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Bernoulli's equation and the contact binaries

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Abstract. In a recent paper Bernoulli's equation has been so interpreted as to mean that the Jacobi energy is constant over the surface of a stationary contact binary. We argue that Bernoulli's result can not be interpreted in this way and show that such a constraint on the Jacobi energy is in general not compatible with stationarity.

We also show that, under isentropic conditions, a surface potential gradient can not cause a steady net flow of material across the system's surface plus a deeper return flow.

An apparent paradox raised by 'stationary solutions' for AB Andromedae can be resolved when it is recognized that Bernoulli's equation does not provide a simple recipe for calculating mean potential differences between contact components.

Key words: stars: binaries: close – hydrodynamics

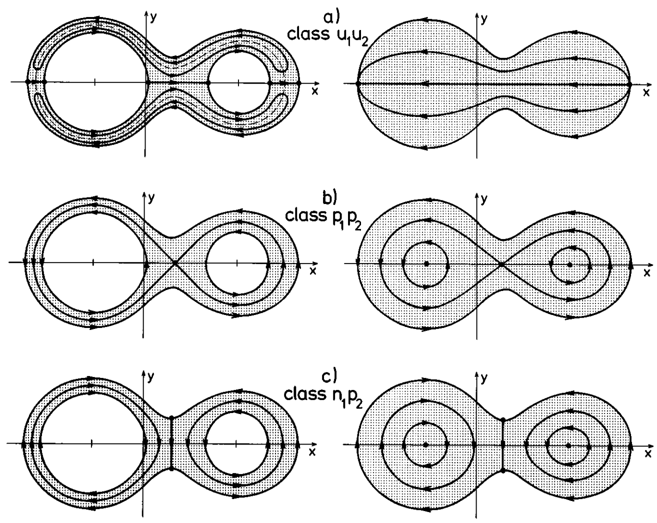

Fig. 1. Streamlines in the orbital plane (left) and on the northern surface (right) for various velocity fields. The grey-shaded region is turbulent and, for the most part, isentropic. The dashed line denotes the reversing surface and the dots denote stagnation points. Figure reproduced from K95 by courtesy of the author.

1. Introduction

Stationary models of contact binaries with large streaming velocities (∼ 200 km/sec) over the system's surface have recently been proposed (Kähler 1995, hereafter K 95). These models are characterised by an appreciable variation of Roche potential over the surface of the system.

The basic assumption underlying this work is that Bernoulli's equation may be interpreted in such a way that the Jacobi energy:

$$\psi = \phi + \frac{1}{2} v^2 \tag{1}$$

is constant over the whole system surface. In the above expression ϕ is the Roche potential and v the velocity in the rotating frame.

We show in Fig. 1 the various flow patterns considered. Despite the differences in topology, the common feature is that ψ is always assumed to be a surface invariant. For example, in the case shown in Fig. 1b, the vanishing of the velocity at the poles would cause the poles of the two components to lie on the same potential surface.

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A subsidiary assumption is made that the velocities in the secondary are relatively small. This ensures, since ψ is constant on the system's surface, that the primary's surface lies on a lower average potential than does the secondary's surface. The overall situation of motions plus potential differences is assumed to be self-maintaining i.e. compatible with stationarity.

For purposes of actual calculation the assumption concerning the Jacobi energy is applied in the integral form $\overline{\psi}_p = \overline{\psi}_s$, where p, s refer to primary and secondary respectively and averages are formed over the respective component surfaces. These averages are calculated using the pseudospherical approximation and the results collected together in Table 2 of K 95. Since the system AB Andromedae is chosen as test object the parameters in the table (mass, mass ratio, compositi on) are matched to this system.

2. Bernoulli's equation; preliminary remarks

For the following considerations it will be immaterial whether we regard the surface as being an isobar or simply the layer where the pressure vanishes exactly (standard boundary condition in the case of an ideal fluid - see Tassoul, 1978).

