Astron. Astrophys. 326, 161–164 (1997)

ASTRONOMY
AND
ASTROPHYSICS

On the surface condition for stationary contact binaries

H. Kähler

Hamburger Sternwarte, Gojenbergsweg 112, D-21029 Hamburg, Germany

Accepted 4 March 1997

Abstract. The surface condition for stationary contact binaries is discussed, and the criticism of Hazlehurst in the preceding paper is commented.

Key words: stars: binaries: close

1. Introduction

In a previous paper (Kähler 1995, hereafter K95) we investigated the conditions for thermal equilibrium in unevolved contact binaries, allowing for the effects of large-scale mass motions in the common turbulent envelope. On account of these motions the Roche equipotential condition must be replaced by a more general surface condition. Assuming that all streamlines on the surface meet in stagnation points (hereafter assumption A) and neglecting viscosity we derived a surface condition from Bernoulli's equation. Using this condition we showed that rapid mass motions in the primary's envelope are required to establish thermal equilibrium. A preliminary discussion of the system AB And suggested that the observed properties are compatible with the assumption of thermal equilibrium.

The discussion in K95 suffered from two shortcomings. (1) Assumption A is satisfied in the velocity field proposed by Webbink (1977) but violated in the general case (arbitrary velocity field). (2) We failed to notice that rapid mass motions are in conflict with the observed spectral line broadening of typical contact binaries. These points have been recognized and the consequences have been discussed in a recent paper (Kähler 1997, hereafter K97). We showed that in the general case the surface condition derived from Bernoulli's equation remains valid as an approximation. Treating the spectral line broadening in typical systems as an observational constraint we concluded that in a configuration in thermal equilibrium the effects of internal mass motions on the surface condition (and thus the departures from Roche geometry) are small. Several observed systems (including AB And) were analysed. All of them turned out to be in thermal disequilibrium.

The paper K95 and (without reference) also the paper K97 have been criticised by Hazlehurst (1997, preceding paper, hereafter H97). We agree with his results that assumption A may be violated and that rapid mass motions may be in conflict with the observations. These results are not new. We disagree with several other conclusions. For this reason a discussion is necessary.

2. Discussion

Thermal equilibrium is probably not realized in contact binaries, but this point is not essential in a theoretical discussion. Here we assume the existence of a system in thermal equilibrium. We recall some results from previous papers, discuss the surface condition and comment on the criticism of Hazlehurst.

2.1. Basic equations

Total mass $M = M_1 + M_2$, separation A of the components and angular velocity ω are connected by Kepler's law

$$\omega^2 = \mathrm{G}M/A^3. \tag{1}$$

The (gravitational plus rotational) potential Ψ in a corotating coordinate system can (at the surface of the system) be closely approximated by the Roche potential Ψ_R. We define a normalised Roche potential Φ by

$$\Psi_\mathrm{R} = -\frac{\mathrm{G}M}{A}\Phi. \tag{2}$$

In a spherical treatment of the components (Kähler 1986) the normalised Roche potential Φ_i at the surface of the component i is

$$\Phi_i = \frac{\lambda_i^2}{3} + \frac{\mu_i}{\lambda_i} + \frac{(1-\mu_i)(3-\mu_i)}{2}, \tag{3}$$

where $\lambda_i = R_i/A$ and $\mu_i = M_i/M$ are, respectively, normalised radius and normalised mass of the component i. Let Φ_{L_j} be the normalised potential in the Lagrangian point j. The degree of contact F_i of the component i is defined by

$$F_i = (\Phi_i - \Phi_{\mathrm{L}_1})/(\Phi_{\mathrm{L}_2} - \Phi_{\mathrm{L}_1}), \tag{4}$$

and the contact condition for the component i is

$$0 < F_i \leq 1. \tag{5}$$

2.2. The Roche equipotential condition

If the influence of internal mass motions can be neglected, i.e. if strict hydrostatic equilibrium can be assumed, the surface condition is the Roche equipotential condition $\Phi_1 = \Phi_2$ which can be written in the form

$$F_1 - F_2 = 0. \tag{6}$$

In reality mass motions are always present. Their influence on the surface condition is small and the Roche equipotential condition is approximately satisfied if

$$|F_1 - F_2| \ll 1 \tag{7}$$

or equivalently

$$|\Phi_1 - \Phi_2| \ll \Phi_{L_1} - \Phi_{L_2}. \tag{8}$$

2.3. Properties of the velocity field on the surface

The velocity field of the internal mass motions on the surface Σ is symmetrical with respect to the orbital plane. The intersection C of the orbital plane and Σ is therefore either a closed streamline, or it consists of streamlines and stagnation points.

