

# Unstable waves in winds of magnetic massive stars

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**Abstract.** We use a luminous fast magnetic rotator model to analyze the influence of a magnetic field on the linear waves induced in the wind of a massive star by the radiative instability. We show that a twisted magnetic field can drive a strong wind with a wind efficiency  $(\dot{M}v_\infty)/(L/c) > 1$  even without multiple scattering. The radiation amplified waves in the wind are modified by the twisted magnetic field so that they can enhance the wind and lead to overestimates for  $\dot{M}$  and  $v_\infty$ . Finally we argue that the spin down time might be consistent with the lifetime derived from the mass loss rate within the uncertainties regarding the stellar structure. Therefore our model may help to explain high, observed values for  $\dot{M}$  and  $v_\infty$  without being ruled out by the spin down problem.

**Key words:** stars: atmospheres – stars: magnetic fields – stars: mass-loss – stars: rotation – stars: early-type

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## 1. Introduction

The wind of massive stars is primarily driven by the interaction with stellar radiation in many lines (Lucy & Solomon 1970). Based on this concept, the theory of Castor, Abbott, and Klein (1975 hereafter CAK) can explain many aspects of hot star winds. But the basic model of CAK fails to describe the strong winds observed in some massive stars, including Wolf-Rayet stars. Beside multiple scattering effects (Gayley et al. 1995) a rotating magnetic field as driving force were proposed (Friend & MacGregor 1984, Poe et al. 1989, Maheswaran & Cassinelli 1992, and Biermann & Cassinelli 1993) to overcome this problem.

We emphasize, that the observations of nonthermal radio emission from OB (Biegging et al. 1989) and Wolf-Rayet stars (Abbott et al. 1986) give clear evidence for non negligible magnetic fields on these stars. On the other hand direct observations give only rather high limits on the magnetic field (Landstreet 1982), especially for very early-type stars. Landstreet's sample contains only one O-star (O9.5), which is not massive enough to form a Wolf-Rayet star later. Further indirect hints for a strong

magnetic field come from radio observations of supernova explosions (Biermann & Cassinelli 1993, Biermann et al. 1995, Biermann 1997). More observations (e.g. Ignace et al. 1995) are necessary, before our model can be verified or ruled out.

Previous models for rotating magnetic stars suffer from one serious problem: A sufficient enhancement of the wind is connected to a severe loss of angular momentum. The star would spin down faster than is allowed from stellar evolution. But Maheswaran & Cassinelli (1994) argued that a massive star could even reach the Wolf-Rayet phase as a fast magnetic rotator. Magnetic fields may also play an important role in rotationally compressed winds (Cassinelli et al. 1995).

Another important feature of hot star winds is the instability of the radiation force, which leads to a highly perturbed wind. The stability and propagation of linear hydrodynamical waves in the radiation field of the star were analyzed by Owocki & Rybicki (1984 hereafter OR, and 1985), who unified prior work by MacGregor et al. (1979) and Abbott (1980). They found that the linear waves grow very rapidly and therefore cannot be neglected in an overall model. The resulting waves can influence the stellar wind in two ways: (i) They can gain momentum and energy from the radiation field, transport it, and later dissipate it into the wind. This can help to drive the unperturbed wind (Koninx 1992) e.g. by increasing the real  $\dot{M}$  at the base of the wind, where shocks might not have formed yet. (ii) Waves can also steepen into shocks and influence the observation of fundamental wind parameters: The *observed* terminal velocity  $v_\infty$  is inferred from the blue edge of P-Cygni lines. This *maximal* velocity has to be compared with the unperturbed  $v_\infty$  plus the velocity amplitude of the waves or, if *outward* running waves have already steepened into outward running shocks, to the unperturbed  $v_\infty$  plus the shock speed. The last scenario is more probable due to the high amplification rates found in OR. Krolik & Raymond (1985) and MacFarlane & Cassinelli (1989) analyzed the structure of a single outward running shock shell in a nonmagnetic wind and found that the shock shell is driven by the radiation field to a velocity much higher than the velocity of sound. The *observed* values for the mass loss rate  $\dot{M}$  are influenced by waves in two ways: (i)  $\dot{M}$  is inferred from observed values of the density  $\rho$  by  $\dot{M} \sim \rho r^2 v_\infty$ . An overestimated  $v_\infty$  therefore leads to an overestimated  $\dot{M}$ . (ii) The values for  $\rho$  are inferred from observed radio or UV fluxes. A matter distribution

