Astron. Astrophys. 326, L37–L40 (1997)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

On the possibility of a tidally excited low-frequency g^+-mode in 51 Peg

B. Willems***, T. Van Hoolst******, P. Smeyers, and C. Waelkens**

Instituut voor Sterrenkunde, Katholieke Universiteit Leuven, Celestijnenlaan 200 B, B-3001 Heverlee, Belgium

Received 11 July 1997 / Accepted 8 August 1997

Abstract. The possibility of a resonant excitation of a free oscillation mode with an observed period of 4.2311 d in the star 51 Pegasi by the tidal action of a low-mass companion is investigated.

A free oscillation mode with the observed period is shown to correspond to a g^+-mode of a high radial order. For the determination of the tidal displacement in a component of a close binary system of stars near resonance between a dynamic tide and a free oscillation mode of the star, expressions established by Smeyers et al. (in preparation) are used. For the numerical calculations, a standard solar model is used.

It results that, in a favourable case, the relative difference between the frequency of the tidally excited oscillation mode and the frequency of the resonant dynamic tide must be smaller than 10^{-8} for the radial velocity variations to be of the order of $10\,\mathrm{m\,s^{-1}}$. The probability for such a close resonance is less than 10^{-5}. Since 51 Peg is not the only member of its class, it then seems unlikely that forced oscillations caused by a low-mass companion are responsible for the radial velocity variations that were reported for 51 Peg.

Key words: Stars: oscillations – Stars: individual: 51 Peg

1. Introduction

In 1995, Mayor & Queloz reported the discovery of the first extra-solar planet orbiting a normal main sequence star. The presence of a Jupiter-mass companion moving in a nearly circular orbit at only 0.05 AU of the star 51 Pegasi, with a period of 4.23 d, was inferred from periodic variations in the star's radial velocity. The discovery of Mayor & Queloz was soon confirmed by Marcy & Butler (1995).

Stellar pulsation and spots crossing the stellar disk were suggested as alternative explanations for the observed radial velocity variations, but Mayor & Queloz (1995) provided arguments against both. By using new photometric data and high-resolution spectra, Marcy et al. (1997) were also able to reject spot-induced radial velocity variations, radial pulsations, and nonradial pulsations of degree $\ell \geq 4$.

Since the discovery of Mayor & Queloz, several more low-mass companions in close orbits around solar-type stars have been found. Current theories on the formation of giant planets do not account for the formation of Jupiter-mass companions in such close orbits around stars. Inward migration of a planet formed at a much larger distance from the star ($\sim$ 5 AU) has been proposed as a possible explanation of the close orbits (Lin et al. 1996).

Recently, Gray (1997) presented high-spectral-resolution observations showing intrinsic shape variations in the spectral lines, with the same period and an amplitude comparable to that of the radial velocity variations reported by Mayor & Queloz. Since the reflex motion of an orbiting planet does not alter the shape of spectral lines, Gray attributed the line profile variations to an as yet unidentified nonradial stellar oscillation mode.

In a response on the World Wide Web, Mayor et al. (1997) suggested that a gravity mode of high radial order might be excited by the tidal force exerted by the planet. Such an excitation is possible when the planet's mean orbital motion or one of its multiples viewed in a frame of reference which is corotating with the star, is close to resonance with a free oscillation mode of the star.

In this Letter, we examine whether a g^+-mode with an observed period of 4.2311 d can be resonantly excited in the star, considered at a fixed stage of evolution, by the tidal action of a low-mass companion.

Send offprint requests to: B. Willems

* Research Assistant of the Fund for Scientific Research - Flanders (Belgium) (F.W.O.)

** Postdoctoral Fellow of the Fund for Scientific Research - Flanders (Belgium) (F.W.O.). *Present address:* Koninklijke Sterrenwacht van België, Ringlaan 3, B-1180 Brussel, Belgium

2. Observations

Marcy et al. (1997) reviewed the orbital parameters of 51 Pegasi based upon Doppler measurements which were made from 1995 October to 1996 August. The authors found an orbital period of 4.2311 ± 0.0005 d, a velocity amplitude $K_1 = 55.9 \pm 0.8\ \mathrm{m\,s^{-1}}$, and an eccentricity $e = 0.012 \pm 0.01$. The nearly sinusoidal velocity variations are consistent with an almost circular orbit.

