

Using distant type Ia supernovae to measure the cosmological expansion parameters^{*}

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Abstract. A comprehensive analysis of two recent collections of distant Type Ia supernovae is presented. The method used is both simple and rigorous. Correcting the absolute magnitudes for the measured decline rates yields in nearly all cases a dispersion consistent with measurement uncertainties, showing from these data that Type Ia supernovae are remarkably reliable standardized candles. With 26 supernovae between $0.01 < z < 0.1$ and a cosmological sample of six supernovae between $0.35 < z < 0.45$, the Cepheid-based Type Ia supernova absolute magnitude scale leads to a Hubble constant $H_0 = 60 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and a deceleration constant $q_0 = 0.385 \pm 0.36$. We note the implications of these results for the age of the universe both with and without the introduction of a cosmological constant. This leads to further constraints on the mass density Ω_M of the universe when comparison is made with the age of the universe derived from globular clusters.

Key words: supernovae: general – cosmology: observations – distance scale

1. Introduction

There is increasing evidence that Type Ia supernovae (SNe Ia), properly selected and corrected, are the most suitable standard candles of sufficient luminosity to provide direct access to the dynamics of universal expansion. Branch & Tammann (1992) have reviewed the subject and there have been further discussions of spectroscopic criteria by Branch et al. (1993) and of color criteria by Vaughan et al. (1995). Phillips (1993) and Hamuy et al. (1995) have noted the dependence of the absolute magnitude on the decline rate, thereby allowing corrections to be made to decrease the intrinsic dispersion.

Hamuy et al. (1996) have recently published a collection of 29 distant Type Ia supernovae from the Calán/Tololo (CT)

SN survey measured using modern photometric methods. This collection encompasses and updates with revised measurements a smaller sample of 13 SNe Ia published earlier (Hamuy et al 1995). They infer for each of them the apparent B and V magnitudes at maximum light as well as Δm_{15} , the decline in magnitude during the first 15 days beyond maximum. In addition, they present estimated uncertainties in these quantities. The measured redshifts z of the host galaxies ranged from 0.01 to 0.10. From the B–V colors, three of the supernovae appear abnormally reddened, leaving 26 that are apparently normal SNe Ia. We combine these with six of the seven cosmological ($z \gtrsim 0.1$) supernovae recently published by Perlmutter et al. (1996) in a new global analysis aimed at investigating the reliability of SNe Ia as standard candles and measuring the cosmological expansion parameters: the Hubble constant H_0 and the deceleration constant q_0 . Since these supernovae are the best-measured distant sample available, both the investigation of the standard candle hypothesis and the determination of H_0 are less subject to peculiar velocity and other problems that have complicated previous efforts. The combined analysis simultaneously yields H_0 , q_0 , and a parameter b introduced to standardize the supernovae. We then generalize the treatment to incorporate a cosmological constant Ω_Λ in a spatially flat universe, and compare the age of the universe derived from these measurements of the expansion parameters with that inferred from the oldest globular clusters in the Galaxy.

2. Method

The following expression (see, e.g., Weinberg 1972) is used to obtain H_0 :

$\log H_0 =$

$$\frac{M_B - m_B + 52.38}{5} + \log \frac{1 - q_0 + q_0 z - (1 - q_0)\sqrt{1 + 2q_0 z}}{q_0^2} \quad (1)$$

Here the log term contains the deceleration constant q_0 , related to the present mass density by $\Omega_M = 2q_0$ when the cosmological constant $\Omega_\Lambda = 0$. This term incorporates in a homogeneous space the deceleration of the Hubble flow over cosmological distances, reducing nearby to $\log z$. The number 52.38

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absorbs the velocity of light and the parsec scales, while m_B is the apparent blue magnitude and M_B the absolute blue magnitude at maximum light.

