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Gravitational microlensing of large sources including shear term effects

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Abstract. The standard deviation δm of the variations in magnitude due to microlensing is calculated for the case when the angular radius (θ_s) of the source is much larger than that of the Einstein ring ($\theta_\circ$) for the microlens, ($\theta_s \geq 5\theta_\circ$). An external shear γ different from zero is included in the calcuations. We find that a shear term usually has the effect of reducing the value of δm, and in all cases δm is smaller than the analytical approximation for the case γ = 0 given by Refsdal and Stabell (1991), (RS). Thus the approximation can still be used to estimate an upper limit of the source size if δm is available from observations. We argue that microlensing effects on the radio core of QSO 0957+561 may be of importance for the modelling of that system, and hence for the value of the Hubble parameter, even if the core radius is as large as one light year.

Key words: quasars: general – QSO 0957+561 – gravitational lensing

1. Introduction

The most spectacular effects of microlensing occur when the angular radius of the source (θ_s) is much smaller than that of the Einstein ring ($\theta_\circ$) for the microlens, and this case has therefore been investigated theoretically in most detail up till now. Although microlensing of large sources ($\theta_s \geq 5\theta_\circ$) may cause significant effects (RS), it has been much less considered till now. The reason for this may be the long timescales for these effects when the lenses have about one solar mass. However, when comparing flux ratios of the different images in multiply lensed quasars, the timescale is of no importance.

For large sources ($\theta_s \geq 5\theta_\circ$) RS derived a usefull analytical approximation for the standard deviation of the variations in magnitude due to microlensing:

$$\delta m_a = 2.17\sqrt{|\kappa|}\ \frac{\theta_\circ}{\theta_s} \tag{1}$$

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Here κ is the optical depth for microlensing. The shear γ was assumed to be zero, and the surface luminosity was constant over the source. One lens plane with a random star distribution was also assumed. The variations in the magnitude m are mainly due to fluctuations in the average surface mass density projected in front of the source caused by Poisson fluctuations in the number of stars. Numerical calculations showed that for κ-values equal to 0.1 and 0.4 the standard deviation δm was always less than δm_a given by Eq. (1), (up to 20% for sources with $\theta_s \approx 5\theta_\circ$), the difference getting smaller for larger sources, hence;

$$\delta m < \delta m_a \tag{2}$$

We have carried out more extensive calculations for various values of κ and $\theta_s/\theta_\circ$ and also included shear terms ($\gamma \neq 0$). Since, for the large sources we are considering, $\theta_s \geq 5\theta_\circ$), the timescale of variability is normally rather long ($\tau > 50\sqrt{M/M_\odot}$ years), we shall not here be simulating lightcurves, but concentrate on determining by numerical simulations the standard deviation δm in the magnitude variation caused by microlensing. This is particularly important in the case of lensed quasars with multiple images, since δm can then often be estimated by measuring flux ratios between the images in different wavebands, or the equivalent widths of an emission line in the different images, see discussion in Section 4. A few details on the numerical method is briefly commented upon in Section 2, and the results are presented in Section 3.

2. Numerical method

The calculations were carried out with conventional backwards ray tracing techniques, for details we refer to earlier work (Kayser et al. 1986 (KRS), Schneider and Weiss 1987, Wambsganss et al. 1990). Since we are primarily interested in the standard deviation δm of the microlensing variation, we found it most convenient to generate a new star field in the lens plane each time a magnitude was calculated (i.e. for each loop). We usually calculated N_m = 1000 independent magnitudes (1000 loops) for each case with a given set of parameter values ($\theta_s/\theta_\circ, \kappa, \gamma$). If we should have calculated a light curve with 1000 "independent" points, we would in some cases have had to introduce a star field of more than 10^9 stars, a difficult requirement, imposing heavy demands on the computer memory. Also, the correction for the not exact independence of the points in a lightcurve is difficult to estimate.

