

The pair production spectrum from photon-photon annihilation

M. Böttcher and R. Schlickeiser

Max-Planck-Institut für Radioastronomie, Postfach 20 24, D-53010 Bonn, Germany

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Abstract. We present the first completely analytical computation of the full differential γ - γ pair production rate from compact radiation fields, exact to 2nd order QED, and use this result to investigate the validity of previously known approximations.

Key words: plasmas – radiation mechanisms: non-thermal – gamma-rays: theory

1. Introduction

The discovery of high-energy γ -radiation from extragalactic compact objects has motivated many authors to consider the effects of γ -ray absorption by γ - γ pair production, eventually inducing pair cascades. The relevance of γ - γ pair production to astrophysical systems has first been pointed out by Nikishov (1962). The first investigation of the γ - γ absorption probability of high-energy photons by different soft photon fields, along with some useful approximations, can be found in Gould & Schröder (1967).

The energy spectrum of injected electrons and positrons due to this process has been studied by several authors (e. g., Bonometto & Rees 1971, Aharonian et al. 1983, Zdziarski & Lightman 1985, Coppi & Blandford 1990). In most astrophysically relevant cases, simple approximations can be used for this purpose, without much loss of accuracy. These usually rely on the high-energy photon having much higher energy than the soft photons and thus dominating the energy input and determining the direction of motion of the center-of-momentum frame of the produced pairs. Bonometto & Rees (1971) used basically the same technique as we do, but restricted their analysis to the case $\epsilon_1 \gg \epsilon_2$, and did not solve the problem analytically. Two recipes to calculate the full energy-dependence of the injected pairs have been published (Aharonian et al. 1983 and Coppi & Blandford 1990), but here the reader is still left with integrations to be carried out numerically.

It is the purpose of this paper to derive the full energy-spectrum of pairs, injected by γ - γ pair production, exact to second order QED for the case of isotropic radiation fields. In

Sect. 2, we give a short overview of the kinematics which are used in Sect. 3 to calculate the pair injection spectrum. In Sect. 4, we compare our results to well-known approximations and specify the limitations of the various approximations. Our analysis is easily generalized to non-isotropic radiation fields. The derivation presented here is widely analogous to the derivation of the pair annihilation spectrum, given by Svensson (1982).

2. Kinematics

We consider an isotropic photon field $n_{ph}(\epsilon)$ where $\epsilon = h\nu/(m_e c^2)$ is the dimensionless photon energy in a rest frame which we call the laboratory frame. The Lorentz invariant scalar product of the four-momenta $\epsilon_{1,2}$ of two photons having energies $\epsilon_{1,2}$ colliding under an angle of cosine $\mu = \cos \theta_\mu$ in the laboratory frame is then given by

$$\underline{\epsilon}_1 \cdot \underline{\epsilon}_2 = \epsilon_1 \epsilon_2 (1 - \mu) = 2 \epsilon_{cm}^2. \quad (1)$$

Here, ϵ_{cm} is the photon energy in the center-of-momentum frame. In order to allow for the possibility to create an electron-positron pair, conservation of energy implies $\epsilon_{cm} = \gamma_{cm}$, and the condition $\gamma_{cm} \geq 1$ determines the pair-production threshold. γ_{cm} is the Lorentz factor of the electron/positron in the cm frame where the produced electrons move with speed $\pm \beta_{cm} c$ and $\beta_{cm} = \sqrt{1 - 1/\gamma_{cm}^2}$. The definition of the angle variables needed in this calculation is illustrated in Fig. 1.

The cm frame moves relative to the laboratory frame with velocity $-\beta_c c$ and Lorentz factor $\gamma_c = (1 - \beta_c^2)^{-1/2}$. The four velocity of the laboratory frame ($\beta_c c$ in the cm frame) is denoted by \underline{v}_L . The Lorentz factors of the produced pairs in the laboratory frame are related to the cm quantities by

$$\gamma_\pm = \gamma_{cm} \gamma_c (1 \pm \beta_{cm} \beta_c u). \quad (2)$$

Evaluating the Lorentz invariant scalar product

$$\underline{\epsilon}_{1,2} \cdot \underline{v}_L = \epsilon_{1,2} = \epsilon_{cm} \gamma_c (1 \pm \beta_c z) \quad (3)$$

in the laboratory and the cm-frame, respectively, we find

$$\gamma_c = \frac{E}{2 \epsilon_{cm}}, \quad \text{where} \quad E = \epsilon_1 + \epsilon_2, \quad (4)$$

