

Cosmological parametrization of gamma ray burst intensity distribution

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Abstract. Using three parametrizations of the gamma ray burst count data comparison is made to cosmological source models. While simple models can fit $\langle V/V_{max} \rangle$ and faint end slope constraints, the addition of a logarithmic count range variable describing the curvature of the counts shows that models with no evolution or evolution power law in redshift with index less than 10 fail to satisfy simultaneously all three descriptors of the burst data. The cosmological source density that would be required for a fit is illustrated.

Key words: gamma rays: bursts – cosmology: theory

1. Introduction

Gamma ray bursts present an astrophysical puzzle as to their mechanism, source, and even locality. Despite knowledge of their existence for almost 25 years and over 1500 detected bursts, we still do not know if they belong to the realm of the solar system, galaxy, or high redshift cosmology, a state that would be unthinkable for other initially mysterious sources such as quasars. In the absence of a sufficiently robust clue from their time structure, energy spectrum, or a non gamma ray counterpart we turn to their spatial distribution. Their angular distribution on the sky is isotropic to 1% and no distance parametrization is known for them. We therefore have recourse only to the number-flux relation, $\log N - \log S$, although for gamma ray bursts the peak count rate in the detector, C , is used instead of the energy flux S .

Sect. 2 discusses the count distribution from the point of view of properties of the detector, including threshold bias, and of the cosmology, e.g. geometry. Sect. 3 introduces three parametrizations of the distribution for comparison of theoretical source models with the data. The main, new results of this paper are illustrated by Fig. 1 showing the lack of agreement between the data and those cosmological models with no evolution or evolution power law in redshift and by Fig. 2 showing what form of cosmological source distribution would be required to fit the data.

2. Count distribution

In analogy with other wavelength regimes (e.g. radio) of astrophysics, one considers the data set composed of values of the peak counts C and the count detection threshold C_{lim} . One wants to find the number distribution $N(C)$ in the hopes of this giving a clue to the source model. (For gamma ray bursts the count rate C is used instead of the energy flux S). An advantage often accrues to using a relative variable C/C_{lim} , in that many instrumental biases cancel out in such a ratio (see, for example, Schmidt, Higdon, and Hueter 1988). From this basic relative variable one can form the “volume fraction” variable $V/V_{max} = (C/C_{lim})^{-3/2}$. However the effect of variation of the threshold value on recovering the entire distribution $N(C)$ is problematic. Another difficulty is the nontrivial physical, i.e. spatial, interpretation of variables like this.

2.1. Source and detector variation

First we consider to what extent $v = V/V_{max}$ reflects the true source space distribution rather than effects arising from variations in source or detector properties. In Sect. 2.2 difficulties from the spatial geometry itself are discussed.

Define the relation between detected counts C , source count luminosity L , and “distance” r according to the usual inverse square law,

$$r \equiv (L/4\pi C)^{1/2}. \quad (1)$$

When C is replaced by C_{lim} this likewise defines the maximum “distance” D at which a source of (count) luminosity L could be seen by a detector with count threshold C_{lim} . Only when the source and detector properties, i.e. L and C_{lim} , are known can $v = (r/D)^3$ be interpreted in terms of a true distance or spatial measure. A range of L and C_{lim} around some fiducial values will cloud our understanding of the true source radial distribution. [Note that other variables could have a similar effect on interpreting r as a distance. For example, a distribution of source beaming properties, rather than isotropic emission, would affect the 4π factor in (1); energy spectrum variations enter the conversion of L from an energy luminosity to a count

rate; detector efficiency or sky coverage fluctuations alter the effective maximum volume, etc.]