Marcy et al. (1997) also listed measured properties of 51 Peg. The star is a solar like star of spectral type G2-3 V, with an age of 8.5 Gyr, a radius between $1.1\,R_\odot$ and $1.3\,R_\odot$, and a rotation period of 33 ± 4 d.

If one adopts a mass of $1.0\,M_\odot$ for the primary, the orbital parameters lead to a minimum mass of $M_2 \sin i = 0.445 \pm 0.01\,M_{\rm Jup}$ for the planetary companion, where i is the unknown orbital inclination, and $M_{\rm Jup}$ the mass of Jupiter.

3. Partial dynamic tides near resonance with a free oscillation mode of a star

In an ongoing investigation, Smeyers et al. (in preparation) derived expressions for the tidal displacement in a component of a close binary system of stars near resonance between a partial dynamic tide and a free oscillation mode of the component. In this investigation, the stellar component is assumed to be static and to rotate uniformly with an angular velocity Ω around an axis normal to the orbital plane. The effects of the Coriolis force and the centrifugal force are neglected, so that the star's free oscillation modes correspond to those of a non-rotating, spherically symmetric equilibrium star.

The tides generated by the companion are assimilated to those of a point mass and are treated as forced, linear, isentropic oscillations of the star. The expressions for the resonantly excited mode are established in the linear approximation by means of a two-variable expansion procedure (Kevorkian & Cole 1981).

We denote the mass and the radius of the star by M_1 and R_1, respectively, and the mass of the point mass by M_2. Let a be the semi-major axis, e the orbital eccentricity, n the mean orbital motion, and τ a time of periastron passage of the companion's relative orbit. Furthermore, we denote by $\boldsymbol{r} = (r, \theta, \phi)$ a system of spherical coordinates with respect to an orthogonal frame of reference that is corotating with the star.

The tide-generating potential $W(\boldsymbol{r}, t)$ can be expanded in a Fourier series as

$$W(\boldsymbol{r}, t) = -\frac{G\,M_1}{R_1} \sum_{\ell=2}^{4} \sum_{m=-\ell}^{\ell} \sum_{k=-\infty}^{\infty} c_{\ell,m,k} \left(\frac{r}{R_1}\right)^{\ell} Y_\ell^m(\theta, \phi)\, \exp[\mathrm{i}\,(m\,\Omega + k\,n)\,t - \mathrm{i}\,k\,n\,\tau], \tag{1}$$

where G is the gravitational constant, and $Y_\ell^m(\theta, \phi)$ the spherical harmonic of degree ℓ and azimuthal number m. The coefficients $c_{\ell,m,k}$ are given by

$$c_{\ell,m,k} = \frac{(\ell - |m|)!}{(\ell + |m|)!}\, P_\ell^{|m|}(0) \left(\frac{R_1}{a}\right)^{\ell-2} \frac{1}{\left(1 - e^2\right)^{\ell-1/2}} \frac{1}{\pi} \int_0^{\pi} (1 + e\,\cos v)^{\ell-1}\, \cos(k\,M + m\,v)\, dv \tag{2}$$

(Polfliet & Smeyers 1990). Here $P_\ell^{|m|}(x)$ is an associated Legendre function of the first kind, and v and M are the true and the mean anomaly of the companion in its relative orbit.

The coefficients $c_{\ell,m,k}$ obey the property of symmetry $c_{\ell,-m,-k} = c_{\ell,m,k}$. For $\ell = 2$, they depend exclusively on the orbital eccentricity e. The only non-zero coefficients $c_{2,m,k}$ are those associated with $m = -2, 0, 2$.

