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Simulations of cosmic ray cross field diffusion in highly perturbed magnetic fields

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Abstract. The process of particle cross-field diffusion in high amplitude Alfvénic turbulence is considered using the method of the Monte Carlo particle simulations. We derive the cross-field diffusion coefficient $\kappa_\perp$ and the parallel diffusion coefficient $\kappa_\parallel$ in the presence of different 1-D, 2-D and 3-D turbulent wave field models and discuss the $\kappa_\perp$,$\kappa_\parallel$ and $\kappa_\perp \kappa_\parallel$ variations with the wave amplitude. $\kappa_\perp$ is compared also to the respective derived values of the magnetic field diffusion coefficient. We note substantial differences in the cross-field diffusion efficiency at the same perturbation amplitude, depending on the detailed form of the turbulent field considered. Vanishing of $\kappa_\perp$ at 1-D and 2-D turbulent fields is confirmed. For some types of the turbulent magnetic field an initial regime of sub-diffusive transport appears in the simulations. The derived values of $\kappa_\parallel$ are weakly dependent on the wave form and scale as the inverse of the squared perturbation amplitude up to our highest non-linear amplitudes.

Key words: cosmic rays – MHD-turbulence – acceleration of particles – diffusion

1. Introduction

The problem of particles motion in a perturbed magnetic field plays an important role in the transport processes of galactic cosmic rays and in particle acceleration at shock waves. In spite of large progress achieved since the paper of Jokipii of 1966 there are a number of issues which still are poorly understood. In particular the diffusion of particles across the magnetic field requires further study. To date all the quantitative analytical derivations of the cross-field diffusion coefficient, $\kappa_\perp$, in turbulent magnetic fields are limited to the quasi-linear approach, valid for small amplitude field perturbations, $\delta B \ll B_0$ [1]. Let us describe some basic findings in that matter.

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[1] In general we use the standard notation as described in Appendix A. One should note the use of the non-standard $\Omega^{(o)} \equiv qB_o/\gamma mc$.

The first considerations by Jokipii (1966, 1967, 1971) used the Fokker-Planck equation to describe particle motion in terms of the magnetic-field perturbations' power spectrum. Jokipii showed that scattering at the small-scale magnetic inhomogeneities drives pitch-angle diffusion, allowing for transverse guiding-centre diffusion across the field lines. For particle distribution close to isotropy the particle distribution averaged over pitch-angle satisfies the diffusion equation with the diffusion tensor expressible in terms of the correlation function of irregular magnetic field. If the fluctuating field depends only on z-coordinate (= direction of the mean magnetic field), the Fokker-Planck coefficients for pitch-angle diffusion and for cross-field diffusion take the form, respectively,

$$\frac{\langle \Delta\mu^2 \rangle}{\Delta t} = \frac{(1-\mu^2)}{|\mu| v} \frac{Z^2 e^2}{\gamma^2 m_o^2 c^2} P_{xx}(k = \Omega^{(o)}/\mu v) \quad , \qquad (1.1)$$

and

$$\frac{\langle \Delta x^2 \rangle}{\Delta t} = \frac{\langle \Delta y^2 \rangle}{\Delta t} =$$
$$\frac{\mu v}{B_o^2} P_{xx}(k=0) + \frac{(1-\mu^2)}{2|\mu| v} \frac{v}{B_o^2} P_{zz}(k = \Omega^{(o)}/\mu v) \quad . \qquad (1.2)$$

At present one learned that the cross-field diffusion terms (1.2) must vanish for the 1-D turbulence model (Giaclone & Jokipii 1994), see also discussion below. However, as this expressions were widely used for discussion of the cross-field diffusion process, and as the interpretation of the terms in (1.2) is mostly valid for 3-D turbulence, we repeate these expression after the original paper. The coefficients (1.1,2) describe particle scattering by fluctuations that are resonant with the particle's gyromotion in the averaged magnetic field $\mathbf{B}_0$. In coefficients (1.2) describing cross-field diffusion one can note an additional non-resonant term $\propto P_{xx}(k=0)$. It represents the tendency of particles to follow the meandering or random walk of magnetic field lines. For the large and small gyro-radius limits Jokipii obtained a very simple form of the perpendicular diffusion coefficient. At lower energies the resonant term is very small compared with the non-resonant one because the power spectrum falls off sharply toward high frequencies. Thus for $r_g \ll L_c$

$$\kappa_\perp = \frac{1}{4}\frac{v}{B_o^2} P_{xx}(k=0) \quad , \tag{1.3}$$

and for $r_g \gg L_c$

$$\kappa_\perp = \frac{1}{2}\frac{v}{B_o^2} P_{xx}(k=0) \quad . \tag{1.4}$$

The derivations presented by Jokipii break down for particles moving with pitch-angles $\mu \cong 0$. To avoid this difficulty, Jokipii (1975) considered explicitly the situation when particles moved perpendicularly to the mean magnetic field of the form:

$$\mathbf{B}(\mathbf{r}) = B_0\mathbf{e}_z + \delta B(x,y)\,\mathbf{e}_z \quad . \tag{1.5}$$

Then, for the typical two-dimensional power-law wave spectrum ($P_{ij}(k) \propto k^{-\alpha}$ he obtained a simple formula:

$$\kappa_\perp \cong v_\perp L_c \frac{\langle\delta B^2\rangle^{1/2}}{B_o}\left(\frac{r_g}{L_c}\right)^{\alpha+\frac{1}{2}} \quad . \tag{1.6}$$

