Astron. Astrophys. 326, 1228–1234 (1997)

ASTRONOMY AND ASTROPHYSICS

Nonlocal communication in self-organising models of solar flare occurrence

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Received 31 October 1996 / Accepted 23 May 1997

Abstract. Recent avalanche models of solar flare occurrence which consider the magnetised solar atmosphere to be in a state of self-organised criticality have concentrated mainly on the magnetic field as the means by which individual energy release sites communicate with neighbouring sites. This, however, neglects the possibility of remote triggering of energy release via fast particles. In this paper we augment the standard avalanche models of flare occurrence to include this possibility of 'non-local' communication between sites. Non-local communication with just one or two remote sites does not observably modify the flare size distribution (this quantity remains consistent with observations). Non-local triggering of too many remote sites can, however, destroy the self-organised critical state. Our simulations also produce more general conclusions on the robustness of the self-organised critical state.

Key words: Sun: flares – Sun: corona – Sun: magnetic fields – Sun: X-rays, gamma rays

1. Introduction

Lu & Hamilton (1991; hereafter LH) introduced a new way of looking at solar active region evolution. They suggest that the magnetised solar atmosphere is in a state of self-organised criticality (SOC) as a result of competition between two factors: the external driver of the convection zone velocity field; and the internal re-arrangement which occurs when the field gradient locally becomes great enough for the field connectivity to change suddenly, releasing energy, in reconnection. Energy release will initially occur only in some very localised region, but field gradients in other, adjacent regions will have changed after the energy release event, and these regions may themselves become unstable to energy release in consequence. Communication between energy release sites thus allows energy-releasing events ('flares') of various sizes to occur, just as sandpile avalanches (Bak et al. 1987), earthquakes (e.g. Carlson & Langer 1989) and other sorts of dynamical readjustments of driven systems may occur. Moreover, simulations on a discrete grid (LH; Lu et al. 1993; Vlahos et al. 1995; Galsgaard 1996) suggest that the 'flare' size distribution will be close to that observed for actual solar flares (Dennis 1985; Crosby et al. 1993; Biesecker 1994).

The magnetic field in this picture plays a dual role. It is both the medium in which the energy releases take place, and the means by which information about one energy release site travels to and influences others. Lu (1995a, b) attempts to delineate general conditions for self-organising behaviour in continuous fields. He draws on this ingredient of LH type models and proposes that a 'local conservation law' must always be present in fields that can self-organise, in order to ensure communication between energy release sites. Heyvaerts et al. (1977), however, realised that explaining flare impulsive phases with 2-D current sheets necessitated rapidly co-ordinated energy release and particle acceleration in several such sheets, and proposed "dissipation spreading" in this fashion as a means by which this might occur. Norman & Smith (1978) discussed related ideas. For our purposes here, the two important, distinguishing features of dissipation spreading are its non-locality (which distinguishes it from an 'avalanche' spreading through an increasing region, as in the original LH models) and its rapidity (communication may take place at up to the speed of light, as opposed to the Alfvénic timescales implicit in the LH picture). The latter attribute may help to make big enough flares with rapid enough impulsive phases.

In introducing this additional possibility of 'non-local' communication, we have various physical possibilities in mind. For instance, particles accelerated at one energy release site will, depending on the field line topology, travel to other potential sites in the grid, enhance the local resistivity by generating, particularly, ion-acoustic turbulence (which may result in an effective resistivity enhanced by up to 10^5 over the classical value - Papadopoulos 1976), and thus increase the likelihood of reconnection taking place there. Recall that Priest (1982), for instance, lists a local increase in resistivity as one of the three ways in which reconnection may be triggered. Another possi-

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bility is that intense, coherent electromagnetic radiation from particles accelerated at one location might alter particle distribution functions at other locations, for instance as proposed by Sprangle & Vlahos (1983) - this could similarly result in enhanced resistivity and thus enhanced likelihood of reconnection. The topology of the field would play a crucial role here also, but in a different way (by determining those locations at which energy release, and second or higher harmonic gyroresonance absorption are both possible). More generally, waves in various modes of propagation may remotely influence transport properties, particularly resistivity, via various sorts of spatially localised, resonant absorption. Without studying some particular scenario in detail, all that we can say - and for these purposes, all that we need to say - is that energy release at one location may rapidly and remotely trigger energy release at other locations, by a variety of physical mechanisms.

