Astron. Astrophys. 327, 167–172 (1997)

ASTRONOMY
AND
ASTROPHYSICS

The RV-variations of γ Coronae Borealis – a frequency analysis

H. Lehmann[1], **G. Scholz**[2], **G. Hildebrandt**[2], **and S. Yang**[3]⋆

[1] Thüringer Landessternwarte Tautenburg, Karl-Schwarzschild-Observatorium, Sternwarte 5, D-07778 Tautenburg, Germany (lehm@tls-tautenburg.de)
[2] Astrophysikalisches Institut Potsdam, Telegrafenberg A 27, D-14473 Potsdam, Germany (gscholz; ghildebrandt@aip.de)
[3] Dept. of Physics & Astronomy, University of Victoria, P.O. Box 3055, Victoria, BC, Canada V8W 3P6 (yang@uvastro.phys.uvic.ca)

Received 21 April 1997 / Accepted 26 May 1997

Abstract. We summarize a longer campaign of spectroscopic observations of the suspected Maia variable star γ CrB . Previously published data are complemented by two-longitudinal observations. We investigate 1262 CCD spectra for short-term radial velocity variations. Results show that A0 stars can pulsate with periods quite similar to those of the well-known δ Scu stars. From a frequency analysis of the data we can derive at least three periods with significant contributions to a multiple frequency model. The probable rotational period of the star is in the order of 0.9 d, and the observed short-term periods are arranged in a triplet around 0.1 d. The observed relation of the frequencies points to the occurrence of rotationally split nonradial pulsations.

Key words: stars: early-type – stars: individual: γ CrB – stars: oscillations – δ Scu

1. Introduction

γ CrB is a bright visual binary of spectral type A0 V. It is one of two stars for which we found an unambigous variation of its radial velocity in our observing campaign for possible Maia variable stars in 1992 to 1994 (Lehmann et al. 1995, hereinafter Paper I). Although a time scale of short-term variations of about 0.1 d could be derived, γ CrB showed a nonuniform behaviour. We observed times with periodical changes of RV as well as times with nearly constant RV. To decide whether the observed variations could be explained by a time-stable multiple frequency model giving rise to an amplitude modulation, or are of irregular nature, we investigated the star for radial velocity variations again. An interim report on the first new observations was given in a research note in 1996 (Lehmann, Scholz & Hildebrandt 1996, hereinafter Paper II). Again we observed changes in the radial velocity only during certain phases. To overcome the problem of the 1 d aliasing, we made observations at two different longitudes in 1996 (1 d corresponds to a possible rotational time scale of γ CrB and could be responsible for a rotational splitting of pulsational modes as discussed in Paper I). The coordinated observational runs included two sites, the Thüringer Landessternwarte Tautenburg (TLS) and the Dominion Astrophysical Observatory (DAO).

Table 1. Spectroscopic parameters

site	spectral region in Å	dispersion in Å/pixel	resolving power	mean S/N
TLS	6220 − 6500	0.28	11000	300
DAO	6150 − 6750	0.15	21000	300

2. Observations and reduction

We have a set of 1262 CCD spectra of γ CrB at our disposal, taken in a time span of about 400 d and covering the spectral region around the Si II doublet 6347/6371 Å. During the time of coordinated observations 384 spectra were taken over 9 nights with the coudé spectrograph of the 2 m telescope of the TLS and 74 spectra over 6 nights with the coudé spectrograph of the 1.22 m telescope of the DAO. Table 1 lists the parameters of the spectra obtained. The coordinated observations enclosed two groups of nights, one with 4 nights and one with 6 nights. The entire set of radial velocities (Fig. 1) will be published elsewhere (Lehmann et al. 1997).

Radial velocities have been measured for the two strongest lines in the wavelength region covered, the Si II doublet 6347/6371 Å. The blends in the wings of both lines are due to telluric contributions, so do not permit the determination of line centers by direct integration or gaussian fits over the entire line profiles. Our determination of the line centers is based on the central parts of the two lines. The width of the central part was chosen to be 1.8 Å, which is the mean FWHM of the lines if we fit their centers by gaussians.