Normally the Bernoulli equation:

$$\psi + \int \frac{dP}{\rho} = \text{constant} \tag{2}$$

would be interpreted in the sense that ψ, the Jacobi energy, is constant *along surface streamlines*. If, in contrast to this, ψ is assumed to be constant over the whole surface, this can only mean that Bernoulli's equation is being given a quite different interpretation - indeed one that does not follow automatically from fluid dynamics. Whether or not this has serious consequences depends, in particular, on the rôle of the Coriolis forces.

Webbink (1977) considered the Coriolis forces to be fairly small, whereas Nariai (1976) regarded them as possibly significant. It suffices however to note that both authors presented streamline diagrams corresponding to the case in which the Coriolis forces are neglected. The flow field is seen to be symmetrical about the line of centres and the surface streamlines appear to diverge from one distal point on this line and to converge on the other. Assuming the fluid at these points to be stagnant, it follows that the 'constant' in Eq. (2) will indeed be the same for all streamlines.

This argument holds, however, only if the Coriolis forces are zero. As correctly pointed out by Nariai, if these forces are significant (not necessarily large) then they 'break symmetry'. The above argument can then no longer be used, since a change of topology may occur. It is therefore better not to make any special assumptions concerning the Coriolis forces and to treat this problem ab initio. We shall now do this and ask whether the assumption of constant ψ over the system surface is really compatible with stationarity.

3. The Jacobi energy assumption

We here investigate the assumption (Eq. (19) of K 95):

$$\psi \quad = \text{constant over system surface} \tag{3}$$

where ψ is the Jacobi energy as defined in Eq. (1) above. For easier comparison with K 95 we shall, as there, define the surface to be an isobar and shall, as there, neglect viscosity. The equation of motion under stationary conditions:

$$\frac{1}{2}\,\mathrm{grad} v^2 - \boldsymbol{v} \times curl\boldsymbol{v} + 2\boldsymbol{\Omega} \times \boldsymbol{v} = -\,\mathrm{grad}\Phi - \frac{1}{\rho}\mathrm{grad}P \tag{4}$$

integrates to give, on the surface:

$$\psi = \Phi + \frac{1}{2}v^2 = \int (\boldsymbol{v} \times curl\boldsymbol{v} - 2\boldsymbol{\Omega} \times \boldsymbol{v}) \cdot d\boldsymbol{s} + \text{constant} \tag{5}$$

where $d\boldsymbol{s}$ is an infinitessimal displacement along the surface. It then follows from Eq. (5) that constancy of ψ over the surface requires that:

$$(\boldsymbol{v} \times curl\boldsymbol{v} - 2\boldsymbol{\Omega} \times \boldsymbol{v}) \cdot d\boldsymbol{s} = 0 \tag{6}$$

for an arbitrary small displacement $d\boldsymbol{s}$ along the surface. This means that the vector inside the brackets must be directed normal to the surface i.e.

$$(\boldsymbol{v} \times curl\boldsymbol{v} - 2\boldsymbol{\Omega} \times \boldsymbol{v}) \times \hat{\boldsymbol{n}} = 0 \tag{7}$$

with $\hat{\boldsymbol{n}}$ a unit vector perpendicular to the surface.

Using the vector identity

$$(\boldsymbol{A} \times \boldsymbol{B}) \times \boldsymbol{C} = (\boldsymbol{C} \cdot \boldsymbol{A})\boldsymbol{B} - (\boldsymbol{C} \cdot \boldsymbol{B})\boldsymbol{A} \tag{8}$$

we find that:

$$\begin{aligned}&\{(2\boldsymbol{\Omega} + curl\boldsymbol{v}) \times \boldsymbol{v}\} \times \hat{\boldsymbol{n}} \\ &\quad = (2\boldsymbol{\Omega} + curl\boldsymbol{v}) \cdot \hat{\boldsymbol{n}}\boldsymbol{v} - (2\boldsymbol{\Omega} + curl\boldsymbol{v})\hat{\boldsymbol{n}} \cdot \boldsymbol{v}\end{aligned} \tag{9}$$