The absolute magnitude of SNe Ia has been measured by Saha et al. (1994, 1995) through the Hubble Space Telescope determination of distances to two nearby galaxies, IC 4182 and NGC 5253. They measured the periods of many Cepheid variable stars in each galaxy, used the currently accepted period-luminosity relation of Madore & Freedman (1991) to determine the absolute luminosity of each Cepheid, and, by comparing this with its apparent magnitude, established its distance. These galaxies were chosen because they contained three prototypical SNe Ia whose light curves had been recorded with the techniques of the day: SN 1937C in IC 4182 and SN 1895B and SN 1972E in NGC 5253. More recently these have been augmented by four more calibrated SNe Ia: SN 1989B in NGC 3627 from Tanvir et al. (1995), SN 1960F in NGC 4496A and SN 1981B in NGC 4536 by Saha et al. (1996a, 1996b), and SN 1990N in NGC 4639 by Sandage et al. (1996). Hamuy et al. (1996) have refitted the light curves for four of these SNe (1937C, 1972E, 1981B, and 1990N) using the same procedure that they used to fit their distant SNe to obtain m_B and Δm_{15} . We use their resulting values for M_B and merge them with the M_B values for the three others compiled by Branch et al. (1996a). Together these give a mean absolute magnitude of $M_B = -19.48 \pm 0.07$.

Following the evidence of Phillips (1993) and Hamuy et al. (1995) that supernovae whose light curves fall more rapidly have lower peak luminosities, we have modified the absolute magnitudes of the supernovae in the following manner:

$$M_B = -19.48 + b(\Delta m_{15} - 1.05). \quad (2)$$

Here b is a decline rate parameter to be varied in a least squares fit. The form of this expression is chosen to yield $M_B = -19.48$ and to fit the mean $\Delta m_{15} = 1.05 \pm 0.04$ of 13 SNe Ia compiled by Branch et al. (1996a), all of which come from blue galaxies similar to the parent galaxies of the seven Cepheid-calibrated supernovae¹.

Knowing M_B , m_B and z we use Eqs.(1) and (2) to evaluate H_0 for each supernova with b and q_0 as parameters. The uncertainty in H_0 for each supernova is obtained by combining in quadrature the quoted errors on m_B and Δm_{15} with an uncertainty in the luminosity distance due to a possible peculiar motion $\delta v = 400$ km/s of the host galaxy with respect to the Hubble flow. Thus²

$$\delta H_0 = H_0 \sqrt{\left(\frac{\ln 10}{5}\right)^2 [\delta m_B^2 + (b\delta\Delta m_{15})^2] + \left[\left(\frac{1}{z} + \frac{1 - q_0}{2}\right) \delta z\right]^2} \quad (3)$$

The uncertainty in absolute magnitude calibration is common to all the supernovae so is not, at this point, included. A

¹ This value for Δm_{15} is in good agreement with the weighted mean of 1.04 ± 0.03 for the four SNe fitted by Hamuy et al. (1996).

² This is an exceedingly good approximation for δH_0 to lowest order in q_0 , differing from the exact expression at the 10^{-5} level for all SNe considered in this paper.

weighted average of the quantities $H_0 \pm \delta H_0$ for each of the 32 supernovae is calculated. In this way a least-squares value for H_0 along with its uncertainty and a χ^2 for the fit is obtained with b and q_0 as parameters. These parameters are then varied to search for a χ^2 minimum.

