R' = \left(\frac{3M}{4\pi\rho'} \right)^{\frac{1}{3}} = 0.029$$

This zeroth order approximation is useful to compare with calculations with different forms of artificial viscosity. The initial conditions also have the advantage that no artificial viscosity is needed to prevent interparticle penetration. A small initial pressure is sufficient to decelerate the particles to zero and prevent them to move through the origin. The results from the test with modified artificial viscosity can therefore not only be compared with the analytical approximation Eq. (25), but also with a model without any artificial viscosity. The energy curves from such a comparison are shown in Fig. 3. In Fig. 4 the density distributions at time $t = 0.5$ are compared.

The heating in the case with standard artificial viscosity, Eq. (6), starts immediately, because it depends on the relative velocities, while the heating without artificial viscosity and with

the modified artificial viscosity is negligible until $t \approx 0.3$. The modified artificial viscosity has a little less steep energy curve compared with the case without artificial viscosity, but gives an almost a compressed gas as without artificial viscosity as seen in Fig. 4. The true density distribution is not known, but the zeroth order approximation from Eq. (25) gives approximately the same size as the model without artificial viscosity.

5.3. The Evrard gravitational collapse

The initial conditions are those from Evrard (1988), which is a isotherm sphere at rest with a thermal energy density of $u = 0.05$, radius $R = 1$ and mass $M = 1$. Initially 8192 particles are distributed uniformly in a sphere with radius $R = 1$ on a slightly disturbed cubic centered grid with a density distribution of

$$\rho(r) = \frac{M}{2\pi R} \frac{1}{r}. \tag{27}$$

Standard, Eq. (6), and modified artificial viscosity, Eq. (13) and (16) are tested with this model and compared with each other.

Due to the cold initial state the sphere begins to collapse. Since the central part is more dense than the outer layers, the collapse is more rapid around the origin. A high central pressure and density is build up, and the central parts starts to expand at $t \approx 0.8$. Where the expanding parts meet the infalling gas, an outward propagating shock front forms. Eventually the gas reach a virial equilibrium.

The total energies are shown in Fig. 5a and b. The curves are more shallow in Fig. 5a, where standard viscosity was used, than in Fig. 5b. In Fig. 6 the velocity, density and pressure distributions are plotted with standard artificial viscosity at $t = 0.8$ when the shock is formed. This can be compared with the results from modified artificial viscosity for the same quantities which are shown in Fig. 7. The main difference between these models are the sharper gradients with modified artificial viscosity. This is an effect of the ability to compress the gas with modified artificial viscosity. The infalling gas is allowed to move inwards without deceleration until it meets the shock front.

6. Conclusions

In SPH two forms of artificial viscosity is necessary, the bulk and von Neumann-Richtmeyer viscosity. The standard recipe does however induce smoothening of the velocity differences and heating of the gas. Another problem is that standard artificial viscosity prevents compression of a gas.

To partly overcome some of these problems, I propose a modification of the artificial viscosity. The von Neumann-Richtmeyer artificial viscosity is used only in supersonic regions

where the gas is not under compression. The bulk viscosity is modified to be calculated from an integrated mean of the velocities in the neighbourhood of the particle, instead of calculating the viscous interaction from each neighbour separately. Tests cases have been performed to test these abilities of the proposed modified form of artificial viscosity, and to compare them with standard artificial viscosity.

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