Send offprint requests to: H. Lehmann

⋆ Guest investigator, Dominion Astrophysical Observatory, Herzberg Institute of Astrophysics, National Research Council of Canada

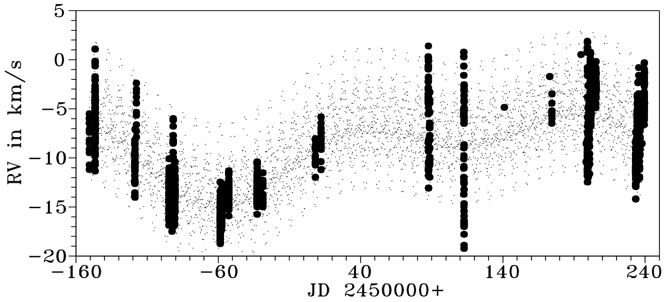

Fig. 1. Long and short-term RV-variations of γ CrB. The shaded area is covered by the multiple frequency fit.

To determine a possible difference in the RV zero points of the two instruments we measured in all spectra four sharp telluric O_2 lines. As a result we found no systematic difference between the instruments but we had to correct the radial velocities of the Si II doublet by a small amount for different nights, ±1.2 km/s at maximum. Within single nights the changes in RV measured from the O_2 lines were always below the mean error of measurement of 0.6 km/s.

3. Period analysis

3.1. Long-term variation

In Paper II we reported on an observed long-term RV variation with a time scale of about 200 d. A period search in the entire data set now available gives about 400 d, and if we take into account also the first harmonic we get 370 d or about 1 year (see Fig. 1). The suspicion of a possible telluric contribution also in the line centers of the Si II doublet was the reason for a very careful reinvestigation of all possible correlations between measured RV values of the Si II doublet and the telluric O_2 lines. We can observe a weak correlation between the strength of the O_2 lines and the strength of the Si II doublet as well as a weak correlation between the strength of the O_2 lines and the radial velocity of the Si II doublet. However, these correlations can only be derived using the entire data set. Within single nights the RV of the O_2 lines are nearly constant and the strength of the O_2 lines shows no correlation with any of the other values.

Fig. 2 explains this behaviour. We have drawn the RV of the Si II doublet versus the RV of the telluric O_2 lines for which we applied here the same heliocentric correction as for the stellar lines. There is a large scale correlation but no correlation within the groups of observations or within single nights. In other words, the large amplitude of the variation of the heliocentric correction (±21 km/s for the TLS observations of γ CrB) gives rise to a measureable long-term variation even if we have only a very small telluric contribution in the Si II doublet. So, we may conclude the following. First, we can make corrections to the measured RV on a long-term scale according to an assumed telluric contribution which varies only on the time scale of the variation of the heliocentric correction. This correction can be done by pre-whitening the data for a period of 365 d. And second, the short-term variations with a time scale of some hours are neither affected by the telluric contribution nor by our long-term pre-whitening.

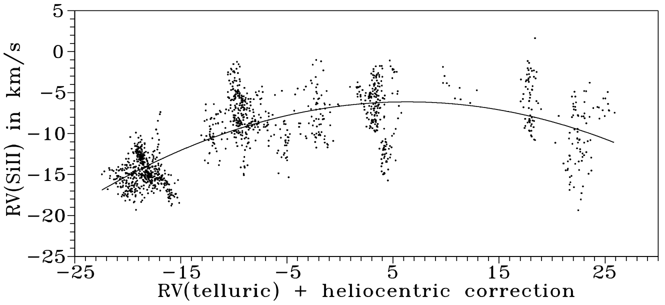

Fig. 2. Measured RV of the Si II doublet versus RV of the telluric O_2 lines. Here, the heliocentric correction had been added to the RV of the telluric lines.

Table 2. Results of multiple frequency search.

	ω_1	ω_2	ω_3
period, harmonics	$P_1,\ 2\,P_1,\ 4\,P_1$	P_2	P_3
period in d	0.44499	0.12714	0.09889
K in km/s	3.0, 1.2, 1.1	1.7	1.6

3.2. Short-term variations

To avoid the problems of the 1 d aliasing described in Paper II we investigated firstly the two groups of coordinated observations. The period search of this subset of RVs gives two significant periods: one of about 0.45 d, and after pre-whitening the data for this first period a second of 0.127 d.

In a next step we determined all significant periods in the entire data set by successive pre-whitening of the data. Again we find the 0.445 d (including also the first harmonic) and 0.127 d periods and as a third period 0.0989 d which corresponds to the sum of the frequencies belonging to the first two periods. In the residuals after pre-whitening for all of the three periods there remains only one peak smaller than 1 km/s for the 1 d aliasing.