The distribution function $p(v)$ is the fraction of detectable sources lying in a (pseudo) volume interval:

$$p(v)dv = \mathcal{N} dV/N_{tot}, \quad (2)$$

where \mathcal{N} is the source number density and N_{tot} is the total number of detectable sources, normalizing the distribution. Given dispersion in a property intrinsic to the source, say L , and one intrinsic to the detector, say C_{lim} , how is the distribution function affected relative to the standard candle, standard camera case? In other words, how are the distributions that include the property dispersions, denoted $[p(v; L)]_C$, $[p(v; C_{lim})]_L$, and $[p(v)]_{L,C}$, related to the fiducial case $p(v; L, C_{lim})$? The subscript notation indicates a weighted average with respect to that quantity, i.e. an integration over its variation.

The procedure is most familiar in the case of a luminosity function $\Phi(L)$, with $\mathcal{N} = n(V)\Phi(L)dL$ and n the spatial number density. One cannot simply write the integration over the dispersion as

$$[p(v; C_{lim})]_L = \int dL \Phi(L)p(v; C_{lim}, L), \quad (3)$$

because of the dependence in the normalizing factor N_{tot} . There is uniform weighting not in terms of the source fraction intrinsically existing at luminosity L , i.e. $\Phi(L)$, but in the fraction *detectable*, as the definition of N_{tot} states. That is,

$$\begin{aligned} [p(v; C_{lim})]_L &= \int dL \Phi(L)n(V)(dV/dv) / \\ &\quad \int dL \Phi(L) \int dV n(V) \\ &= \int dL \Phi(L)N_{tot}(L, C_{lim})p(v; C_{lim}, L) / \\ &\quad \int dL \Phi(L) \int dV n(V) \\ &= \int dL \psi(L, C_{lim})p(v; C_{lim}, L). \end{aligned} \quad (4)$$

Two alternative views of this procedure are equivalent: it is either a number weighting [by $\Phi(L)N_{tot}(L)/N_{tot}$] of the variable $p(v; C_{lim}, L) = N_{tot}^{-1}(L)n(V)(dV/dv)$, as in the last line, or a uniform weighting [by $\Phi(L)$] of the fundamental variable $N_{tot}^{-1}n(V)(dV/dv)$, as in the first line.

The case of dispersion in the threshold C_{lim} is completely analogous since again this variable enters in the normalization factor. Both dispersions together simply involve a double integration over the variables. In this case $\Phi(L)$ is joined by the analogous $f(C_{lim})$ and the weighting function ψ is generalized to

$$\chi(C_{lim}, L) = f(C_{lim})\Phi(L)N_{tot}(L, C_{lim}) / \int dC_{lim}f(C_{lim}) \int dL \Phi(L) \int dV n(V). \quad (5)$$

The effect is to introduce a coarse grained averaging or diffusivity over the distribution of v , smearing out sharp features. Even worse, it can act as a redistribution mechanism, shifting high count rates (low v), for example, to low. This is very similar to the case of amplification bias from gravitational lensing in flux limited surveys (cf. Schneider 1987). The observed faint end slope of -1 in $N(C)$ could be obtained from a steeper true source distribution if there existed a broad (flat f) threshold variation. Petrosian (1993; also see Caditz and Petrosian 1993) deconvolves the threshold variations from the count data using a nonparametric maximum likelihood statistical method to derive a true faint end slope for $N(C)$ in the range -1.6 to -1.8 . The distribution is still reasonably fit by a broken power law, with a bright end slope of -2.5 . See Sect. 3 for further discussion.

2.2. V/V_{max} properties

Because of its historical importance in the gamma ray burst debate we briefly consider the volume fraction variable $v \equiv V/V_{max}$, concentrating on its implications for the geometry of the source distribution, particularly cosmologically. The comoving volume element is

$$dV = (1+z)^3 r_a^2 dr_{pr} d\omega, \quad (6)$$

where z is the redshift, r_{pr} the proper distance, r_a the angular diameter distance, and $d\omega$ the solid angle element. Since observations indicate that bursts are distributed isotropically, $d\omega$ will simply contribute a factor 4π .