perturbed by waves or shocks generates higher fluxes, which lead to higher values for  $\rho$  if the perturbations are not taken into account properly (Abbott et al. 1981 and Hillier 1991). Shocks are also a good source for the observed X-rays from hot stars. Lucy (1982) developed a heuristic model consisting of a chain of outward running forward shocks mainly in order to describe the X-ray emission from hot stars. But Owocki et al. (1988) found in a nonlinear time-dependent calculation, that waves in the stellar wind are dominantly running inward and steepen into inward running, reverse shocks. In this case the arguments about  $v_\infty$  and also Koninx's model do not apply, so that such waves can not significantly help to explain the high values for  $\dot{M}$  and  $v_\infty$  observed in the wind of massive stars.

The aim of this paper is to show in the limit of a linear analysis that the objections of Owocki et al. (1988) do not apply, if a strong magnetic field and rotation are present. Therefore we join the ideas of a rotating magnetic field (Maheswaran & Cassinelli 1992,1994) and of unstable waves (Owocki & Rybicki 1984(OR)) in the wind of a massive star and do a linear stability analysis. We show that the magnetic field increases the number of wave modes and changes their properties significantly. High phase velocities can be achieved due to the high Alfvén velocity. Therefore inward running waves will not be advected outward and will not steepen into reverse shocks anymore, which dominate at large radii. Furthermore outward and inward running waves have the same growth timescale in the short wavelength regime where both modes are unstable. So we can expect outward running waves far from the star and forward shocks in the nonlinear regime. This model does not exclude multiple scattering as described e.g. by Lucy & Abbott (1993) or Gayley et al. (1995). Rather both could be joined to create a model for an even stronger wind.

In Sect. 2 we derive the dispersion relation for radiatively amplified waves in the presence of a magnetic field. In Sect. 3 we discuss our unperturbed wind models. In Sect. 4 we discuss the waves we found for the wind models of Sect. 3. In Sect. 5 we discuss the observational consequences of our model. And in Sect. 6 we describe some conclusions.

## 2. The wave equations

To analyze waves in the wind of hot stars we start from the equations of magnetohydrodynamics for a compressible, nonviscous, perfectly conducting fluid as described in Jackson (1962) and add the analytic description of OR for the influence of the stellar radiation on the plasma. OR was criticized by Lucy (1984) for the neglect of damping due to diffuse radiation. But Owocki & Rybicki (1985) showed that this effect reduces the amplification rate only by approximately 50% at one stellar radius and 20% at infinity. In this initial paper we emphasize the effect of the magnetic field. Therefore we choose the simple description of OR instead of the more exact but also more involved description of Owocki & Rybicki (1985) or the description of Gayley & Owocki (1995), who analyzed the instability in opti-

cally thick winds. We start with

$$0 = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \quad (1)$$

$$0 = \rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla p + \frac{\mathbf{B}}{4\pi} \times (\nabla \times \mathbf{B}) - \mathbf{F}_{\text{rad}} \quad (2)$$

$$0 = \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (3)$$

where  $\mathbf{F}_{\text{rad}}$  is the force term due to radiation pressure. Now we express the pressure  $p$  by  $c_s^2 \rho$ , where  $c_s$  is the speed of sound, and replace  $\rho$ ,  $\mathbf{v}$ , and  $\mathbf{B}$  by sums of a equilibrium value and a perturbation. In the comoving reference frame ( $\bar{\mathbf{v}} = 0$ )<sup>1</sup> this is:

$$\rho = \bar{\rho} + \delta\rho(\mathbf{x}, t) \quad (4)$$

$$\mathbf{v} = \delta\mathbf{v}(\mathbf{x}, t) \quad (5)$$

$$\mathbf{B} = \bar{\mathbf{B}} + \delta\mathbf{B}(\mathbf{x}, t) \quad (6)$$

Eqs. 1–3 give to first order in small quantities:

