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The apsidal motion test of stellar structure in relativistic systems

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Abstract. We have compared observational data of apsidal motion rates for relativistic eclipsing binaries with theoretical predictions based on stellar theoretical models. Ten double-lined eclipsing binaries with high quality light curves and radial velocities were selected. The analysis of the data available indicate that the predictions by the General Relativity and the new stellar models are able to explain the shift in the periastron position.

The two versions of the non-symmetrical theory of gravitation were also investigated. The early version of this theory (1984) is not supported when contrasted with recent observational data since it predicts too slow apsidal motion rates. The more recent version (1989) presents several problems in its formulation, like too many degrees of freedom and some parameters for which there is no severe physical constraint. Moreover, such a theory is not able to predict a priori the apsidal motion rates.

Key words: stars: binaries – close: eclipsing – relativity

1. Introduction

Due to the tidal and rotational distortions present in stars which belong to eclipsing binaries there is a secular change in the position of the periastron if the orbit is eccentric. Such distortions can be described as a function of the internal structure constant - hereafter k_2. Moreover, there is a relativistic contribution for the periastron shift in a similar way that occurs with the orbit of the planet Mercury.The resulting apsidal motion can be monitored by observing the time of minima. The apsidal motion test to the stellar structure and evolution has been used to contrast observational data of eclipsing binaries with theoretical predictions of the mass concentration derived from evolutionary models. Given the wide range of masses observed in eccentric binary systems it is possible to investigate the interior of the stars under different physical conditions. This test is also important since the direct comparison between the observed and theoretical predictions of the apsidal motion ate is performed only after previous tests concerning stellar models (like radii, effective temperatures and isochrones) have been done with success. In this sense it is a complementary test to the stellar evolution theory.

We have already performed a comparison between observations and theory for about 20 systems with accurate absolute dimensions and apsidal motion rate determinations (Claret & Giménez 1993a, 1993b). The importance of the good determination of absolute dimensions is obvious since the observed internal structure constant depends on the mass, eccentricity, rotational velocities, and more strongly, on the radius of the components (as R^{-5}). There are many systems showing observational evidence of apsidal motion (see for example Giménez 1994) but lacking a good scanning of time of minima and/or a good determination of masses and radii. This fact limits severely the study on the internal structure constants, although a sufficient number of adequate systems is now available.

The eclipsing binaries which present apsidal motion are also useful to test the predictions of the General Relativity - GR - for the periastron advance. In the papers quoted above we have only analysed the systems for which the relativistic contribution to the total apsidal motion were small. The results from these papers indicated that using new opacity calculations, core overshooting, rotation, improved orbital elements and recent apsidal motion rates the theoretical predictions are in good agree ment with observations. The old problem, that real stars seemed to be more mass concentrated than predicted by theory, was solved or at least minored. As we did not know a priori which was the cause for these discrepancies we have separated the systems presenting high relativistic contributions in order to avoid these disagreements with the theory to be attributed to relativistic effects. In this way we have divided our investigation in two parts: one concerning the non-relativistic systems (Claret & Giménez 1993ab) and the present work probing the relativistic ones. As the name indicates, the relativistic systems are those for which the advance of the periastron predicted by the GR is comparable with the classical contribution. Moreover, the separation in two classes is justified given that the observations for some relativistic systems, DI Her and AS Cam, do not seem to be in agree nt with GR predictions. An alternative theory for the periastron shift was presented by Moffat (1984, 1989) and, in principle, it was able to explain such systems. However, such

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theory presents some problems which will be discussed later in this paper (Sect. 4).

The magnitude of the discrepancy found in DI Her (see for example Guinan & Maloney, 1985) demands an analysis in a separate paper (Claret 1997b). For this binary we have very good absolute dimensions determination and the discrepancy can not be attributed to errors in the radii and masses. Even if we take the mass point model to represent DI Her the disagreement remains. Another interesting case is AS Cam. But this system will not be analysed because it does not present the basic requirements this kind of investigation (good absolute dimension determination). Moreover, there are some discussions on the observed apsidal apsidal motion rate (Krzesinski et al. 1990; Maloney et al. 1991).

In this paper we perform a comparison of the observational data with the theoretical predictions concerning apsidal motion rates for the relativistic systems which fulfil the basic requirements. In the following sections we will discuss the astrophysical data for the eclipsing binaries used in the analysis, and the equations which govern the apsidal motion rate are presented. Next the observed astrophysical parameters are contrasted against the theoretical ones (including ages). Finally, we compare the observed apsidal motion rates with those predicted theoretically including the results based on the work by Moffat.

2. The relativistic systems - a brief description

The systems used in this investigation were selected following the quality of their spectroscopic and photometric elements. We only consider the systems with good determination of masses, radii and orbital parameters including the apsidal motion rates. Of course, within the group of eclipsing binaries with high quality absolute dimensions, it is not easy to find many systems showing apsidal motion. And within this reduced sub-group it is more difficult to separate those for which the relativistic contribut on is significant. In spite of this restriction we have selected 10 systems. Some of them were used because some authors have classificated them as “problematic” ones as for example AG Per, Y Cyg (Hegedus, 1989, Moffat 1984). In the following sub-sections we will comment briefly on each system separately. The astrophysical properties for each individual system are given in Table 1 [1].

The HR diagram for these systems is shown in Fig. 1. Most of the systems are young ones although some of them are well evolved and are close to the TAMS. In Figs. 2-12 we present the results for the comparison between the observed radii and theoretical ones provided by stellar models. The isochrones for the 10 systems were computed using the new stellar models by Claret (1995, 1997a) and Claret & Giménez (1995). For each system we have derived the theoretical values for k_2 (see Sect. 3).

2.1. V1143 Cyg

This double-lined eclipsing binary presents a high eccentricity ($e \approx 0.54$) and it is a good candidate to test the predictions by the GR. The relativistic contribution is more than 50% of the observed apsidal motion. The masses were determined by Andersen et al. (1987) and are around 1.4 $M_\odot$. The rotational velocities are 18±2 and 28±3 km s^{-1} for the primary and secondary respectively. Comparing the astrophysical data for V1143 Cyg with the theoretical models we can conclude that it is an young system (recall that the dotted line in Figs. 2-12 denotes the ZAMS).

The apsidal motion rate was determined by several authors. Giménez & Margrave (1985) determined that $\dot{\omega}_{obs}$=0.00071±.00004 degrees per cycle while Khaliullin (1983), using a different method, found 0.00073±0.0008 in the same units. More recently, Burns et al. (1996) have monitored V1143 Cyg and they have determined that $\dot{\omega}_{obs}$=0.00074±.00015 per cycle. All values are consistent although the error in the last case is about 3 times the other ones. The corresponding period of apsidal motion U is $1.02\times10^4\pm2.1\times10^3$ years.