The surface Σ is topologically equivalent to a sphere. If the stagnation points on Σ are isolated, their number n is even and the minimum number is 2. This follows from the facts that the Poincaré index of a stagnation point is either 1 or -1 and that the sum over all indices (the Euler characterictic of a sphere) is 2.

If $n = 2$, the field of streamlines is topologically equivalent either to the system of parallels of latitude on a sphere, or to the system of meridians. In the latter case assumption A is satisfied since all meridians meet in the poles. Since C corresponds to a meridian, the two stagnation points are in the orbital plane. The reversing-layer fields discussed by Nariai (1976), Webbink (1977) and Zhou & Leung (1990) are of this type.

2.4. Bernoulli's equation

Neglecting viscosity in the Navier-Stokes equations for steady motions and assuming constancy of the pressure on Σ we found that the Jacobi energy $\Psi + v^2/2$ (where v is the velocity of the mass motions in a rotating coordinate system) is constant on a streamline on Σ (Bernoulli's equation). Since C consists of streamlines we have

$$\Psi + v^2/2 = \text{ constant} \quad \text{on } C, \tag{9}$$

i.e. the Jacobi energy is constant on the equator. If assumption A is satisfied, the Jacobi energy is the same for all streamlines and we obtain the stronger result

$$\Psi + v^2/2 = \text{ constant} \quad \text{on } \Sigma, \tag{10}$$

i.e. the Jacobi energy is constant over the surface.

2.5. The rôle of the Coriolis forces

In Sects. 2 and 3 of H97 it is claimed that Eq. (10) is based on assumptions concerning the Coriolis forces. This is incorrect. Eq. (10) is based on assumption A and three other assumptions which are not controversial (stationarity, neglect of viscosity, constancy of the pressure on Σ). Assumption A concerns only the topology of the velocity field on Σ.

To be more specific, in Sect. 2 of H97 it is claimed that (in a treatment of a reversing-layer field) the argument leading to Eq. (10) holds 'only if the Coriolis forces are zero'. Symmetry arguments are used to 'prove' this claim. Concerning these arguments, recall that a stationary velocity field is part of a stable solution of a system of differential equations and boundary conditions. Let the Coriolis terms in the Navier-Stokes equations be multiplied by a factor θ. Taking $\theta = 1$ we have equations which are physically relevant. Taking $\theta = 0$ we ignore the influence of the Coriolis forces. Treating θ as a continuous parameter we have a formal mathematical problem. Suppose that for $\theta = 0$ a stable reversing-layer solution exists. The field is symmetrical about the line of centres. In this case Hazlehurst admits that Eq. (10) is satisfied.

Since a stable solution depends continuously on the parameters, for sufficiently small positive values of θ there is also a stable solution. The symmetry about the line of centres is broken but the topology of the field is preserved for continuity reasons. Assumption A remains therefore valid. The field has two stagnation points on C. In the case that $\theta = 1$ can be reached in this way we obtain a solution in which the Coriolis forces are fully taken into account, and in which Assumption A is satisfied. Eq. (10) is then also satisfied. Hazlehurst did not show that this case can be excluded. His claim - that the absence of Coriolis forces is necessary for the validity of Eq. (10) in a reversing-layer field - is therefore incorrect.

2.6. The Jacobi energy assumption

The purpose of Sect. 3 in H97 is to show that the assumption of constant Jacobi energy over the whole surface is not admissible since in conflict with stationarity. For this purpose it is tentatively assumed that Eq. (10) is satisfied. The discussion leads to an equation (Eq. (13) in H97) from which several conclusions are drawn. One conclusion is that Bernoulli's equation can not be used for 'proving' constancy of the Jacobi energy since implicit assumptions concerning the Coriolis forces (or the symmetry of the field about the line of centres) are involved. This conclusion is erroneous as shown in the previous subsection.