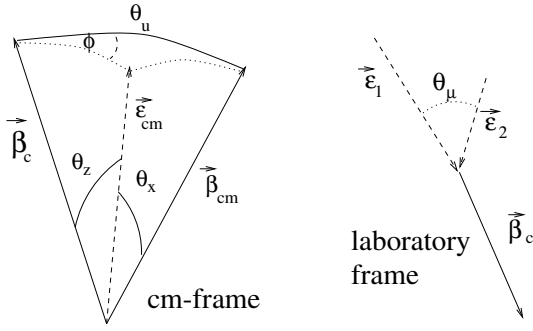


Fig. 1. Definition of the angles in cm and laboratory frame. $\epsilon_{1,2}$ denotes the direction of motion of an incoming photon, $\beta_{cm,\pm}$ is the direction of motion of the produced electron and positron in the cm and the laboratory frame, and β_c characterizes relative motion of the laboratory and the cm-frame, respectively.

and

$$z = \cos\theta_z = \frac{\epsilon_1 - \epsilon_2}{N}, \quad \text{where} \quad N = \sqrt{E^2 - 4\epsilon_{cm}^2}. \quad (5)$$

Inserting Eq. (4) into Eq. (2) and using energy conservation ($\epsilon_{cm} = \gamma_{cm}$) fixes the angle cosine u to

$$u = u_0 \equiv \frac{E - 2\gamma_-}{\beta_{cm} N}. \quad (6)$$

The differential cross section for γ - γ pair production (see Eq. [11]) depends on

$$x = \cos\theta_x = uz + \sqrt{1-u^2}\sqrt{1-z^2}\cos\phi. \quad (7)$$

3. The pair yield

The differential yield of produced pairs is calculated as

$$\begin{aligned} & \dot{n}(\gamma_-) \\ &= \frac{c}{4} \int_0^\infty d\epsilon_1 n_{ph}(\epsilon_1) \int_{\max\{\frac{1}{\epsilon_1}, \gamma_- + 1 - \epsilon_1\}}^\infty d\epsilon_2 n_{ph}(\epsilon_2) \int_{-1}^{1 - \frac{2}{\epsilon_1 \epsilon_2}} d\mu (1 - \mu) \frac{d\sigma}{d\gamma_-} \end{aligned} \quad (8)$$

where

$$\frac{d\sigma}{d\gamma_-} = \oint d\Omega_{cm} \frac{d^2\sigma}{d\Omega_{cm} d\gamma_{cm}} \frac{d\gamma_{cm}}{d\gamma_-}. \quad (9)$$

The differential cross section has been evaluated by Jauch & Rohrlich (1959):

$$\frac{d^2\sigma}{d\Omega_{cm} d\gamma_{cm}} = \delta(\epsilon_{cm} - \gamma_{cm}) \frac{d\sigma}{d\Omega_{cm}}, \quad (10)$$

where

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{1}{2\pi} \frac{3}{16} \sigma_T \frac{\beta_{cm}}{\epsilon_{cm}^2}.$$

$$\cdot \left\{ -1 + \frac{3 - \beta_{cm}^4}{2} \left(\frac{1}{1 - \beta_{cm}x} + \frac{1}{1 + \beta_{cm}x} \right) - \frac{1}{2\epsilon_{cm}^4} \left(\frac{1}{[1 - \beta_{cm}x]^2} + \frac{1}{[1 + \beta_{cm}x]^2} \right) \right\}. \quad (11)$$

We may express the solid angle element $d\Omega_{cm} = du d\phi$. Using Eq. (2), we find

$$\delta(\epsilon_{cm} - \gamma_{cm}) \frac{d\gamma_{cm}}{d\gamma_-} = \delta(u - u_0) \frac{2}{N\beta_{cm}}. \quad (12)$$