2.2. VV Pyx

The masses of this system were determined by Andersen et al. (1984) who found equal components with m=2.098±0.018 $M_\odot$. The radii of both stars are 2.167±0.020 in solar units. This is a well evolved system as one can see in Fig. 3.

The eccentricity of the orbit is 0.0956±0.0009 and the orbital period is 4.6 days. The rotational velocities are 23±3 km s^{-1} and are close to the synchronization values. Using a serie of time of minima Andersen et al. have found that $\dot{\omega}_{obs}$=0.00142±.00045 degrees per cycle. This corresponds to an apsidal motion period of $3.2\times10^3\pm1\times10^3$ years. The relativistic contribution is 0.00052±.00001 degrees per cycle which represents 37% of the observed value.

2.3. BW Aqr

The period of this system is 6.7 days and the eccentricity is 0.17. An spectroscopic orbit was determined by Imbert (1987). The derived masses are close of those determined by Clausen (1991). The last author re-analysed the CORAVEL observations and using the information of the light curves found that the mass of the primary is 1.39±0.02 while for the secondary he derived 1.49±0.02 in solar units. The radii are 1.79±0.04 and 2.06±0.04 respectively. BW Aqr is a very interesting object und the evolution point of view because core overshooting seems to be necessary to explain its position on the HR diagram. The fit showed in Fig. 4 is not as perfect as in other cases. This means that such a system may present a chemical composition a little bit larger than that for the Sun.

The rotation of the components are characterized by the following values of Vsini: 9.6±0.3 and 13.5±0.8 km s^{-1} respectively for the primary and secondary. As pointed out by Clausen

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[1] References to Table 1: **1** Burns et al. 1996, **2** Andersen et al. 1987, **3** Andersen et al. 1874, **4** Imbert 1987, **5** Clausen 1991, **6** Popper 1984, **7** Claret et al. 1995, **8** Clausen et al. 1977, **9** Andersen & Giménez 1986, **10** Giménez & Quintana 1992, **11** Hill & Holmgren 1995, **12** Simon et al. 1994, **13** Giménez & Clausen 1994, **14** Andersen et al. 1983, **15** Giménez et al. 1986, **16** Clausen et al. 1986, **17** Holmgren et al. 1995

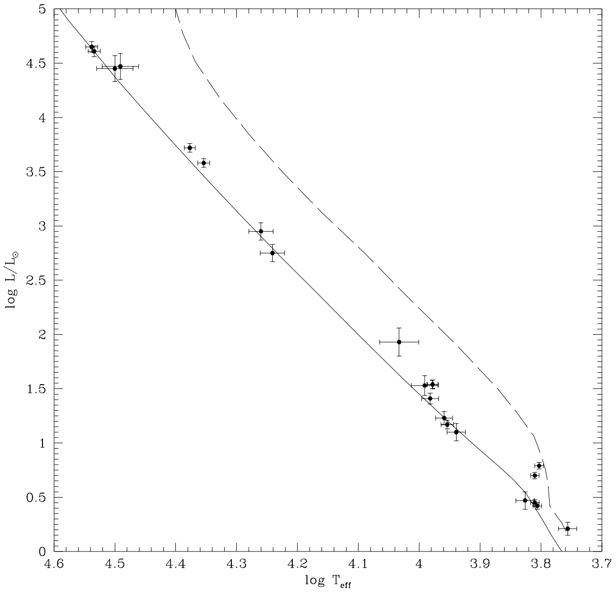

Fig. 1. HR diagram for the system shown in Table 1. Continuous line denotes ZAMS while TAMS is represented by a dashed one.

this system present one curious particularity since its components rotate slower than predicted by synchronism or pseudo-synchronism.

Through a serie of observations of time of minima, for example Gronbech et al. (1987) and Khaliullin & Kozureva (1986), Clausen established the apsidal motion as 0.0009 degrees per cycle. The relativistic correction is of order of 0.00032 in the same units.

2.4. *EK Cep*

This is one of the most interesting eclipsing binaries for many reasons. For example: one of the components is still in the pre main-sequence. In addition, its absolute dimensions are well determined (Popper, 1987), presents apsidal motion (Giménez & Margrave, 1985) and in the last years the LiI resonance doublet was investigated yielding to a lithium depletion of the order of 0.3 dex (Martín & Rebolo, 1993). These characteristics make of EK Cep an interesting laboratory to test evolutionary models. This system is represented in in Fig. 5 in a different way due to its peculiarities. We have to calculate stellar models during the pre main-sequence phase and the radii and k_2 are given as functions of time. It is gratifying to note that all observational constraints were perfectly fulfilled for EK Cep simultaneously (Claret et al. 1995).

The apsidal motion rate was improved (Claret et al. 1995) using recent observations of time of minima by the following authors: Khaliullin (1983), Hill & Ebbighausen (1984), Caton et al. (1989), Diethelm (1992), Caton & Bruns (1993), Diethelm (1993), Lacy & Fox (1994) and Pajdosz (1993). The eccentricity was taken to be 0.109 (Tomkin 1983). Considering the absolute dimensions from Popper and using the above data Claret et al. 1995 found that the apsidal motion is 0.00101±0.00015 degrees per cyc

2.5. *V1647 Sgr*

This early A-type system is compounded by two stars with masses 2.19±0.04 and 1.97±0.03 M$_\odot$ (Clausen et al. 1977; Andersen & Giménez 1985). The respective radii are 1.83±0.02 and 1.67±0.02 in solar units. Its orbit is highly eccentric (e=0.4130) and the period is 3.28 days. The apsidal motion period U=590 years and the contribution due to the General Relativity to the advance of periastron is about 15% of the total.

2.6. *V477 Cyg*

The relativistic contribution to the apsidal motion of this system is about 10% of the total value. The eccentricity is high, 0.307, and the orbital period is 2.35 days. In this system Giménez & Quintana (1992) used a version of the synthetic light curve code EBOP which takes into account the change of the position of periastron. Re-analysing the light curves by O'Connel (1970) and the radial velocity curves by Popper (1968) they were able to determine the masses for each component as 1.79±0.12 and 1.35±0.07 M$_\odot$. The corresponding radii are 1.57±0.05 and 1.27±0.04 R$_\odot$. By inspecting Fig. 7 we can conclude that this is a young system and the observed eccentricity are compatible with the circularization time.

2.7. *Y Cyg*

We have recently got two determinations of masses and radii for this massive system (Simon et al. 1994, Hill & Holmgren 1995). The masses and radii are of the same order although in the last case the associated errors are larger. These masses are about 17 M$_\odot$ and the radii 6 R$_\odot$ for both components. We have studied this star using the two possibilities, let us say, case *a* and case *b* (see Table 1).