We agree with the conclusions that the circulation is retrograde, that the quantity c_i defined in K95 (the coefficient decribing the correlation between the orbital motion and the internal mass motions in the component i) must be negative (at the surface), and that contact binary models with non-negative values of c_1 or c_2 are inconsistent. Concerning negative values of c_1 and c_2, we agree with the result that Eq. (13) in H97 imposes restrictions and that in general (i.e. for arbitrary negative values of the c_i) the restriction will not be satisfied. In fact, it is trivial that

models for arbitrary values of c_1 and c_2 are generally inconsistent. The real velocity field in a stationary contact configuration is not known. For this reason we have to investigate all possible fields for given physical parameters (mass, angular momentum and composition) or given observational constraints, i.e. an infinity of fields. By necessity almost all of them are inconsistent. Usually only the real field is self-consistent.

The discussion leads to the final conclusion (Sect. 4 in H97) that 'the constancy of the Jacoby energy over the whole surface is not an admissible assumption'. This conclusion is erroneous. It is conceivable that the Jacobi energy assumption is not admissible, but this has not been shown. Indeed, Hazlehurst showed that models with non-negative values for the c_i are inconsistent (which is interesting) and that models with arbitrary values for the c_i are in general inconsistent (which is trivial), but he was not able to show that consistent models do not exist.

In fact, with this 'conclusion' Hazlehurst is in conflict with his own statement that the existence of a thermally driven reversing-layer field is 'entirely compatible with stationarity' (Sect. 4) or 'always possible' (Sect. 8). In the thermally driven reversing-layer fields discussed by Nariai (1976), Webbink (1977) and Zhou & Leung (1990) the Jacobi energy assumption is satisfied since Assumption A is satisfied.

2.7. The surface condition

If assumption A is satisfied, Eq. (10) can be used to derive a surface condition as shown in K95. The result is

$$\Phi_1 - \frac{\lambda_1^2}{3}\eta_1 = \Phi_2 - \frac{\lambda_2^2}{3}\eta_2, \tag{11}$$

where η_i is defined by

$$\langle v^2\rangle = \frac{2}{3}\omega^2 R_i^2 \eta_i \tag{12}$$

and the angle brackets denote an average over the surface of the component i. In the absence of mass motions ($\eta_1 = \eta_2 = 0$) the surface condition reduces to the Roche equipotential condition.

In the general case (arbitrary velocity field) an approximate surface condition can be derived from Eq. (9) as shown in K97. Eq. (11) remains valid as an approximation if the definition of η_i is modified, i.e. if the average over the surface of the component i is replaced by the average over the equator.

In unevolved stationary contact systems the mass motions in the primary's envelope must be much larger than in the secondary's envelope. For simplicity henceforth mass motions in the secondary will be neglected, as in the models in K95 (and as in Sect. 5 of H97). Writing η instead of η_1, the final result for the approximate surface condition is

$$\Phi_1 - \Phi_2 = \frac{\lambda_1^2}{3}\eta \tag{13}$$

with

$$\overline{v^2} = \frac{2}{3}\omega^2 R_1^2 \eta, \tag{14}$$

where the bar denotes an average over the primary's equator. Uncertainties in this surface condition will be discussed in Sect. 2.9.

Table 1. Critical values η_c for different values of the mass ratio q and the primary's degree of contact F_1

q	F_1					
	0	0.2	0.4	0.6	0.8	1
0.2	1.18	1.15	1.11	1.08	1.04	1.01
0.4	2.48	2.30	2.24	2.13	2.01	1.90
0.6	3.69	3.46	3.23	2.99	2.78	2.57
0.8	4.19	3.87	3.56	3.26	2.97	2.68

2.8. A criterion for approximate Roche geometry

Here η will be treated as a formal parameter defined by Eq. (13). Taking into account Eq. (8) we see that the departures from Roche geometry are small when

$$|\eta| \ll \eta_c \quad \text{with} \quad \eta_c = \frac{3}{\lambda_1^2}(\Phi_{L_1} - \Phi_{L_2}), \tag{15}$$

where η_c is determined by the mass ratio q and the primary's degree of contact F_1. Some values are listed in Table 1. They show that in contact binaries with typical parameters the departures from Roche geometry are negligible when $|\eta| \ll 1$ and small when $|\eta| < 0.3$. For an illustration cf. Fig. 2 in K95.