Achatz et al. (1991) re-derived the Fokker-Planck equation for charge particle transport in a slab turbulence superimposed on homogeneous magnetic field, involving all phase-space variables. In contrast to the previous papers they included dispersive effects of the waves by considering whistler-mode waves in addition to the Alfvén waves. They confirmed the previous results of Jokipii that diffusion perpendicular to the magnetic field is solely due to the waves with zero wave vectors. The same result was obtained in a different way by Achterberg and Ball (1994), who studied the requirements for efficient electron acceleration in young supernova remnants, where the shock is expanding into the progenitor's stellar wind with the magnetic field lines forming a tightly-wound spiral. Then the intersection point between the shock and the magnetic field line moves along the mean magnetic field at a speed exceeding c. To allow the shock wave to accelerate electrons to GeV energies required to account for the observed radio emission, efficient particle diffusion across the magnetic field is necessary. In the considered situation relativistic particles can be scattered by resonant low-frequency MHD waves. The waves with a wave-number component perpendicular to the magnetic field $k_\perp$, contribute to the s-th resonance with a weight $J^2_{s\mp1}(k_\perp v/\Omega^{(o)})$, where J_n is the n-th order Bessel function. For $k_\perp v_\perp/\Omega^{(o)} \le 1$ the dominant contribution comes from the $s = \pm1$ resonances. These resonant waves produce a stochastic change in particle momentum as well. The net diffusion process in particle momentum is accompanied by guiding centre shift across the field. Achterberg and Ball interpret wave-particle interactions with the Melrose (1980) quantum mechanical formalism, where the low-frequency MHD waves are described as a collection of quanta with energy $\hbar\omega$ (where ω is the wave frequency), momentum $\hbar\mathbf{k}$ and occupation number $N(\mathbf{k})$. They regard the diffusion process as caused by emission or absorption of quanta changing the particle energy by an amount $\Delta E = \hbar\omega$ and the particle momentum by $\Delta\mathbf{p} = \hbar\mathbf{k}$. In the case of the turbulence symmetry around $\mathbf{B}_0$, $< k_x^2 >=< k_y^2 >= k_\perp^2/2$, they estimated the diffusion tensor components as:

$$\kappa_\parallel \approx \frac{v^2}{3\nu_s} = \varepsilon^{-1}\kappa_B \quad , \tag{1.7}$$

$$\kappa_\perp \approx \frac{\varepsilon}{2}\left(\frac{k_\perp}{k_\parallel}\right)^2 \kappa_B \quad , \tag{1.8}$$

where $\varepsilon \approx \nu_s/\Omega^{(o)}$, ν_s is the effective pitch-angle scattering frequency and the Bohm diffusion coefficient $\kappa_B = \frac{1}{3}r_g^2\Omega^{(o)}$. One may note that the waves with $k_\perp = 0$ do not contribute to particle transport across the magnetic field.

Vanishing of the cross-field diffusion for turbulence models involving 1- or 2-dimensional perturbation fields was proved by Giacalone and Jokipii (1994). They demonstrated that if one or two ignorable coordinates appear in the magnetic field description, the ions are effectively tied to the magnetic lines of force, independent of the turbulent field's amplitude.

The process leading to particle diffusion perpendicular to the average magnetic field due to field line wandering (or braiding) has been considered in some forms since the first papers of Jokipii and Parker in 60th. Achterberg & Ball (1994) discuss the case with long-wavelength perturbations leading to stochastic excursions of magnetic field lines Bansverse to $\mathbf{B}_0$. After travelling the correlation distance L_c in the time t_c along the field line the particle can jump to the neighbouring, statistically independent, patch of field lines. Application of this model yields the perpendicular diffusion coefficient

$$\kappa_\perp = D_m\left(\frac{L_c}{t_c}\right) \quad , \tag{1.9}$$

where D_m is the field lines' diffusion coefficient. Recently, a regime of sub-diffusive transport and of compound diffusion in the presence of 'braided' magnetic field was discussed by Duffy et al. (1995). In the sub-diffusive motion the average square displacement of a particle perpendicular to $\mathbf{B}_0$ grows as a square-root of diffusion time. The compound diffusion combine wandering of field lines and diffusion of particles along and across the local field. This problem was discussed by Kirk et al. (1996) for the issue of cosmic ray acceleration at perpendicular shock waves (see also Giacalone & Jokipii (1996) for numerical modelling).

In the present paper we performed numerical Monte Carlo simulations yielding the cross-field and the parallel diffusion coefficients in a set of 1-D, 2-D and 3-D finite amplitude turbulence patterns composed of superimposed Alfvén waves (Sect. 2). The results are presented and discussed in Sect. 3. We show substantial differences in the cross-field diffusion efficiency for the same perturbation amplitude, depending on the detailed form of the turbulent field considered. Vanishing of $\kappa_\perp$ at 1-D and 2-D turbulent fields is confirmed. For some types of the turbulent magnetic field an initial regime of sub-diffusive transport may occur. The derived values of $\kappa_\parallel$ are weakly dependent on the wave form and scale as the inverse of the squared perturbation amplitude up to our highest non-linear amplitudes. A short summary is provided in Sect. 4. The relation between the parallel diffusion coefficient and the momentum diffusion coefficient in highly turbulent magnetic fields is considered in Michałek & Ostrowski (1996).