The observed apsidal motion rate is 0.0618 degrees/cycle (Holmgren et al. 1995). Y Cyg presents the smaller period of apsidal motion of the sample ($\approx$ 48 years). It was used by Moffat (1984) to calibrate his gravitation theory and it was quoted by Hegedüs (1989) as a "problematic" system.

2.8. *AG Per*

This is another fast system. Its period of apsidal motion is around 76 years. The rotational velocities are given by Olson (1984) and are 94 and 70 km s^{-1} for the primary and secondary respectively. Although the relativistic contribution is less than 10% such a system was used by Moffat (1984) to calibrate his alternative gravitation theory and it was also included as "problematic" in the list by Hegedüs (1989). Recently Giménez & Clausen (1994) have studied AG Per and the derived masses are 5.36±0.16 and 4.90±0.13 M$_\odot$ while the respective radii are found to be 2.99±0.07 and 2.60±0.07.

AG Per is a very young system and presents the inherent difficulties to determinate its age. The grid with Z=0.02 and

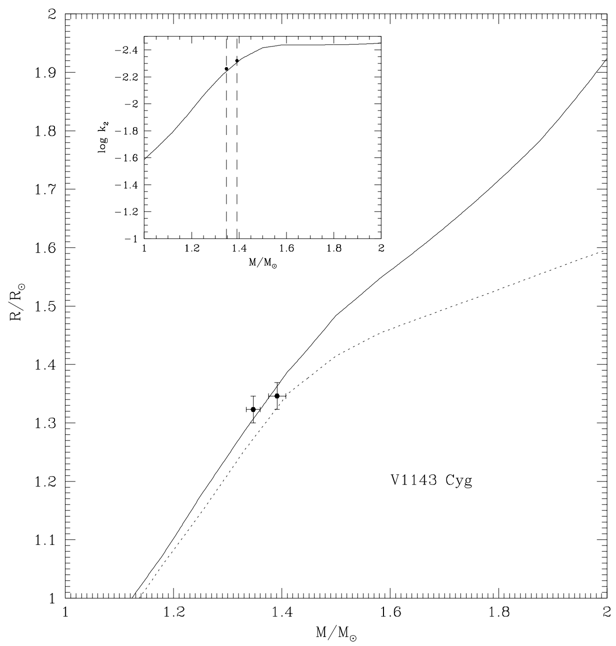

Fig. 2. Theoretical predictions for the radii and apsidal motion constant k_2 for V1143 Cyg. Dotted line represents the ZAMS.

Y=0.28 does not fit both stars well. As commented by Giménez & Clausen (1994) there are other young metal-deficient B-type systems which present a similar behaviour. We have used for AG Per the grid Z=0.01 and X=0.723. For this combination the isochrones are closer to the position of AG Per in the log $M \times$ log R diagram. The corresponding isochrones are for log τ=7 4 and 7.5 (see Fig. 10).

2.9. *QX Car*

The masses of the components of QX Car were determined by Andersen et al. (1983). They derived 9.27±0.12 and 8.48±0.12 M$_\odot$ for the primary and secondary respectively. Its orbit presents an eccentricity of 0.278 and the orbital period is 4.48 days. Some years after this paper, Giménez et al. (1986) using a new set of time of minima established an apsidal motion period of 360 years. The relativistic contribution for this system is 0.00147 degrees per cycle which corresponds to 10% of th total.

2.10. *V451 Oph*

Another system whose relativistic contribution is of the order of 10%. However, the high quality of the masses and radii determination (Clausen et al. 1986) justified its presence in our sample. The eccentricity of the orbit is 0.0125 and U=180 years. The rotational velocities are characterized by 41±7 and 30±5 km s^{-1}. The masses of the components of this binary are 2.78 and 2.36 M$_\odot$ while the radii are 2.64 and 2.03 in solar units.

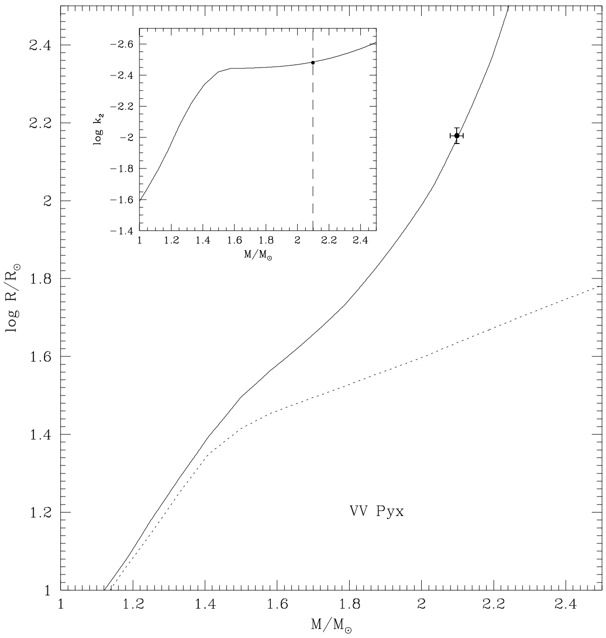

Fig. 3. The same as in Fig. 2 for VV Pyx.

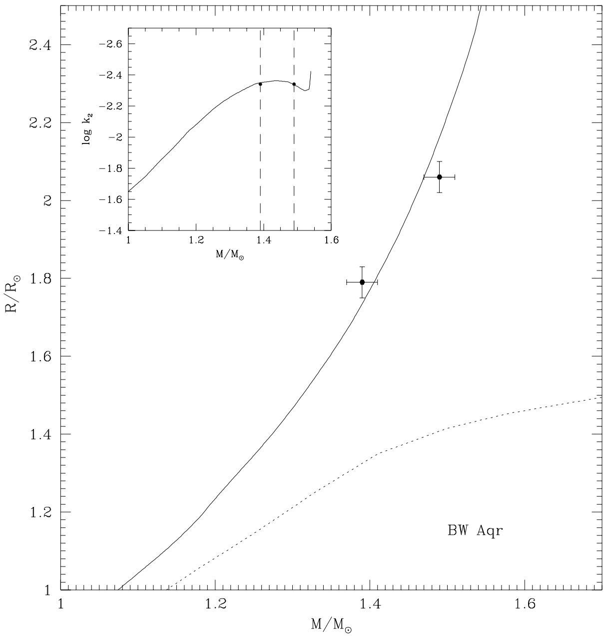

Fig. 4. The same as in Fig. 2 for BW Aqr.

3. The equations of the apsidal motion

If the observed apsidal motion rate ($\dot{\omega}_{obs}$) is given in degrees/cycle the corresponding period U can be written as

$$U = \frac{360P}{\dot{\omega}_{obs}} \tag{1}$$

where P is the orbital period. One can not measure directly the internal structure constant - ISC - for the individual components.