This enables us to carry out the u -integration in Eq. (9) immediately. If we write the denominators in Eq. (11) as

$$1 \pm \beta_{cm}x = a_\pm + b_\pm \cos\phi \quad (13)$$

with

$$a_\pm \equiv 1 \pm u_0 z \beta_{cm}, \quad b_\pm \equiv \pm \sqrt{1 - u_0^2} \sqrt{1 - z^2} \beta_{cm}, \quad (14)$$

we find

$$\begin{aligned} & \frac{d\sigma}{d\gamma_-} = \frac{3}{8} \sigma_T \frac{1}{N\epsilon_{cm}^2} \\ & \cdot \frac{1}{2\pi} \int_0^{2\pi} d\phi \left\{ -1 + \frac{3 - \beta_{cm}^4}{2} \left(\frac{1}{a_+ + b_+ \cos\phi} + \frac{1}{a_- + b_- \cos\phi} \right) - \frac{1}{2\epsilon_{cm}^4} \left(\frac{1}{[a_+ + b_+ \cos\phi]^2} + \frac{1}{[a_- + b_- \cos\phi]^2} \right) \right\} \\ & = \frac{3}{8} \sigma_T \frac{1}{\epsilon_{cm}} \\ & \cdot \left\{ -\frac{1}{N\epsilon_{cm}} + \frac{3 - \beta_{cm}^4}{4} (G_+ + G_-) - \frac{1}{8\epsilon_{cm}^2} (F_+ + F_-) \right\} \end{aligned} \quad (15)$$

where

$$G_\pm = \frac{1}{\sqrt{\epsilon_1 \epsilon_2 + \epsilon_{cm}^2 c_\pm}}, \quad (16)$$

$$F_\pm = \frac{d_\pm - 2\epsilon_{cm}^2}{(\epsilon_1 \epsilon_2 + \epsilon_{cm}^2 c_\pm)^{3/2}}, \quad (17)$$

with

$$c_\pm \equiv (\epsilon_{1,2} - \gamma_-)^2 - 1, \quad (18)$$

$$d_\pm \equiv \epsilon_{1,2}^2 + \epsilon_1 \epsilon_2 \pm \gamma_- (\epsilon_2 - \epsilon_1) \quad (19)$$

and we used the integrals

$$\int_0^{2\pi} \frac{d\phi}{a + b \cos\phi} = \frac{2\pi}{\sqrt{a^2 - b^2}}, \quad (20)$$

$$\int_0^{2\pi} \frac{d\phi}{(a + b \cos \phi)^2} = \frac{2\pi a}{(a^2 - b^2)^{3/2}} \quad (21)$$

and the identity

$$a_{\pm}^2 - b_{\pm}^2 = 4 \frac{\epsilon_1 \epsilon_2 + \epsilon_{cm}^2 c_{\pm}}{N^2 \epsilon_{cm}^2} \quad (22)$$

which follows from Eqs. (5), (6) and (14). Now, inserting Eq. (15) into Eq. (8) yields the exact expression for the differential pair injection rate. Using Eq. (1) we transform the μ integration into an integration over $d\epsilon_{cm}$. This leads us to

$$\begin{aligned} \dot{n}(\gamma_{\pm}) &= \frac{3}{4} \sigma_T c \int_0^{\infty} d\epsilon_1 n_{ph}(\epsilon_1) \cdot \\ &\cdot \int_{\max\{\frac{1}{\epsilon_1}, \gamma_{\pm} + 1 - \epsilon_1\}}^{\infty} d\epsilon_2 n_{ph}(\epsilon_2) \frac{1}{(\epsilon_1 \epsilon_2)^2} \int_{\epsilon_{cm}^L}^{\epsilon_{cm}^U} d\epsilon_{cm} \epsilon_{cm}^2 \cdot \\ &\cdot \left\{ -\frac{1}{N \epsilon_{cm}} + \frac{3 - \beta_{cm}^4}{4} (G_+ + G_-) - \frac{1}{8 \epsilon_{cm}^2} (F_+ + F_-) \right\} \quad (23) \end{aligned}$$

which can be calculated analytically. The integration limits follow from $1 \leq \epsilon_{cm} \leq \sqrt{\epsilon_1 \epsilon_2}$ and the condition $\gamma_{\pm} \lesssim \gamma_{cm} \gamma_c (1 \pm \beta_{cm} \beta_c)$ which yields

$$\epsilon_{cm}^U = \min \{ \sqrt{\epsilon_1 \epsilon_2}, \epsilon_{cm}^* \}, \quad \epsilon_{cm}^L = \max \left\{ 1, \epsilon_{cm}^{\dagger} \right\} \quad (24)$$

where

$$\begin{aligned} \left(\epsilon_{cm}^{*,\dagger} \right)^2 &= \\ \frac{1}{2} \left(\gamma_{\pm} [E - \gamma_{\pm}] + 1 \pm \sqrt{(\gamma_{\pm} [E - \gamma_{\pm}] + 1)^2 - E^2} \right). \quad (25) \end{aligned}$$