As known from gravitational lensing, when dealing with a line of sight toward a source one must use an angular diameter distance that incorporates the fact that this direction is clear of density inhomogeneities that might appreciably (i.e. detectably) lens and alter the flux. These are known as clumpy universe distances and are parametrized by the ratio of smoothly distributed matter density to the total (Dyer and Roeder 1973, Ehlers and Schneider 1986). When this ratio is unity the angular distances agree with the standard Friedmann ones. But calculations indicate that models using smooth vs. totally clumpy distances deviate by less than 20% in $\langle V/V_{max} \rangle$, so we will only present results for the smooth distances. In general we find that a flat or open clumpy model acts intermediate between the smooth cases $\Omega = 0$ and $\Omega = 1$.

Given the source functions $\Phi(L)$ and $n(z)$ and the cosmological model, one can determine the distribution and moments of v for comparison with data:

$$\begin{aligned} \int_{v_1}^{v_2} dv p(v) &= N_{tot}^{-1} \int_0^\infty dL \Phi(L) \\ &\quad \times \int_{z(L, v_1)}^{z(L, v_2)} dz (dr_{pr}/dz) r_a^2 (1+z)^3 n(z), \\ N_{tot} &= \int_0^\infty dL \Phi(L) \\ &\quad \times \int_0^{z(L)} dz (dr_{pr}/dz) r_a^2 (1+z)^3 n(z), \end{aligned}$$

$$\langle v \rangle = N_{tot}^{-1} \int_0^\infty dL \Phi(L) D^{-3}(L) \times \int_0^{z(L)} dz (dr_{pr}/dz) r_a^2 r_l^3 (1+z)^3 n(z). \quad (7)$$

Here $z(L, v)$ is the inversion of $r_l(z) = D(L)v^{1/3}$ with $z(L) = z(L, 1)$ and D following from (1) with $C = C_{lim}$. The BATSE detector team of the Compton Gamma Ray Observatory reports $\langle v \rangle = 0.33 \pm 0.01$.

[Note that in contrast to the quasar V/V_{max} test (Schmidt 1968), uniformity of source spatial distribution does not imply $\langle v \rangle = 1/2$ and vice versa. To see this we write the z integral of (7) in general functional terms and apply a theorem from integral calculus.

$$\langle v \rangle = \int_0^Z dz f(z)v(z) / \int_0^Z dz f(z); \quad (8)$$

in (7) we have $f(z) = (n/N_{tot})dV/dz$. Now (8) has the form of a weighted average of $v(z)$ where v takes the values 0 and 1 at the limits. However, the theorem states that if the average of a function with that behavior equals 1/2 over all intervals, i.e. for all values Z , then the differential of the function must equal the weighting function, $f = dv/dz$. Using $v = (r_l/D)^3$ and Eq. (6) we see this is not generally true for n uniform, so for gamma ray bursts the properties of $\langle v \rangle = 1/2$ and uniformity are not equivalent. As a specific example, take the model $\Omega \ll 1$, $n(z) = n_0(1+z)^4$: one obtains $\langle v \rangle = 1/2$ for all z , since here coincidentally $f = dv/dz$. In the quasar test, however, v is defined as $\int_0^V dV / \int_0^{V_{max}} dV$, so $dv/dz = (\int_0^{V_{max}} dV)^{-1} dV/dz$, which does equal f for n uniform.]

Quasilocal cosmological models, those with detection depth $z \ll 1$, can be severely constrained by calculating the volume relation to first order in redshift:

$$\langle V/V_{max} \rangle = (1/2)[1 - (3/7)(1 - \beta/4)z]. \quad (9)$$

Here β is the density evolution index in $n(z) = n_0(1+z)^\beta$, i.e. $\beta = 0$ corresponds to no evolution, and this first order result is independent of Ω . Obtaining $\langle V/V_{max} \rangle = 0.33$ would require $z = 0.79$ or negative evolution, the first violating our low redshift assumption and the second requiring excessive evolution at low redshifts (e.g. $\beta = -28$ at $z = 0.1$ in order to give the value 0.33).