2. Description of simulations

In our Monte Carlo simulations the following procedure is applied. Test particles are injected at random positions into turbulent magnetized plasma and their trajectories are followed by integrating particle equations of motion in space and momentum. By averaging over a large number of trajectories one derives the diffusion coefficients for turbulent wave fields[2]. In the simulations we usually used 500 particles in an individual run. The presented cross-field diffusion coefficients were derived with diffusion coefficients along the X and Y axes as

$$\kappa_\perp = \frac{1}{2}\left(\frac{<\Delta x^2>}{2t} + \frac{<\Delta y^2>}{2t}\right) \quad , \qquad (2.1)$$

and the parallel diffusion coefficient as

$$\kappa_\parallel = \frac{<\Delta z^2>}{2t} \quad . \qquad (2.2)$$

In the expressions above $<\Delta x^2>$, $<\Delta y^2>$ and $<\Delta z^2>$ are particle dispersions along the respective axes and t is the integration time. We use this formal definition for $\kappa_\perp$ even if the transport is not of diffusive character. Then this coefficient characterizes the change of particle dispersion perpendicular to mean magnetic field during simulations.

In the case of high amplitude waves, there are no analytic models available reproducing the turbulent field structure. Because of that and limited computational power we use only approximate model of real, 3-D turbulence. In spite of it, our numerical simulations are able to reproduce mean feature of existing magnetic fields. In next paper, which is in preparation, we consider more realistic model of MHD turbulence. Three of the models, in which the turbulence is regarded as a superposition of Alfvén-like waves, are described below. The perturbed field structures obtained are explicitly divergence-free. In the simulations, for any individual particle a separate set of wave field parameters is selected. So, all averages taken over the particles involve also averaging over multiple magnetic field patterns. Examples of magnetic field lines for the three models described below are presented at Fig. 1.

A. Linearly polarized plane waves

In the model we take a superposition of plane Alfvén waves propagating along the z-axis, in the positive (forward) and the negative (backward) direction. Two planar polarizations, along the x and y axes, are considered. In the computations 24 sinusoidal waves, 12 of each polarization, are taken into account. The wave parameters - wave vectors k and wave amplitudes δB_0 - are drawn in a random manner from the flat wave spectrum. The magnetic field fluctuation vector $\delta\mathbf{B}^{(i)}$ related to the wave 'i' is given in the form:

$$\delta\mathbf{B}^{(i)} = \delta\mathbf{B}_0^{(i)} \sin(k^{(i)}z - \omega^{(i)}t - \Phi^{(i)}) \quad . \qquad (2.3)$$

The dispersion relation for Alfvén waves, $\omega^2 = V_A^2 k^2$, provides the respective ω parameters for any individual wave. The sign of ω is determined by selecting the wave velocity V at, randomly, $\pm V_A$, but with the same number of waves moving in any direction within any selected range of wave vectors. The wave field with the same number of positive and negative waves in any wave-vector range is called the 'isotropic wave field' or 'isotropic turbulence'. The electric field fluctuation related to the particular wave is given as $\delta\mathbf{E}^{(\mathbf{i})} = -\mathbf{V}^{(\mathbf{i})} \wedge \delta\mathbf{B}^{(\mathbf{i})}$. For selecting any individual set of wave parameters the following procedure is applied. Wave vectors, expressed in units of the resonance wave vector, $k_{res} \equiv 2\pi/r_g(<B>, p_0)$, for a particle with initial momentum $p = p_0$ moving in the *mean* magnetic field $<B>$, are drawn in a random way from the respective ranges: $2.0 < k < 8.0$ for 'short' waves, $0.4 < k < 2.0$ for 'medium' waves and $0.08 < k < 0.4$ for 'long' waves. Four waves are taken from each range for every polarization plane. The respective wave amplitudes are drawn in a random manner so as to keep constant

$$\Big[\sum_{i=1}^{24}(\delta B_0^{(i)})^2\Big]^{1/2} \equiv \delta B \quad , \qquad (2.4)$$

where δB is a model parameter, and, correspondingly in all wave-vector ranges

$$\Big[\sum_{i=1}^{8}(\delta B_0^{(i)})^2\Big]^{1/2} = \Big[\sum_{i=9}^{16}(\delta B_0^{(i)})^2\Big]^{1/2} = \Big[\sum_{i=17}^{24}(\delta B_0^{(i)})^2\Big]^{1/2} = \frac{\delta B}{\sqrt{3}} \quad . \qquad (2.5)$$

The mean magnetic field for sinusoidal Alfvén waves (with $\delta\mathbf{B}\cdot\mathbf{B} = 0$) is given as $<B> = \sqrt{B_0^2 + \delta B^2/2}$. In the present work we consider a wide range of amplitudes δB, between 0.1 and 2.0.

B. Wave packets

The above plane wave model represents 1-D turbulence. Therefore, as an extension to higher turbulence dimensions we consider waves in the form of wave packet in a direction perpendicular to the propagation direction. In the present case one can use the formula (2.3) for $\delta\mathbf{B}^{(i)}$, where the phase parameter is subject to modulation on the scale comparable to the corresponding wavelength. Two types of modulation were taken into consideration. For the x-components in (2.3): B1.) the sinusoidal 'smooth' modulation is given as

$$\Phi_x^{(i)}(y) = \sin(k_y^{(i)}y) \quad , \qquad (2.6)$$

[2] In these simulation we use the following units: $1/\Omega^{(o)}$ for time, $c/\Omega^{(o)}$ for distance and $c^2/\Omega^{(o)}$ for spatial diffusion coefficients, and often substitute short form δB for $\delta B/B_0$.