Table 1. Astrophysical parameters for the relativistic systems

Name	m_1	m_2	R_1	R_2	e	$\dot{\omega}_{obs}$	$\dot{\omega}_{GR}$	P	V_{r1}	V_{r2}	Ref.
V1143 Cyg	1.391	1.347	1.346	1.323	0.540	0.00074	0.00039	7.64	18	28	1, 2
	0.016	0.013	0.023	0.023	0.003	0.00015	0.00001		2	3	
VV Pyx	2.098	2.098	2.167	2.167	0.0956	0.00142	0.00052	4.60	23	23	3
	0.018	0.018	0.020	0.020	0.0009	0.00045	0.00001		3	3	
BW Aqr	1.39	1.49	1.79	2.06	0.17	0.0009	0.00032	6.72	9.6	13.5	4, 5
	0.02	0.02	0.04	0.04	0.01	0.0001	0.00001		1.3	0.8	
EK Cep	2.02	1.12	1.58	1.32	0.109	0.00101	0.00044	4.43	23	10.5	6, 7
	0.01	0.01	0.015	0.015	0.003	0.00015	0.00001		2	2	
V1647 Sgr	2.19	1.97	1.83	1.67	0.4143	0.00546	0.00077	3.28	80	70	8, 9
	0.04	0.03	0.02	0.02	0.0005	0.00006	0.00001		5	5	
V477 Cyg	1.79	1.35	1.57	1.27	0.307	0.00661	0.00073	2.35	64	50	10
	0.12	0.07	0.05	0.04	0.003	0.00018	0.00005		11	9	
Y Cyg^a	17.5	17.3	6.0	5.7	0.1458	0.0618	0.00285	3.00	147	138	11, 17
	0.4	0.3	0.3	0.3	0.0023	0.0003	0.00007		10	10	
Y Cyg^b	17.57	17.04	5.93	5.78	0.142	0.0618	0.00283	3.00	147	138	12, 17
	0.27	0.26	0.07	0.07	0.002	0.0003	0.00005		10	10	
AG Per	5.36	4.90	2.99	2.60	0.0710	0.0264	0.00161	2.03	95	71	13
	0.16	0.13	0.07	0.07	0.0010	0.0022	0.00005		23	9	
QX Car	9.27	8.48	4.29	4.05	0.278	0.01222	0.00148	4.48	120	110	14, 15
	0.12	0.12	0.06	0.06	0.003	0.00022	0.00003		10	10	
V451 Oph	2.78	2.36	2.64	2.03	0.0125	0.0120	0.0096	2.20	41	30	16
	0.06	0.05	0.03	0.03	0.0015	0.0007	0.0002		7	5	

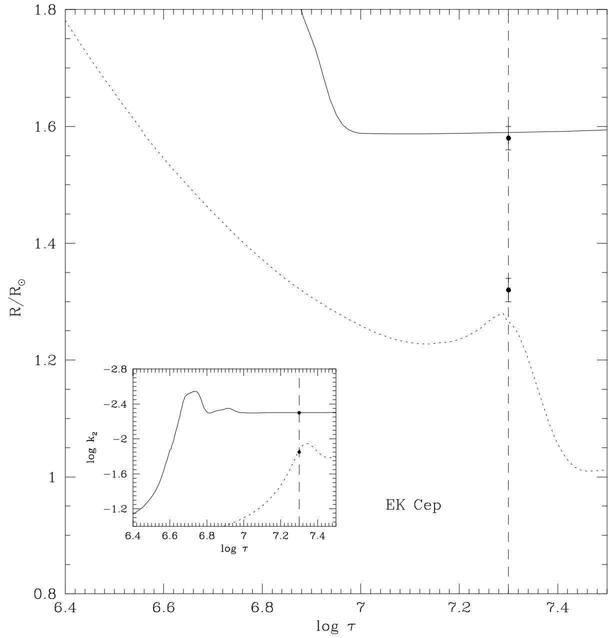

Fig. 5. Theoretical radii and log k_2 as a function of the time for the components of EK Cep. Primary is denoted by continuous line and the secondary is represented by a dotted line.

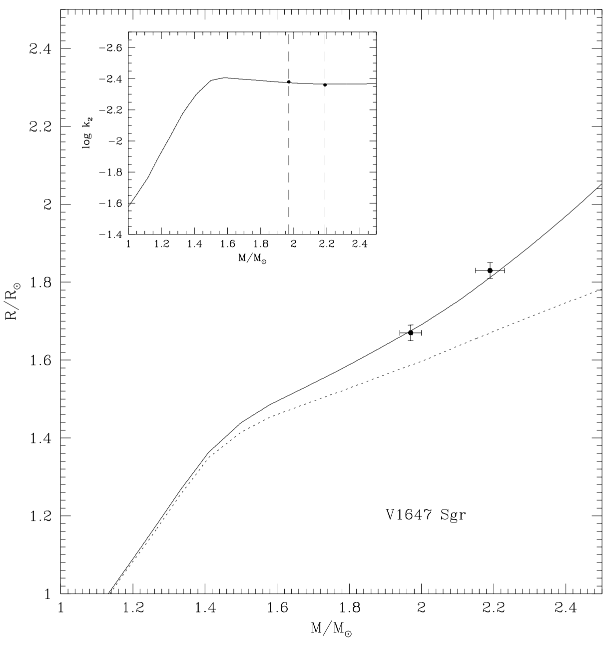

Fig. 6. The same as in Fig. 2 for V1647 Sgr.

The observed parameter connected with the structure of the stars that we can measure is the period of apsidal motion. In this way we have

$$\frac{P}{U} = c_{21}k_{21} + c_{22}k_{22} \tag{2}$$

where k_{2i} are the internal structure constant for the component i. The c_{2i} for each individual component is evaluated through the following expression:

$$c_{2i} = \left[\left(\frac{\Omega_i}{\Omega_K}\right)^2 \left(1 + \frac{m_{3-i}}{m_i}\right) f(e) + \frac{15 m_{3-i}}{m_i} g(e)\right] \left(\frac{R_i}{A}\right)^5 \tag{3}$$

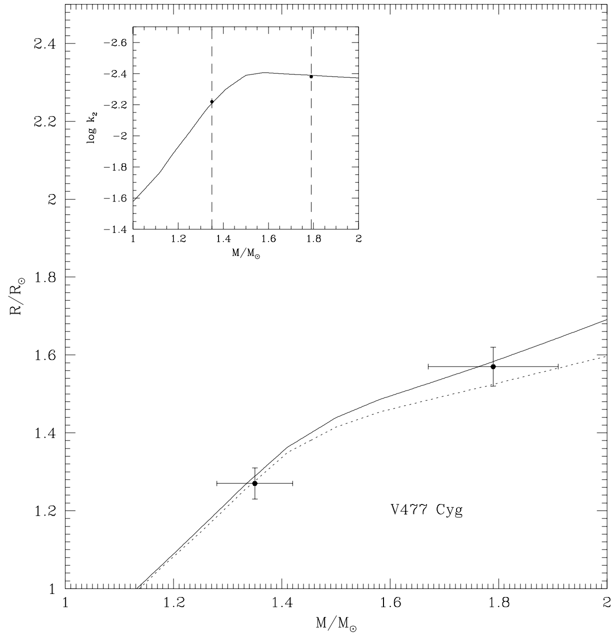

Fig. 7. The same as in Fig. 2 for V477 Cyg.