Next we tried to fit a multiple frequency model to the entire data set. We considered the fixed 365 d period and its first harmonic, the two short-term periods derived from the coordinated observations and up to 3 harmonics as well as all sums and differences of the short-term frequencies and their harmonics. The computer program used a least squares fit of a sum of sine-waves. As results we first get a measure for the reduction in the sum of squares (ratio of the rms of the residuals after subtracting the model to the rms of the original data). And second, we obtained the half-amplitudes of the contributions of any of the several periods to the observed variation. The applied procedure allowed an iterative optimization of periods and a stepwise exclusion of non-significant contributions or combinations of frequencies.

Table 2 gives the results. The periods represent the optimum solution tested for a large range (0.07-10 d), ω_1 is the difference

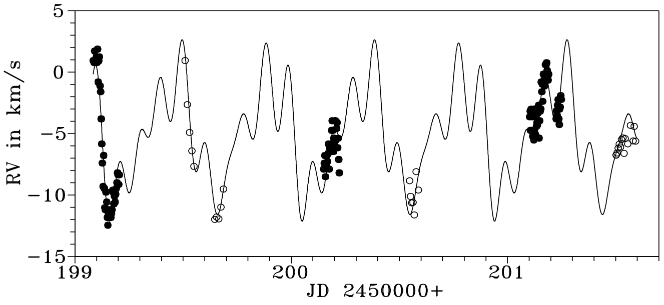

Fig. 3. Results of multiple period search shown for one of the groups of coordinated observations. TLS observations are marked by dots, DAO observations by open circles. The continuous curve represents the multiple frequency fit, which follows from all of the observations.

Table 3. Results of multiple frequency search listed as harmonics of a fundamental frequency ω_0 which corresponds to a period of 0.88998 d

	ω_1	ω_3	ω_6	ω_7	ω_8
P [d]	0.44499	0.22249	0.12714	0.11125	0.09889
K [km/s]	3.0	1.2	1.7	1.1	1.6

of ω_3 and ω_2. Most remarkable is the ratio of the frequencies ω_1, ω_2, ω_3 which is exactly 1:3.5:4.5. Taking into account also the obtained 1st and 3rd harmonics of ω_1 our periods correspond to the 1st, 3rd, 6th, 7th and 8th harmonics of a fundamental period of 0.88998 d. We tested this by a special period search including one fundamental mode and up to 12 harmonics and we obtained again 0.88998 d as the optimum solution with significant amplitudes for all the harmonics given above and amplitudes below 0.5 km/s for the fundamental period itself and all of the other harmonics. A fit including only the significant harmonics gives exactly the same results as listed in Tab. 2. For a comparison we give the results again in the new notation in Tab. 3.

Fig. 3 shows an example of the model fit to the RV values. The different contributions to the observed variations are presented in Fig. 4. In all panels the underlying period is the period derived for the entire data set. Amplitudes and phases are optimized to fit the RV values of the particular group, however. This allows a comparison of amplitudes and phase shifts between the different groups. The panels in the columns 2-4 show the contribution of one single frequency after pre-whitening of the data for all of the other frequencies. The panels in column 1 show the variation due to ω_1 and ω_3 folded with the ω_1-period.

4. Discussion

4.1. Results of measurement

First, we want to compare the derived periods with previous results given in Papers I, II. The radial velocities determined in Paper I were based on the Mg II 4481 Å line measured on photographic spectra. This line should be free of any telluric contribution. In the 1992/93 part of the spectral data set we found a well-defined short-term RV-variation with a period of about 0.1 d. In Paper I we tried to fit a multiple frequency model to the data and chose from a larger set of possible periods of RV amplitude modulation a period of about 1 d which could be also the rotational period of γ CrB. Now, based on our large data set and the coordinated observations, we can certainly exclude 1 d, which is an alias period. In Paper II which reported on the first part of our CCD observations of the Si II doublet we found a period of 0.9931 d (1 d alias) and a second period of 0.305 d which is almost exactly the first alias of the 0.445 d period derived from the entire data set. The observed long-term RV-variation we have to attribute to a small contribution of telluric lines in the Si II doublet which should not affect the short-term variations as pointed out in section 3.1.