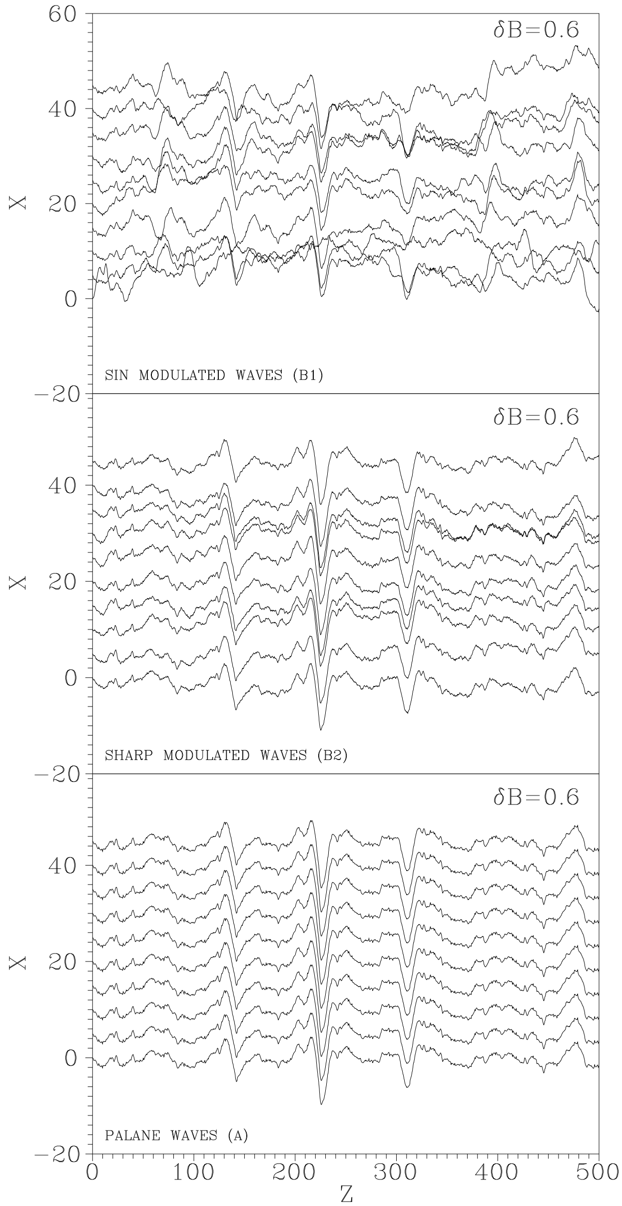

Fig. 1. Examples of magnetic field lines in different field models. We show field line projections at the XZ plane. The same set of wave vectors along the mean magnetic field (k_z) was used for the models presented.

and B2.) the 'sharp-edged' modulation as

$$\Phi_x^{(i)}(y) = y\ mod\ (1/k_y^{(i)}) \qquad . \qquad (2.7)$$

The y-components can be obtained from the above formulae by interchanging x and y. Vectors $k_x^{(i)}$ and $k_y^{(i)}$ are drawn in a random manner from the respective wave-vector range for the wave 'i'.

Testing the simulation scheme

The 4th order Runge-Kutta integration code with constant integration step was used. We performed ordinary tests for preserving the particle pitch-angle and energy in uniform magnetic fields. Then we checked continuity and smooth shape of derived trajectories in perturbed magnetic fields. The accuracy of long time integrations was checked by repeating computations with higher accuracy parameter. The particle positions and momenta obtained coincided to 10^{-6} or better. For static non-uniform magnetic field configurations particle energy was conserved with high accuracy. We also performed a number of standard checks of the applied random number generator. Good test of the accuracy of our computations is provided by performing derivations of the cross-field diffusion in the 1-D or 2-D turbulence. In agreement with the analytic theory of Giacalone & Jokiipi (1994) the respective 'diffusion coefficients' decay with time as t^{-1}, while any computation errors should lead to numerical diffusion and a spurious constant value for $\kappa_\perp$ (see below).

Magnetic field line diffusion coefficient

Since classical papers of Jokipii and Parker that appeared in 60s it is known that the particle cross-field diffusion is possible if magnetic field lines are subject to diffusive wandering. In order to verify the relation of diffusion coefficients $\kappa_\perp$ derived in our simulations to this process, we derived independently the magnetic field diffusion coefficients D_m for all considered field models and all field amplitudes. For these derivations we selected randomly 100 pairs of magnetic field lines and a separate set of perturbations for each pair. For any given pair starting from nearby points at randomly selected position we derived the evolution of the line perpendicular separation Δr as a function of the length s measured along the line. Then, by averaging over all trajectory pairs the corresponding value of the field diffusion coefficient was obtained

$$D_m = \frac{<(\Delta r)^2>}{2s} \qquad . \qquad (2.8)$$

Examples of such derivations are illustrated at Fig. 6 below.