Let the partial dynamic tide with frequency $\sigma_{\rm T} = m\,\Omega + k\,n$ in the corotating frame of reference be close to resonance with the star's free oscillation mode g^+ of radial order N, with eigenfrequency $\sigma_{\ell,N}$, which is associated with the spherical harmonic $Y_\ell^m(\theta, \phi)$. The components ξ_r, ξ_θ, ξ_ϕ of the Lagrangian displacement with respect to the local orthonormal coordinate basis $\partial/\partial r$, $(1/r)(\partial/\partial\theta)$, $[1/(r\sin\theta)](\partial/\partial\phi)$ can be expressed as

$$\left.\begin{aligned}
(\xi_r)_{\ell,m,N}(\boldsymbol{r}) &= \xi_{\ell,N}(r)\, Y_\ell^m(\theta, \phi),\\
(\xi_\theta)_{\ell,m,N}(\boldsymbol{r}) &= \frac{\eta_{\ell,N}(r)}{r} \frac{\partial}{\partial\theta} Y_\ell^m(\theta, \phi),\\
(\xi_\phi)_{\ell,m,N}(\boldsymbol{r}) &= \frac{\eta_{\ell,N}(r)}{r} \frac{1}{\sin\theta} \frac{\partial}{\partial\phi} Y_\ell^m(\theta, \phi).
\end{aligned}\right\} \tag{3}$$

In the first approximation in the small expansion parameter $\left(\sigma_{\ell,N} - \sigma_{\rm T}\right)/\sigma_{\ell,N}$, the components of a resonantly excited displacement field are given by

$$\left.\begin{aligned}
(\xi_r)_{\rm T}(\boldsymbol{r}, t) &= \frac{\sigma_{\ell,N}}{\sigma_{\ell,N} - \sigma_{\rm T}} \left(\frac{R_1}{a}\right)^3 \frac{M_2}{M_1}\, c_{\ell,m,k}\\
&\quad Q_{\ell,N}\, \xi_{\ell,N}(r)\, P_\ell^{|m|}(\cos\theta)\, \cos\beta_{m,k}(\phi, t)\,,\\
(\xi_\theta)_{\rm T}(\boldsymbol{r}, t) &= \frac{\sigma_{\ell,N}}{\sigma_{\ell,N} - \sigma_{\rm T}} \left(\frac{R_1}{a}\right)^3 \frac{M_2}{M_1}\, c_{\ell,m,k}\\
&\quad Q_{\ell,N}\, \frac{\eta_{\ell,N}(r)}{r} \frac{\partial P_\ell^{|m|}(\cos\theta)}{\partial\theta} \cos\beta_{m,k}(\phi, t)\,,\\
(\xi_\phi)_{\rm T}(\boldsymbol{r}, t) &= -\frac{\sigma_{\ell,N}}{\sigma_{\ell,N} - \sigma_{\rm T}} \left(\frac{R_1}{a}\right)^3 \frac{M_2}{M_1}\, c_{\ell,m,k}\\
&\quad Q_{\ell,N}\, \frac{\eta_{\ell,N}(r)}{r} \frac{m}{\sin\theta}\, P_\ell^{|m|}(\cos\theta)\, \sin\beta_{m,k}(\phi, t)\,,
\end{aligned}\right\} \tag{4}$$

where

$$\beta_{m,k}(\phi, t) = m\,\phi + \sigma_{\rm T}\,t - k\,n\,\tau, \tag{5}$$

$$Q_{\ell,N} = \frac{G\,M_1}{R_1^{\ell+1}} \frac{1}{\sigma_{\ell,N}^2} \frac{\int_0^{R_1} \ell r^{\ell-1} \left[\xi_{\ell,N} + (\ell + 1)\,\eta_{\ell,N}/r\right] \rho\, r^2\, dr}{\int_0^{R_1} \left[\xi_{\ell,N}^2 + \ell(\ell + 1)\,\eta_{\ell,N}^2/r^2\right] \rho\, r^2\, dr}. \tag{6}$$

The quantity $Q_{\ell,N}$ is dimensionless and depends solely on the star's free oscillation mode g^+ of degree ℓ and radial order N. The numerator is often referred to as the overlap integral for the oscillation mode (see, e.g., Press & Teukolsky 1977).