Since all detected frequencies involved in the RV-variation are multiples of one frequency ω_0 they cannot cause an amplitude modulation with a time scale larger than $P_0 \sim 0.89$ d. So, the observed time intervals without measurable changes in radial velocity can be explained only by a variation of the amplitudes of the individual contributions themselves. The width of the sinusoidal belts in the phase diagrams, which is much larger than the mean error of RV measurement of 0.6 km/s, should be caused by a variable amplitude of the RV variations and, possibly, by spontaneous phase shifts of the different contributions. A check for such phase shifts is difficult. In the data set outside the coordinated observations we found only one group of spectra which has a time sampling suitable for a multiple frequency fit including all the periods derived for the entire data set. The results for this group are shown in Fig. 4, row d. From the phase diagrams we see that the 0.445 d changes are in phase with the two groups of coordinated observations. For the other two periods there is a phase shift of about 0.15, however.

The phase curve for the 0.445 d variation is much more structured than for the other periods. This structure seems to be variable and could not be removed by introducing any additional frequency. The contribution of P_7 is less significant than those of P_6, P_8 and can be clearly recognized only in the entire data set.

The ratio of the frequencies ω_6 and ω_8 of 0.78 is in good agreement with the ratio predicted for the fundamental and first overtone mode of radial pulsations of δ Scuti stars (e.g. Breger 1979). But radial pulsation does not explain the observed relationship between the period lengths of all the periods found.

4.2. Stellar parameters and the period of rotation

γ CrB is classified in the Bright Star Catalogue (Hoffleit & Warren 1991) as B9 IV+A3 V. Percy (1969) estimated a mass of the primary of 1.9 $M_\odot$ from binary motion and parallax. The values underlying his estimation are very rough, however. In Tab. 4 we have collected the parameters of γ CrB known from literature. Although the orbital elements derived by Hartkopf et al. (1989) are very precise, the uncertainty of the parallax of γ CrB is - as usual - the main source of trouble in the determination of the masses. Equation 1 gives the total mass in solar units as derived

