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Age of the universe constrained from the primordial nucleosynthesis in the Brans-Dicke theory with a varying cosmological term

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Abstract. The age of the universe is investigated from a viewpoint of the primordial nucleosynthesis in the Brans-Dicke model with a varying Λ term. It is shown that the age can be long enough compared with that of the globular clusters using two critical quantities recently reported from the HST observations: the Hubble constant, and deuterium abundances. Then it is shown that the present rate of variation in the gravitational "constant" can be predicted.

From the observational constraint in the primordial nucleosynthesis, the baryon density relative to the critical density is found to be $\Omega_b = 0.047 - 0.14$ from the observation by Songaila et al. (1996) and $\Omega_b = 0.16 - 0.22$ from that by Tytler et al. (1996), where the critical density is $\rho_c = 6.0 \times 10^{-30}$ g cm^{-3} for $H_0 = 80$ km s^{-1} Mpc^{-1} and a parameter characteristic to the Brans-Dicke theory. It is concluded that most of the matter consists of the non-baryonic dark matter if the universe is flat.

Key words: cosmology: dark matter – early universe – general: nuclear reactions – nucleosynthesis – abundances

1. Introduction

A big bang theory based on the Friedmann model has succeeded in a natural way to explain the expansion of the universe, the spectrum of the cosmic microwave background radiation, and the origin of the light elements. Therefore the model is called the standard model (see e.g. Weinberg 1972, Rees 1995, Schramm 1996). In spite of this success, the standard model seems still not to be perfect from the two observational points. First so called the *age problem* revives sometimes due to the still uncertain value of the Hubble constant ; Freedman et al.(1994) deduced the Hubble constant at the present epoch $H_0 = 80 \pm 17$ km s^{-1} Mpc^{-1} from the Hubble space telescope observations though the latest estimate (Feast & Catchpole 1997) suggests a lower value compared with that of Freedman et al. On the otherhand, the age of the oldest globular cluster (M92) in our galaxy was estimated to be $\simeq 17 \pm 2$ Gyr (Chaboyer et al. 1992). More recently, Sommer-Larsen (1996) has got the age of the universe to be 17 ± 2 Gyr from the study of disk galaxies. In a framework of the Friedmann model, the present age of the universe is $t_0 = 2/(3H_0)$ if the universe is flat. This leads to be $t_0 \simeq 8$ Gyr for $H_0 = 80$ km s^{-1} Mpc^{-1}. Though this value may be overestimated, generally speaking, it would be desirable for a cosmological model to give a longer age than that of a standard model.

Second, we must be careful when we compare the model with the observation of the light elements. For example, observations of deuterium in quasar absorption systems have given contradicting results between the analysis of Songaila et al. (1994, 1996) and that of Tytler et al. (1996). Therefore there occurs a claim that the observation by Songaila et al. (1994) seems to be incompatible with the primordial helium abundances calculated from the standard model (e.g. Hata et al. 1996, Kernan & Sarkar 1996). However more recent observational analysis by Songaila et al. (1996) is consistent with the standard model and now high D/H is not absolutely required though it is still credible (Hogan 1997). It should be noted that the primordial helium abundance is also under ambiguity (Olive et al. 1996).

To lenghen the age, a cosmological "constant" Λ is often introduced for convenience (e.g. see the review by Weinberg 1989). Though the introduction of Λ has been suggested by the inflation theory (e.g. see the review by Liddle 1996), compared with Λ during the inflation, its present value is too small. In this sense, it is natural to include a time varying cosmological term. However it is shown that a simple power law decrease in the cosmological term, $\Lambda \propto t^{-\alpha}$, is difficult to be compatible with the constraint obtained from the primordial nucleosynthesis (Birkel & Sarkar 1996). On the other hand, among the gravitational theories still surviving but for the general relativity, the most promising one is the Brans-Dicke (hereafter BD) theory (Brans & Dicke 1961). It was shown that there is a possibility that the inflation field corresponds to a scalar field included in BD theory (García-Bellido & Linde 1994). BD theory naturally yields

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the variation in the gravitational "constant" G. Degl'Innocenti et al. (1996) have estimated the present rate of the variation to be $-35 \times 10^{-12}\mathrm{yr}^{-1} \lesssim \dot{G}/G \lesssim 7 \times 10^{-12}\mathrm{yr}^{-1}$. This estimate might also give a constraint to the time variation of G.