The size of the star field was in each case kept constant, and for each loop the stars were randomly distributed in the lens plane. The number of stars, however, was not constant for the different loops, but followed a Poisson distribution such that the expected number of stars gave the "correct" value of κ. Only then will the number of stars projected in front of the source be Poisson distributed in each case (given set of parameter values). With a constant number of stars the distribution would have been binomial, since the number of stars is finite.

From standard statistics we find that with $N_m(\gg 1)$ independent values of the magnitude m, it is possible to estimate δm with an accuracy (standard deviation) of

$$\delta_1(\delta m) = \sqrt{\frac{1}{2N_m}}\,\delta m \tag{3}$$

With our chosen value of $N_m = 1000$ we then get $\delta_1(\delta m) \approx 0.022\delta m$.

Another source of statistical error in the determination of δm is caused by the finite number of traced rays, N_r, which hit the source. With a regular grid of rays traced through the lens plane ($N_r \gg 1$), it is found (KRS) that $\delta N_r \approx {}^4\sqrt{N_r}$. It is then easy to show that this gives rise to a standard deviation in δm equal to $\delta_2(\delta m) = 1.085 N_r^{-3/4}$ and that the combined (total) standard deviation is

$$\delta(\delta m) = \sqrt{\delta_1(\delta m)^2 + \delta_2(\delta m)^2} \tag{4}$$

We usually chose N_r such that $\delta(\delta m) < 0.03\delta m$. In a few cases for the largest sources $\delta(\delta m)$ approaches $0.05\delta m$.

3. Results

a) $\gamma = 0$
For the case $\gamma = 0$ we have calculated δm for three different source sizes ($\theta_s/\theta_\circ = 5, 10, 30$) and a large range of κ-values. We have plotted in Fig. 1 the values of δm together with δm_a as a function of κ. We see that δm is always smaller than δm_a for the same value of $\theta_s/\theta_\circ$ and κ. Except for κ-values close to one, where the amplification gets very large and δm approaches zero, compare Deguchi and Watson (1987), δm_a is a good approximation for large sources. As a rule of thumb, δm_a represents a reasonable approximation to δm for $\theta_s > 5\theta_\circ$. The deviation of δm from δm_a is then less than 30% for $|\kappa - 1| > 0.1$.

The underlying assumption for the derivation of Eq.(1) was that the variation in m was due to Poisson fluctuations in the number of stars projected in front of the source. In order to test this, we have investigated the correlation between m and the actual number of stars (N) projected in front of the source. The results are shown in Fig. 2, where we have plotted the correlation coefficient $CORR(-m, N)$ as a function of $\theta_s/\theta_\circ$, for three different values of κ with $\gamma = 0$. We see that the correlation increases

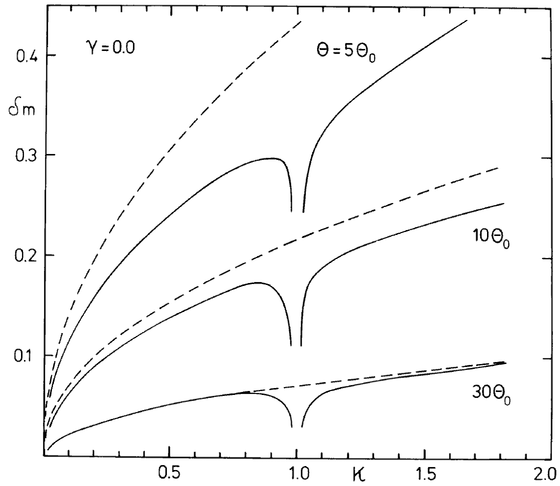

Fig. 1. The standard deviation δm (for $\gamma = 0$) of the mean apparent magnitude from the numerical calculations and δm_a according to the analytical formula, Eq. (1), (dashed line)