Table 1. Values of the coefficients $c_{2,m,k}$ for $e = 0.012$ and $k = 1, 2, \ldots, 10$. Blank spaces correspond to values smaller than 10^{-14} in absolute value.

k \ m	-2	0	2
1	$-7.5\ 10^{-04}$	$-9.0\ 10^{-03}$	$4.5\ 10^{-09}$
2	$1.2\ 10^{-01}$	$-1.6\ 10^{-04}$	$1.1\ 10^{-10}$
3	$5.2\ 10^{-03}$	$-2.9\ 10^{-06}$	$2.0\ 10^{-12}$
4	$1.5\ 10^{-04}$	$-5.0\ 10^{-08}$	$3.3\ 10^{-14}$
5	$3.8\ 10^{-06}$	$-8.6\ 10^{-10}$	
6	$8.6\ 10^{-08}$	$-1.5\ 10^{-11}$	
7	$1.8\ 10^{-09}$	$-2.5\ 10^{-13}$	
8	$3.8\ 10^{-11}$		
9	$7.6\ 10^{-13}$		
10	$1.5\ 10^{-14}$		

4. Application to 51 Peg

In our investigation, we adopted the following values for the star's radius, mass, and rotation period: $R_1 = 1.2\,R_\odot$, $M_1 = 1.0\,M_\odot$, $P_{\rm rot} = 33$ d. We restricted ourselves to $\ell = 2$, since the amplitudes of the components of the tidal displacement fall of as $\left(R_1/a\right)^{\ell+1}$.

The values of the non-zero coefficients $c_{2,m,k}$ are listed in Table 1 for the orbital eccentricity $e = 0.012$, and $k = 1, 2, \ldots, 10$. For a fixed value of m, the coefficients decrease rapidly in absolute value as k increases. The largest of the coefficients is $c_{2,-2,2}$.

Let $\sigma_{\rm obs}$ be the frequency corresponding to the observed period of 4.2311 d. If we neglect the frequency splitting due to the rotation of the star, the frequency in the corotating frame of reference is given by

$$\sigma_{\rm co} = \sigma_{\rm obs} + m\,\Omega. \tag{7}$$

The associated periods for $m = 2, 0, -2$ are 3.3675 d, 4.2311 d, and 5.6902 d, respectively.

We determined the eigenfrequencies of low-frequency g^+-modes in the relevant range of periods by means of an asymptotic approximation formula suitable for stars with a convective envelope [Tassoul 1980, Eq. (124)]. For the determination of the dimensionless part in the asymptotic expression, we used a standard solar model without helium diffusion. The relative errors of the asymptotic eigenfrequencies are of the order of 0.2 %.

The periods of the g^+-modes of radial orders $N = 250$ and $N = 425$ are 3.3524 d and 5.6921 d, respectively. Thus, an oscillation mode with an observed period of 4.2311 d corresponds to a g^+-mode of a high radial order. For larger values of the degree ℓ, the radial orders of the g^+-modes in the relevant range of periods are even higher.

Next, we determined the radial parts of the radial and the transverse components of the Lagrangian displacement for various g^+-modes of the standard solar model, by integrations of the full fourth-order system of governing differential equations. The logarithms of the quantities $Q_{2,N}\,\xi\left(R_1\right)/R_1$ and $Q_{2,N}\,\eta\left(R_1\right)/R_1^2$ are represented in Fig. 1 as a function of the radial order N of the mode, for all modes of radial orders $N = 1, \ldots, 50$ and for modes of higher radial orders that are a multiple of 50. Both quantities are dimensionless and independent of the normalization adopted for the determination of the free oscillation mode.