Recalling these points, MacKinnon et al. (1996), hereafter Paper I, introduced a simple, illustrative, 1-D stochastic cellular automaton model, which includes little more than the possibility that roughly similar, 'elementary' events may trigger one another. They showed that qualitative, and probably quantitative agreement with the observed form of the flare size distribution could be obtained (see also Macpherson & MacKinnon 1997). Thus more primitive models can be constructed, involving no local conservation law, which nonetheless self-organise and agree with observations. In particular, while each energy release site has 'neighbours' which it can influence, these need not be physically adjoining, and dissipation spreading is included as a possibility.

To investigate further the implications of dissipation spreading for SOC models, we here augment the simulation procedure of LH by including the possibility of 'non-local' communication between lattice sites. We augment the simulations in a way which hopefully represents the sorts of physical mechanisms of non-local communication outlined above.

Before presenting these calculations and their results, we first recall the details of the LH models and the various assumptions which go into these simulations (cf. Galsgaard 1996). We then proceed to the details of our own model and the adaption of the standard LH type simulation to include explicit non-local communication between sites. After presenting the results of our simulations, we demonstrate that a local conservation law is not, in fact, an essential ingredient of SOC by constructing a self-organising model which relies purely on non-local communication. This last model is closer in spirit to LH type models than that of Paper I was, and is obviously not bound to Alfvénic timescales. All of this work is presented in Sect. 2, and Sect. 3 discusses its significance.

2. A self-organising model inclusive of nonlocal communication

2.1. Details of the standard SOC model

In the original picture of LH, a vector field $\mathbf{V}$ is randomly incremented at sites of a 3-D lattice. The local gradient at any one site i, $\nabla|\mathbf{V_i}|$, is defined as the difference between the local field and the average of its six nearest neighbours $\mathbf{V_{nn}}$,

$$\nabla|\mathbf{V_i}| = \mathbf{V_i} - \frac{1}{6}\sum_{nn}\mathbf{V_{nn}} \tag{1}$$

The lattice grid is said to be stable if the magnitude of the gradient is everywhere less than a specified critical value V_{cr}. The system is driven by adding a random vector $\delta\mathbf{V}$, with $|\delta\mathbf{V}| \ll V_{cr}$, to a randomly selected position on the grid. If no sites become unstable due to this addition, another random vector is added to another randomly selected site until the gradient at some site exceeds the threshold. Once this happens, the field $\mathbf{V}$ at each unstable grid point is adjusted according to the following relaxation rules

$$\mathbf{V_i} \rightarrow \mathbf{V_i} - \frac{\mathbf{6}}{\mathbf{7}}\nabla|\mathbf{V_i}|, \quad \mathbf{V_{nn}} \rightarrow \mathbf{V_{nn}} + \frac{\mathbf{1}}{\mathbf{7}}\nabla|\mathbf{V_i}| \tag{2}$$

These adjustments reduce the (local) $\nabla|\mathbf{V}|$ below the threshold, but open up the possibility of neighbouring gradients now exceeding the threshold. Thus an 'avalanche' may take place. If we identify $\mathbf{V}$ with the magnetic field $\mathbf{B}$, we may have a discrete analogue for the behaviour of the solar coronal magnetic field. The relaxation rules followed by Lu et al. (1993), Vlahos et al. (1995) and Galsgaard (1996) allow some generalisations of Eq(s). (2) but retain the same feature of local conservation of the field. The boundaries of the simulation grid are held constant and any instabilities reaching the boundary are considered to diffuse out of the system.

Following the schemes outlined in LH, Lu et al. (1993), Vlahos et al. (1995) and Galsgaard (1996), we have carried out our own numerical simulations, on a $40 \times 40 \times 40$ grid, to confirm that our avalanche model produces results consistent with the previous work listed above. We obtained power-law frequency distributions for total energy (event size) and peak flux with indices of -1.44 ± 0.02 and -1.78 ± 0.05 respectively, in close agreement with both the observed values and with previous SOC simulations of other authors. Moreover, the power-law behaviour persists over almost 5 orders of magnitude for total event size, and over 3 orders of magnitude for the peak flux distribution.

We comment briefly on the robustness of this avalanche model of self-organisation, in particular with respect to different initial states of the simulation grid. In LH, the authors describe driving the simulation grid to the SOC state from an initially stable configuartion. Alternatively, however, it is also possible to start the simulation from an 'over-critical' initial state, where instabilities are present from the start and we allow the grid to relax 'down' into its SOC state. The qualitative behaviour in the initial relaxation phase will be important and instructive for what follows.