Since material can not move across the surface we have:

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{v} = 0 \tag{10}$$

so that Eqs. (7), (9) and (10) give:

$$(2\boldsymbol{\Omega} + curl\boldsymbol{v}) \cdot \hat{\boldsymbol{n}} = 0 \qquad or\, \boldsymbol{v} = 0 \tag{11}$$

Hence for any enclosed region lying in the surface we have:

$$\int (2\boldsymbol{\Omega} + curl\boldsymbol{v}) \cdot d\boldsymbol{A} = 0 \qquad (\boldsymbol{v} \neq 0) \tag{12}$$

which gives:

$$\oint \boldsymbol{v} \cdot d\boldsymbol{s} = -2\Omega S \qquad (\boldsymbol{v} \neq .0) \tag{13}$$

where S denotes the area projected on to the equatorial plane by the thus enclosed surface region.

The first point to note is that Eq. (13) can not be satisfied by *any* flow field in which the velocity is symmetrical about the line of centres. Hence the argument of Sect. 2, which involves implicit assumptions concerning the Coriolis forces, can not be used for 'proving' constancy of ψ. Such a priori assumptions are not permissible, at least not for present purposes.

Next, we see from Eq. (13) that, if the validity of Eq. (3) is assumed, as in K 95, then a necessary condition is that a large 'negative' (i.e. retrograde) axial circulation be present. A measure of this axial circulation is the quantity c(c_1 primary, c_2 secondary) defined in K 95, with $c < 0$ for retrograde motions.

Our result means that models with $c > 0$ and with $c = 0(\boldsymbol{v} \neq 0)$ *which have been constructed assuming the correctness of Eq. (3)*, as in Table 2 of K 95, are not self-consistent.

In the case of retrograde motions ($c < 0$) the situation is as follows: Eq. (13) imposes on the parameters of K 95 (specifically c and η) a restriction additional to the stellar structure requirements involved in constructing Table 2 of that paper. In general (i.e. for arbitrary c) it will not be possible for the data of this table to satisfy this extra restriction. Hence also models in the table with $c < 0$ can not be considered in general to be both self-consistent and stationary.

We therefore conclude that a stationary contact binary will not in general be able to satisfy Eq. (3) i.e. that the Jacobi energy will not, in general, be constant over the system surface.

4. The reversing-layer model

We shall consider in this section the situation shown in Fig. 1 a. In regarding the flow-field shown there as being a possible choice for a stationary contact system, the author (K 95) concludes that there appears to be 'no principal difficulty'.

The argument is essentially an energetic one; according to Eq. (3) (assumed to be valid) the component with the higher velocities (the primary) must lie on the lower potential. This means that material will tend to 'fall in' from the secondary on to the primary via the surface layers. Stationarity requires however the return of this material via the deeper layers and a reversing layer, separating the oppositely-directed streams, must form. The kinetic energy which was gained by the infall from the secondary is re-used to push the material back to where it started from.

Now although, as we have seen (Sect. 3), constancy of the Jacobi energy over the whole surface is not an admissible assumption, nevertheless along a connecting equatorial streamline constancy of ψ can be assumed (Bernoulli's equation). This leads us to concentrate our discussion on the equatorial plane and its immediate neighbourhood, and for this purpose the diagram on the left of Fig. 1a appears adequate.