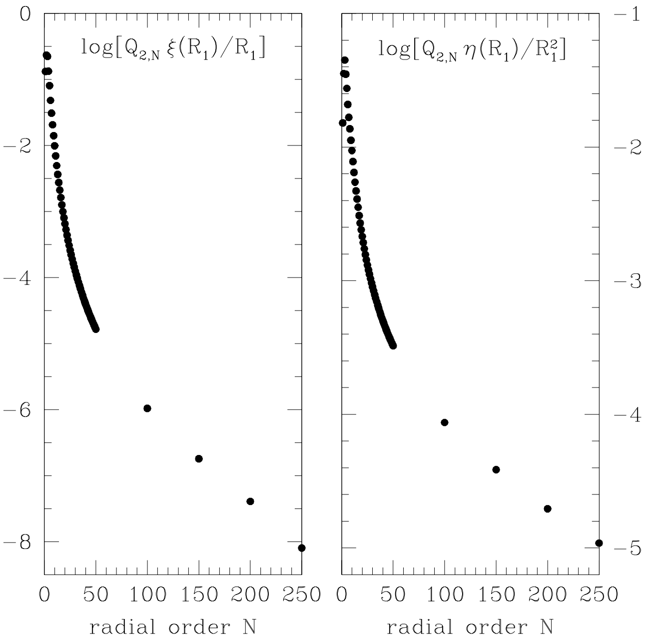

Fig. 1. The logarithms of the quantities $Q_{2,N}\,\xi\left(R_1\right)/R_1$ and $Q_{2,N}\,\eta\left(R_1\right)/R_1^2$ for g^+-modes of a standard solar model, as a function of the radial order N of the mode.

From $N = 3$ on, the quantities $Q_{2,N}\,\xi\left(R_1\right)/R_1$ and $Q_{2,N}\,\eta\left(R_1\right)/R_1^2$ decrease rapidly as the radial order N increases. The decrease is slower for higher-order modes. Except for the lower-order modes, the quantities $Q_{2,{\rm N}}\,\xi\left(R_1\right)/R_1$ are smaller than the corresponding quantities $Q_{2,N}\,\eta\left(R_1\right)/R_1^2$. This could be anticipated since for higher-order g^+-modes, the ratio of the radial component to the transverse component of the Lagrangian displacement at the star's surface is proportional to the square of the dimensionless eigenfrequency of the mode (Smeyers & Tassoul 1987).

As an example, we considered the favourable case of a tidally excited g^+-mode associated with $m = -2$ and $k = 2$. The relevant period in the corotating frame of reference is then 5.6902 d. With respect to an inertial frame of reference, the condition for resonance takes the form $k\,n \simeq \sigma_{\rm obs}$. Hence, the g^+-mode with the observed period of 4.2311 d would be excited in 51 Peg by a companion with an orbital period equal to 8.4622 d. Neglecting the mass of the planet, one derives from Kepler's third law that the semi-major axis of the relative orbit is equal to 0.08 AU.

From the relation

$$\left(M_2\,\sin i\right)^3 \simeq \frac{1}{G}\,M_1^2\,\frac{K_1^3}{n}\left(1-e^2\right)^{3/2}, \tag{8}$$

it results that $M_2 \sin i = 0.561\,M_{\rm Jup}$. If part of the measured radial velocity variations would be due to the forced oscillation, the mass of the companion would even be smaller. Following Mayor & Queloz (1995), we used a lower limit of 0.4 for $\sin i$. Even if one takes into account a possible misalignment as large as 10°, the mass of the companion is still smaller than $2.5\,M_{\rm Jup}$, so that $M_2/M_1 \leq 2.4\;10^{-3}$.

Finally, from Fig. 1, it can be seen that the quantities $Q_{2,N}\,\xi\left(R_1\right)/R_1$ and $Q_{2,N}\,\eta\left(R_1\right)/R_1^2$ will be smaller than 10^{-8} and 10^{-5}, respectively.

One obtains the components of the velocity at the points of the star's surface by multiplying the components of the Lagrangian displacement at these points by $\sigma_{\rm T}$. For the example considered, one has for the radial part of the transverse components of the velocity field at the star's surface

$$\sigma_{\rm T}\left(\frac{R_1}{a}\right)^3 \frac{M_2}{M_1}\, c_{2,-2,2}\, Q_{2,N}\, \frac{\eta_{2,N}\left(R_1\right)}{R_1} \leq 1.1\;10^{-8}\,{\rm m\,s^{-1}}.$$

The radial part of the radial component of the velocity field at the star's surface is even smaller.