In the present paper, we show that not only the *age problem* but the observed primordial abundances are explained by the Brans-Dicke theory with a varying Λ term (hereafter BDΛ) which is a function of a scalar field ϕ. This BDΛ theory was formulated by Endo & Fukui (1977). As for primordial nucleosynthesis, Arai et al. (1987) have already shown that BDΛ theory can explain the observed primordial abundances. However, their calculation was done under the assumptions of the characteristic parameter of $\omega = 6$ and two species of neutrinos, and they missed their calculations to relate with the *age problem*. Furthermore it should be clarified what is a relation between BDΛ universe and the dark matter. Therefore it must be worthwhile to investigate BDΛ theory with the use of a new value of $\omega \geq 500$ (Will 1984), input physics, the recent observations of light elements and the observed constraint of the variation in G.

We describe the characteristic feature of BDΛ universe in Sect. 2. Primordial nucleosynthesis and constraints to the baryon density are shown in Sect. 3. The age of BDΛ universe is presented in Sect. 4. Discussion and conclusions are given in Sect. 5.

2. Characteristic feature of the Brans-Dicke theory with a varying Λ term

The basic equations of the Brans-Dicke theory with a $\Lambda(\phi)$ term are the same as those of Arai et al. (1987). For completeness, we describe the necessary equations.

The field equations are written as follows:

$$R_{ij} - \frac{1}{2}g_{ij}R + g_{ij}\Lambda = \frac{8\pi}{\phi}T_{ij} + \frac{\omega}{\phi^2}(\phi_{,i;j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}) + \frac{1}{\phi}(\phi_{,i;j} - g_{ij}\Box\phi)\,, \tag{1}$$

$$R - 2\Lambda - 2\phi\frac{\partial\Lambda}{\partial\phi} = \frac{\omega}{\phi^2}\phi_{,i}\phi^{,i} - \frac{2\omega}{\phi}\Box\phi. \tag{2}$$

We employ the Robertson-Walker metric which assumes the homogeneous and isotropic universe (e.g. Weinberg 1972):

$$ds^2 = dt^2 - a(t)^2\left\{\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right\}, \tag{3}$$

where $a(t)$ is the scale factor and k is the curvature constant. Hereafter we set $c = 1$. We can get the equation of motion with the use of the Robertson-Walker metric from the (0,0) component of Eq. (1). Let x be a scale factor normalized to the present value of a ($x = a/a_0$), then the equation of motion is

$$\left(\frac{\dot{x}}{x}\right)^2 + \frac{k}{x^2} - \frac{\Lambda}{3} - \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\dot{x}}{x}\frac{\dot{\phi}}{\phi} = \frac{8\pi}{3}\frac{\rho}{\phi}, \tag{4}$$

where ρ is the density of the fluid.

We assume the simplest case of the coupling between the scalar and the matter field:

$$\Box\phi = \frac{8\pi}{2\omega+3}\mu T_i^i\,, \tag{5}$$

where μ is a constant. Assuming the perfect fluid for T_{ij}, Eq. (5) reduces to

$$\frac{d}{dt}(\dot{\phi}x^3) = \frac{8\pi\mu}{2\omega+3}(\rho - 3p)x^3, \tag{6}$$

where p is the pressure.

A particular solution of Eq. (2) is obtained from Eqs. (1) and (5):

$$\Lambda = 2\pi(1-\mu)T_i^i/\phi = 2\pi(\mu-1)(\rho-3p)/\phi. \tag{7}$$

For $\mu = 1$, the original Brans-Dicke theory is obtained.

We express the gravitational "constant" G as follows

$$G = \frac{1}{2}(3 - \frac{2\omega+1}{2\omega+3}\mu)\frac{1}{\phi}. \tag{8}$$

This is an extension of the original form given by Brans and Dicke (1961). It should be noted that BDΛ model reduces to the Friedmann model if ϕ = constant, $\mu = 1$, and $\omega \gg 1$.