$$0 = \frac{\partial \delta\rho(\mathbf{x}, t)}{\partial t} + \bar{\rho} \nabla \cdot \delta\mathbf{v}(\mathbf{x}, t) \quad (7)$$

$$0 = \bar{\rho} \frac{\partial \delta\mathbf{v}(\mathbf{x}, t)}{\partial t} + c_s^2 \nabla \delta\rho(\mathbf{x}, t) + \frac{\bar{\mathbf{B}}}{4\pi} \times (\nabla \times \delta\mathbf{B}(\mathbf{x}, t)) - \delta\mathbf{F}_{\text{rad}} \quad (8)$$

$$0 = \frac{\partial \delta\mathbf{B}(\mathbf{x}, t)}{\partial t} - \nabla \times (\delta\mathbf{v}(\mathbf{x}, t) \times \bar{\mathbf{B}}) \quad (9)$$

We assume now, that the radiative force acts only in radial direction and depends only on the radial velocity. These equations can be reduced to a single equation for  $\delta v$ :

$$0 = \frac{\partial^2 \delta v(\mathbf{x}, t)}{\partial t^2} - c_s^2 \nabla \cdot \delta\mathbf{v}(\mathbf{x}, t) + \bar{\mathbf{v}}_A \times \nabla \times [\nabla \times (\delta\mathbf{v}(\mathbf{x}, t) \times \bar{\mathbf{v}}_A)] - \frac{1}{\bar{\rho}} \frac{\partial \delta v_r(\mathbf{x}, t)}{\partial t} \frac{\partial F_{\text{rad}}}{\partial v_r} \mathbf{e}_r \quad (10)$$

with the vectorial Alfvén velocity  $\bar{\mathbf{v}}_A = \bar{\mathbf{B}}/\sqrt{4\pi\bar{\rho}}$ . If we use the result of OR for the linear perturbation of the radiative force, we can get a dispersion relation from this wave equation for plane waves:

$$\delta\mathbf{v}(\mathbf{x}, t) = \delta\mathbf{v} e^{i\mathbf{k} \cdot \mathbf{x} - i\omega t}. \quad (11)$$

Eq. 10 then becomes:

$$0 = -\omega^2 \delta\mathbf{v} + (c_s^2 + v_A^2)(\mathbf{k} \cdot \delta\mathbf{v})\mathbf{k} + (\bar{\mathbf{v}}_A \cdot \mathbf{k})[(\bar{\mathbf{v}}_A \cdot \mathbf{k})\delta\mathbf{v} - (\bar{\mathbf{v}}_A \cdot \delta\mathbf{v})\mathbf{k} - (\mathbf{k} \cdot \delta\mathbf{v})\bar{\mathbf{v}}_A] - \frac{\Omega k_r}{\bar{\chi} + i k_r} \omega (\delta\mathbf{v} \cdot \mathbf{e}_r) \mathbf{e}_r \quad (12)$$

Here we used the analytic form of the perturbation of the radiation force derived in OR.

$$\Omega = \sqrt{\frac{2c^{\alpha_{\text{CAK}}-1/2}}{1 - \alpha_{\text{CAK}}} \frac{\bar{v}_r}{L_{\text{Sobo}}}} \quad (13)$$

<sup>1</sup> Variables marked with a  $\bar{\phantom{x}}$  refer to the unperturbed wind.

is their combined amplification rate of all lines.

$$\bar{\chi} = \sqrt{\frac{2c^{\alpha_{\text{CAK}}-1/2}}{1 - \alpha_{\text{CAK}}} \frac{1}{L_{\text{Sobo}}} = \frac{\Omega}{\bar{v}_r}} \quad (14)$$

is their mean blue-edge absorption strength with the empirical parameter  $c \approx 1.6$ . And  $\bar{v}_r$  is the radial velocity of the unperturbed wind in the rest frame of the star.  $L_{\text{Sobo}} = v_{\text{th}}/(\partial\bar{v}_r/\partial r)$  is the Sobolev length. Eq. 12 is a vector equation, which is linear in  $\delta\mathbf{v}$ . We can think of it as a generalized eigenvalue problem:

$$(\mathbf{A}(\mathbf{k}, \bar{v}_A, c_s^2) - \omega\mathbf{B}(k_r, \Omega, \bar{\chi}) - \omega^2\mathbf{I})\delta\mathbf{v} = 0 \quad (15)$$