3. Results and discussion

Our main results, presented at the figures, are discussed below. First, at Fig. 2, examples of the derived *formal* cross-field diffusion coefficients (2.1) versus integration time are presented for the considered Alfvénic turbulence models. One can note that in accordance with Giacalone & Jokipii (1994) the cross-field diffusion falls of as $\propto t^{-1}$ for the plane wave model (A) because the magnetic-field vector is dependent only on the single spatial co-ordinate 'z'. For 2-D turbulence model we used the model B1 involving only single polarization waves ($\delta B_x = \delta B_x(y,z) \neq 0$, $\delta B_y = 0$). In this situation particle diffuse across the magnetic field along the x-axis only, while $\kappa_y = 0$. In more complicated 3-D situations represented by the full models B1 and B2 the diffusion coefficient $\kappa_\perp$ does not vanish. An important feature is seen at Fig. 2 that the value of $\kappa_\perp$ depends much on the assumed shape of magnetic field perturbations – for the same amplitude

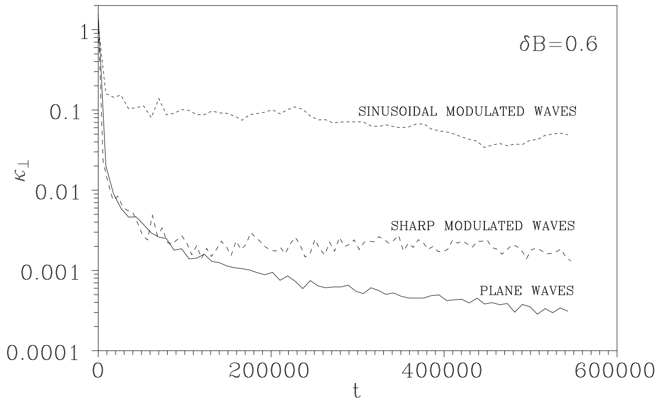

Fig. 2. Examples of the simulated $\kappa_\perp$ versus integration time t in the case of $\delta B = 0.6$ for three considered Alfvén wave turbulence models.

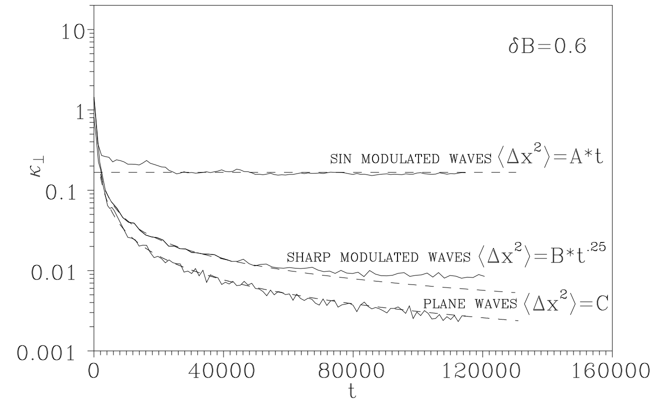

Fig. 3. The initial part of the $\kappa_\perp$ versus t relation from Fig. 2. The power-law fits of the cross-field particle dispersion $< \Delta x^2 >$ are presented for the model A and for the initial part of the curve for the model B2 (A, B and C – constants). A constant fit is provided for the model B1.

and a similar form of modulation in models B1 and B2 the diffusion coefficient values can differ more than by an order of magnitude. If we refer to Fig. 1, it is clear why this difference occurs – magnetic field lines are much more random and 'diffusive' in the model B1 than in B2. However, we would like to point out that appearance of such a pronounced difference was not evident at all from the assumed analytic modulation form for these models. Later, at Fig. 6 one will see that this large difference is observed up to very high amplitudes of turbulent magnetic field. Our field models are compound of a big number sinusoidal waves with uncommensurate wave vectors. Period of a such turbulence is practically unlimited and it is impossible to find by Fourier transformation all wave modes presented in them. But Fourier transformation applied for a finite segment of turbulence shows that the spectrum of the waves is dominated by very long waves. These modes should be responsible for diffusion of magnetic field lines.

Another difference between the two field models considered can be observed at Fig. 3, where the initial period of evolution from Fig. 2 is expanded along the horizontal axis[3]. For the model B2 (sharp-edge modulated waves), the regime of sub-diffusive transport across the mean magnetic field is discovered on this time scale, with $\kappa_\perp$ decreasing in the beginning as approximately t^{-s} with $0 < s < 1$, and later flattens to reach a constant level at large t (Fig. 2). This time evolution of particle cross-field dispersion differs from the one expected for ordinary diffusion with a very short initial free-flow phase followed immediately by the phase with $\kappa_\perp$ fluctuating near some constant value. The observed behaviour reflects the restraining influence of stochastic particle trapping by large amplitude magnetic waves. The analogous transport property of particle sub-diffusion with $s = 0.5$ was analysed for braided magnetic field by Duffy et al. (1995) and Kirk et al. (1996), for the case of cosmic ray acceleration at perpendicular shock waves. Studying the initial part of the curve for the model B1, we can observe a small range of time with analogous sub-diffusive evolution of particle distribution. In this case the ordinary particle cross-field diffusion is much larger than in the case B2 and particles decorrelate from any given 'trap' much earlier. We performed also some numerical experiments in order to understand the role of various wave distributions. If the long waves from the range (0.08, 0.4), are removed from the spectrum the sub-diffusive behaviour for the field model B2 is much reduced. It proves that the phenomenon is caused by the long distance correlations introduced by these waves. On the other hand if the short waves – with wave vectors from the range (2.0, 8.0) – are removed the initial sub-diffusive period increases and the final $\kappa_\perp$ decreases. A more detailed discussion of particle transport in the presence of realistic wave spectra will be provided in a separate publication (Michałek & Ostrowski, in preparation).