In typical unevolved systems stationarity requires $\eta > 5$ and thus large departures from Roche geometry. For the system AB And this was shown in K95. We investigated also several other observed systems, with the same result.

2.9. Uncertainties in the surface condition

The purpose of Sect. 5 in H97 is to show that the surface condition is useless since the uncertainties involved are too large. For the mass motions on the primary's surface, differential rotation with a law $v = f(\rho)$ is assumed, where ρ is the distance from the primary's rotation axis. The Navier-Stokes equations are used to calculate the potential difference $\Delta\Psi = \overline{\Psi}_1 - \Psi_2$, where the bar denotes an average over the primary's surface. In the case of the rotation law

$$v = -\rho(1 - \rho^2/R_1^2)^p\omega \qquad (0 < p < \infty) \tag{16}$$

$\Delta\Psi$ turns out to be positive. This is said to be 'in clear contradiction with the prediction' of the surface condition.

Making use of Eq. (13) it can be verified that the potential difference corresponds to $-0.5 < \eta < 0$ for arbitrary positive values of p. Since $|\eta/\eta_c| < 0.2$ for typical values of mass ratio and degree of contact, the departures from Roche geometry are small. The surface condition gives $\eta = 0$ since $v = 0$ on the equator. In this approximation the Roche equipotential condition is satisfied. Accordingly, the surface condition is adequate at least as a rough approximation.

Actually the rotation law (16) is unrealistic. The energy transfer between the components requires mass motions between the components. A field with no motions on the equator (and very slow motions in a belt around the equator) is highly artificial.

More realistic (apart from reversing-layer fields) is the simple field described by the rotation law

$$v = a\omega\rho \quad \text{with} \quad a = \text{constant}. \tag{17}$$

The Navier-Stokes equations give a potential difference corresponding to

$$\eta = a + 2a^2. \tag{18}$$

The surface condition (14) leads to the approximation

$$\eta \simeq \eta_s = 3a^2/2. \tag{19}$$

which is admittedly rough. Nevertheless, in the case of $|a| \geq 1$ we have

$$\eta_s = F\eta \quad \text{with} \quad 0.5 \leq F \leq 1.5 \tag{20}$$

which shows that the effects of rapid motions are correctly predicted apart from a factor not far from unity. The effects of slow motions ($|a| \ll 1$) are small and thus reasonably approximated by the surface condition.

In summary, the surface condition is admittedly rough (unless assumption A is satisfied) but nevertheless sufficient to estimate the effects of rapid mass motions.

2.10. Concluding remarks

We are now prepared to comment on the final conclusion in H97 that 'models based upon the Jacobi energy constraint can not be used for the purpose of deciding whether a given observed system is stationary or not'. If the surface condition is applied to observed systems, assumed to be unevolved, it turns out that rapid mass motions ($\eta > 5$) in the primary's envelope are required which are in conflict with the observations. This argument (K97), based on the Jacobi energy constraint, excludes stationarity in unevolved systems.

In H97 it is argued that the surface condition is a rough approximation and that even a wrong sign for the potential difference can be obtained. This is correct but not essential. A wrong sign can be obtained if the motions are slow, but in this case the potential difference is small. Stationarity requires a large potential difference and thus rapid motions. In this case the sign is correct and the approximation gives the correct potential difference apart from a factor not far from unity. This is fully sufficient for the purpose of showing that rapid motions in the primary are required.

Summarizing, observed systems (if unevolved and stationary) require a large potential difference, and this difference requires rapid motions in the primary which are in conflict with the observations. Observed systems, if unevolved, are therefore not stationary. This result is not affected by the uncertainty in the surface condition. Further arguments against stationarity (also in evolved systems) are given in K97.

References

Hazlehurst J., 1997, A&A (preceding paper)
Kähler H., 1986, A&A 157, 329
Kähler H., 1995, A&A 294, 497
Kähler H., 1997, A&A 318, 171
Nariai K., 1976, PASJ 28, 587
Webbink R.F., 1977, ApJ 215, 851
Zhou D.Q., Leung K.C., 1990, ApJ 355, 271

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.