We can find  $\omega$  and  $\delta\mathbf{v}$  numerically. Then  $\delta\mathbf{B}$  and  $\delta\rho$  follow from Eqs. 7 & 9:

$$\delta\rho = \frac{\bar{\rho}}{\omega}(\mathbf{k} \cdot \delta\mathbf{v}) \quad (16)$$

$$\delta\mathbf{B} = \frac{1}{\omega}[(\mathbf{k} \cdot \delta\mathbf{v})\bar{\mathbf{B}} - (\bar{\mathbf{B}} \cdot \mathbf{k})\delta\mathbf{v}] \quad (17)$$

Although Eq. 15 is very involved in the general case, we can find an analytical solution for a simplified situation:

$$\mathbf{k}(r) = k\mathbf{e}_r \quad (18)$$

$$\bar{\mathbf{v}}_A(r) = \bar{v}_A\mathbf{e}_\phi \quad (19)$$

The latter approximation is quite accurate far away from the star. In this limit we find

$$\omega = -\frac{\Omega k}{2(\bar{\chi} + ik)} \pm \sqrt{\left(\frac{\Omega k}{2(\bar{\chi} + ik)}\right)^2 + (c_s^2 + \bar{v}_A^2)k^2}. \quad (20)$$

In the long wavelength limit ( $k \ll \bar{\chi}$ ), where the waves are stable, this leads to

$$\omega = \left[ -\frac{\bar{v}_r}{2} \pm \sqrt{\frac{\bar{v}_r^2}{2} + c_s^2 + \bar{v}_A^2} \right] k. \quad (21)$$

In the case of a weak magnetic field with  $\bar{v}_r \gg c_s \gtrsim \bar{v}_A$ , this resembles Abbott's (1980) result for stable radiative-acoustic waves with a fast inward and a slow outward mode. In the case of a strong magnetic field we have  $\bar{v}_r \approx \bar{v}_A \gg c_s$ . This leads to higher phase velocities for both modes and reduces the relative difference between inward and outward waves. Additionally inward running waves are not advected outward by the average wind motion any more, because their phase velocity is higher than the velocity of the unperturbed wind. In the short wave length limit ( $k \gg \bar{\chi}$ ) we find

$$\omega = i\frac{\Omega}{2} \pm \sqrt{-\frac{\Omega^2}{4} + (c_s^2 + \bar{v}_A^2)k^2}. \quad (22)$$

These waves propagate, if  $\Omega$  is less than  $2k\sqrt{c_s^2 + \bar{v}_A^2}$ , with the same phase velocity inward and outward. The amplification rate is also the same for both modes.

In the case of no magnetic field Eq. 15 leads to

$$0 = \omega^3 + \frac{\Omega k_r}{\bar{\chi} + ik_r}\omega^2 - c_s^2\mathbf{k}^2\omega - \frac{\Omega k_r}{\bar{\chi} + ik_r}c_s^2(k_\phi^2 + k_\theta^2). \quad (23)$$

For  $k_\theta = k_\phi = 0$  this reproduces the result for isothermal waves found in OR. For oblique waves there is a third wave mode.