3. Results

The 26 CT supernovae (Hamuy et al. 1996) are combined with six cosmological supernovae of Perlmutter et al. (1996) and fit simultaneously. (The seventh cosmological supernova SN 1992bi, for which there was neither spectroscopic Type Ia identification nor color confirmation of being non-reddened, was eliminated due to clear evidence of a lower luminosity when compared with the other six, as will be discussed later.) With these 32 SNe we obtain a minimum reduced χ^2 of 1.47 to be compared with 0.98 expected for 32 data points and 3 parameters. This results in a low confidence level (CL) of 0.05. Note that, in contrast to earlier supernovae analyses, we have not assigned any intrinsic uncertainty to the SNe absolute magnitudes apart from a dependence on Δm_{15} . Previous work generally assumed there to be a 0.2 mag intrinsic uncertainty in the supernova magnitude to be added to the measurement uncertainty. If we were to follow this procedure by adding in quadrature 0.2 mag intrinsic uncertainty then the minimum reduced χ^2 would fall to 0.48, resulting in an exceedingly high CL = 0.99, so that amount of added uncertainty is clearly excessive. A more reasonable additional uncertainty, 0.1 mag, yields a reduced $\chi^2 = 0.96$ or a CL = 0.53. This maximum CL is obtained for $H_0 = 59.9$ and the parameters $b = 0.75$ and $q_0 = 0.37$.

An alternative, perhaps more appropriate, modification of the uncertainties is suggested by inspection of the residuals obtained from the difference between the measured and the fitted value of H_0 divided by the uncertainty δH_0 for each of the supernovae. Fig. 1a reveals that although the bulk of the supernovae satisfactorily follow a gaussian distribution, two of the CT SNe lie at more than 2.75 standard deviations from their best-fit expectations whereas for this sample of 32 SNe only 1/5 of an event is expected to have such a large residual. These two events (1992bh and 1992bp) contribute nearly all of the observed excess χ^2 . With this in mind, if we add in quadrature to the two SNe with large residuals an intrinsic uncertainty of 0.2 mag of unknown origin, then a satisfactory minimum reduced $\chi^2 = 0.99$ is obtained (CL = 0.47). For this minimum $H_0 = 60.4 \pm 0.8$ (statistical uncertainty) with $b = 0.78$ and $q_0 = 0.385$. Fig 1b shows that the residuals so obtained follow a reasonable gaussian expectation. We adopt this fit as a basis for further discussions. The 1σ statistical uncertainties in b and q_0 for this fit are ± 0.23 and ± 0.39 respectively. These errors are sufficiently large to easily encompass alternative treatments of the data to correct for the excess χ^2 .

In connection with uncertainties, we note that Eq. (3) has been written with no correlation between m_B and Δm_{15} . This is because no correlations have been presented by the observers. These correlations can in principle be either positive or negative, leading to an increase or a decrease in δH_0 . For example, if

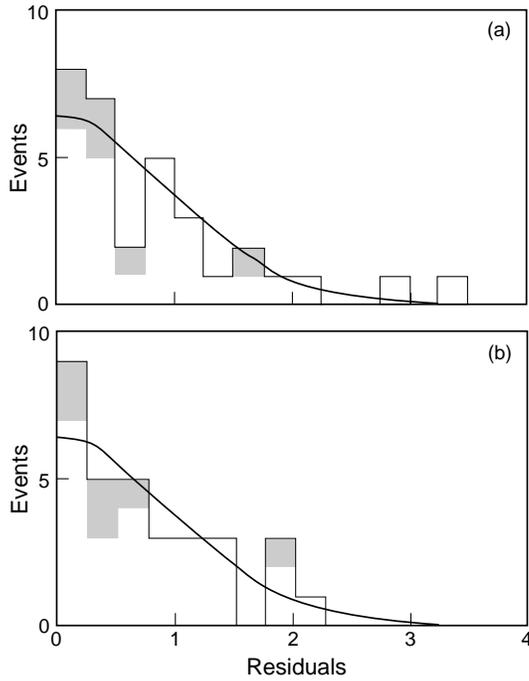


Fig. 1a and b. Distribution of residuals (the difference between the measured and the fitted value of H_0 divided by the uncertainty δH_0) for the 32 supernovae. The six cosmological SNe are shown shaded. **a** residuals with the quoted uncertainties. **b** after adding 0.2 mag uncertainty in quadrature to the two SNe having the largest residuals

the galactic background subtraction is shifted within its uncertainty, it may affect both m_B and Δm_{15} and contribute to their estimated uncertainties in a correlated way. Thus a decrease in the subtracted galactic background would increase the apparent supernova signal and, although in the first approximation would not affect Δm_{15} , if the entire light curve is fit with a lower background, plausibly it would decrease Δm_{15} . Correlated in this way, δH_0 would diminish. Other scenarios can lead to a positive correlation and a larger δH_0 . If such correlations are found to be significant, they should be reported with future results of observations.