Using the integrals 2.271.4, 2.271.5, 2.272.3, 2.272.4, and 2.275.9, of Gradshteyn & Ryzhik (1980), we find as final result for the differential pair yield

$$\begin{aligned} \dot{n}(\gamma_{\pm}) &= \frac{3}{4} \sigma_T c \int_0^{\infty} d\epsilon_1 \frac{n_{ph}(\epsilon_1)}{\epsilon_1^2} \int_{\max\{\frac{1}{\epsilon_1}, \gamma_{\pm} + 1 - \epsilon_1\}}^{\infty} d\epsilon_2 \frac{n_{ph}(\epsilon_2)}{\epsilon_2^2} \cdot \\ &\cdot \left\{ \frac{\sqrt{E^2 - 4\epsilon_{cm}^2}}{4} + H_+ + H_- \right\} \Bigg|_{\epsilon_{cm}^L}^{\epsilon_{cm}^U}, \quad (26) \end{aligned}$$

where for $c_{\pm} \neq 0$ we have

$$\begin{aligned} H_{\pm} &= -\frac{\epsilon_{cm}}{8 \sqrt{\epsilon_1 \epsilon_2 + c_{\pm} \epsilon_{cm}^2}} \left(\frac{d_{\pm}}{\epsilon_1 \epsilon_2} + \frac{2}{c_{\pm}} \right) \\ &+ \frac{1}{4} \left(2 - \frac{\epsilon_1 \epsilon_2 - 1}{c_{\pm}} \right) I_{\pm} \end{aligned}$$

$$+ \frac{\sqrt{\epsilon_1 \epsilon_2 + c_{\pm} \epsilon_{cm}^2}}{4} \left(\frac{\epsilon_{cm}}{c_{\pm}} + \frac{1}{\epsilon_{cm} \epsilon_1 \epsilon_2} \right) \quad (27)$$

and

$$\begin{aligned} I_{\pm} &= \\ \begin{cases} \frac{1}{\sqrt{c_{\pm}}} \ln \left(\epsilon_{cm} \sqrt{c_{\pm}} + \sqrt{\epsilon_1 \epsilon_2 + c_{\pm} \epsilon_{cm}^2} \right) & \text{for } c_{\pm} > 0 \\ \frac{1}{\sqrt{-c_{\pm}}} \arcsin \left(\epsilon_{cm} \sqrt{-\frac{c_{\pm}}{\epsilon_1 \epsilon_2}} \right) & \text{for } c_{\pm} < 0. \end{cases} \quad (28) \end{aligned}$$

For $c_{\pm} = 0$ we find

$$\begin{aligned} H_{\pm} &= \left(\frac{\epsilon_{cm}^3}{12} - \frac{\epsilon_{cm} d_{\pm}}{8} \right) \frac{1}{(\epsilon_1 \epsilon_2)^{3/2}} \\ &+ \left(\frac{\epsilon_{cm}^3}{6} + \frac{\epsilon_{cm}}{2} + \frac{1}{4\epsilon_{cm}} \right) \frac{1}{\sqrt{\epsilon_1 \epsilon_2}}. \quad (29) \end{aligned}$$

4. Comparison to approximations

Now, we use the exact expression, given in Eq. (26) to specify the regimes of validity and the limitations of various approximations. The first detailed computation of the pair production spectrum was presented by Bonometto & Rees (1971). Based on the neglect of the energy input of the soft photon, they basically follow the same procedure as described above, but do not carry out the angle-integration (integration over ϵ_{cm} in our formalism) analytically. In the case $\epsilon_1 \gg \epsilon_2$, it is in very good agreement with the exact result, but its evaluation is even more time-consuming than using the latter. For this reason, we will not consider it in detail, but concentrate on approximations which really yield simpler expressions than the exact one.