Fig(s). 1 and 2 demonstrate the behaviour across the 3-D grid of the average field value and the average magnitude of field gradient, respectively, as the simulation is driven. We plot these values for three cases, a sub-critical initial state, a marginally critically initial state and finally from an initial over-critical

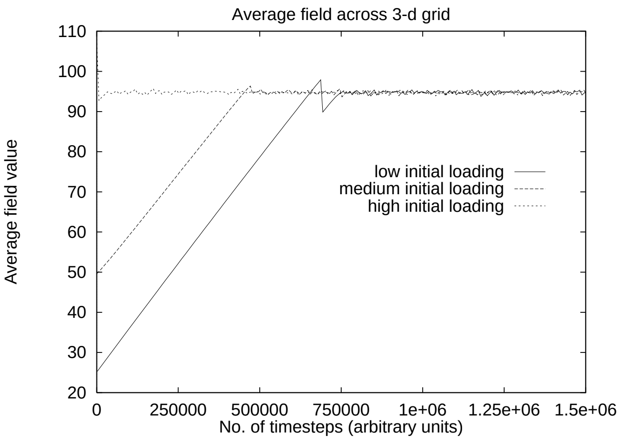

Fig. 1. Average field against time for different initial loadings.

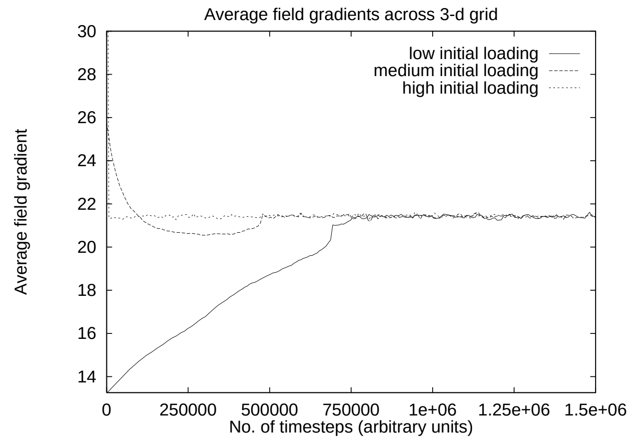

Fig. 2. Average field gradient against time for different initial loadings.

state. These figures confirm that the SOC state is insensitive to the initial conditions as all three cases tend to the same average field and average field gradient values. Obviously, frequency distributions of event sizes and durations are only compiled once the critical state has been achieved.

The final important point to notice from Fig(s). 1 and 2 is the near constancy of the average field and field gradients throughout the SOC state. Galsgaard (1996) notes that the presence of constant tension in the simulation grid is one of the necessary conditions for avalanche behaviour to be observed. In addition, however, we note that even when the largest avalanches are occurring in our simulation, at no time does a fluctuation of greater than approximately 1% of the average field or field gradient occur. Thus the 'stored' tension is considerably greater than that released during any one avalanche. This feature proves important when compared to the models which include non-local communication and so we return to a discussion of this point later.

2.2. The effect of varying the critical threshold across the grid

In Paper I we noted the possibility that circumstances at different locations on the Sun (or within one active region) may vary considerably, resulting in differing conditions of criticality at different locations. Physically, we expect electron and ion temperatures, densities, and levels of turbulence to vary from one place to another, and we would also expect the value of V_{cr} to depend on these. We include this possibility now by re-running our simulation but with V_{cr} redefined to be a function of position $V_{cr}(x, y, z)$. We allow $V_{cr}(x, y, z)$ to take random values between 0.5 to 1.5 times the constant V_{cr} used in Sect. 2.1. This random distribution of $V_{cr}(x, y, z)$ has mean value $\bar{V}_{cr} \approx V_{cr}$. Other authors have mentioned that the self-organised critical (SOC) state is insensitive to the (constant) value of the threshold (Lu et al. 1993). Also LH and Lu et al. (1993) reported that small ($\pm 20\%$) spatial variation of V_{cr} had no effect on the SOC state. The present work considers a much greater range of V_{cr} than any previous. However, re-running the simulation box produced frequency distributions for event size which were entirely consistent within the error bars of the distributions for the constant V_{cr} case. Hence variations in the critical condition across the simulation grid do not affect the results of the SOC state.