**Table 1.** Fundamental parameters of our standard star

$M$	$23M_\odot$
$L$	$1.7 \times 10^5 L_\odot$
$R$	$8.5R_\odot$
$T$	60000K
$\alpha_{\text{CAK}}$	0.56
$k_{\text{CAK}}$	0.28

**Table 2.** Results for the unperturbed wind models

Model	A	B	C
$B_{r0}$ [G]	0	500	500
$\Omega/\Omega_{\text{crit}}$	0	0	0.92
$v_\infty$ [km s <sup>-1</sup> ]	1146	1146	784
$\dot{M}$ [ $10^{-6}M_\odot$ yr <sup>-1</sup> ]	0.6	0.6	17
$R_{\tau=2/3}$ [ $R_\odot$ ]	8.5	8.5	11
$(\dot{M}v_\infty)/(L/c)$	0.2	0.2	3.8

### 3. The unperturbed wind models

To analyze the effect of these waves we construct three wind models for a standard massive star. Table 1 gives the fundamental parameters of our standard star. The first model (model A) is the standard analytic CAK wind for a star without magnetic field, rotation, or pressure. In this model we reproduce the previous result of dominantly inward running waves found by Owocki et al. (1988). In the second model (model B) we add a radial magnetic field. Since this magnetic field is parallel to the natural stream lines of the wind of a nonrotating star, this field does not change the velocity profile of the unperturbed wind. But it changes the microphysics for waves. The last model (model C) is a luminous fast magnetic rotator model. The rotating magnetic field provides an additional driving force, which changes the properties of the wind drastically. The terminal velocity decreases and the mass loss rate increases. The wind efficiency  $(\dot{M}v_\infty)/(L/c)$  is 3.8, which is much higher than for a purely radiatively driven wind in the single scattering limit, where the efficiency can not be higher than unity. In spite of the known spin down problem we choose a strong magnetic field and a high rotation rate in order to emphasize the influence of these parameters. For this model we use the equations derived by Biermann & Cassinelli (1993), which do not assume that the magnetic field is radial close to the star. This leads to a smaller and more accurate value for the Alfvénic radius  $R_A$ , which controls the angular momentum loss of the star. Table 2 gives the magnetic field, rotation rate, and the resulting values for the above mentioned unperturbed wind models. In Sect. 5 we will discuss how these values and their observation can be influenced by waves. In this paper we do not discuss the influence of the waves back on the unperturbed wind as Koninx (1992) did. A model with rotation but without magnetic field is not included, because this model has the same microphysics for

waves as model A. Just  $\dot{M}$  and the velocity dependence on  $r$  are different. Model C differs from model B in the local conditions by the fact that, due to the rotational twist, the magnetic field and the direction of the radiative force are not parallel anymore. Even at the base of the wind  $B_\phi$  is approximately  $-1.67B_r$ .