Were there no problems with threshold bias one could fit simple cosmological models with greater depth to the data, obtaining the values $\langle V/V_{max} \rangle = (0.4, 0.35, 0.3)$ at detection limits $z = (0.7, 1.3, 2.2)$ for $\Omega = 1$ and at $z = (0.7, 1.5, 3.3)$ for $\Omega = 0$, with no evolution. Excellent approximations to the cosmological model source counts of (7) over the redshift range of interest, including evolution, are given to 5% by $\langle v \rangle = 0.5y^{-b}$ where $y = 1+z$ and $b = 0.36 - 0.12\beta$ for $\Omega = 0$ and $b = 0.43 - 0.15\beta$ for $\Omega = 1$. Not only the mean but the binned distribution (first line of Eq. 7) is fit admirably.

3. Cosmological parametrization

However, two problems exist with using solely the V/V_{max} distribution to describe the source counts. One, the information from high counts, including the break away from the $-5/2$ slope, is scrunched into a small region due to transformation to the $(C/C_{lim})^{-3/2}$ variable. Two, the threshold variation problem of C_{lim} also warps the fundamental $N(C)$ distribution, as clearly shown by Petrosian's (1993) deconvolution.

The definition of the differential source counts is

$$N(C) = \int dV n(V) \int dL \Phi(L) \delta(C - L/4\pi r_l^2) = \int dV 4\pi r_l^2 n(V) \Phi(4\pi r_l^2 C). \quad (10)$$

In the case of uniformity (i.e. sources distributed homogeneously in Euclidean space) the volume element $dV = 4\pi r^2 dr$ and $r_l = r$ so the integral can be transformed to the form

$$N(C) = (4\pi)^{-1/2} n C^{-5/2} \int_0^\infty dx x^4 \Phi(x^2). \quad (11)$$

Although the bright end slope is matched, there is no possibility of a break in the power law with respect to C , regardless of luminosity function. So a luminosity function alone cannot give the observed behavior of the bright end slope of $-5/2$ turning over to a shallower faint end slope. There are only four possibilities for the cause of a change in slope. Changes can occur by varying the geometry, i.e. dV , by evolution, i.e. n or $L(V)$, and by combinations of these with each other or with the luminosity function $\Phi(L)$.

Initially neglect the luminosity function; assume all sources have identical luminosity L . Including the appropriate delta function for $\Phi(L)$ in (10) gives

$$N(C) = 2\pi n_0 C^{-1} [y^{\beta-1} r_l^3 (dr_{pr}/dr_l)]_Y, \quad (12)$$

with the bracketed term evaluated at $y = Y$ implicitly defined by $L = 4\pi C r_l^2(Y)$. The faint slope index becomes constant in the asymptotic, i.e. high redshift, regime with (negative) values found to be $(6 + \beta)/4$ or $(3 + 2\beta)/4$ for $\Omega = 0$ or 1.

The effect of cosmology is that as more distant sources are considered the slope of the $\log N - \log C$ relation becomes shallower (greater than $-5/2$), i.e. there are fewer faint sources than expected because of the limited volume available: because of expansion the younger universe was smaller. Positive density evolution (more sources at higher redshift: $\beta > 0$), however, acts to counteract this. At the end of this section we examine how unreasonable the density run has to be to match the source counts. Positive luminosity evolution increases Y and steepens (shallows) the slope for $\Omega = 0$ (1). The effects are opposite because the two models' volume-redshift dependences are very different.

If the BATSE data are accepted at face value (i.e. without threshold deconvolution), the faint end slope is found to be in the range -0.8 to -1.0 . In this case low density cosmological models are poor fits since substantial evolution is needed to

achieve such a shallow slope (by redshifts of a few rather than asymptotically). Were this extreme evolution in fact to exist, however, the source counts would not resemble a broken power law but a *very* gradual turnover. Constraints such as $\langle V/V_{max} \rangle$ from Sect. 2.2 add to the difficulty, even for $\Omega = 1$ models.