Now since in K 95 the fluid is assumed to be isentropic, and hence barotropic, taking the 'curl' of Eq. (4) leads to the result:

$$curl\,(2\,\mathbf{\Omega}\,\times\,\boldsymbol{v}\,-\,\boldsymbol{v}\,\times\,curl\,\boldsymbol{v})\,=0 \tag{14}$$

It will now be convenient to use Cartesian coordinates referred to the mass-centre of the system. In the equatorial plane v_z and the z- derivative of v_x and v_y vanish. Hence taking the vertical component of Eq. (14) we find:

$$2\Omega\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)+\frac{\partial}{\partial x}(v_x w_z)+\frac{\partial}{\partial y}(v_y w_z)=0 \tag{15}$$

where

$$w_z=\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}\right) \tag{16}$$

is the z - component of the vorticity. Since the other components of the vorticity vanish, we shall simply refer to w_z as the 'vorticity'.

Introducing now the total time derivative

$$\frac{D}{Dt}\;=\;v_x\,\frac{\partial}{\partial x}+v_y\,\frac{\partial}{\partial y}\;+\;v_z\,\frac{\partial}{\partial z}\;=\;v_x\frac{\partial}{\partial x}\;+\;v_y\,\frac{\partial}{\partial y} \tag{17}$$

and using the equation of continuity:

$$div\,(\rho\boldsymbol{v})=0 \tag{18}$$

we find, after some reduction:

$$\frac{D}{Dt}\left(\frac{2\Omega}{\rho}+\frac{w_z}{\rho}\right)=\left(\frac{2\Omega}{\rho}+\frac{w_z}{\rho}\right)\frac{\partial v_z}{\partial z} \tag{19}$$

It can be confirmed by substitution or otherwise that this equation has the solution:

$$\frac{2\Omega+w_z}{\rho\partial z/\partial z_o}=\text{constant on streamlines} \tag{20}$$

where now z $(x_o, y_o, z_o,$ t) denotes the height of a particle above the equatorial plane i.e. we have changed from an Eulerian to a Lagrangian notation.

The interpretation of Eq. (20) is as follows. The numerator represents the total vorticity, including the contribution of rotation. Hence the numerator gives the number of (vertical) vortex lines, per cm^2. The denominator gives the mass/cm^2 between streamlines at any time. Since the ratio of these quantities is conserved we deduce that the number of lines of vorticity is conserved. Such lines can be neither created nor destroyed. Thus Eq. (20) is essentially Kelvin's circulation theorem, or more accurately Taylor's extension (Taylor 1921) of that theorem. Taylor appears to have been mainly interested in the incompressible case (rotating liquids) for which however Eq. (14) is valid also.

Let us now return to the model shown in Fig. 1a. In the equatorial plane there is a line (shown dashed) which follows the intersection of the 'reversing surface' with this plane. No motions are occurring along this line.

It is apparent from the figure that in the whole neighbourhood of the reversing line the shear is so large that the term in Ω in Eq. (20) can be neglected. Hence:

$$\left(\frac{w_z}{\rho\partial z/\partial z_o}\right)_{primary}\simeq\left(\frac{w_z}{\rho\partial z/\partial z_o}\right)_{secondary} \tag{21}$$

since in this region there are connecting streamlines.

However since the secondary velocities are treated as being small compared with the primary ones we have

$$(w_z)_{primary}>>(w_z)_{secondary} \tag{22}$$

so that Eq. (21) can only be satisfied if the 'column density' (denominator) on the secondary side is much smaller. In this case however mass conservation *on connecting streamlines* would become impossible, since much more material would be flowing along the primary than along the secondary portion of these streamlines. If, on the other hand, we insist upon mass conservation and stationarity, then vortex lines must be generated at a great rate as material comes into the primary and destroyed just as quickly as material leaves this star. This however would be incompatible with Eq. (14).

It is therefore apparent that a circulation of the above type is not possible in a stationary situation. The features are a) large potential differences (specifically, at the equator) b) a reversing layer and c) isentropic conditions.

It is important to note that the failure of the reversing-layer model in the present context is a direct consequence of its 'dynamical' (i.e. potential-driven) character. The situation can be likened to that of a perpetual, self-maintaining waterfall operating in a closed circuit. The upper and lower levels of this waterfall correspond to the secondary and primary surfaces respectively.