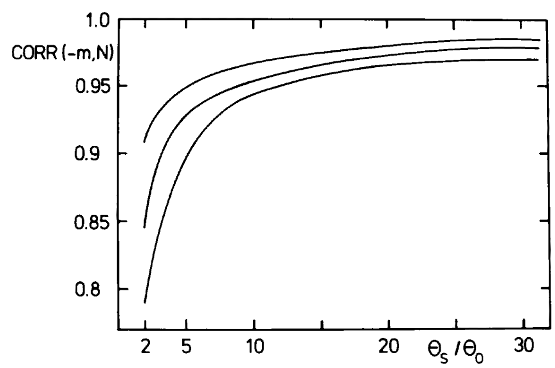

Fig. 2. The correlation between $-m$ and the number of stars N projected in front of the source for $\gamma = 0$. Results are plotted for $\kappa = 0.1$ (upper curve), $\kappa = 0.4$ and $\kappa = 0.8$ (lower curve)

with increasing source size and is larger than 0.9 already for $\theta_s \approx 5\theta_\circ$, confirming very well the underlying assumption.

b) $\gamma \neq 0$
For the case $\gamma \neq 0$ we have not succeeded in deriving a simple analytical formula similar to Eq. (1).

Some of the results from our numerical calculations are plotted in Figs. 3 and 4.
The standard deviation δm and the analytical approximation δm_a given by Eq. (1), are plotted as a function of κ for $\theta_s = 5, 10, 30\theta_\circ$ and $\gamma = 0.2, 0.4$.

It is generally found that also when shear terms are included, δm_a represents an upper limit for δm. Except for a narrow interval around $\kappa = 1$, we even find that the value of δm is smaller than for $\gamma = 0$. For a large range of γ-values we see however that the effect of the shear is rather small. An obvious exception is of course the "forbidden" intervals around $\kappa = (1 \pm \gamma)$ where the amplification again gets very large and δm approaches zero. Eqs.(1) and (2) can therefore still be used to

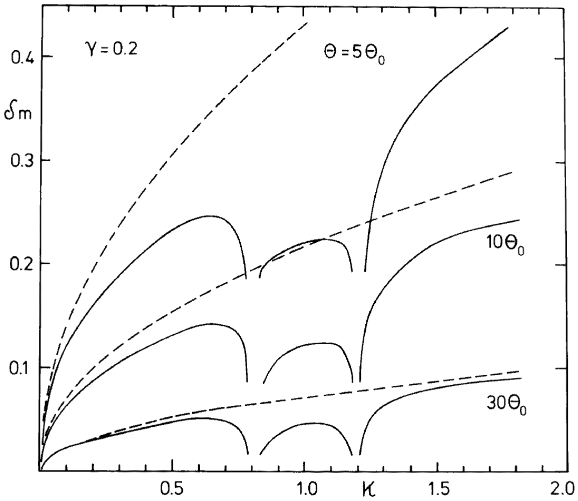

Fig. 3. The standard deviation δm for the case γ = 0.2 together with δm_a given by Eq. (1), (dashed)

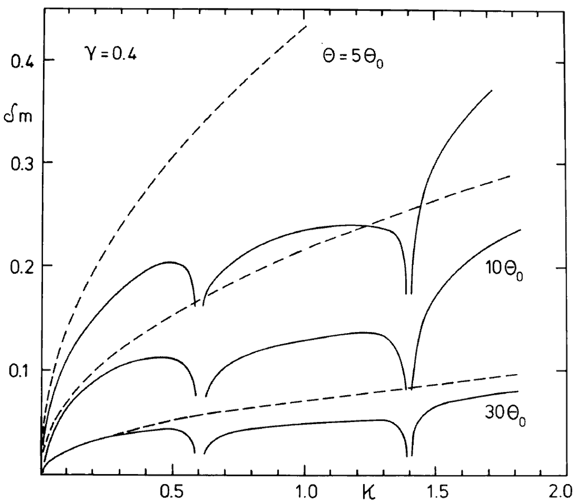

Fig. 4. The standard deviation δm for the case γ = 0.4 together with δm_a given by Eq. (1), (dashed)

estimate an upper limit of the source size if δm is available from observations:

$$\theta_s < 2.17\sqrt{|\kappa|}\ \theta_\circ/\delta m \tag{5}$$

4. Astrophysical applications

4.1. Flux ratios

The effects discussed above may be of some importance for the determination of $H_\circ$ by means of the time delay in QSO 0957+561A,B because of microlensing effects on the radio core. Our argument is based on the theoretical modelling by Grogin and Narayan (1996) who use radio observations of Garrett et al. (1994). One of the constraints of their model is the gradient along the jet (A image), $\dot{M}_2$, of the eigenvalue M_2 of the relative magnification matrix transforming the A image into B. The observed value of $\dot{M}_2$ is $(2.6 \pm 0.8)10^{-3} mas^{-1}$ measured upward along the A jet, corresponding to a difference in M_2 between core and jet (Component 5 in image A at an angular distance of $48mas$ from the core) of -0.13 ± 0.04. We find that an error of 0.04 in $M_2 (M_2 \approx -0.63$ in the core) amounts to an error of about 0.07 mag in the core, and vice versa since according to Garret et al. "the change in the relative magnification from the jet to the core is mainly caused by the spatial derivative of M_2". Therefore the uncertainty of $\dot{M}_2$ should be changed:

$$\dot{M}_2 = (2.6 \pm 0.8\sqrt{1 + (\frac{\delta m}{0.07})^2}\ )\ 10^{-3} mas^{-1} \tag{6}$$

Hence, microlensing effects are important if δm is about 0.07 or larger. We must here take into account that the core images of both component A and B are affected, so that the relevant analytical approximation for the standard deviation is that of the difference between the two images

$$\delta m_a = 2.17\sqrt{|\kappa_A| + |\kappa_B|}\ \frac{\theta_\circ}{\theta_s} \approx 2\frac{\theta_\circ}{\theta_s} = 2\frac{\zeta_\circ}{R_s} \tag{7}$$

where we have assumed $|\kappa_A| + |\kappa_B| = 1$, and $\zeta_\circ$ is the Einstein radius projected on to the source plane.

With a typical lens mass of $M = 0.5M_\odot, H_\circ = 65km\ s^{-1}Mpc^{-1}$ and density parameter $\Omega = 1(\Lambda = 0)$, we find $\zeta_\circ = 0.012pc = 0.04l.y.$. In view of our numerical results we roughly estimate the standard deviation δm in Eq. (6) to be about 20% smaller than δm_a in Eq.(7), i.e.

$$\delta m \approx 0.8\delta m_a \approx \frac{0.07}{R_s(l.y.)} \tag{8}$$

We thus see from Eq.(6) that the estimated uncertainty in $\dot{M}_2$ (one standard deviation) is significantly increased if R_s is about one l.y. or less ($R_s/\zeta_\circ = \theta_s/\theta_\circ < 25$), and that this could lead to a change in the best-fit model of Grogin and Narayan (1996) and thus a change in the estimated value of $H_\circ$.

In view of the observed variability of the radio core of more than 20% in one year (5 months in the source system), a radius of less than one l.y. does not seem unrealistic. It is also interesting to note that the discrepancy between the observed and the model-predicted value of $\dot{M}_2$ in the best-fit model og Grogyn and Narayan is rather large (nearly two standard deviations).

We finally note that the relative importance of microlensing effects will be greater when more accurate radio observations are available.

4.2. Emission lines

A constraint on the source size can be obtained by measuring the equivalent widths of an emission line in the different images of a quasar. An interesting example of this has been published by Lewis et al. (1996) for QSO 2237+0305. Their data from 1991 on the CIV line for the four images give an estimated value of the standard deviation δm = 0.45. This is most likely entirely due to microlensing effects on the continuum and allows us to put an upper limit on the source size. Assuming typical κ-values of 0.5, we get from Eqs. 7 and 8 a two standard deviation upper limit for the angular size of the continuum source:

$$\theta_s < 7\theta_\circ \tag{9}$$

For this particular quasar we arrived at a similar conclusion by analyzing the lightcurve (see Refsdal and Stabell (1993)). Note however, that this system is very special in that the lensing galaxy is very close (z_d = 0.039), so that the microlensing timescale is much shorter than for other known systems. For these systems a light curve covering about 50 years or more would probably be required, so that the method using the equivalent widths is the only one applicable at the moment.

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