To the statistical error of ± 0.8 in H_0 coming from the least-squares fit must be added an error in H_0 arising from the statistical uncertainties in b and q_0 . The combined error from these three sources is ± 2.0 . The dominant contribution to δH_0 arises however from the uncertainty δM_B in the absolute magnitude calibration. As can be seen from Eq.(1), these quantities are related through $\delta H_0 = 28 (H_0/60)\delta M_B$. Following Branch et al. (1996a), we assign 0.15 mag uncertainty to the Cepheid zero point, along with 0.07 mag uncertainty in the seven Cepheid-calibrated SNe and 0.04 mag uncertainty in their mean Δm_{15} . Combining these absolute magnitude calibration errors in quadrature yields $\delta M_B = 0.17$ or $\delta H_0 = \pm 4.7$ for the Cepheid calibration uncertainty, thus leading to a total quadrature-added uncertainty in H_0 of ± 5.1 .

Fig. 2 displays the Δm_{15} dependence clearly apparent in these data. Here we have plotted the values of H_0 obtained for

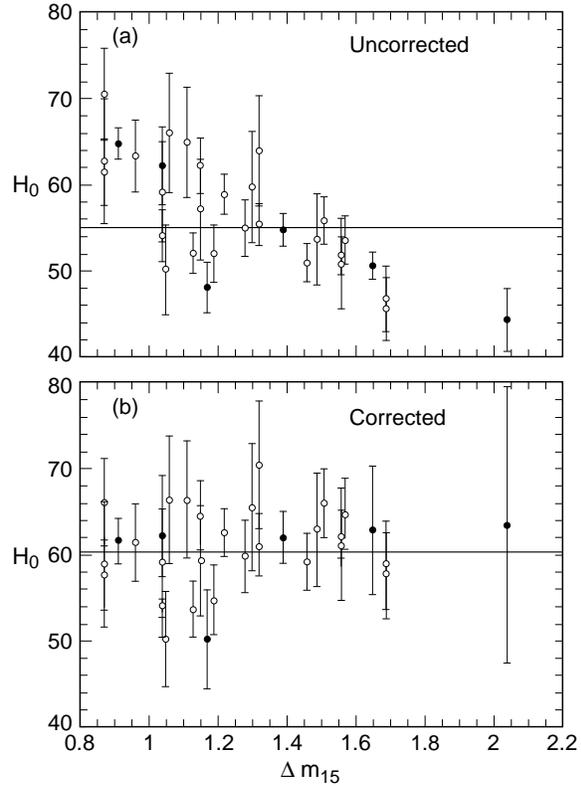


Fig. 2a and b. The dependence of H_0 on Δm_{15} for two values of the slope parameter b calculated for the 32 supernovae using Eqs. 1 & 2 (with $q_0=0.385$). Here Δm_{15} is the decline in magnitude of the supernova luminosity during the first 15 days beyond maximum: **a** H_0 obtained for these SNe with b set to zero. The very low confidence level (CL = 10^{-10}) of this best-fit value $H_0 = 55.0$ shows that a Δm_{15} dependence is required to yield a consistent value for H_0 . **b** H_0 obtained with $b = 0.78$, yielding the lowest χ^2 and resulting in good fit to $H_0 = 60.4$. Note that both the 26 CT supernovae and the six cosmological supernovae call for essentially the same value of b