At Fig. 4 we can see the simulated values of $\kappa_\perp$, and $\kappa_\parallel \cdot \kappa_\perp$ versus the wave amplitude δB. For small wave amplitudes $\kappa_\parallel \cdot \kappa_\perp$ expected from analytic estimates (1.7,8) is

$$\kappa_\perp \cdot \kappa_\parallel = \kappa_B^2 \left(\frac{< k_\perp >}{< k_\parallel >} \right)^2 \quad . \tag{3.1}$$

We derived the ratio of mean wave vectors (2.3,6,7) squared for our models B1 and B2 by averaging wave vectors with the perturbation amplitude as a weight function. One should note at this point that the range of phase angles given by (2.6) and (2.7) is smaller than 2π and thus amplitude of perpendicular waves – along axes x and y – is smaller than for the waves along the mean field or z-axis. The corresponding ratios $< k_\perp > / < k_\parallel >$ are 0.67 for the sinusoidal modulation model B1 and 1.3 for the sharp modulation model B2. They differ from each other by a factor of 2, making the expressions (1.8) and (3.1) incompatible with the simulation results. The difference appears in the values of $\kappa_\perp$ (for B1 $\kappa_\perp$ is much larger than that given by (1.8)) and the ratio of these coefficients for models B1 and B2 differs more than an order of magnitude while the ratio of $< k_\perp > / < k_\parallel >$ by a factor 2 or 3 only. At smaller wave amplitudes, for both considered wave spectra, there is tendency – partly masked by substantial statistical errors in the simulations – for the products

[3] We used 20 times more particles in these simulations with respect to Fig. 2.

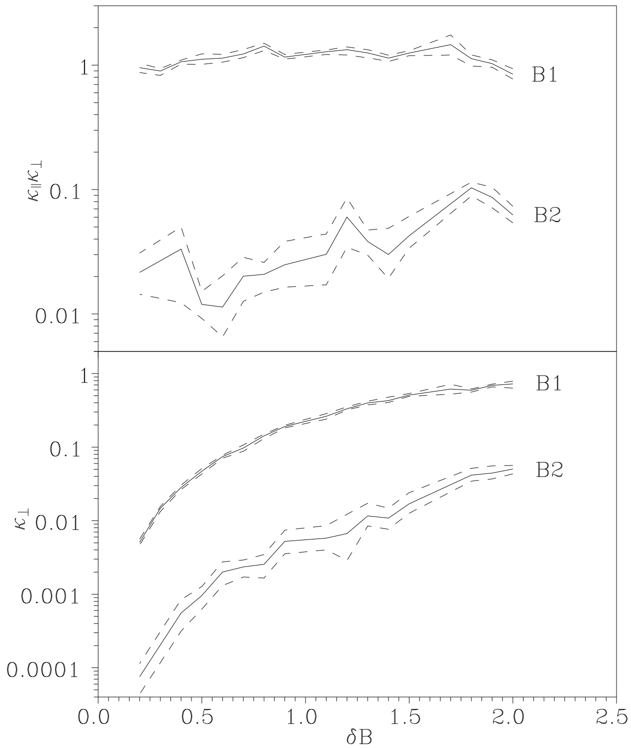

Fig. 4. The simulated values of $\kappa_\perp$ and $\kappa_\parallel \cdot \kappa_\perp$ versus δB for the wave models B1 and B2. Solid lines join the results obtained using our fitting procedure. The adjacent dashed lines provide information about errors as they join the maximum (or respectively minimum) values of the quantity measured within the range used for fitting.

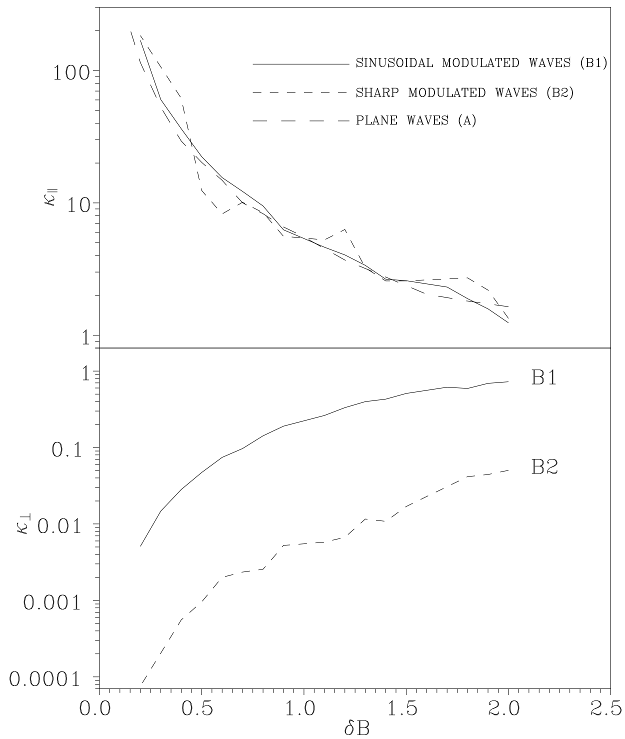

Fig. 5. The simulated values of $\kappa_\parallel$ and $\kappa_\perp$ versus δB for the 3-D turbulence models (B1) and (B2). For comparison the curve for $\kappa_\parallel$ is also provided for the 1-D model A, where $\kappa_\perp$ = 0.