If, on the other hand, the circulation were to be thermally driven and caused by an entropy gradient (Nariai 1976, Martin + Davey 1995) then the existence of a reversing layer would be entirely compatible with stationarity.

5. A differential-rotation test

The discussion of Sect. 1 requires that we next test the Jacobi energy constraint in the surface-averaged form $\overline{\psi_p} = \overline{\psi_s}$ or, equivalently:

$$\overline{\Phi}_p - \overline{\Phi}_s = -\frac{1}{2}\overline{\upsilon_p^2} + \frac{1}{2}\overline{\upsilon_s^2} \tag{23}$$

To make this test worthwhile we shall at the same time consider the more general possibility:

$$\overline{\Phi}_p - \overline{\Phi}_s = -\frac{1}{2}\overline{\lambda^2\upsilon_p^2} + \frac{1}{2}\overline{\lambda^2\upsilon_s^2} \tag{24}$$

where λ^2 is a weighting function (with $\overline{\lambda^2} = 1$) dependent on latitude only. For example, if λ^2 were a delta function, with maximum at the equator, then Eq. (24) would simply become Eq. (23) with the r.m.s. velocities replaced by their equatorial values. This would reflect, at least qualitatively, the constancy of the Jacobi energy along connecting equatorial surface streamlines. More generally, λ^2 might be taken to be large on a finite 'equ atorial strip' and small elsewhere.

Let us now allow the primary motions to take the form of a differential rotation, those in the secondary being assumed to be relatively small. For simplicity we shall assume a primary rotation law of the form:

$$\boldsymbol{\upsilon} = \upsilon_\phi(\rho)\hat{\boldsymbol{\phi}} \tag{25}$$

where ρ, z, ϕ are cylindrical coordinates with the primary centre as origin. The velocities are measured in the rotating frame. It is readily confirmed that there is no conflict between Eq. (14) and Eq. (25); rotation laws of the above form were considered in a contact binary context by Whelan (1972).

Since we wish to test the applicability of Eq. (24) (of which Eq. (23) is a special case) we must now calculate the *actual* potential differences. We shall do this in the pseudospherical approximation, as used in K 95 and elsewhere; we shall comment upon the use of this approximation later.

Returning to Eq. (5) we note that, since the direction of integration can be chosen arbitrarily, we can find the difference in ψ between any two points on the primary's surface by alternately moving along and perpendicular to the streamlines. We then find from Eqs. (5) and (25) that ψ depends on latitude only and is given by

$$\psi(\rho) = \psi(R) - \int_\rho^R 2\Omega\,\upsilon_\phi\,d\rho - \int_\rho^R \frac{\upsilon_\phi}{\rho}\frac{d}{d\rho}(\rho\upsilon_\phi)\,d\rho \tag{26}$$

Since at the equator ψ must be the same for both stars, and the secondary velocities are to be neglected we have

$$\psi(R) = \Phi_s \tag{27}$$

so that, using the definition of ψ we obtain:

$$\Phi(\rho) - \Phi_s = -\frac{1}{2}\upsilon_\phi^2(\rho) - \int_\rho^R 2\Omega\upsilon_\phi\,d\rho - \int_\rho^R \frac{\upsilon_\phi}{\rho}\frac{d}{d\rho}(\rho\upsilon_\phi)\,d\rho \tag{28}$$

The above expression can be simplified somewhat to give:

$$\Phi(\rho) - \Phi_s = -\int_\rho^R 2\Omega\upsilon_\phi d\rho - \int_\rho^R \frac{\upsilon_\phi^2}{\rho}\,d\rho - \frac{1}{2}\upsilon_\phi^2(R) \tag{29}$$

We must now comment upon the use of the pseudospherical approximation. It is apparent that the figure of the primary must adjust in order that Eq. (29) be satisfied and that, in general, the result will be a non-spherical surface. However, the approximation has only been used for calculating the integral appearing in Eq. (5); this being done, Eq. (29) then tells us how the adjustment must occur. For example, the first term on the R.H.S. causes the figure to become more oblate than Roche for positive, and more prolate for υ_ϕ negative (retrograde motions).