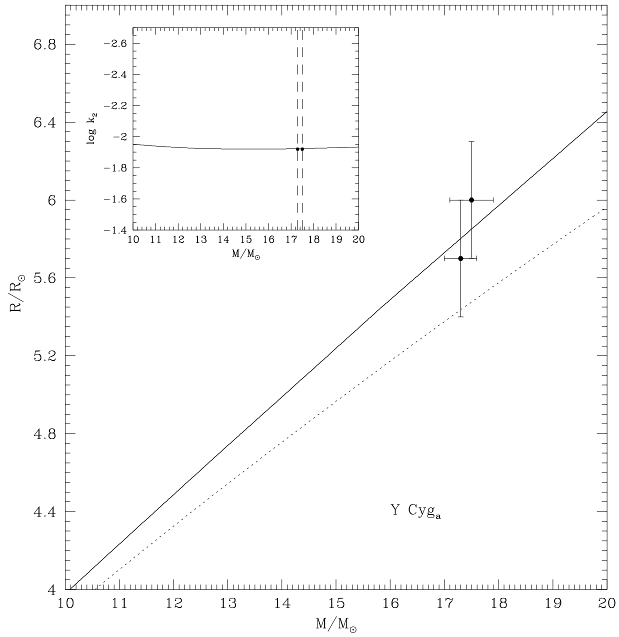

Fig. 8. The same as in Fig. 2 for Y Cyg (case a).

where the auxiliary functions f(e) and g(e) are given by

$$f(e) = (1 - e^2)^{-2} \tag{4}$$

$$g(e) = \frac{(8 + 12e^2 + e^4) f(e)^{2.5}}{8} \tag{5}$$

In the above equations m is the mass of the component, R_i the stellar radius, A the semi-major axis, Ω_i is the angular velocity of the component i, e is the eccentricity and Ω_K is the keplerian angular velocity. Note that the first term in Eq. 3

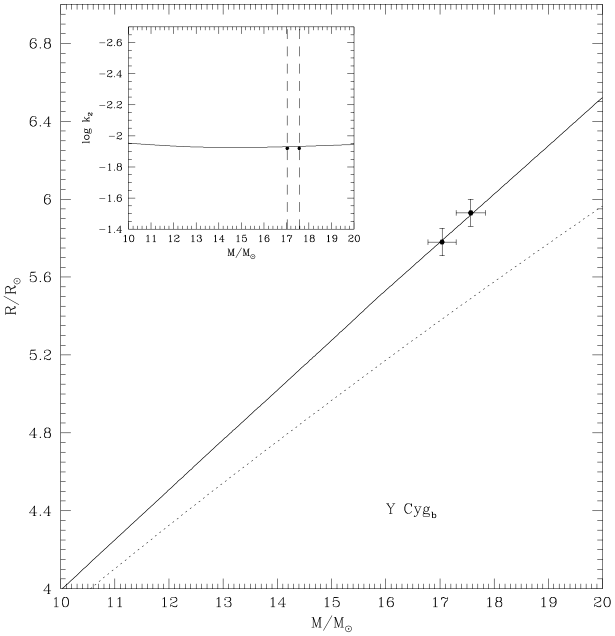

Fig. 9. The same as in Fig. 2 for Y Cyg (case b).

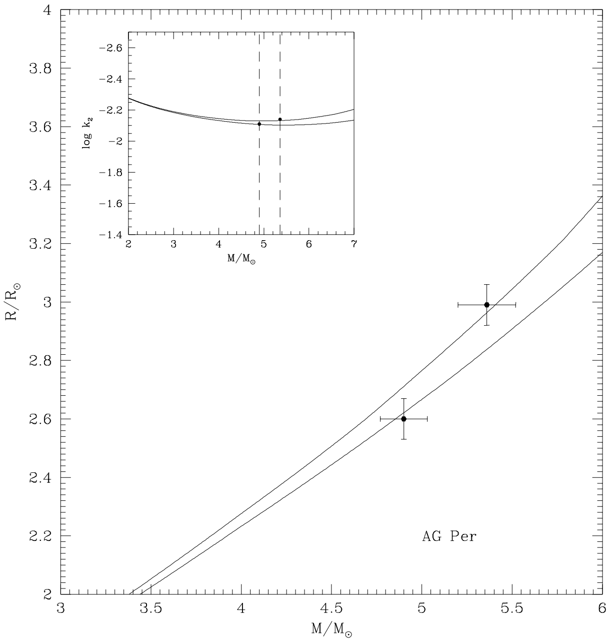

Fig. 10. Isochrones for AG Per (log τ=7.5 and 7.4 respectively.)

is due to the rotational distortion while the second describes the tidal contribution. For some systems the quantity Ω_i is not available. In this case we often assume that the system is pseudo-synchronized, i.e., it is synchronized at the periastron where the tidal forces achieve the maximum. A convenient expression can be found in Kopal (1978) and can be written as

$$\Omega_P^2 = \frac{(1 + e)}{(1 - e)^3} \Omega_K^2 \tag{6}$$

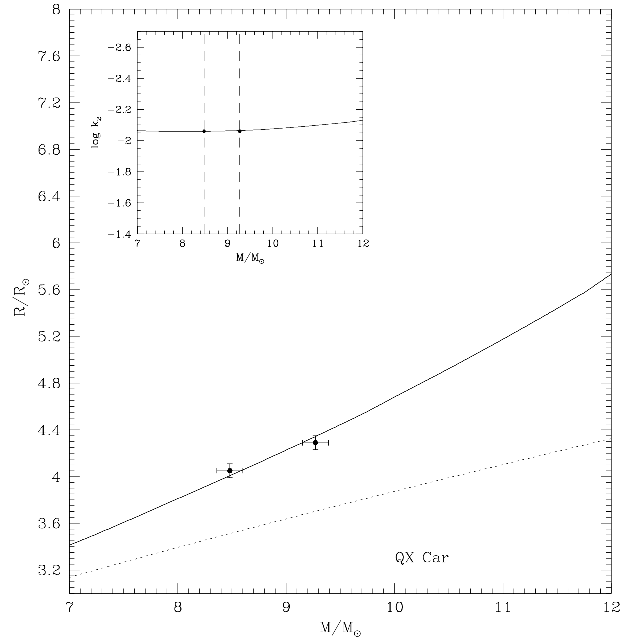

Fig. 11. The same as in Fig. 2 for QX Car.