From these components, we infer that the tidal excitation of the g^+-mode by the companion would lead to radial velocities not larger than

$$\frac{\sigma_{\ell,N}}{\sigma_{\ell,N}-\sigma_{\rm T}}\;10^{-7}\,{\rm m\,s^{-1}}.$$

Hence, for the radial velocities to be of the order of $10\,{\rm m\,s^{-1}}$, and thus for the radial velocity variations to be measurable, the relative difference $\left(\sigma_{\ell,N}-\sigma_{\rm T}\right)/\sigma_{\ell,N}$ between the frequency of the free oscillation mode and the forcing frequency has to be smaller 10^{-8}. Since the relative difference between consecutive eigenfrequencies of the solar model in the relevant range of radial orders is of the order of 10^{-3}, it follows that the probability of the resonant excitation of the g^+-mode with a period near 5.6902 d is smaller than 10^{-5}.

With its age of 8.5 Gyr, 51 Peg is already slightly more evolved than the Sun. Its central condensation is therefore larger than for the solar model considered here. As a result, the eigenfrequencies of the free g^+-modes of 51 Peg will be larger than those of the Sun and still higher overtones will be needed for the resonance to occur. We therefore believe that the very small probability found above also applies to 51 Peg.

5. Conclusions

We have investigated the possibility of a resonant excitation of a g^+-mode with an observed period of 4.2311 d in the star 51 Peg. To this end, we used expressions derived by Smeyers et al. for the tidal displacement in a component of a close binary system of stars near resonance between a partial dynamic tide and a free oscillation mode of a star. For the numerical calculations, we adopted a standard solar model.

By applying an asymptotic approximation formula appropriate for stars with a convective envelope, we found that a free oscillation mode with an observed period of 4.2311 d corresponds to a g^+-mode of a high radial order. Furthermore, we determined the overlap integrals for a large number of free oscillation modes g^+ from the radial and the transverse components of the Lagrangian displacement, obtained by integrating the full fourth-order system of governing differential equations.

By means of expressions (4) for the components of a resonantly excited displacement field, we estimated the radial parts of the velocity variations at the star's surface that are associated with the excited mode. We conclude that, in the favourable case of $m = -2$ and $k = 2$, the relative difference between the frequency of the tidally excited oscillation mode and the frequency of the resonant partial dynamic tide must be smaller than 10^{-8} for the radial velocity variations to be of the order of $10\,{\rm m\,s^{-1}}$. The probability for such a close resonance is less than 10^{-5}. When one takes into account that 51 Peg is not the only member of its class, it then seems unlikely that forced oscillations caused by a low-mass companion are responsible for the radial velocity variations that were reported for 51 Peg.

Acknowledgements. The authors would like to thank Dr. J. Christensen-Dalsgaard for the use of his standard solar model and oscillation code.

References

Gray, D.F., 1997, Nature 385, 795

Kevorkian, J., Cole, J.D., 1981, Perturbation Methods in Applied Mathematics, Springer, New York

Lin, D.N.C., Bodenheimer, P., Richardson, D.C., 1996, Nature 380, 606

Marcy, G.W., Butler, R.P., 1995, IAU Circ. No. 6251

Marcy, G.W., Butler, R.P., Williams, E., Bildsten, L., Graham, J.R., Ghez, A.M., Jernigan, J.G., 1997, ApJ 481, 926

Mayor, M., Queloz, D., 1995, Nature 378, 355

Mayor, M., Queloz, D., Marcy, G.W., Butler, R.P., 1997, http://cannon.sfsu.edu/˜williams/planetsearch/nrp.html

Polfliet, R., Smeyers, P., 1990, A&A 237, 110

Press, W.H., Teukolsky, S.A., 1977, ApJ 213, 183

Smeyers, P., Tassoul, M., 1987, ApJS 65, 429

Smeyers, P., Willems, B., Van Hoolst, T., in preparation

Tassoul, M., 1980, ApJS 43, 469

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.