4.1. δ -function approximation for power-law spectra

Probably the simplest expression for the pair spectrum injected by γ -rays interacting with a power-law of energy spectral index $\alpha' = \alpha - 1$ ($n(\epsilon_2) \propto \epsilon_2^{-\alpha}$) is based on the assumption $\epsilon_1 \gg \epsilon_2$ and on the well-known fact that photons of energy ϵ_1 interact most efficiently with photons of energy $\epsilon_2 = 2/\epsilon_1$ which motivates the approximation for the γ - γ opacity of Gould & Schröder (1967) using a δ function approximation for the cross section,

$$\sigma_{\gamma\gamma}(\epsilon_1, \epsilon_2) \approx \frac{1}{3} \sigma_T \epsilon_2 \delta \left(\epsilon_2 - \frac{2}{\epsilon_1} \right)$$

Since in this approach pair production takes place only near the pair production threshold ($\gamma_{cm} \approx 1$), the produced pairs have energies $\gamma_{\pm} \approx \gamma_{\mp} \approx (\epsilon_1 + \epsilon_2)/2$ (Bonometto & Rees 1971). The resulting pair injection spectrum is therefore

$$\dot{n}(\gamma) \approx \eta(\alpha') c \sigma_T \frac{n_{ph}(\epsilon_0)}{\epsilon_0} n_{ph} \left(\frac{1}{\epsilon_0} \right) \quad (30)$$

(e. g. Lightman & Zdziarski 1987) where $\epsilon_0 \equiv \gamma + \sqrt{\gamma^2 - 1}$ (which, of course, reduces to 2γ for $\gamma \gg 1$) and $\eta(\alpha')$ is a numerical factor, depending only on the spectral index of the

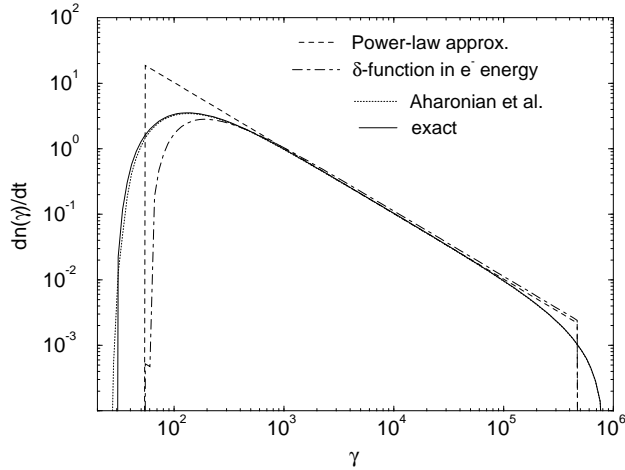


Fig. 2. Differential pair injection rate (arbitrary units) for the interaction of a power-law from $\epsilon_1 = 10^2 - 10^6$, photon spectral index $\alpha_\gamma = 1.5$, with a soft power-law from $\epsilon_2 = 10^{-7} - 10^{-2}$, $\alpha_s = 1.5$.

soft photon distribution. This approximation yields useful results, if the power-law photon spectra extend over a sufficiently wide range ($\epsilon_U/\epsilon_L \gtrsim 10^4$) and if for every high-energy photon of energy ϵ_1 there is a soft photon of energy $\epsilon_2 = 2/\epsilon_1$. Else, the injection spectrum calculated with Eq. (30) cuts off at the inverse of the respective cutoff of the soft photon spectrum, seriously underpredicting the injection of pairs of higher or lower energy, respectively, where the injection spectrum declines smoothly. Nevertheless, these pairs can still carry a significant fraction of the injected power. Eq. (30) fails also to describe the injection of low-energetic pairs in case of a high lower cut-off of the γ -ray spectrum even if soft photons of energy $\epsilon_2 = 1/\epsilon_1$ are present. For example, in the case of the interaction of a power-law spectrum extending from $\epsilon_1 = 10^2 - 10^6$ with a soft power-law spectrum extending from $\epsilon_2 = 10^{-7} - 10^{-2}$ Eq. (30) overpredicts the injection of pairs slightly above $\gamma = 50$ by an order of magnitude and cuts off below this energy. A similar problem arises at the high-energy end of the injection spectrum. An example for this fact is shown in Fig. 2. In contrast, the approximation (30) can well be used to describe the injection of pairs at all energies if both photon fields extend up to (and down to, respectively) $\epsilon_{1,2} \sim 1$. For more general soft photon distributions which are different from a power-law (e. g. a thermal spectrum) the analogous δ -function approximation has first been introduced by Kazanas (1984).