2.3. Remote communication between flaring elements

To include the possibility of remote communication between sites on the Sun, as discussed in Sect. 1, we make additions to the redistribution laws described by Eq(s). (2). The calculation of the field gradients depends on the nearest neighbours of each site. Thus in making the first modifications to include nonlocal effects we do not completely remove the reliance on the local conservation law. What we do, however, is to allow each unstable site to influence a number of remote sites in the simulation box in such a way as to reduce the critical value of V at those remote sites. Thus an instability at one location in the grid may trigger flaring behaviour remotely at other sites. We may expect the occurrence of larger events, with consequent 'hardening' of the distribution or even, in an extreme case, destruction of the SOC state. Remotely triggered energy release, mediated by fast particles, has been observationally diagnosed (e.g. Nakajima et al. 1985). On such occasions, spatially resolved observations in various wavelengths were available, and the events were widely enough separated that they could be distinguished. Such physical occurrences must commonly take place over smaller distances, remaining undiagnosed. For instance, one might speculate quite legitimately, albeit without much hope of confirmation or refutation, that each of the various UV brightenings seen in a particular flare by Cheng et al. (1981) corresponded to distinct energy release events separated by distances on the scale of the active region. Thus in calculating the frequency distributions for this model we have no particular reason for counting nonlocally triggered events as separate, new events, and regard all field readjustment and energy release subsequent to instability onset at one point as part of the event we identify with that initial instability. (In other words, we suppose that the Sun and solar flares are viewed with a non-imaging, whole-Sun instrument,

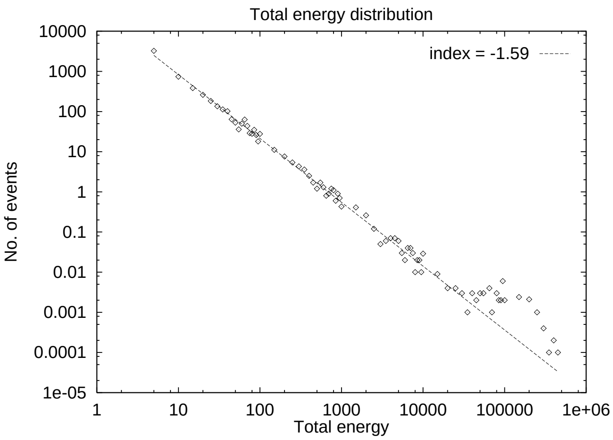

Fig. 3. Frequency distribution for total energy with 3 remotely triggered 'neighbours'. The distribution starts to exhibit a bimodal tendency with a steeper low energy distribution with an excess of large events. The power-law fit is restricted to include only the lower energy part of the distribution.

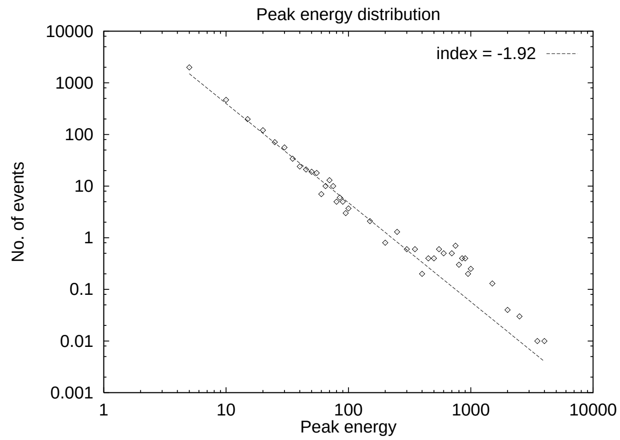

Fig. 4. Frequency distribution for peak energy with 3 remotely triggered 'neighbours'. As above, the bimodal tendency is in evidence with the excess of large events and the power-law fit is restricted to the low energy half of the distribution.

e.g. a hard X-ray spectrometer like HXRBS or BATSE.) This follows the same convention as LH and Lu et al. (1993) in that the statistics of each avalanche event are calculated from the time of the initial instability until the complete grid returns to a stable configuration, irrespective of whether the energy release branched out into unconnected regions on the grid or remained in one compact area.