We next calculate the average potential over the primary surface, again evaluating our integrals as if the surface were spherical. We find after partial integration and collecting terms:

$$\overline{\Phi}_p - \Phi_s = \int_0^R \left[(\Omega + \upsilon_\phi/\rho)^2 - \Omega^2\right]\left\{\sqrt{1-(\rho/R)^2} - 1\right\} \rho d\rho - \frac{1}{2}\upsilon_\phi^2(R) \tag{30}$$

In order to proceed further, we must introduce some explicit rotation law. Since however we are only concerned with *testing* Eq. (24) we have considerable freedom in this matter, although we will clearly prefer those laws for which the Rayleigh stability criterion:

$$\frac{d}{d\rho}\{(\Omega + \upsilon_\phi/\rho)^2\rho^4\} > 0 \tag{31}$$

is satisfied. We choose:

$$\Omega + \upsilon_\phi/\rho = \{1 - (1 - \rho^2/R^2)^p\}\,\Omega \tag{32}$$

(with $p > 0$) which is a "solar-type" rotation, with the equator rotating more rapidly than the pole. It is readily confirmed that Eq. (31) is satisfied.

Substituting from Eq. (32) in Eq. (30) we find after integration that for $0 < p < \infty$:

$$\overline{\Phi}_p - \Phi_s > 0 \tag{33}$$

in all cases. This is in clear contradiction with the prediction of Eq. (23) and Eq. (24), for υ_s small, quite irrespective of the choice of the weighting function λ^2. The conflict remains even when the regions near the equator are given an extremely high weighting. In terms of the 'equatorial strip' argumentation given at the beginning of this section we can say that the Jacobi energy function does indeed supply information concerning the equatorial potential; l is however not representative of the potential at other points of the primary's surface.

It has been suggested to us by Dr. Kähler that the performance of Eq. (24) might be somewhat better when the motions in the rotating frame correspond to a constant angular velocity (uniform non-synchronism). Let us consider the example

$$v_\phi = 2\Omega\rho \tag{34}$$

which gives r.m.s. velocities roughly corresponding to those in K 95. We then find that, irrespective of whether we use Eq. (23) or Eq. (24), the error committed in predicting the potential difference is never small. Even for the most favourable choice of λ (high equatorial weighting) the correction amounts to 70% of the quantity to be corrected.

The significance of the failure of Eq. (24) to make accurate or even reliable predictions can be understood in the following way. Validity of Eq. (24) would have meant that some of the results based upon Eq. (23) could still have been used, provided that the underlying variables were subjected to an appropriate renormalization. Failure of Eq. (24) means however that no such remedy is possible; the structure of Eq. (30) is quite different from that of Eqs. (23) and (24). Models based upon Eq. (23) are therefore fundamentally self-inconsistent; moreover it is clear from a close comparison of Eqs. (23) and (30) that Eq. (23) can not even be used as a first approximation for the purpose of calculating mean potential differences between contact components.

The conclusions of this and the previous sections are of relevance for understanding why a paradox seems to have arisen regarding age-zero models of AB Andromedae; we shall discuss this aspect later in Sect. 7.

6. Discussion of neglected effects

In this paper we have, following K95, expressly neglected viscosity. This not only permits easier comparison but, we believe, offers the most favourable conditions for the possible maintenance of streaming velocities of 200 km/sec. Indeed it is difficult to see how viscosity - if effective - could improve matters in the purely mechanical situation envisaged in Sect. 4; thermally-driven circulation (where viscosity can play a positive rôle) is of course a somewhat different matter.