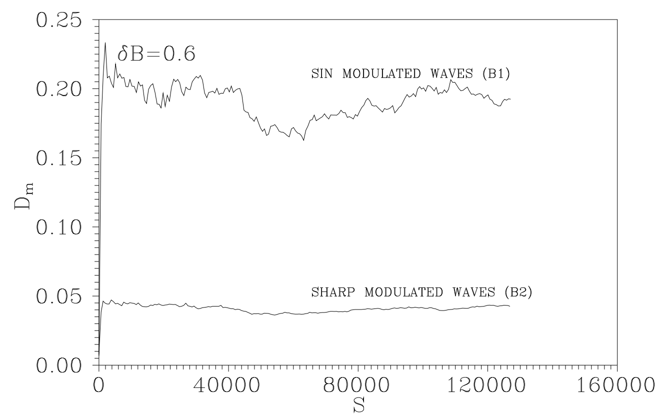

Fig. 6. Simulated values of the magnetic field diffusion coefficient D_m versus distance along the field line s. Results for the models B1 and B2 are presented for δB = 0.6. For 1-D model A the diffusion coefficient vanishes, D_m = 0.

$\kappa_\perp \cdot \kappa_\parallel$ to increase with growing δB. At larger amplitudes the trend seems to start reversing, but the curve fluctuations prevent us from making any definite statement in that matter.

The presented simulations were performed for large wave amplitudes, as contrasted to the 'small' amplitudes considered within the quasi-linear analytic approach. Therefore, in most cases the resonance character of wave-particle interaction is lost very soon – the particle pitch-angle (with respect to $\mathbf{B}_0$) changes substantially at the distance comparable to the wave-length of a 'resonant' perturbation, $\sim r_g$. During the simulations of perpendicular diffusion we noticed that this weakening of resonances, appearing as non-statistical shifts in particle positions in the phase-space co-ordinates, is much more effective – occurs at smaller δB – than in the simulations of $\kappa_\parallel$ or the momentum diffusion coefficient (cf. Michałek & Ostrowski 1996). One can explain this fact by noting that the particle diffusion along the magnetic field is controlled in higher degree by regular changes of pitch-angle μ, while the cross-field diffusion in our simulations occurs due to magnetic field line diffusion (cf. Fig. 6) and fluctuations of μ at an individual gyro-cycle.

This feature is seen also in Fig. 5, where we compare $\kappa_\parallel$ and $\kappa_\perp$ derived at different amplitudes δB. One may note that parallel diffusion coefficients are weakly (if at all) dependent on the turbulence model assumed and the main parameter influencing the value of $\kappa_\parallel$ is the wave amplitude. It is possible to make quite accurate power-law fits to these data and the resulting relations coincide within the fitting accuracy with the quasi-linear relation $\kappa_\parallel \propto (\delta B)^2$. Having in mind that our small-amplitude values for $\kappa_\parallel$ are approximately equal to those derived with the quasi-linear formulae (Michałek & Ostrowski 1996), it follows that the quasi-linear expressions may provide reasonable estimates of $\kappa_\parallel$ for any wave amplitude. This, at first glance unexpected result can be qualitatively explained in following way. At small wave amplitudes the perturbations of particle momentum are $\propto \delta B$ and combine in a random way to yield

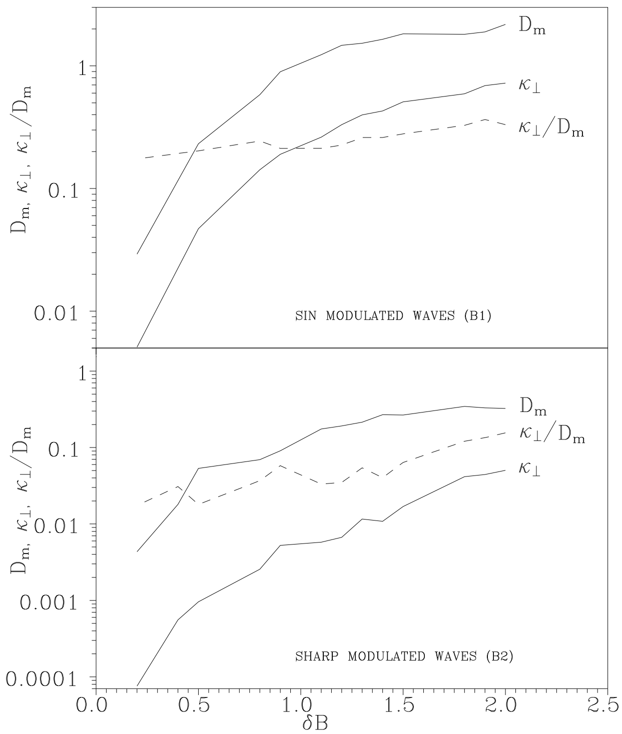

Fig. 7. Simulated values of the coefficients D_m, $\kappa_\perp$, and their ratio $\kappa_\perp/D_m$ versus the wave amplitude δB. Results are presented for the models B1 and B2. For 1-D model A both diffusion coefficients vanish.

the scattering efficiency $\propto (\delta B)^2$ (cf.Drury 1983). For large wave amplitudes even a single interaction with a magnetic field perturbation can scatter particle momentum at 180°. However, in this case the diffusion process is not determined by the product of the large-angle scattering frequency times the squared mean distance required for a single scattering, which is proportional to $(\delta B)^{-1}$ for $\delta B > B_0$. Instead, the efficiency of particle transport to the region of uncorrelated magnetic field is essential here. If a particle interacts with a single magnetic perturbation, the successive modifications of its trajectory are not random. As efficiency of such decorrelation process is proportional to the perturbation amplitude, the effective scattering efficiency is again $\propto \delta B$. In our perturbation models this proportionality continues from small δB up to large perturbation values, at all intermediate wave amplitudes. One should remember that the above far reaching conclusion about validity of the formally quasi-linear expression was proved to hold for particular models for the magnetic field turbulence may be invalid at more general conditions.