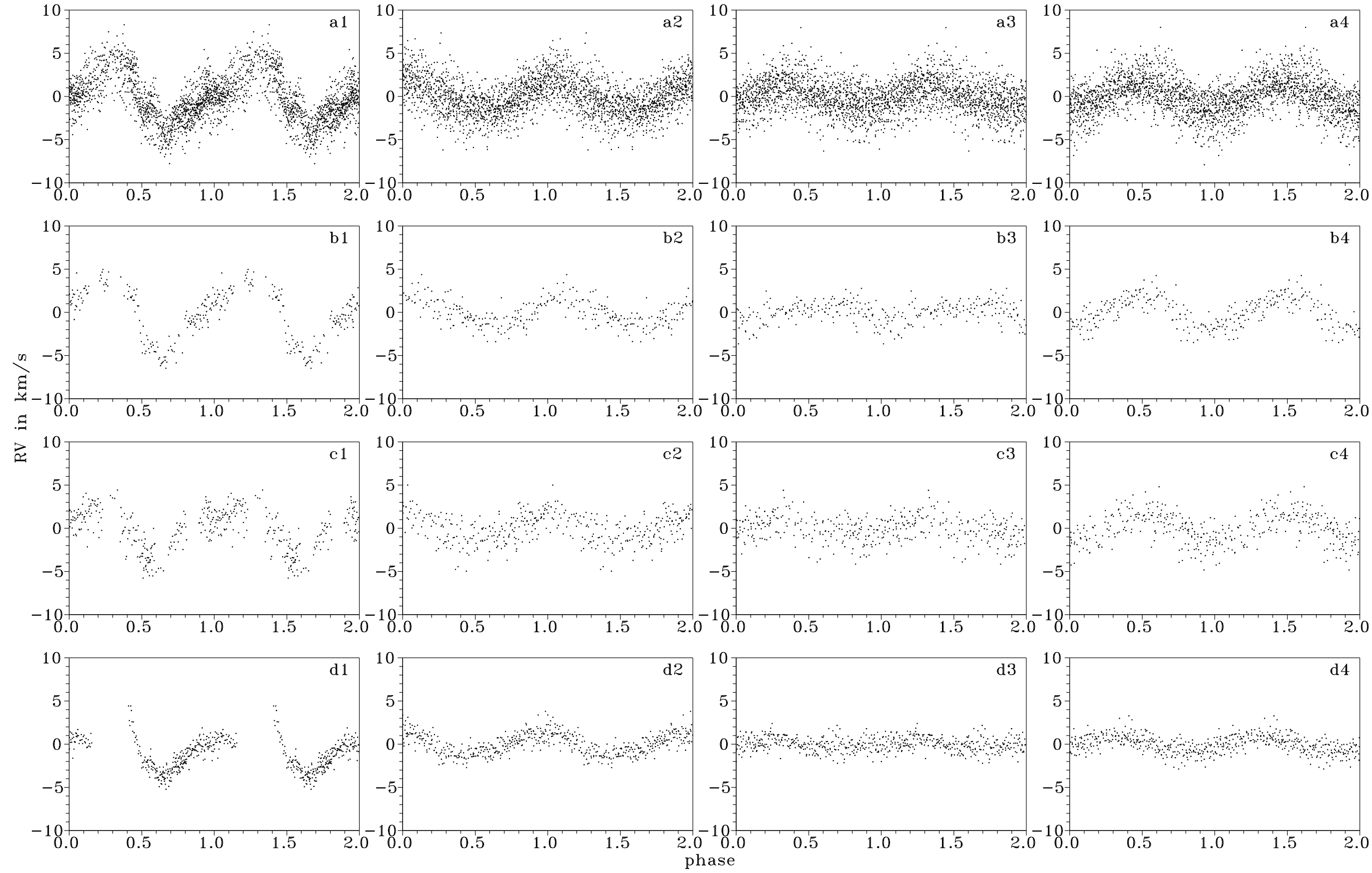

Fig. 4. Phase diagrams for the contributions of different frequencies. Row **a**: entire data set (Fig. 1), rows **b, c**: the two groups of coordinated observations centered at JD 2 450 200 (6 nights, Fig. 3) and at JD 2 450 235 (6 nights), row **d**: an early group of TLS observations centered at JD 2 449 912 (4 nights). Columns **1** to **4** correspond to the frequencies ω_1, ω_6, ω_7, ω_8 of Tab. 3. Each column of panels shows the contribution of one frequency to the observed RV variation after pre-whitening for all of the other frequencies.

from binary motion, semi-major axis a and parallax Π are in arcseconds, the period P is in years

$$M_1 + M_2 = \frac{1}{P^2}\left(\frac{a}{\Pi}\right)^3 \quad (1)$$

Considering the error of the parallax of $0.''005$ given by van Altena (1995), it follows a possible range of the total mass of the binary from 1.5 to 5.0 $M_\odot$.

To estimate probable stellar parameters of the binary we can use the difference in magnitude of the binary components of about 1.4 mag. Under the assumption that the mass-luminosity relation has an exponent of 3.8 (McCluskey & Kondo 1972), we get a mass ratio of $M_1/M_2 = 1.4$. Adopting further 5 solar masses as an upper limit of the total mass of both components, estimated from the above quoted values of the parallax and orbital motion, we get an upper limit of about 3 $M_\odot$ for the primary and 2 $M_\odot$ for the secondary. These correspond to main sequence stars of spectral types near to A0 and A4, respectively.

From the mean FWHM of the Si II lines of 1.8 Å we derived a $v \sin i$ of 100 km/s in good agreement with other authors listed in Tab. 4. With $R = 2.4\,R_\odot$ it follows an upper limit of the rotational period of 1.2 d. A lower limit of about 0.3 d can be estimated from the limit of stability of the star with respect to centrifugal forces.

4.3. Non-radial pulsations

As a conclusion, from our estimation of the rotational time scale, we can interpret the observed 0.445 d period as well as the above mentioned 0.89 d period as periods of rotation, which gives rise to two different scenarios of rotationally split non-radial pulsations (see Paper I).

The splitting of a frequency ω_l of a non-rotating star into several equidistant modes $\omega_{l,m}$ of a rotating star is described by (Ledoux 1951)

$$\omega_{lm} = \omega_l - m\,(1 - C_{nl})\,\omega_r, \quad -l \geq m \leq +l \quad (2)$$

where l is the angular and m is the azimuthal quantum number, and ω_r is the angular velocity of rotation. The constant C_{nl} depends on the stellar equilibrium structure and on the mode considered. In general it satisfies $C < 0.1$ (Shibahashi & Saio 1985).