With regard to Bernoulli's equation, an extra term appears when viscosity is present. Nevertheless, the restriction that Bernoulli's equation (now including viscosity) is valid on streamlines only still remains in force. Just as we saw in Sect. 5 that the inviscid Bernoulli equation (as incorporated in Eq. (24)) is no substitute for the Euler equations, so we would expect in general that the Bernoulli equation including viscosity is no substitute for the full Navier-Stokes equations.

The effects of radiative conductivity have also not been considered in this paper. Conductivity - unlike viscosity - does not lead (explicitly) to additional terms in the Bernoulli equation; however, radiative conductivity can cause departures from barotropy, and we must ask whether these departures can affect the conclusions drawn in Sect. 4.

Departures from barotropy due to the above cause will only be important in the extreme outer layers and, possibly, at the base of the convection zones. A 'typical' streamline as used in the argumentation of Sect. 4 passes however through neither of these regions so that the assumption of barotropy should be good, at least within the framework of the basic postulate of equal entropies within the adiabatic parts of the convection zones.

There is perhaps one important difference of detail which must be considered when viscosity and/or thermal conductivity are present. This is that the zero-pressure form of the boundary condition can not be used and must be replaced by the general isobaric form; we have already considered this as a possible alternative in Sect. 2 and actually used this form in Sect. 3.

In summary, we do not anticipate that the inclusion of viscosity and radiative conductivity will change the qualitative conclusions of this paper.

7. AB Andromedae

A comparison is made in K 95 between the photometric properties of the system AB And. and the data relating to 'stationary' isentropic age-zero models in Table 2 of that paper. The author then comes to the conclusion that AB And is a stationary system - or at least that stationarity is strongly indicated.

This however creates a new problem; since high streaming velocities ($\sim$ 200 km/sec) are assumed to be needed for stationarity why have these not shown up in spectroscopic investigations (e.g. Hrivnak 1988) of this object?

Now we have seen in Sects. 3 and 5 that the data in Table 2 of K 95 do not correspond to self-consistent stationary solutions. This means that the overall situation changes completely. A particular system can no longer be classified as stationary/non-stationary depending on whether (after fitting mass, mass ratio and composition) its properties do/do not fit in with the data of this table.

Assuming AB And. to be unevolved and isentropic, there thus appears to be no longer any reason to classify the system as being stationary; if it is non- stationary (thermal imbalance) then the high streaming velocities become unnecessary and the spectroscopic paradox disappears.

Finally, we note that the removal of this particular 'age-zero' paradox does not in any way strengthen the argument that this is an age-zero system. Indeed, Hrivnak (1988) remarks that the space motion of AB And. is relatively large and 'consistent with the system belonging to the intermediate to old disk population.'

8. Conclusion

The assumption that the Jacobi energy:

$$\psi = \Phi + \frac{1}{2}v^2 \tag{35}$$

is constant over the surface of a contact binary, while being compatible with Bernoulli's equation, does not follow from that equation. It was found that systems underlying the above constraint on the Jacobi energy can not in general be both stationary and self-consistent.

We therefore did not use the above assumption when investigating the problem of specific interest; potential-driven circulation under isentropic conditions. In this situation material is imagined to flow from one star to the other over the surface and back again via the deeper layers. Application of the fluid-dynamical equations showed such a flow to be impossible. On the other hand a thermally-driven circulation caused by entropy gradients (see Martin & Davey 1995) is always possible.

Use of the condition $\bar{\psi}_p = \bar{\psi}_s$ leads not only to inconsistency; the equation can not even be used to obtain a reasonably accurate estimate of the mean potential differences between contact components and can even lead to results of the wrong sign. The difficulty is not removed by using weighted averages for the surface velocities or by confining the averaging procedure to an 'equatorial strip'.

Finally, we conclude that models based upon the Jacobi energy constraint can not be used for the purpose of deciding whether a given observed system is stationary or not. Acceptance of this basic conclusion automatically leads to a resolution of the (apparent) spectroscopic paradox raised by the system AB Andromedae.

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