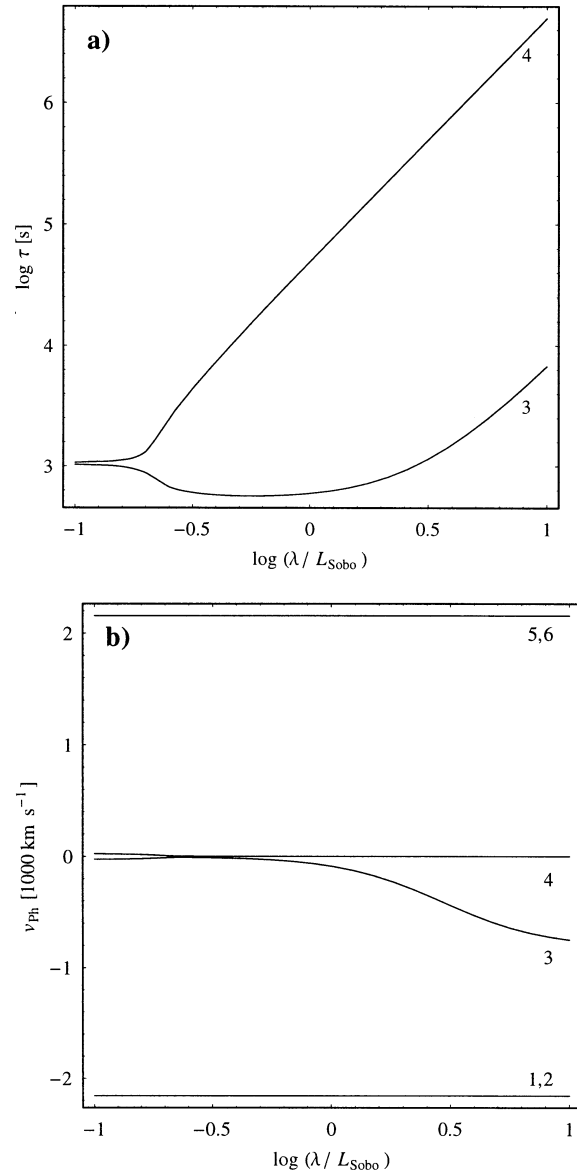
#### 4. Numerical results for waves

Since our unperturbed wind model is limited to the equatorial plane we limit our discussion for waves to the same plane. We have still two free parameters then: The wavelength and the azimuthal angle of  $\mathbf{k}$ . In the first section we discuss radial waves. This will show all important properties of this wave model. In the second section we discuss briefly the influence of the azimuth angle of  $\mathbf{k}$ .

##### 4.1. Radial waves

Figures 1a&b and 2a&b show numerical results for wind model A&B. The sonic wave modes<sup>2</sup> found for model A are identical to the slow magnetosonic modes of model B. For model B we find additionally four fast wave modes. These modes, the fast magnetosonic and the Alfvénic modes, have the same phase velocity  $v_{\text{Ph}} \approx \bar{v}_A$ , because both,  $\mathbf{B}$  and  $\mathbf{k}$ , are parallel to  $\mathbf{e}_r$ . But these wave modes are stable and show no dependence on wavelength. Therefore we concentrate our discussion for model A&B on the slow waves: Fig. 1a and b shows the dependence of the waves on the wavelength for  $r = 2R$ , plotted relative to the Sobolev length, which is the relevant length scale for radiative wave amplification in the model of OR, we use here.  $R = 8.5R_\odot$  is the stellar radius. In the long wavelength limit we find stable waves with a high phase velocity inward and a low phase velocity outward. This reproduces Abbott's (1980) result of radiative-acoustic waves. In the short wavelength limit we find the same amplification timescales and phase velocities (except the direction) for both modes. In this limit we would expect to see both wave modes in the wind. But the most interesting case is the bridging case  $\lambda \approx L_{\text{Sobo}}$ . Here we find the shortest amplification timescale for all wavelengths. Inward waves are two orders of magnitude faster in amplification than outward waves. They have also a higher phase velocity. This resembles the result of Owocki et al. (1988), who did a nonlinear calculation and found that primarily the inward running waves steepen into reverse shocks, which are advected outward. The aim of this paper is to argue that this scenario changes if a magnetic field *and* rotation are involved. Fig. 2a and b shows the radial dependence of the wave with  $\lambda = L_{\text{Sobo}}(r)$ . The phase velocity for the slow magnetosonic modes is approximately the velocity of sound, so that inward running waves are advected outward in the supersonic part of the wind. The amplification of the waves is strongest close to the star, where the Sobolev length is short.

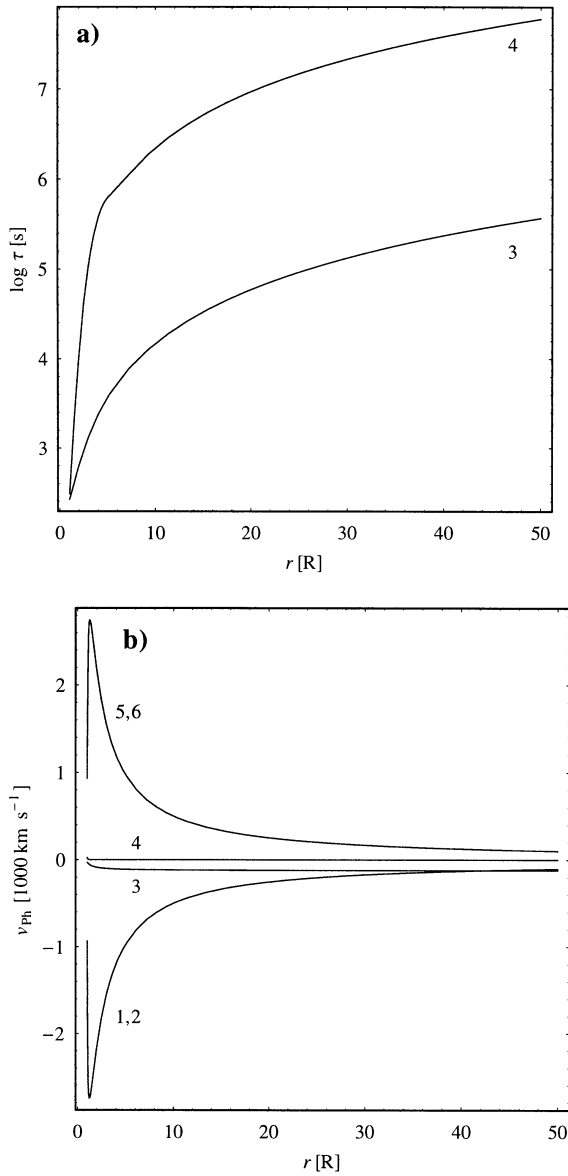
<sup>2</sup> Ordered by the radial phase velocity we denote in our figures the fast magnetosonic modes with 1&6, the Alfvénic modes with 2&5, and the slow magnetosonic modes with 3&4. The sonic modes of model A, which are identical to the slow magnetosonic modes of model B, are denoted with 3&4 as well.