the 32 SNe vs. Δm_{15} for the case $b = 0$ in Fig. 2a and for the case $b = 0.78$ in Fig. 2b (both for $q_0 = 0.385$), along with their best-fit values of H_0 . Two things are evident from these figures. First, the uncorrected data shown in Fig. 2a for both sets of data clearly call out for a Δm_{15} dependence since the best-fit common value $H_0 = 55.0$ is manifestly a poor fit to the data with a reduced χ^2 of 3.66, while the best fit with $b = 0.78$ shown in Fig. 2b results in a satisfactory confidence level³. Brighter supernovae result in slow-decliners with smaller values of Δm_{15} . Secondly, the simple phenomenological parametrization of Eq.(2) seems from Fig. 2b to adequately describe this dependence within the

³ There is a bias associated with χ^2 minimization that favors a larger b parameter. This comes about because of the appearance of b in the Eq.(3) expression for δH_0 , leading to a lower χ^2 for a larger b . The bias is inconsequential for the CT data where in most cases the uncertainty due either to δz or Δm_B dominates the determination of δH_0 , but this will not be the case for cosmological SNe (where the uncertainty due to peculiar motion becomes unimportant) if uncertainties in Δm_{15} are dominant.

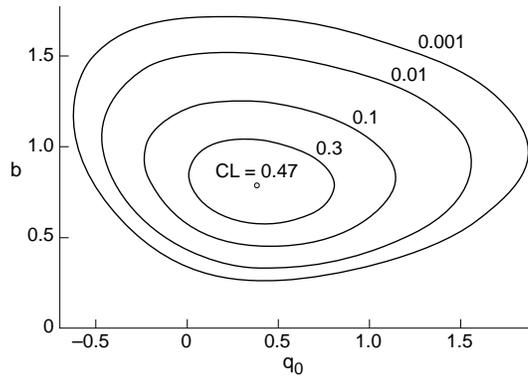


Fig. 3. The confidence level of fits to 32 supernovae as a function of slope parameter b and deceleration parameter q_0 , showing that these parameters are essentially uncorrelated

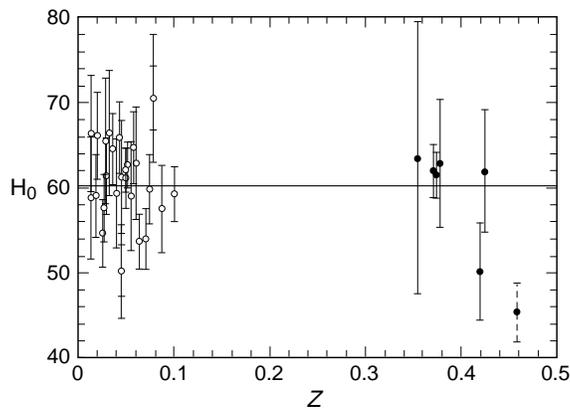


Fig. 4. The Hubble constant vs. redshift z calculated from measurements of each of the 32 supernovae using Eq.(1) with $q_0 = 0.385$ and $b = 0.78$. SN 1992bi, shown dashed, has been excluded from the fit

uncertainties of the data over a Δm_{15} interval of at least one, representing a luminosity range of more than a factor of two. Note from these figures that for these data H_0 and b are correlated with $\delta H_0 / \delta b \approx 6$. In Fig. 3 the confidence level of the fit is shown as a function of b and q_0 . From this figure it can be seen that these two parameters are essentially uncorrelated.

Fig. 4 is a plot of H_0 vs. z , showing that a common $H_0 = 60.4$ yields a good fit to the two data sets when, as in Fig. 2, two of the CT SNe (at $z = 0.045$ and 0.079) have their measured uncertainties augmented with an intrinsic uncertainty of 0.2 mag as indicated by the double error bars in this figure. Here we also plot the cosmological SN 1992bi of Perlmutter et al. (1996). Shown dashed, it falls at $H_0 = 45.5 \pm 3.4$, more than four standard deviations below the best-fit value, so that a dimming of about 0.5 magnitudes either by extinction or due to the supernova not being of Type Ia are much more likely reasons for the poor agreement than is a 4σ fluctuation. Since neither of these possibilities for SN1992bi is contradicted by direct observations, it is best discarded so as not to bias any fits obtained when combining it with the other six SNe. For

example, using SN 1992bi alone, along with the 26 CT SNe, would lead to a negative $q_0 = -0.52$.