To solve Eq. (4), the density and the pressure must be given:

$$\rho = \rho_m + \rho_r, \tag{9}$$

$$p = p_r = \rho_r/3, \tag{10}$$

where $\rho_m = \rho_{m0}x^{-3}$, $\rho_r = \rho_{r0}x^{-4}$ and the subscript "0" means the present value. The solution of Eq. (6) was obtained by Arai et al. (1987):

$$\dot{\phi} = (At + B)/x^3, \tag{11}$$

where $A = 8\pi\mu\rho_{m0}/(2\omega+3)$ and B is an integral constant (see also Weinberg 1972). They found that there exist solutions with $B < 0$ which means $\dot{\phi} < 0$ in the early period of the universe shorter than t_* for $\omega = 500$:

$$t_* < -\frac{B}{A} = 63\left(\frac{-B}{10^{-22}\ \mathrm{g\ s\ cm^{-3}}}\right)\left(\frac{\rho_{m0}}{10^{-30}\ \mathrm{g\ cm^{-3}}}\right)\ \mathrm{yr}.$$

Long after the epoch t_*, all solutions are converging to the solution with $B = 0$. These solutions are crucial for our case and are discussed in Sect. 3. Hereafter we use the normalized values of B and H_0: $B^* = B/(10^{-22}$ g s cm^{-3}) and $h = H_0/(100$ km s^{-1} Mpc^{-1}). The coupled equations (4), (7), and (11) can be solved numerically specifying the critical quantities: as for global quantities, $G_0 = 6.6726 \times 10^{-8}$ dyn cm^2 g^{-2}, $H_0 = 80$ km s^{-1} Mpc^{-1}, and $T_{r0} = 2.7$ K; as for microscopic quantities, the number of neutrino species is three (e.g. ALEPH Collab. 1990) and the neutron life time is 887 s (Particle Data Group 1994). In Eq. (7), μ is a free parameter. Noting that $\Lambda < 0$ if $\mu < 1$, and $\phi G < 0$ if $\mu > 3$ and $\omega \gg 1$. If $\mu = 3$, the age of the universe will become too short (see Sect. 4). Therefore it is reasonable to set $\mu = 2$ as done by Arai et al. (1987).

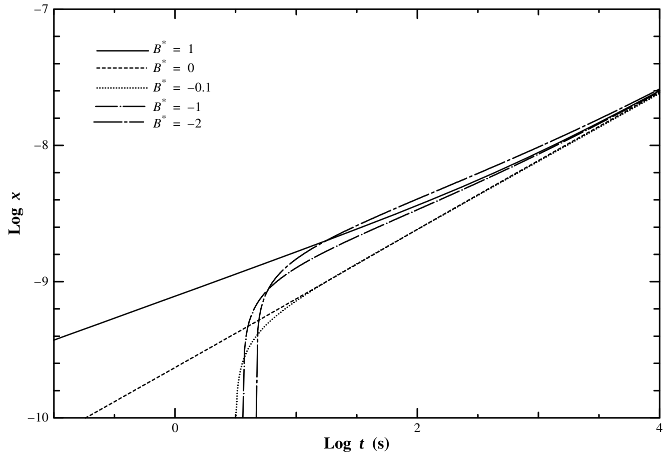

Fig. 1. Expansion scale x as a function of time in models with B^* = $1, 0, -0.1, -1$, and -2 for ρ_{m0} = 10^{-30} g cm^{-3}.

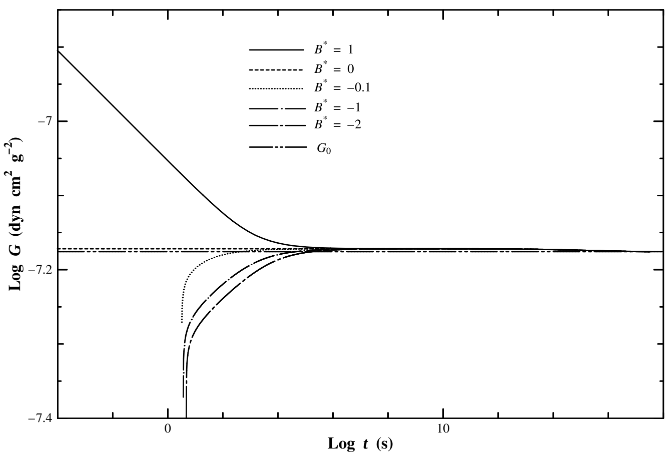

Fig. 2. Time variation of G for the same models as in Fig. 1.