To put these constraints on a firmer footing we consider three parametrizations of the cosmological $\log N - \log C$ distribution for comparison with the data. Two are the slope of the differential counts and the logarithmic range of counts between given values of the slope. The virtue of using slopes and logarithmic ranges lies in the ability to neglect questions of absolute values for the number of bursts or the peak count rate, made difficult by detector dependent effects. (Of course once a model achieves a fit under these conditions then the normalization is important in recovering the physical values of source luminosity and density). We retain $\langle V/V_{max} \rangle$ as a third parameter, both for historical reasons and as a complete, rather than point, measure of the distribution. Although these are not independent, e.g. knowledge of the slope at every point allows reconstruction of the (unnormalized) distribution and hence any of the other measures, there is sufficient difference in the areas of emphasis each places on the counts that it is useful to consider all three.

We elaborate the source model to include a version of the cosmological k-correction, taking into account the finite energy window through which the bursts are observed and the redshift of the source spectrum. The relation between the peak count rate C and the received flux f_ν is

$$C = \int d\nu S(\nu) (f_\nu/h\nu), \quad (13)$$

where $S(\nu)$ is the window frequency response and h is Planck's constant. The k-correction occurs in transforming the specific luminosity L_ν to f_ν ,

$$f_\nu = (1+z) \frac{L[\nu(1+z)]}{4\pi r_l^2} \frac{d[\nu(1+z)]}{d\nu}, \quad (14)$$

where the initial factor of $1+z$ accounts for the time dilation since C measures a count rate not number. Adopting the spectral models $L_\nu = L_0(\nu/\nu_0)^{-\alpha}$ and S flat between ν_- and ν_+ yields

$$C = \left[L_0 \nu_0^\alpha \int_{\nu_-}^{\nu_+} d\nu \nu^{-1-\alpha} / 4\pi h(\nu_+ - \nu_-) \right] \times (1+z)^{2-\alpha} r_l^{-2}(z), \quad (15)$$

generalizing (1). This differs from the usual inverse square distance law by the $1+z$ factors; one comes from bandwidth dilation, one from time dilation, and the α dependence from the spectral shift through the observing window. The advantage of using differential logarithmic parameters is that the entire group of constants in brackets can be neglected.

Suppose all sources are identical, i.e. same L_0 and α , but with evolution possible in the source population, as powers of $1+z$ with β the index for density evolution and γ for luminosity (L_0). Generalizing (12),

$$N(C) = 4\pi n_0 C^{-1} Y^{\beta-1+\delta} (dr_{pr}/dy) |d(y^\delta r_l^{-2})/dy|^{-1}$$

$$\equiv AC^{-1}f(Y), \quad (16)$$

where $\delta = \gamma + 2 - \alpha$ and derivatives are evaluated at $y = Y$, the solution to a modified Eq. (1) [or (15)]: $C = \kappa Y^\delta r_l^{-2}(Y)$.

Taking advantage of the form of (16) in solving for the wanted parameters, the (negative) slope of the cumulative distribution $N(> C)$ is

$$s_c \equiv -\frac{d \ln N(> C)}{d \ln C} = \frac{C N(C)}{N(> C)} = f(Y) / \int dr_{pr} y^{-1} r_l^2. \quad (17)$$

This can be seen by direct evaluation of $\int dC C^{-1} f(Y)$ or by using that $N(> C)$ is just the effective ‘‘volume’’ out to $Y(C)$ to obtain the denominator. The differential slope is

$$s_d \equiv -\frac{d \ln N(C)}{d \ln C} = 1 - \frac{C}{f} \frac{df}{dy} \frac{dy}{dC}. \quad (18)$$

Note that because of the bending of the source counts caused by cosmology the relation $s_c = s_d - 1$ does not generally hold, as it would in the scale free case.