4.2. δ -function in electron energy

Using the full cross section for γ - γ pair production as given by Jauch & Rohrlich (1959) instead of the δ -function approximation adopted in Eq. (30) does not reduce the limitations of this power-law approach significantly, but other soft photon distribu-

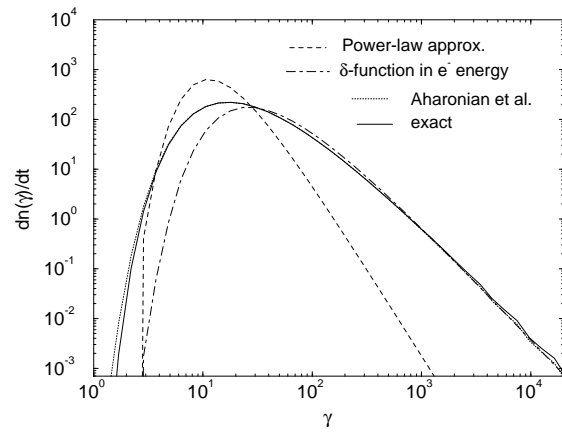


Fig. 3. Differential pair injection rate (arbitrary units) for the interaction of a power-law γ -ray spectrum from $\epsilon_1 = 5$ to 10^6 , $\alpha_\gamma = 1.5$, with a thermal blackbody spectrum of temperature $\Theta = 10^{-2}$.

tions can be treated more successfully with this approximation which in the limit $\epsilon_1 \gg \epsilon_2$ reads

$$n(\gamma) \approx 2c \frac{n_{ph}^{(1)}(2\gamma)}{(2\gamma)^2} \int_{1/2\gamma}^1 d\epsilon_2 \frac{n_{ph}^{(2)}(\epsilon_2)}{(\epsilon_2)^2} \int_{\epsilon_{cm}^L}^{\epsilon_{cm}^U} ds s \sigma_{\gamma\gamma}(s) \quad (31)$$

where $s = \epsilon_{cm}^2$ and the limits $\epsilon_{cm}^{U/L}$ are given by Eqs. (24) and (25). Here, we have assumed that the produced electron and positron have energy $\gamma_- = \gamma_+ = \epsilon_1/2$. This approach works equally well for power-law photon fields, but in contrast to Eq. (30), it tends to underpredict the injection of low-energetic pairs. The same is true for the interaction of γ -ray photon fields with thermal soft photon fields where the high-energy tail of the injection spectrum is described very accurately (a few % error) by Eq. (31). The accuracy of this approximation improves with decreasing lower cut-off of the γ -ray spectrum. E. g., the injection due to a power-law γ -ray spectrum from $\epsilon_1 = 2 - 10^6$ interacting with a thermal spectrum of normalized temperature $\Theta = 0.1$ is described by Eq. (31) with a deviation of less than 30 % from the exact result down to $\gamma \sim 1$. For $\gamma > 2$, the deviation was much less than 10 %. We show an example for the latter situation in Fig. 3.

4.3. Approximation by Aharonian et al.

A very useful approximation to the pair injection spectrum for all shapes of the soft photon spectrum under the condition $\epsilon_2 \ll 1 \lesssim \epsilon_1$ has been found by Aharonian et al. (1983). They use a different representation of the pair production cross section and end up with a one-dimensional integral over $k = \sqrt{\epsilon_1^2 + \epsilon_2^2 + 2\epsilon_1\epsilon_2\mu}$ which is equivalent to our ϵ_{cm} integration in Eq. (21). They solve this integration analytically after simplifying the integrand and the integration limits according to the assumptions mentioned above. The resulting injection spectrum is