The model initially follows the standard set-up of LH, Lu et al. (1993), Vlahos et al. (1995) and Galsgaard (1996) with a constant threshold value across the grid. After a site becomes unstable, however, not only is the instability relaxed locally in the normal way, but a random site elsewhere in the grid has its threshold lowered by a random amount, up to a maximum of $V_{cr}/2$. Hence the avalanche can continue both locally and branch out remotely if this random new site is now unstable in the next timestep. After one timestep, all remotely lowered thresholds are returned to their original value unless another instability causes them to be lowered again. This lowering of the threshold represents a change in conditions (e.g. onset of anomalous resistivity).

We consider first the case where each unstable site only communicates remotely with one other randomly selected site in the simulation grid. We can treat this as the minimal case of a SOC system with nonlocal communication included. The important questions are: whether this makes any difference to the stability and robustness of the simulation; and whether we can differentiate between the results from this model and the original simulation. The development of the SOC state is found to follow a very similar pattern to Fig(s). 1 and 2. Evidently the SOC state is not significantly altered by including remote triggering of one site. Looking at the frequency distributions for peak flux and total energy, we find that the distributions 'harden' very slightly, with new best fit power-law indices of 1.41 ± 0.02 for the total event size distribution and 1.67 ± 0.06 for the peak flux index. Thus the effect is more noticeable in the peak flux index. These changes can be explained by an enhanced avalanche effect occurring when the system becomes highly stressed. When the number of unstable sites in any one timestep grows, so too does the number of remote sites whose threshold is lowered, allowing further triggering of instabilities.

We next allow 3 remote sites to be affected by any one unstable site. A critical state is apparently reached although with a noticeably lower average stress ($V_{av} \approx 85$) than for the standard (purely local) case. The frequency distributions produce power-law behaviour for flare energy (E) and peak activity (P) for small values of E and P. However, the distributions start to show a more significant bimodal form with an excess of events occurring at large E and P and a steeper low energy distribution. This is due to the larger number of sites which can now be triggered from one initial instability. Once a large event gets underway, it can grow quicker than a standard, locally propagating, avalanche and hence the likelihood of large events is greatly enhanced. Figs. 3 and 4 show this effect with the power-law fits being restricted to the low end of the distribution and excluding the obvious excess in large E and P.

Finally, when we increase the number of remotely affected sites to 6, to equal the number of locally affected points, we find that the SOC state is never reached. As soon as the system approaches a highly stressed configuration, the size of an instability grows rapidly, because of the much greater number of remote sites which can be affected. Events occur which are large enough to drain the simulation grid of all its built up resources. A long time interval then occurs before the grid builds up sufficient stress to allow the next large stress dissipation event. This behaviour is shown clearly in Fig. 5. It is difficult to see how such temporal behaviour might be reconciled with the observed evolution of active regions. Moreover, the (time-averaged) flare size distribution in this case bears little resemblance to that actually observed, having local maxima at both small and large event sizes.

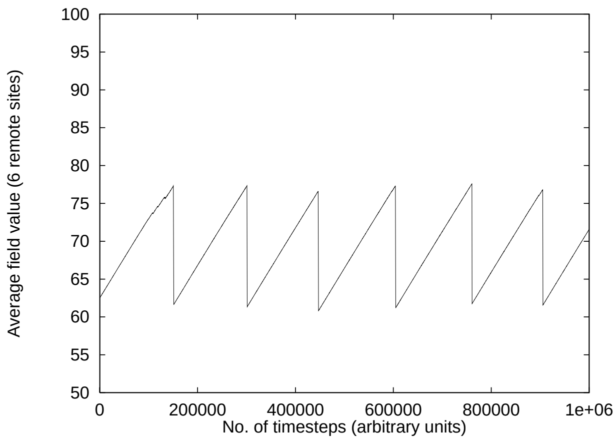

Fig. 5. Excessively large events caused by strong remote communication leads to a dissipation of all available energy in the box and a destruction of the SOC state.

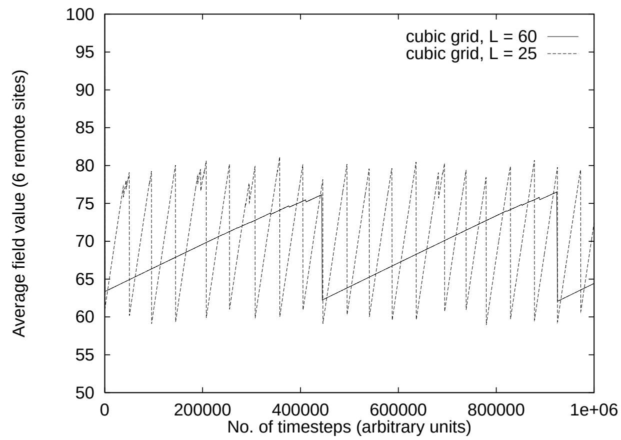

Fig. 6. Simulations of SOC with 6 additional nonlocal remote sites for different sized grids scales the time interval between large events by a factor roughly proportional to the change in grid size.