In lower panel of Fig. 5 variation of $\kappa_\perp$ with the amplitude δB is presented. It should be noted that the large difference between cross-field diffusion for two considered turbulence models presented earlier for a single δB = 0.6 (Fig. 2, 3) extends in the wide range of amplitudes. One can observe a slow diminishing trend in the difference between models B1 and B2 for large δB. However, even at our highest δB the difference of one order of magnitude is preserved. We believe that the evidence of possible large differences in $\kappa_\perp$ at 'similar' wave models with the same amplitudes, as well as that those differences will not be substantially less for large δB, are the most important findings of this paper. Comparison of these results with the values for field line diffusion coefficients (Fig. 7) shows that the value of the cross field diffusion coefficient reflects the amount of magnetic field lines' diffusion. However, the local cross-field scattering due to resonant waves increases slightly with δB as indicated by a slow increase of $\kappa_\perp/D_m$.

4. Final remarks

We considered particle transport in the perturbed magnetic field of Alfvén-like waves with amplitudes ranging from small ones up to highly non-linear ones. As a test for the present simulations the analytical derivations of Giacalone & Jokipii (1994) of vanishing particle cross-field diffusion in the 1-D turbulent fields and vanishing of such diffusion in the direction of varying magnetic field for 2-D turbulence models were confirmed. For the considered 3-D turbulence models we proved the possibility of substantial - by more than one order of magnitude at the same δB - difference in $\kappa_\perp$ between at first glance similar models and showed that such a difference does not disappear for $\delta B > 1$. Analyzing our simulations we conclude that reason underlying this difference is the more uniform modulation pattern in our model B2 with respect to B1, as illustrated at Fig. 1. From Fig. 7 one can see that the value of the $\kappa_\perp$ is closely related to the value of magnetic field line diffusion coefficient D_m. The growth of wave amplitude is accompanied by slight increase in the ratio of $\kappa_\perp/D_m$. This corresponds to the relative increase of the particle cross-field scattering due to particle-wave interactions relative to the diffusion caused by magnetic field line wandering.

The simulated values of the parallel diffusion coefficient are nearly independent of the considered perturbation model and approximately coincide for all wave amplitudes. We note that they scale as $(\delta B)^{-2}$. As such a relation is expected from the quasi-linear derivations of $\kappa_\parallel$, and our value of $\kappa_\parallel$ approximately equals to the quasi-linear value at small wave amplitude, thus the quasi-linear relation is expected to provide a reasonable estimate for the diffusion coefficient at any δB. However, one should note that this statement is proved only for the three wave models considered.

In the simulations we observed the sub-diffusive behaviour of particle cross-field transport at short time scales due to long-range correlations preserved along particle trajectories. We note that these correlations and the sub-diffusive propagation are more substantial in the case of small cross-field diffusion of the model B2. In the model B1, with much larger $\kappa_\perp$, correlations decay very quickly and the sub-diffusive range of evolution is almost not observed. With the sub-diffusive form of cross-field transport particle dispersion grows for short times much more rapidly than the dispersion derived for the ordinary diffusion with the asymptotic ($t \to \infty$) value of the diffusion coefficient. It may be of importance for a number of astrophysical phenomena, where particle transport across the magnetic field plays a role at short time scales. Our simulations with either the short waves or the long waves removed show that the sub-diffusive transport is particularly important for low energy particles, with most part of the perturbations' power contained in waves longer than the respective resonant waves. On the other hand, the particles with large energies are not expected to exhibit the sub-diffusive transport phenomenon. An example of such behaviour is considered by Kirk et al. (1996) for cosmic ray acceleration at perpendicular shock waves.

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Appendix A: summary of notation

$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$ – magnetic induction vector
$\mathbf{B}_0$ – regular component of the background magnetic field (B_0 = 1 in the simulations)
$\delta\mathbf{B}$ – turbulent component of the magnetic field
c – light velocity (c = 1 in simulations)
E – electric field vector
$\gamma \equiv (1 - v^2/c^2)^{-1/2}$ – the Lorentz factor
k – wave-vector
$\kappa_\perp$ – transverse (cross-field) diffusion coefficient
$\kappa_\parallel$ – parallel diffusion coefficient
L_c – longitudinal correlation length of $\delta\mathbf{B}$
m – particle mass (m = 1 in simulations)
ω – wave frequency
$\Omega \equiv qB/\gamma mc$ – particle angular velocity
$\Omega^{(o)} \equiv qB_o/\gamma mc$
p – particle momentum vector
$P_{ij}(k)$ – power spectrum of $\delta\mathbf{B}$
q – particle charge
r_g – particle gyro-radius
Θ – the momentum pitch-angle with respect to $\mathbf{B}_0$ ($\mu \equiv \cos\Theta$)
$\mathbf{v} \equiv c^2\mathbf{p}/\varepsilon$ – particle velocity vector
V_A – the Alfvén velocity in the field B_0
ε – particle energy

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