**Fig. 1a and b.** Amplification timescales **a** and phase velocities (in the frame of the unperturbed wind) **b** versus wavelength for model A&B and  $r = 2R$ . In model A only modes 3&4 exist. Modes missing in **a** are stable. Mode 3 with  $\lambda \approx L_{\text{Sobo}}$  has the shortest amplification timescale and therefore will dominate the wind. These inward running waves are advected outward with  $\bar{v}_r = 811 \text{ km s}^{-1}$  and steepen into reverse shocks

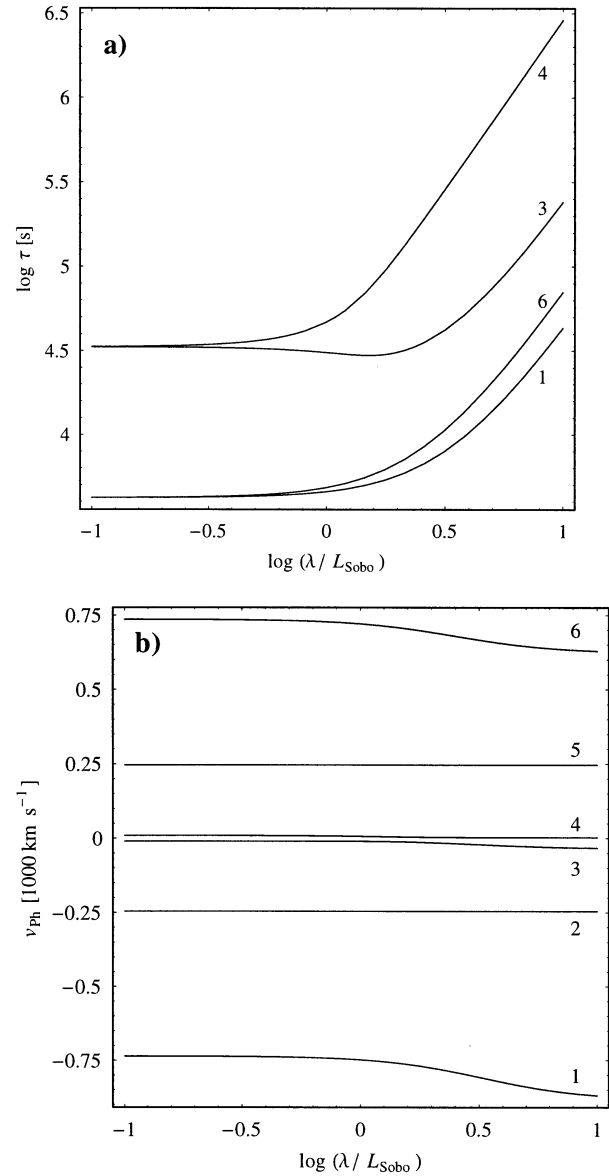
The phase velocity of the fast modes goes with  $r^{-1}$  for large radii since  $B = B_r \sim r^{-2}$ . This might lead to a small velocity for outward running shocks at large radii.