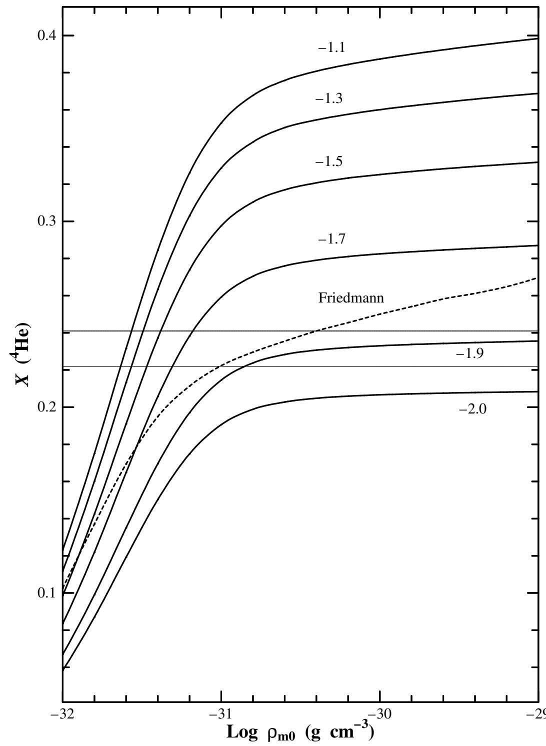

Fig. 3. Helium mass fractions synthesized for relevant values of B^*. The short-dashed line refers to the Friedmann model. The region between the thin lines indicates the observational determination.

Computational results of BDΛ model with several values of B^* are shown in Figs. 1 and 2 for ω = 500, h = 0.8, and ρ_{m0} = 10^{-30} g cm^{-3}. Fig. 1 shows the expansion scale as a function of time . We show in Fig. 2 the time variation of G. The dependence on B^* becomes insensitive for $t > 10^8$ s. Note that the effect of Λ remains appreciable till the present epoch as G does and this is the same tendency as the constant Λ model like the Lemaître model.

3. Primordial nucleosynthesis

Once values of B^* and ρ_{m0} are specified, nucleosynthesis has been calculated with the use of the nuclear reaction network (Hashimoto & Arai 1985) coupled to BDΛ model explained in Sect. 2. As is done by Arai et al. (1987), we have determined the value of B^* from the observed ^{4}He abundances because ^{4}He abundance is very sensitive to B^* as seen from Fig. 3. It is shown that models with $B^* \geq 0$ are ruled out due to over production of ^{4}He. We show in Fig. 4 the results of the primordial nucleosynthesis with B^* = -1.9. It is remarkable that the variation of ^{2}H abundance against ρ_{m0} is very mild in BDΛ models compared with that in the Friedmann model, because available neutrons to transform ^{2}H into ^{4}He becomes less abundant if $B^* \leq 0$. As the consequence, BDΛ model can be consistent with observations for both ^{4}He (Olive et al. 1996) and the recent observational results of deuterium (Songaila et al. 1994, 1996, Tytler et al. 1996). Adding the observational constraint of ^{7}Li by Copi et al. (1995), we finally obtain the following ranges to the present baryon density for B^* = -1.9:

$$2.8 \times 10^{-31} \leq \rho_{m0} \leq 8.1 \times 10^{-31} \text{ g cm}^{-3}$$

(Songaila et al. 1996) , (12)

$$9.5 \times 10^{-31} \leq \rho_{m0} \leq 1.3 \times 10^{-30} \text{ g cm}^{-3}$$

(Tytler et al. 1996) . (13)

These ranges are rather higher density side than the range in the Friedmann model as found by Arai et al. (1987). The consequences of this result will be discussed in Sect. 4. As is shown by Birkel & Sarkar (1996), a simple power law decrease of $\Lambda \propto t^{-\alpha}$ will cause a trouble to explain primordial abundances due to the constraint of the radiation dominant era if $\alpha \leq 1.75$. This is not the case of BDΛ model; it is crucial that the Λ term plays only a minor role during the epoch of the nucleosynthesis because $\Lambda \propto \rho_m/\phi$ which is much less than ρ_r/ϕ (see Eq. (4)).

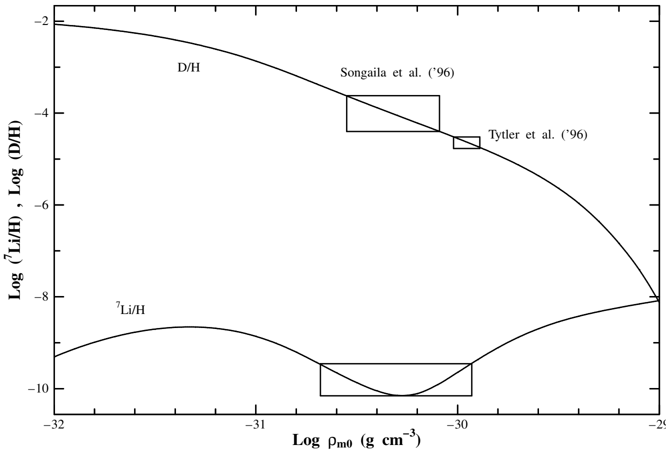

Fig. 4. Predicted primordial abundances versus the present baryon density ρ_{m0} with $B^{*} = -1.9$.