The volume fraction parameter takes its usual definition in terms of counts relative to threshold, $\langle V/V_{max} \rangle = \langle (C/C_{lim})^{-3/2} \rangle$, but note that it is now even further from interpretation as a physical volume than before. Not only is $(C/C_{lim})^{-3/2}$ not the volume element (6) relative to maximum detectable volume (as it would be in the Euclidean case), but it is not even the previous distance ratio $(r_l/D)^3$ because of the k-correction and time dilation factors.

Finally, define a count range parameter R by

$$R(s_d) = \log[C(s_d = 2.25)/C(s_d)]. \quad (19)$$

This statistic measures how quickly in count space the geometric and evolutionary effects become appreciable. It is interpreted as the range *to* the highest count rate C where $N(C)$ has slope s_d *from* where the slope begins to deviate significantly (10%) from the Euclidean value of $5/2$.

By defining the survey depth for each variable – z_v , z_s , and z_r – where the cosmological model behavior fits the observations, we discover as viable those models which agree on this maximum redshift within the data uncertainties. Fig. 1 plots the model results for $\Omega = 1$ and a variety of evolutionary indices.

Using data deconvolved of threshold effects by the method of Petrosian (1993) suggests parameter values in the ranges of $s_d = 1.6 - 1.8$, $R = 1.0 - 1.5$, and (independent of deconvolution) $\langle V/V_{max} \rangle = 0.33 \pm 0.01$. None of the models in Fig. 1 simultaneously satisfy these values within the errors, as clearly quantified by the depth criterion mentioned above. For example, for the $(\beta, \delta) = (-1, 0)$ model to achieve $\langle V/V_{max} \rangle = 0.33 \pm 0.01$ requires $z_v = 1.01 \pm 0.1$ while $s_d = 1.7 \pm 0.1$ gives $z_s = 0.58 \pm 0.13$ and $R = 1.25 \pm 0.25$ needs $z_r = 0.49 \pm 0.13$. While the plot is for $\Omega = 1$, lower densities fit even more poorly.

Recall that the curves incorporate evolution in number density and luminosity, as well as k-correction and time dilation, through the parameters β and δ . To obtain a unique survey redshift, the curves for $\langle V/V_{max} \rangle$ and s_d need to be shifted to

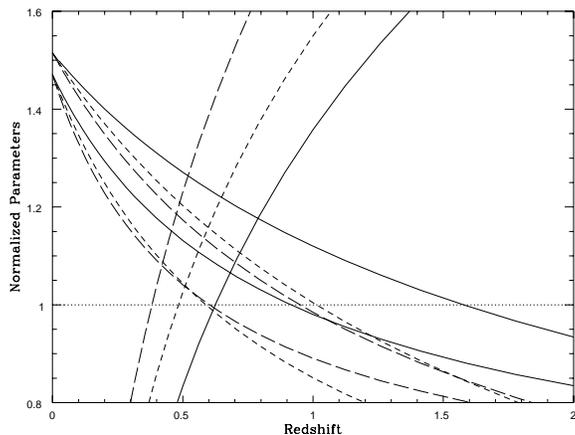


Fig. 1. The statistical parameters, normalized to the center of their observed range, are plotted vs. redshift. Those three curves starting at $z = 0$ with value 1.52 show $\langle V/V_{max} \rangle$, those with value 1.47 show s_d , and the positive slope curves show R . Solid curves have $(\beta, \delta) = (0,0)$, long dashed have $(0,-1)$, and short dashed $(-1,0)$. The intersection of each curve with the value one defines the appropriate depth z_v , z_s , or z_r . A successful model would have all solid curves, say, intersecting the dotted line simultaneously. Of course uncertainty in the data broadens the dotted line, into 1 ± 0.03 for $\langle V/V_{max} \rangle$, 1 ± 0.06 for s_d , and 1 ± 0.2 for R .

smaller redshifts, which can be accomplished by decreasing β or δ ; each affects the results in roughly the same magnitude. A unique depth, within the data uncertainties, is not achieved until $\beta + \delta < -10$, at which point the depth has decreased to $z \approx 0.1$. That requires an extreme amount of evolution in a very small redshift range.