We now turn to a generalization of the phenomenology to include a cosmological constant Ω_Λ (Carroll, Press, & Turner 1992; Goobar & Perlmutter 1995), which affects q_0 through the more general relation $q_0 = \Omega_M/2 - \Omega_\Lambda$. In this situation where both Ω_M and Ω_Λ are present, the luminosity distance related to the log term in Eq.(1) has not been expressed in closed form and is therefore written as an integral over z . For simplicity we limit ourselves to a spatially flat universe where $\Omega_M + \Omega_\Lambda = 1$, the case theoretically preferred for an inflation-based cosmology. The last term in Eq.(1) then becomes (Carroll, Press, & Turner 1992)

$$\log(1+z) \int_0^z [(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda]^{-1/2} dz \quad (4)$$

With Eq.(1) thus modified, and using the 32 supernovae, we follow the same procedure as before to explore χ^2 with b and $\Omega_M (= 1 - \Omega_\Lambda)$ as parameters. The best fit for a spatially flat universe is found with $\Omega_M = 0.89 \pm 0.37$ and $b = 0.78$, yielding a CL = 0.48 for 29 degrees of freedom⁴.

4. Discussion

Fig. 5a shows the confidence level for fits using the six cosmological SNe with b fixed at 0.78 as a function of Ω_M for both the spatially flat universe containing Ω_Λ and for a universe with $\Omega_\Lambda = 0$. For the latter case essentially the entire range $0 \leq \Omega_M \leq 1$ is permitted whereas for the spatially flat universe $\Omega_M \lesssim 0.50$ is excluded at the 1σ level. Fig. 5b shows the age of the universe as a function of Ω_M for both cases. For $H_0 = 60$ and $\Omega_\Lambda = 1 - \Omega_M$, this ranges from about 11 Gyr for $\Omega_M = 1$ to over 20 Gyr for $\Omega_M = 0.1$. Both curves in Fig. 5b should be viewed as having a $\pm 8\%$ error band coming from an uncertainty in H_0 of five units. A recent review (VandenBerg et al. 1996) of stellar ages in globular clusters gives for these objects a maximum age and 1σ error of $15.0^{+2.5}_{-1.5}$ Gyr. Adding about 1 Gyr for the first stars to form would imply a universe age of $16^{+2.5}_{-1.5}$ Gyr as indicated by the shaded area in figure 5b. By comparing Figs. 5a and 5b we may draw the following conclusions. At the present time with only six cosmological supernovae the uncertainties on Ω_M are sufficiently large to allow an $\Omega_\Lambda = 0$ open universe with a small mass density $\Omega_M \approx 0.3$ and still be compatible with the age constraint on the universe. Likewise a flat universe with $\Omega_M \approx \Omega_\Lambda$ would be consistent with this analysis and with the age constraint. On the other hand, a flat universe without a cosmological constant

⁴ Somewhat tighter uncertainties on Ω_M could be obtained in the following manner. Since the 26 CT SNe have only a minor effect upon Ω_M but contribute most of the degrees of freedom in the calculation of the uncertainties, for the error evaluation it is more constraining to fix $b = 0.78$ from the combined fit and use only the six cosmological SNe to evaluate Ω_M and its error. This yields in a spatially flat universe $\Omega_M = 0.93 \pm 0.29(1\sigma \text{ error})$ with a maximum CL = 0.58 for the six cosmological SNe with five degrees of freedom. Following the same procedure for a universe with $\Omega_\Lambda = 0$ results in $\Omega_M (= 2q_0) = 0.84 \pm 0.56$ with the same confidence level.