Having obtained this result for the $40 \times 40 \times 40$ grid, we check also how sensitive this behaviour is to the grid size. Keeping all other parameters constant, we show in Fig. 6 the behaviour of cubic grids of side 25 and 60. We observe exactly the same type of behaviour except, primarily, that the 'period' between the very large events is different. The longer period for the larger grid is explained by the fact that the driving rate, having been kept constant for all the simulations, is effectively slower, by a factor of $(60/25)^3 \approx 14$, for the larger grid. This is comparable to the period between major events being approximately 10 times longer for the 60^3 grid. Comparing with the period between events for the 40^3 grid, we find that an effective driving rate which is $(40/25)^3$ slower and $(60/40)^3$ quicker, respectively, accounts well for the difference in period. Further investigations into the complete dynamics of this behaviour are under way (MacKinnon & Macpherson 1997).

2.4. No local conservation law

Having slowly relaxed the dependence on a purely local conservation law as a means of propagating instabilities across the simulation grid, we now wish to demonstrate the fact that a system can exhibit SOC behaviour while relying purely on non-local communication. Robinson (1994a, b) introduces a model exhibiting SOC analogous to that outlined at the start of Sect. 2 in which a vector field $\mathbf{h}$ is prescribed over a lattice grid. However, instead of a gradient (or "slope") term defining the instability (as in Eq. (1)), he simply defines the magnitude (or "height") of the field, $|\mathbf{h}|$, as the important quantity. Bak et al. (1987) also relied on a height function for their sandpile model rather than a true definition of slope. A physical analogue for such a picture here might involve the local value of $|\mathbf{j}|$, the current density, rather than magnetic field strength.

Following an instability, $|\mathbf{h}|$ is removed from the unstable site and distributed equally to its nearest neighbours, thus conserving $\mathbf{h}$ (locally). There is, however, no reason why the height $|\mathbf{h}|$ needs to be distributed amongst local neighbours since the definition of $|\mathbf{h}|$ is not reliant on the neighbouring values (unlike the LH scenario). Hence we can introduce a similar model in which the redistribution takes place to non-local, randomly chosen sites elsewhere in the grid, with the expectation that a SOC state will still be observed.

Fig. 7 confirms this as we plot the total event size frequency distribution for two cases: the first follows the methodology of Robinson (1994a, b) with a local distribution of the height, the second introduces our purely non-local redistribution. Although the indices of the two distributions are different, both exhibit strong power-law behaviour and are indicative of SOC behaviour. The difference in slopes are explained by effects which relate to the finite size of the simulation grid. In the local transport case, instabilities are propagated through nearest neighbour points and so have to travel through all points between the initial instability and the boundary before allowing certain tension to diffuse out of the system. The event can of course die out before reaching the boundary but this keeps a higher tension in the system allowing larger events to occur. In the non-local case, tension can diffuse out of the grid irregularly due to the random choice of non-local sites, reducing the average grid tension. Moreover, in the local transport case there is a greater chance of sites being influenced by more than one neighbour and so becoming unstable through multiple enhancements from nearby instabilities. The distributions start exhibiting an exponential tail at very large event sizes due to finite grid size effects.

3. Discussion

Our primary aim in this paper has been to investigate the influence of 'non-local' triggering of energy release on the statistical properties of flares. Briefly, our two main conclusions are as follows. First, SOC models consistent with observations can be constructed in which lattice sites communicate with other lattice sites at completely random locations (see also Paper I). Second, augmenting existing (e.g. LH) SOC models by includ-