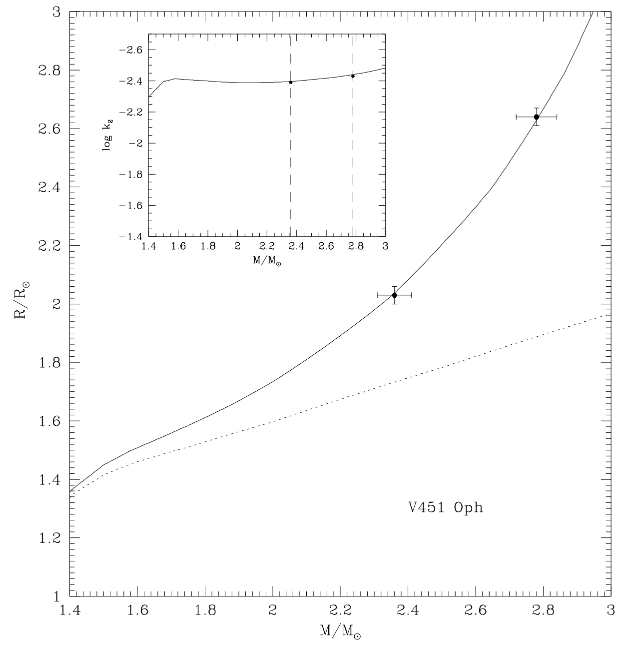

Fig. 12. The same as in Fig. 2 for V451 Oph.

In a recent paper we have shown that the systems compiled by Andersen (1991) seem to be pseudo-synchronized (Claret & Cunha 1997). For comparison we show in Fig. 13 the observed rotational velocities versus the predicted ones at the periastron. The aspect of this figure enables us to conclude that it really is a good approximation to take the angular velocities at the periastron, but only for those systems for which such velocities are not available.

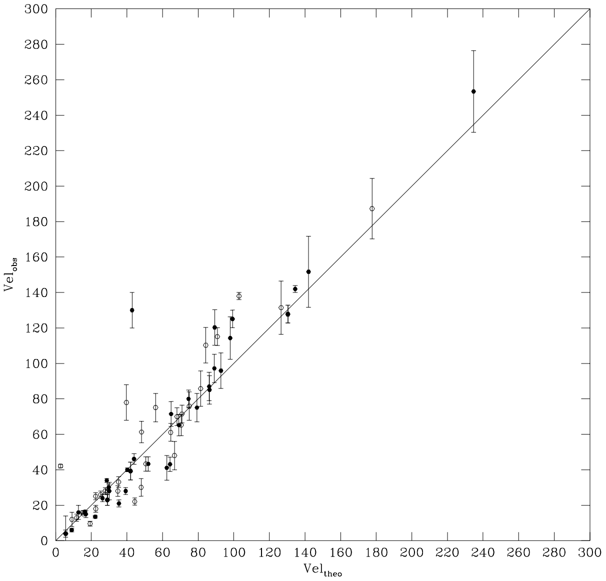

Fig. 13. Observed rotational velocities versus rotational velocities computed at the periastron for selected eclipsing binaries (see references in Andersen 1991)

The observed value of k_2 can be derived using the following expression:

$$\overline{k}_{2obs} = \frac{1}{c_{21} + c_{22}} \frac{P}{U} \tag{7}$$

or

$$\overline{k}_{2obs} = \frac{1}{360\,(c_{21} + c_{22})} \dot{\omega}_{obs} \tag{8}$$

On the other hand the theoretical values of k$_2$ are obtained through integration of the Radau differential equation for each stellar configuration:

$$\frac{a d\eta_j}{da} + \frac{6\rho(a)}{\overline{\rho}(a)}(\eta_j + 1) + \eta_j(\eta_j - 1) = j(j+1) \tag{9}$$

where

$$\eta \equiv \frac{a}{\epsilon_i} \frac{d\epsilon_i}{da} \tag{10}$$

and a and r are related through the parametric equation

$$r = a \left(1 + \sum_{j=0}^{n} \epsilon_j(a) P_j(\theta) \right) \tag{11}$$

In the above expressions a is the mean radius of a given equipotential, $\rho(a)$ is the mass density at the distance a from the center, $\overline{\rho}(a)$ is the mean mass density within a sphere of radius a, $P_j(\theta)$ are the Legendre polynomials and ϵ_j described the amplitude of the distortions.

The theoretical predictions for the ISC for each component - in its corresponding evolutionary status - can be computed using

$$k_j = \frac{j + 1 - \eta_j(R)}{2\left(j + \eta_j(R)\right)} \tag{12}$$

with $\eta_j(R)$ being the value at the surface of the model. Often we only consider the terms for j=2 since the contribution of k_3 and k_4 are very small. For a mass point model we have $k_2 = 0$, that is, $\eta_2(R)$=3. Before contrasting the observed values of k_2 with those predicted theoretically it is necessary to introduce the relativistic contribution (Levi-Civita 1937; Kopal 1978) in Eqs. 7 (or 8). This contribution is independent of the classical one and can be computed using the following formula

$$\frac{P}{U'} = 6.35 \times 10^{-6} \frac{(m_1 + m_2)}{A(1 - e^2)} \tag{13}$$

where the masses m_i and the semi-major axis are given in solar units. Finally, the theoretical weighted $\overline{k}_2$ to be compared with the observed one is

$$\overline{k}_{2theo} = \frac{c_{21} k_{21theo} + c_{22} k_{22theo}}{c_{21} + c_{22}} \tag{14}$$

4. Comparison with the observational data and conclusions

The analysis performed previously (Figs. 2-12) allowed us to perform a direct comparison between the observational data for k_2 and those predicted theoretically. Before doing that, $\dot{\omega}_{obs}$ has to be corrected by the relativistic contribution (or that by Moffat) following

$$\dot{\omega}_{clas} = \dot{\omega}_{obs} - \dot{\omega}_{GR} \tag{15}$$

or

$$\dot{\omega}_{clas} = \dot{\omega}_{obs} - \dot{\omega}_{NST} \tag{16}$$

where the symbol *clas* refers to the classical contribution, *obs* the observed value, *GR* the prediction by GR and *NST* refers to the non-symmetrical theory.

The resulting value of $\dot{\omega}_{clas}$, or the equivalent k_2, will be compared with theoretical predictions.