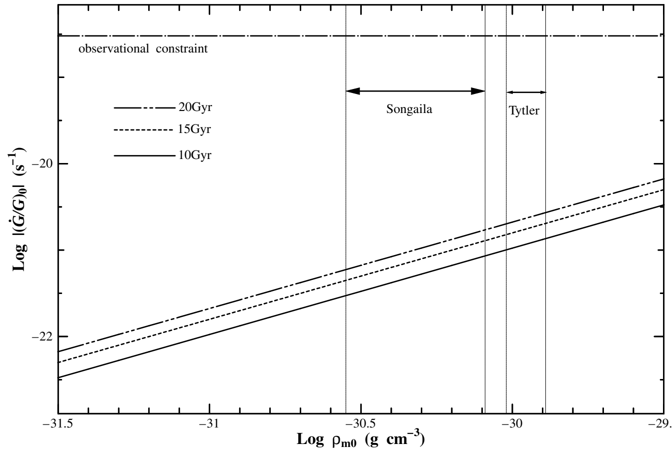

Fig. 5. Relation between the present variation of G normalized to the present G_0 and the ρ_{m0} for t_0 = 10, 15, and 20 Gyr.

4. Age of the universe

The present age of the universe t_0 and the baryon density ρ_{m0} are related each other through Eqs. (8) and (11):

$$\left(\frac{\dot{\phi}}{\phi}\right)_0 = -\left(\frac{\dot{G}}{G}\right)_0 = fG_0\rho_{m0}t_0\ , \tag{14}$$

where $f = 32\pi/(2\omega + 7)$ for $\mu = 2$. Eq. (14) leads to

$$t_0 = 15\left(\frac{\rho_{m0}}{10^{-30}\ \mathrm{g\ cm^{-3}}}\right)^{-1}\frac{(-\dot{G}/G)_0}{10^{-13}\ \mathrm{yr^{-1}}}\ \ \mathrm{Gyr}\ , \tag{15}$$

where $\omega = 500$. Now from the observational constraints of both the variation of G and the present baryon density, we can restrict the age of the universe. A relation between the variation of G and ρ_{m0} is shown in Fig. 5 for three assumed ages of the universe. There we show an upper limit of $-\left(\dot{G}/G\right)_0 = 10^{-12}\ \mathrm{yr^{-1}}$ which is adopted from rather a conservative point of view and the recent observational limits (Dickey et al. 1990, Anderson et al. 1991, Müller et al. 1991, García-Berro et al. 1995). We can see that the present upper limits by Degl'Innocenti et al. (1996) to $\dot{G}$ is too large to yield a constraint to the age. Using the constraint (12) or (13) derived from the primordial nucleosynthesis, t_0 can be greater than 15 Gyr. Therefore as far as BDΛ universe is concerned, we can do without the *age problem*. Taking account of the range (12) we can predict the variation of G from Eq. (15) (see also Fig. 5); for example if t_0 = 20 Gyr, we have

$$3.7 \times 10^{-14} \leq -\left(\dot{G}/G\right)_0 \leq 1.1 \times 10^{-13}\ \ \mathrm{yr^{-1}}\ . \tag{16}$$

It should be highly necessary to determine much more accurate measurements of $\dot{G}$. We note that if $\mu = 3$, then $f = 8\pi$. Thus t_0 becomes too short.

5. Discussion and conclusions

Our caluculational results for $\omega = 500$ are summarized in Table 1. Within the present uncertainty in $\left(\dot{G}/G\right)_0$, we have shown that BDΛ model can stand two crucial tests which should judge validity of the cosmological model. First, $BD\Lambda$ model can explain the recently observed primordial abundances. This is because compared with the result of the Friedmann model, the constraint by ^{4}He abundance has extended to a rather wide range of ρ_{m0}. Consequently, we can say that the deuterium observations do not give a severe constraint anymore. Second, the age of the universe is consistent with that of the age of the globular cluster which could be considered to have the longest age. In other words, we can predict the present rate of the variation of G if the oldest age of the globular cluster is determined.