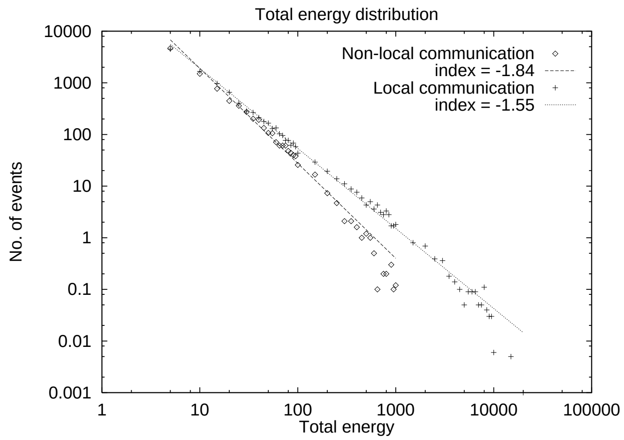

Fig. 7. Power-law event size distributions for a self-organising grid with either local or non-local communication between sites. The difference in indices is explained in the text. The distributions tail off at very large event sizes due to the finite size of the grid. The indices are fitted to the power-law part of the distributions.

ing non-local communication has effects which depend on the magnitude of the remote disturbance, and on the number of remote sites which may be influenced by any one site. Specifically, if communication with too many sites is possible, events large enough to completely relax the grid take place regularly and no SOC state occurs. Although not demonstrated in Sect. 2.3 above, it is obvious that reducing the magnitude of the reduction of V_{cr} would also reduce the consequences of including non-local communication.

Secondary aims were to confirm the insensitivity of LH type simulations to initial conditions, and to substantial spatial variations in **V**. The first of these also proves instructive for understanding LH type models with non-local communication. Specifically, we see the contrast between the temporal behaviour of cases with no (Fig. 1) and 'too much' (Fig. 5) non-local communication. The approach to the SOC state, a mere technical detail in the former case, dominates the evolution of the latter, being (quasi-)periodically re-initialised via the occurrence of a huge event which completely relaxes the grid. Such huge events in turn are able to occur because non-local communication involves a much greater fraction of the grid in flaring. In this respect, the important role of non-local communication is really to increase the number of other sites that any one site can influence.

We must conclude the following. If SOC models with local (MHD) communication really describe the evolution of solar magnetic fields, 'dissipation spreading' must not normally allow any one region to influence more than one or two other, non-adjacent regions. Such comments might ultimately be reinterpreted in terms of field topology in complex active regions.

The occurrence of very large events in some of the cases considered leads us naturally to discuss the occurrence of such events on the Sun, and in other simulations. Lu et al. (1993) found no correlation between the size of simulation avalanche event and the time to the next avalanche (or from the previous one). In the SOC picture, apparently, very large flares may occur very close together in time. If observed, the clear presence of a 'recharging time' in real data would then argue against SOC models. Biesecker (1994) analysed observations of solar flares made by CGRO BATSE, to see if any correlation between flare size and subsequent interval between flares existed in the data. He obtained a null result, consistent with the prediction of Lu et al. (1993). The very robustness of the SOC state to initial loading levels, as investigated in Sect. 2.1, shows that even during the largest events in the simulation run, the drops in the average field and average gradient (or stress) levels amount to no more than around 1% of the equilibrium values. No event in these simulations ever remotely approaches relaxation of the entire grid, as may happen when non-local communication is included. Hence there is obviously enough stored energy in the grid to allow subsequent 'large' events almost immediately, irrespective of the fact that the LH type simulations drive the grid externally at very low levels. Indeed, there would not be a well-defined SOC state if this were not the case. The results of Biesecker (1994) show no apparent requirement for a significant 'recharging time' between flares, and may therefore lead us to the conclusion that the amount of energy released in solar flares must normally be a comparatively insignificant amount compared to the stored free energy in the system.

We may comment that very large flares, for example the June 3, 1982 flare, happen only a few times during a solar activity cycle. One might still entertain the hypothesis that such extreme events represent complete relaxation of the coronal field; certainly there will be too few such events in e.g. Biesecker's (1994) data set for statistical discussion. In addition, the possibility remains, in principle, that we might compensate for the grid dissipating significant energy in a single event by simply increasing the external driving to compensate. This would be equivalent to saying that magnetic fields arising at the photospheric surface are already very highly stressed and can provide (almost) immediate re-stressing of the system (Lu 1995b).

Acknowledgements. The authors would like to acknowledge the financial support of PPARC plus the use of the Starlink computer facilities at the University of Glasgow. We would also like to thank M. Georgoulis and K. Galsgaard for useful discussions and the referee, L. Vlahos, for his suggestions to improve the manuscript.

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