4.1. Using the relativistic correction

Eq. 13 gives us the rate of advance of periastron in degree per cycle. The contribution due to the GR is given in column 8 of Table 1. In order to compute the observed $\overline{k}_2$ we use Eqs. 8 and 13 while for the corresponding theoretical value Eq. 14 was used. Stellar models computed taking into account rotation are more mass concentrated than the standard ones (for details, see Claret & Giménez 1993a). The theoretical values were corrected by the effect of the stellar rotation. Ho ve found that the correction is not too large and it is about 0.02-0.05 in log k_2 and depends on the value of the rotation rate at the surface of the star.

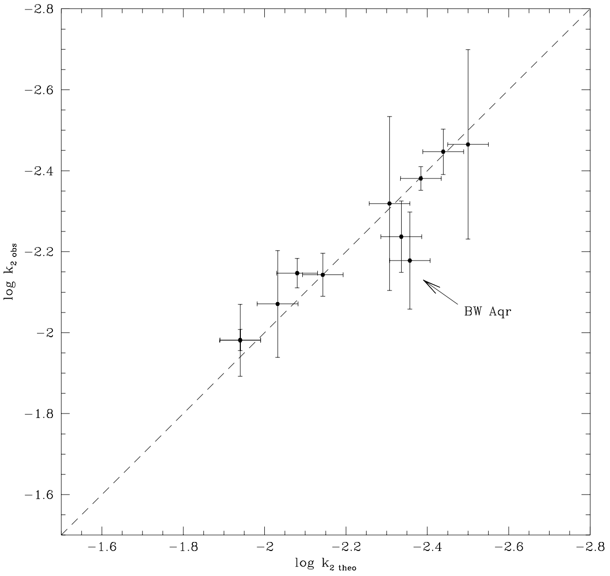

Fig. 14. Theoretical and observed apsidal motion constants. The shift in the periastron was computed correcting $\dot{\omega}_{obs}$ by the prediction of the General Relativity (Eqs. 8, 13 and 15).

The results can be seen in Fig. 14. The old effect - real stars seemed to be more centrally condensed than predicted by the models - is not detected. Indeed the agreement is very satisfactory. The system BW Aqr presents the maximum discrepancy but in the sense that the components seemed to be less mass concentrated than predicted by the models. Fig. 15 shows that the differences $\delta log k_2 \equiv log\overline{k}_{2obs} - log\overline{k}_{2theo}$ do not depend on the masses. There is also n ence of this parameter with log g$_{obs}$ as one can see in Fig. 16. This means that the stellar models are able to predict the apsidal motion in good agreement with the observations. In addition, the correction given by GR is also supported. The last point is very important in the light of the discussions on the validity of the predictions of the advance of the periastron given by the GR. These results are not substantialy changed if one assumes that all the stars of the sample are synchronized at t eriastron instead of using the observed rotational velocities.

4.2. The non-symmetrical theory of gravitation (1984)

In his paper of 1984 Moffat derived an alternative formulation for the shift of periastron. Such formulation is based on a non-symmetrical tensor g$_{\mu\nu}$ and on a non-symmetrical affine connection $\Gamma^{\lambda}_{\mu\nu}$. This theory predicts a slower relativistic apsidal motion rate than that given by the GR. In some cases, this rate can even be reversed. As commented before the old comparisons between observations and theoretical predictions for the apsidal motion indicated that real stars seemed to be more concentrated in mass than predicted by the stellar models. As the equations by Moffat predicted a slower apsidal motion (less concentrated stars) this theory seemed to be adequate to be applied to the "problematic" systems. In fact Moffat (1984) pre-calibrated his equation using systems whose disagreements with respect to

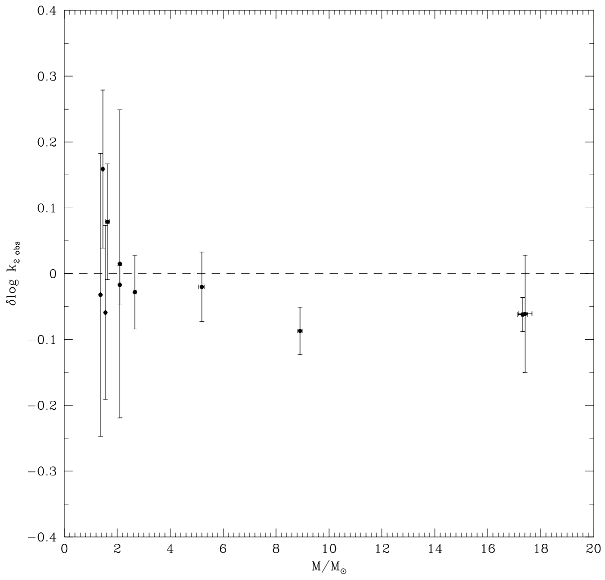

Fig. 15. Discrepancies between the observed and theoretical values of the apsidal motion constant (see text) as a function of $\overline{M}$. This latter value was computed using a similar definition as in Eq. 14. The shift in the periastron was computed correcting $\dot\omega_{obs}$ by the prediction of the General Relativity (Eqs. 8, 13 and 15).

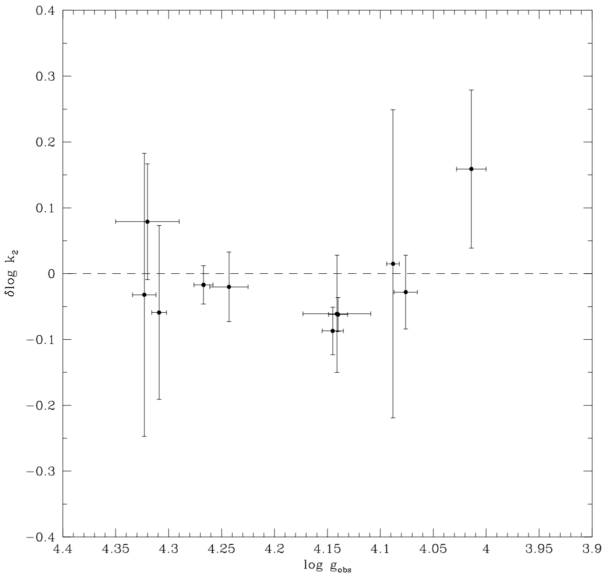

Fig. 16. Differences between the observed and theoretical values of the apsidal motion constant (see text) as a function of observed log g weighted as in Eq. 14. The shift in the periastron was computed correcting $\dot\omega_{obs}$ by the prediction of the General Relativity (Eqs. 8, 13 and 15).

the theory were notorious (DI Her, AS Cam, Y Cyg, etc). His equation to describe the shift of the periastron is

$$\begin{aligned}\left(\frac{d\omega}{dt}\right)_{NST} &= \frac{9.287\times10^{-3}\left(P/365.25\right)(m_1+m_2)^{2/3}\,\lambda_{NST}}{(P/2\pi)^{5/3}(1-e^2)}\\ &= \dot\omega_{GR}\lambda_{NST} \qquad (17)\end{aligned}$$

with

$$\lambda_{NST} = 1 - \frac{4.625\times10^{-35}l^4(1+e^2/4)}{(P/2\pi)^{4/3}(1-e^2)^2(m_1+m_2)^{8/3}} \qquad (18)$$

Note that for some situations - small *l* - the prediction by Moffat is the same as that given by the GR. The parameter *l* is a constant of integration and can be calculated using the following relation:

$$l = [0.2\,(m_1+m_2)+0.08]\times10^9 cm \qquad (19)$$

where the masses are given in solar units.