Finally there remains a problem: what can be predicted about the dark matter if BDΛ universe is correct ? If the universe is flat (k = 0), then using Eq. (4), the present matter density ρ_0 for BDΛ model can be obtained from the following equation

$$\frac{1}{3}\left(\frac{8\pi\rho_0}{\phi_0} + \Lambda_0\right) + \frac{\omega}{6}\left(\frac{\dot{\phi}}{\phi}\right)_0^2 - \left(\frac{\dot{\phi}}{\phi}H\right)_0 = H_0^2. \tag{17}$$

The last two terms on the left side of this equation are small and may be neglected. Note that $1/\phi_0 = 2G_0$ for $\omega \gg 1$. Let us call $\rho_c^{BD\Lambda}$ the critical density of BDΛ model: $\rho_c^{BD\Lambda} = 3H_0^2/(16\pi G_0)$. In BDΛ model, we obtain from Eq. (7) $\lambda_0 = \rho_{m0}/(4\rho_c^{BD\Lambda})$, where $\lambda_0 = \Lambda_0/(3H_0^2)$. Then in analogy with the Lemaître model, we can transform Eq. (17) to the following form:

$$\Omega_0 + \lambda_0 = 1, \tag{18}$$

where $\Omega_0 = \rho_0/\rho_c^{BD\Lambda}$. Since $\rho_c^{BD\Lambda} = 9.4 \times 10^{-30}h^2\ \mathrm{g\ cm^{-3}}$, we have $\lambda_0 = 0.027\left(\rho_{m0}/(10^{-30}\mathrm{g\ cm^{-3}})\right)h^{-2}$. Therefore the contribution of the cosmological term to the closure is less than 10 %. In other words while the cosmological term in the Lemaître model is $3.5 \times 10^{-56}\lambda_0 h^2$, in the $BD\Lambda$ model $\Lambda_0/c^2 = 9.3 \times 10^{-58}\left(\rho_{m0}/10^{-30}\ \mathrm{g\ cm^{-3}}\right)\ \mathrm{cm^{-2}}$. The cosmological term in the BDΛ universe is well below the observational limit if $\lambda_0 > 0.1$ as pointed by Arai et al. (1987).

Table 1. Physical quantities in BDΛ universe

ω B^*	t_0 (Gyr)[a]	$\rho_{m0}/10^{-31}$ (g cm^{-3})		Ω_b	
		Tytler	Songaila	Tytler	Songaila
500 – 1.9	15	9.5 - 13	2.8 - 8.1	0.16 - 0.22	0.047 - 0.14

[a] Note that t_0 depends on both $\left(\dot{G}/G\right)$ and ρ_{m0} as seen in Eq. (15).

It should be noted that for the Friedmann model, ρ_c^{Fr} = $3H_0^2/(8\pi G_0)$ = 1.88 × $10^{-29}h^2$ g cm^{-3} which becomes 1.2 × 10^{-29} g cm^{-3} for h = 0.8. Note also that $\rho_c^{Fr} = 2\rho_c^{BD\Lambda}$. Since the present baryon density is constrained to the range either (12) or (13), the baryon density relative to the critical one is obtained to be $\Omega_b = 0.047 - 0.14$ from the observation by Songaila et al. (1996) and $\Omega_b = 0.16 - 0.22$ from that by Tytler et al. (1996). Since the luminous baryonic matter may contribute to Ω_b less than 1 % (e.g. Jedamzik et al. 1995), BDΛ model needs fair amount of baryonic dark matter. Therefore, for h = 0.6 – 0.9 we conclude that in BDΛ model, we need a fair amount of non-baryonic dark matter if the universe is flat.

BDΛ theory would worth investigating still more because it includes a scalar field which could play a crucial role in the very early epoch of the universe. García-Bellido and Linde (1994) have examined the fluctuation of G using BD theory and the relation with the inflationary model. We will propose that the Λ term should be included in this kind of approach. Though in the present study the role of the Λ term is not clear, it may play a role for the structural formation of the universe because Λ term exerts a repulsive effect during the evolution of the universe. The varying Λ term may become also important on the effects of the gravitational lensing among galaxies. Therefore, we stress that the much more precise measurements of the variation of G is crucially desirable to recognize the role of a scalar field in the universe.

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