The count range parameter proves very useful, giving a measure of the rate of curvature of the count distribution. Without paying heed to this constraint simple, even nonevolutionary, models will seem to fit for depths varying from $z \approx 1 - 7$. However the logarithmic count ranges are then some two times larger than the values indicated by the deconvolved data, i.e. a factor of 60-250 off the observed count rate.

Of course one can always adjust the source spatial distribution in (10) so as to obtain any count behavior desired. Fig. 2 illustrates the spatial density $n(z)$ of gamma ray burst sources required to match the observed count distribution. We adopt two forms for this distribution: a broken power law with bright slope -2.5 and faint slope -1.7 , and a more gradual behavior that interpolates between these two asymptotes. For the cosmology we take models with $\Omega = 0$ and $\Omega = 1$ and incorporate luminosity evolution, k-correction, and time dilation effects through the parameter δ as before. The requisite cosmological density behavior for explanation of the observations is not satisfied by the form $n(z) \sim (1+z)^\beta$, which would be a straight line on the plot with slope β , or by any familiar cosmological source distribution.

4. Conclusion

The gamma ray burst source count plot is similar to the classical Hubble diagram in that (for the cosmological scenario) the cur-

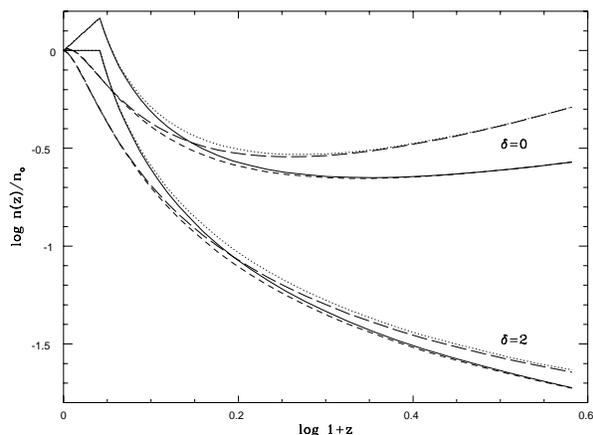


Fig. 2. The cosmological spatial number density derived to match a source counts model is plotted vs. redshift. The solid and dotted lines use a broken $N(C)$ and $\Omega = 0$ and 1, respectively; the short and long dashed lines a gradual $N(C)$ and $\Omega = 0$ and 1, respectively. The break, or turnover, in counts is assumed to lie at $z = 0.1$; it is actually a function of the intrinsic luminosity and spectrum and will scale the horizontal axis accordingly, but should lie close to the value adopted.

vature away from the Euclidean behavior is a direct measure of cosmological effects. The bright end slope of $-5/2$ has a natural interpretation in terms of nearby sources, $z < 0.1$, where geometry and evolution have not yet broken source uniformity. The faint slope value determines a characteristic survey depth, or redshift, as does the weighted average $\langle V/V_{max} \rangle$ of the entire data set, and the observed range in counts between the transition region and the asymptotic behavior.

These three distinct estimates for the depth out to which gamma ray bursts are detected – z_{slope} , z_v , and z_{range} – must be equal for the cosmological model fit. Fig. 1 shows, however, that this does not hold for any of the models considered with power law evolution. Fig. 2 illustrates the run of spatial number density required for cosmological sources to match the observed count distribution, again in disagreement with power law evolution. The models took into account the cosmological characteristics of geometry, k-correction and time dilation, as well as number and luminosity evolution power law in redshift.

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