If we assume 0.445 d for the rotational period of γ CrB, then the three periods given in Tab. 2 are consistent with a non-

Table 4. Stellar parameters of the binary γ CrB taken from the literature.

P in y	92.94 ± 0.58	(9)
a in $''$	0.7353 ± 0.0041	(9)
i in $^\circ$	94.70 ± 0.84	(9)
T	1931.66 ± 0.23	(9)
e	0.484 ± 0.020	(9)
Ω in $^\circ$	111.25 ± 0.61	(9)
ω in $^\circ$	105.24 ± 0.61	(9)
Π in $''$	0.026 ± 0.005 (14) 0.023 (15)	0.027 ± 0.005 (17)
spectral type	A0 IV (5,6,7)	A1 V (3) A0 V (1)
$B-V$	0.01 (7)	0.00 (6,10)
$U-B$	-0.04 (7,10)	-0.03 (6)
$R-I$	-0.02 (10)	
W(4481 Å) in Å	0.42 (1)	
$\triangle m_V$	1.5 (4,5,8,9) 1.4 (7)	1.43 (2) 3.0 (13)
	comp. A	comp. B
spectral type	B9 IV (5,10,12)	A3 V (5,10)
m_v	4.08 (2) 4.2 (7,11)	5.51 (2) 5.6 (7,11)
$v \sin i$ in km/s	112 (16) 100 (1,18)	

References

1 Abt 1995
2 Baize 1989
3 Baglin 1973
4 Bagnuolo 1992
5 Edwards 1976
6 Fernie 1969
7 Garcia 1995
8 Haniff 1989
9 Hartkopf 1989
10 Hoffleit 1991
11 Leroy 1993
12 McAlister 1993
13 Percy 1969
14 Schlesinger 1935
15 Schlesinger 1940
16 Uesugi 1981
17 van Altena 1995
18 this paper

Table 5. Two possible correlations of the observed frequencies with nrp modes, $m = a - k$.

ω_{k-1}	$a = 8$	$a = 3$
ω_1	$\omega_r/2$	$m = +1$
ω_3	$\omega_r/4$	$m = -1$
ω_6	$m = +1$	$m = -4$
ω_7	$m = 0$	$m = -5$
ω_8	$m = -1$	$m = -6$

radial pulsation model with an intrinsic period of pulsation of 0.127 d and a retrograde moving wave observed with a period of 0.099 phase diagram, or a prograde wave if we exchange the two short-term periods. Comparing the 0.445 d period of rotation with the adopted stellar parameters and $v \sin i$, we must see the star nearly pole-on ($i \sim 20^\circ$) and the equatorial velocity of rotation would be about 270 km/s. Such a large inclination of the rotational axis of γ CrB to the orbital plane (the line of sight lies nearly in the orbital plane itself) seems to be unlikely. Although in this model the rotational frequency is naturally the difference of the two frequencies of pulsation, the relation between all three frequencies of 1 : 3.5 : 4.5 cannot be explained and must be taken as accidental.

On the other hand, all observed frequencies can be reduced to a fundamental frequency ω_0 (Tab. 3) with a corresponding period of 0.89 d, which could be the period of rotation of the star. To set the observed frequencies in order to possible modes of non-radial pulsations, we can write Eq. 2 with $\omega_{lm} = k\,\omega_0$ in the form

$$m = a - b\,k, \qquad a = \frac{1}{1-C}\,\frac{\omega_l}{\omega_r}, \qquad b = \frac{1}{1-C}\,\frac{\omega_0}{\omega_r}. \tag{3}$$

Since m and k are integers, a and b have to be (positive) integers, too. From section 4.2. it follows that $b = 1$. In other words, the derived fundamental period P_0 has to be the period of rotation in this model, and the basic non-radial pulsational frequency is (neglegting the factor $1/(1 + C)$) a multiple of the rotational frequency: $\omega_l \simeq a\,\omega_r,\ a = m + k$.

Table 5 gives the interpretation of the observed frequencies in terms of nrp modes. The first model ($a = 8$) favours the observed symmetry in the amplitudes of the two stronger short-term frequencies ω_6 and ω_8 and correlates them with the nrp modes $m = \pm 1$. Here, the intrinsic pulsational period is 0.111 d (corresponding to ω_7) and the RV-variations with frequencies ω_1 and ω_3 are caused directly by the rotation of γ CrB. The second model ($a = 3$) assumes that all observed frequencies are due to rotationally split nrp modes. Here, the intrinsic pulsational period is 0.296 d corresponding to the non-observed frequency ω_2.