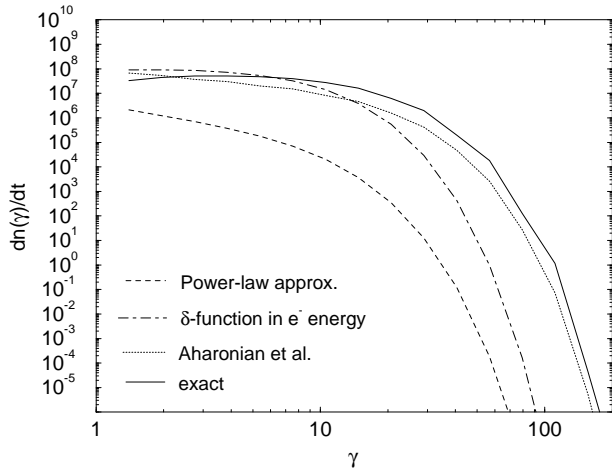


Fig. 4. Differential pair injection rate (arbitrary units) for the interaction of a thermal blackbody spectrum of temperature $\Theta = 5$ with itself.

$$\dot{n}(\gamma) \approx \frac{3}{32} c \sigma_T \int_{\gamma}^{\infty} d\epsilon_1 \frac{n_{ph}^{(1)}(\epsilon_1)}{\epsilon_1^3} \int_{\frac{\epsilon_1}{4(\epsilon_1 - \gamma)}}^{\infty} d\epsilon_2 \frac{n_{ph}^{(2)}(\epsilon_2)}{\epsilon_2^2} \cdot \left\{ \frac{4\epsilon_1^2}{\gamma(\epsilon_1 - \gamma)} \ln \left(\frac{4\epsilon_2\gamma(\epsilon_1 - \gamma)}{\epsilon_1} \right) - 8\epsilon_1\epsilon_2 + \frac{2(2\epsilon_1\epsilon_2 - 1)\epsilon_1^2}{\gamma(\epsilon_1 - \gamma)} - \left(1 - \frac{1}{\epsilon_1\epsilon_2} \right) \frac{\epsilon_1^4}{\gamma^2(\epsilon_1 - \gamma)^2} \right\}. \quad (32)$$

It describes the power-law tail of the pair spectrum injected by power-law γ -ray photon fields perfectly and is much more accurate to the injection of low-energy pairs. Interaction with a power-law soft photon field is reproduced within errors of only a few %. Even if as well the γ -ray as the soft photon spectrum extend to $\epsilon_{1,2} = 1$, the error at $\gamma \sim 1$ increases only to $\sim 10\%$. Problems with this approximation arise if the soft photon spectrum extends up to $\epsilon_2 \sim 1$, but the γ -ray spectrum has a lower cut-off $\epsilon_1 \gg 1$. In this case, the injection of low-energetic pairs is seriously overpredicted by Eq. (32). For power-law soft photon fields, the integration over ϵ_2 in Eq. (32) can be carried out analytically, as was found by Svensson (1987). His Equation (B8) multiplied with the total absorption coefficient $\propto \epsilon^\alpha$ (by which the total injection rate had been normalized to unity) is in perfect agreement with the numerical results according to Eq. (32).

The interaction of power-law γ -ray spectra with thermal soft photon fields is generally described within an error of a few % at all electron/positron energies if the soft photon temperature is $\Theta \lesssim 0.1$, even if the γ -ray spectrum extends down to $\epsilon_1 \sim 1$.

Interestingly, even the interaction of a mildly relativistic thermal photon field ($\Theta \lesssim 3$) with itself (for which Aharonian's approximation was not designed) is reproduced reasonably well, but the result of Eq. (32) differs from the exact injection rate by

a roughly constant factor. When artificially introducing a factor adjusting the high-energy tails of the injection spectra, Eq. (32) overpredicts the injection of low-energetic pairs by a factor of ~ 3 , but for $\gamma \gtrsim 3$ there is very good agreement with the exact result. The deviation becomes more important with increasing photon temperature, and for $\Theta = 5$ the injection of cold pairs is already overpredicted by a factor of > 10 . Fig. 4 illustrates the accuracy of the various approximations for a compact thermal radiation of temperature $\Theta = 5$.

We find that all the statements on soft photon or γ -ray power-law spectra made above are only very weakly dependent on the respective spectral index.

5. Summary and conclusions

We derived the full energy spectrum of injected pairs, produced by γ - γ pair production and compared the result to the various approximations known before. We found that the simplest expressions, based on δ function approximation to the cross section can well reproduce the power-law tail of the injection spectrum, but have problems at low particle energies.

The approximation of Aharonian et al. (1983) was shown to be the most accurate one and even yields useful results in regimes for which this approach was not designed.

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