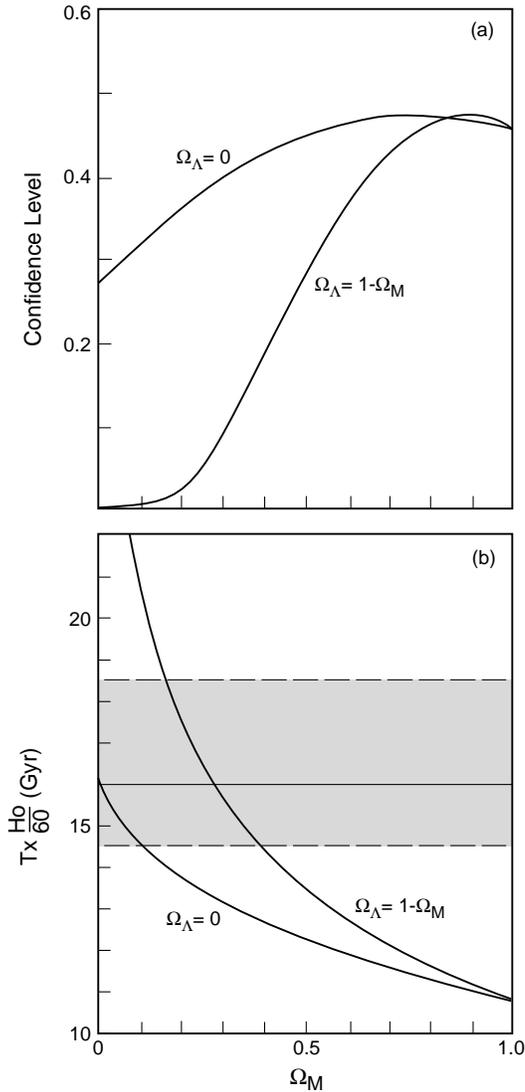


Fig. 5a and b. The confidence level of fits vs. Ω_M , obtained for a universe with $\Omega_\Lambda = 0$ and one for a spatially flat universe with $\Omega_M = 1 - \Omega_\Lambda$. **b** The age of the universe scaled to a Hubble constant $H_0 = 60$ vs. Ω_M for both cases. The shaded region shows the 1σ age range of the universe permitted by the maximum age of stars in globular clusters according to VandenBerg et al. (1996) after allowing 1 Gyr for stars to form

($\Omega_M = 1$), while in good agreement with the expansion parameters obtained here would result in an age of the universe of 10.8 ± 0.9 Gyr and would thus be in conflict with an age determined from globular clusters. The disagreement lies somewhere between 2σ and 3σ , depending on whether one takes the errors on the two ages to be completely anticorrelated or completely uncorrelated respectively⁵.

⁵ An insecure knowledge of the distance scale is the major uncertainty in both the determination of H_0 and the maximum age of globular clusters. The Madore & Freedman (1991) distance scale currently used fixes the distance to the Large Magellanic Cloud to be 18.5 mag ($=50.1$ kpc) and scales all other Cepheid-based distances to this value.