5. Conclusions

Our investigation of the RVs of γ CrB shows that A0 stars are able to pulsate with periods similar to the well-known δ Scu stars. But, an enormous amount of observing time is necessary to secure at least the reality of the existence of pulsations. In spite of our large data set we cannot give a final model of the pulsational processes at the moment. Based on a multiple period finding technique we can explain the unusual RV variations of γ CrB by a multiple frequency model. All observed periods are multiples of a fundamental period of 0.89 d. Starting from the model of rotationally split non-radial pulsations and comparing different scenarios with probable stellar parameters we get two solutions.

Solution 1 ($a = 8$ in Table 5) results in an intrinsic period of pulsation of 0.111 d, rotationally splitted into three components with periods of 0.127 d, 0.111 d and 0.099 d. The period of rotation should be identical with the fundamental period of 0.89 d. To explain the observed active and inactive phases of the RV variation of γ CrB we have to assume an additional variation of the amplitude of the nrp modes. This variation seems to be of irregular nature and can not be explained by an amplitude modulation due to multiple frequencies. The time scale of excitation and damping of the pulsational modes should be of the order of a few days, but there must be a long-term phase correlation of the excited modes. Since we observe this phase correlation also in the RV variation due to rotation itself, we must assume surface inhomogenities which are variable in strength, but keep nearly the same position on the stellar surface over a longer time scale.

The latter addition can be omitted if we assume that all observed frequency contributions are correlated with nrp modes (solution 2, a = 3 in Table 5). In this case, all irregular changes in the RV amplitudes can be directly explained by the spontaneous excitation and damping of nrp modes.

Acknowledgements. We are grateful to Prof. J. B. Hearnshaw for his careful proofreading of the manuscript.

References

Abt H. A., Morrell N. I., 1995, ApJS 99, 135

Baglin A., Breger M., Chevalier C. et al., 1973, A&A 23, 221

Bagnuolo W. G. Jr., Mason B. D., Barry D. J. et al., 1992, AJ 103, 1399

Baize P., Petit M., 1989, A&AS 77, 497

Breger M., 1979, PASP 91, 5

Edwards T. W., 1976, AJ 81, 245

Fernie J. D., 1969, J. R. Astr. Soc. Can. 63, 133

Haniff C. A., Buscher D. F., Christou J. C., Ridgway S. T., 1989, MNRAS 241, 51p

Hartkopf W. I., McAlister H. A., Franz O. G., 1989, AJ 98, 1014

Hoffleit D., Warren Jr W. H., 1991, The Bright Star Catalogue, 5th Revised Ed., Yale Univ. Obs.

Garcia J. R., Cebral J. R., Scoccimarro E. R. et al., 1995, A&AS 109, 201

Leroy J. L., 1993, A&AS 101, 551

Ledoux P., 1951, ApJ 114, 373

Lehmann H., Scholz G., Hildebrandt G., Klose S., Panov K. P., Reimann H.-G., Woche M., Ziener R., 1995, A&A 300, 783

Lehmann H., Scholz G., Hildebrandt G., 1996, A&A 312, 508

Lehmann H., Scholz G., Yang S., 1997, J. of Astron. Data, in preparation

McAlister H. A., Mason B. D. Hartkopf W. I., Shara M. M., 1993, AJ 106, 1639

McCluskey G. E., Kondo Y., 1972, Ap&SS 17, 134

Percy J. R.,1969, Comm. David Dunlap Obs. 250

Schlesinger F., Jenkins L. F., 1935, General Catalogue of Stellar Parallaxes, Yale Univ. Obs. 1935

Schlesinger F., Jenkins L. F., 1940, Catalogue of Bright Stars, Yale Univ. Obs.

Shibahashi H., Saio H., 1985, PASJ 37, 245

Uesugi A., Fukuda I., 1981, Proc. 7th Intern. CODATA Conf., Strasbourg

Van Altena W. F., Lee J. T., Hoffleit D., 1995, The General Catalogue of Trigonometric Stellar Parallaxes, 4th Edition, Yale Univ. Obs. 1995

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.