The non-symmetrical theory of gravitation should also be able to fit the data for the other systems and not only for the systems used in the pre-calibration. We have performed such a test. In Fig. 17 we show the results using the predictions by Moffat (1984). One can conclude that the relativistic corrections given by GR are in better agreement with the observations than those by Moffat. The later predicts too slow apsidal motions and a systematic deviation with respect to k_2 was detected.

4.3. The non-symmetrical theory of gravitation (1989) - NST89

In 1989, Moffat revised his theory. The basic difference concerning apsidal motion with respect to Eq. 17 is that in the parameter λ_{NST} the variable *l* was changed to *K* where

$$K^4 = \left[\left((m_1+m_2)(l_1^2-l_2^2\right) d\right] \qquad (20)$$

with

$$d = \frac{l_1^2}{m_1} - \frac{l_2^2}{m_2} \qquad (21)$$

and l_i^2 is given by

$$l_i^2 = \frac{YM}{2m_p}\left[f_p^2(Z/N)+f_n^2+f_c^2(N_c/N_B)N_r\right] \qquad (22)$$

where *M* is the mass of the star, *Y* is the mass fraction of helium, m_p is the mass of the proton, *Z* is number of protons, *N* is the number of neutrons, N_B is the baryon number, N_c is number of cosmions and N_r=N_B/*N*. The parameters f_p, f_n and f_c are universal coupling constants for protons, neutrons and cosmions respectively.

This formulation depends on parameters which are not well established and also present some inconsistencies as we comment below:

1. Let us examine Eq. 22. If one gives typical numerical values (see Moffat 1989, Table II) one can see that the main contribution to l_i is due to the last term. This term is around 10^6 larger than the other factors and the calibration by Moffat depends strongly on the stellar cosmion number N_c. Although its importance in the NST89 calculations this factor is not well constrained.

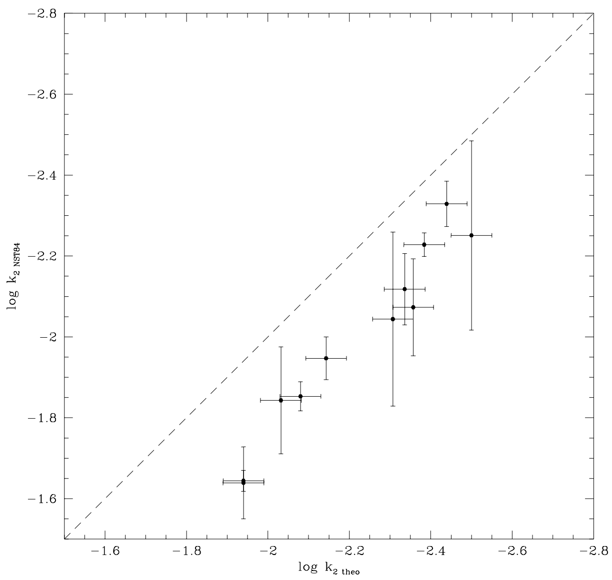

Fig. 17. Theoretical and observed apsidal motion constants. The shift in the periastron was computed correcting $\dot{\omega}_{obs}$ by the prediction of the the non-symmetrical theory (Eqs. 8, 16 and 17). Note the systematic deviation in the predictions given by this theory.

2. Another important restriction to the NST89 calculations is the large number of free parameters present in the formulation.
3. In addition, the differences in the chemical compositions attributed to the components of the systems used in the calibration are a little confuse since the primaries always present a helium content different from the secondaries. This goes against the current theory that in a binary system both stars have the same age and chemical composition. Moreover, NST89 are not able to predict the shift in the periastron position a priori.
4. There is also a dependence on the adopted systems used in the calibration. In fact, the astrophysical parameters used in the 1989 calibration have been changed being substituted by new observations and new advances in the stellar models. In this way, the discrepancies, which are very important in order to to establish the calibration, have decreased with time. Another additional problem was also detected: the quality of the photometric and spectroscopic data used by Moffat. Let us examine with more detail the four systems used in the calibration. Giménez & Clausen (1994) have determined the masses for AG Per as 5.36 and 4.90 $M_\odot$ for the primary and secondary respectively. This means that the recent masses are about 20% larger than previous ones, used by Moffat. The data for AS Cam and mainly for α Vir (see Claret & Giménez 1993a) are not sufficiently accurate to be used in this kind of investigation. Only the astrophysical data for DI Her are accurate enough to test apsidal motion. Of course, the new determinations of $\dot{\omega}$ have also an influence in the calibration.
5. On the other hand, the primaries and secondaries used by Moffat in his calibration follow two different relationship between the mass and the ratio of cosmions to baryons. For example, the mass of the secondary of DI Her is only 0.2% smaller than the mass of the primary of AG Per (given in that paper). However, the corresponding ratio N_c/N_B for the two stars is about 65%.
6. Eqs. 20 and 21 predict that for a mass ratio of 1.0 the correction due to NST89 is the same as that given by GR. In order to test his theory, Moffat (1989) only used systems with q=1.0 (V1143 Cyg, V889 Aql and V541 Cyg) after the calibration. However, among this three systems only one fulfilled the basic requirements for apsidal motion test (V1143 Cyg). Whatever the quality of the data, it is clear that the predictions by NST89 were not really compared with the apsidal motion data: of a total seven systems used, four were used in the calibration while three presented a mass ratio of 1.0. The later give, by definition, the same results as GR and no differential results could be detected. It would be interesting that data for other systems like EK Cep, BW Aqr, AG Per, QX Car, etc are be used.

To summarize, the NST by Moffat (1984) is not able to fit the observations of apsidal motion since it predicts too slow shifts in the periastron position. On the other hand, NST89 formulation depends on too much free parameters; some of them very hard to determine. We have also detected some inconsistency in the calculations of the N_c/N_B ratio. Moreover, NST89 is not able to predict the apsidal motion a priori. The predictions of the General Relativity compare very well with the present data (also with those for non-relativistic systems following Claret & Giménez 1993ab) and it is independent of previous calibrations. Concerning DI Her - GR is not able to predict the correct $\dot{\omega}$ - some possibilities to explain its strange behaviour are investigated in a separate paper (Claret 1997b)

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