The value of the Hubble constant found here, $H_0 = 60.4 \pm 0.8$ (statistics) ± 1.8 (b, q_0 uncertainty) ± 4.7 (Cepheid calibration), rests heavily on the absolute magnitude calibration using Cepheid variables. A change in this absolute magnitude would however merely shift the vertical scales in Figs. 2 and 4, leaving conclusions concerning b, q_0 and the quality of the fit unaltered. In order to reconcile the value found here and in other analyses of SNe Ia (Hamuy et al 1995, 1996; Riess et al. 1995; Tammann & Sandage 1995, Saha et al. 1996; Branch et al. 1996a, 1996b) with the high values ($H_0 \approx 80$) found by other means (Pierce et al. 1994; Freedman et al. 1994), a change of more than one-half magnitude of the SNe Ia absolute magnitude would be required. However, this large discrepancy cannot be even partially attributed to the considerably smaller (0.15 mag) uncertainty in the Cepheid distance scale since both methods rely upon this same scale. The much smaller differences between the H_0 found here and those of other distant SNe Ia analyses that use some or all of the CT data and find H_0 ranging from 55 to 67 arise from different choices of the Cepheid-calibrated absolute magnitude, differences in the value of b, and from use in other analyses of the nearby approximation to Eq.(1)⁶. The particular advantage of using these remote supernovae to measure H_0 is that their great distance makes negligible the effect of peculiar velocity inherent in nearby objects when used to measure the cosmic expansion rate. Incorporating cosmological SNe into the fit as done here for the first time strengthens the case for a low value for H_0 by removing all doubts concerning the effects of peculiar motion on its determination. At the same time it supports the initial assumption of a homogeneous universe, since the best-fit value of H_0 for the six cosmological SNe taken by themselves differs from that for the 26 nearer CT SNe by only 0.5 ± 2.0 units of H_0 (see also Kim et al. (1997)).

This same advantage of using remote supernovae is realized when establishing SNe Ia as standard candles since otherwise peculiar velocity or other distance uncertainties introduce a significant apparent dispersion in the absolute luminosity. The question of reliability of SNe Ia as standard candles appears to be resolved by the data of Hamuy et al. (1996). With these data we are dealing with a homogeneous sample of distant SNe whose relative distances are found within a reasonable certainty through measurement of the parent galaxy redshift and whose

Most of the H_0 uncertainty in this paper comes from lack of precise knowledge of this Cepheid distance scale. Likewise, the uncertainty in age of globular clusters comes largely from uncertainty in the absolute magnitude of RR Lyrae variables found in the globular clusters, in the Galactic field, and also in the Large Magellanic Cloud (see Feast (1997) for recent discussion). This depends even more strongly on the chosen distance scale. Thus, a significant change in this scale would have a substantial effect on both age determinations. The ages are anti-correlated such that the disagreement between the two ages would decrease if distances were to increase.

⁶ The value $H_0=63.3$ found by Hamuy et al. (1996) can be reproduced using their 26 CT SNe (with their $b=0.784$, $v_{pec}=600$, and their quoted errors) if the calibrating absolute magnitude of -19.48 in Eq. 2 is dimmed to -19.37 for $q_0=0.385$ or to -19.33 when using, as they do, the nearby approximation ($q_0=1$).

magnitudes and decline rates are measured in a uniform and systematic way. Some 90 % of these normal, unreddened SNe Ia have a dispersion consistent with measurement errors alone, with the remainder correctable by the addition of 0.2 mag of intrinsic uncertainty of unknown origin. This conclusion is nicely confirmed by the six cosmological SNe Ia of Perlmutter et al. (1996) whose quoted uncertainties alone yield a CL=0.57 with the same b parameter used to standardize all 32 SNe.

The value of q_0 found here is somewhat lower than those of Perlmutter et al. (1996) who present results of an analysis using five of their seven cosmological SNe, having first obtained from Hamuy et al. the magnitude zero point and b parameter with a selected sample of 18 CT SNe. The difference in q_0 between the two analyses (0.385 vs. 0.44) is within statistical uncertainty, but since we are dealing with essentially the same data set, one would have expected closer agreement. The discrepancy can be attributed to inappropriate use of the nearby approximation to the luminosity distance for the CT data which, as can be seen from Eq.(1), is equivalent to setting $q_0 = 1$. This has the effect of lowering the average CT value of H_0 by nearly one unit, thereby requiring a larger q_0 for the cosmological SNe in order to lower the latter values by an equivalent amount when the full expression in Eq.(1) is used for them. This demonstrates the hazards of using the nearby approximation for SNe when z approaches 0.1 and the advantage of the unified treatment presented in this paper.

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