Figures 3a&b and 4a&b show the same plots for model C – with magnetic field and rotation. The crucial point is that the magnetic field and the amplifying stellar radiation are not parallel anymore. For large radii they are even perpendicular. Therefore the fast magnetosonic wave modes, which are most interesting for us, are amplified as well. Figure 3b shows six modes with different phase velocities. Two of them, the Alfvénic modes,



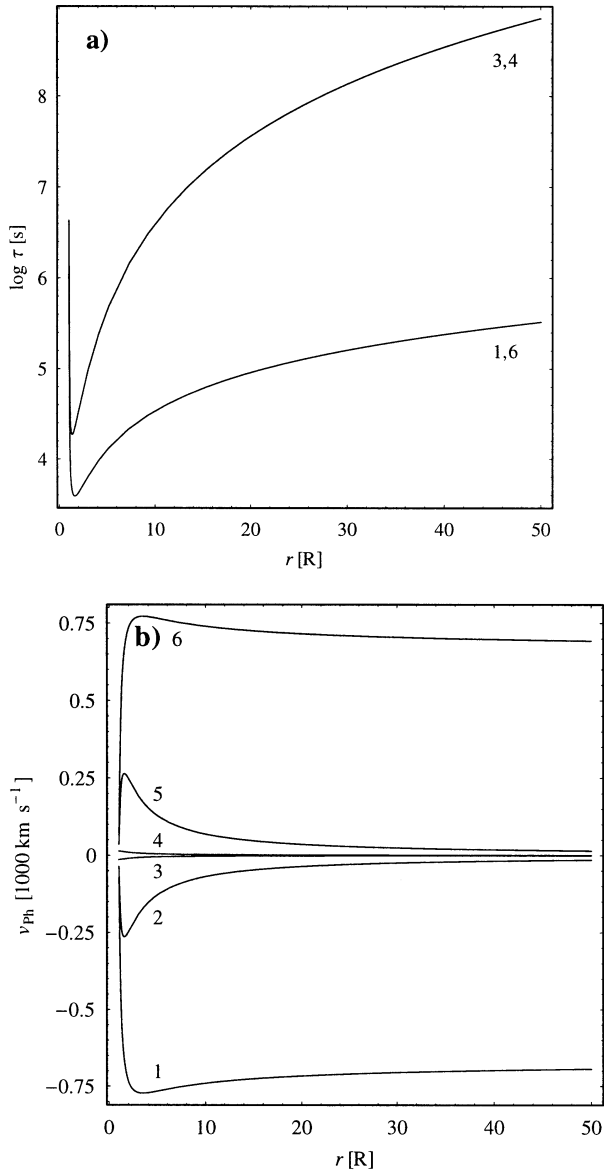
**Fig. 2a and b.** Amplification timescales **a** and phase velocities (in the frame of the unperturbed wind) **b** versus radius for model A&B and  $\lambda = L_{\text{Sobo}}$ . In model A only modes 3&4 exist. Modes missing in **a** are stable. The wave amplification is strongest close to the star, where the radiation field and wind acceleration are strong. Inward running waves originating there will be advected outward and steepen into reverse shocks

show no dependence on wavelength. They are unaffected by the radiation field and therefore stable. Figure 3a shows the amplification timescales for the magnetosonic waves. The fast magnetosonic waves grow approximately one order of magnitude faster than the slow waves. We can therefore expect that these waves will dominate. The crucial point is that they are much faster than the unperturbed wind – especially close to the star. Inward running fast waves will therefore not be advected away from the star. Figure 4b shows that the phase velocity for the fast magnetosonic waves remain high for large radii. This



**Fig. 3a and b.** Amplification timescales **a** and phase velocities (in the frame of the unperturbed wind) **b** versus wavelength for model C and  $r = 2R$ . Modes missing in **a** are stable. In the long wavelength limit all modes are stable. In the short wavelength wavelength limit the fast magnetosonic modes (1&6) grow approximately one order of magnitude faster as the slow magnetosonic modes (3&4). We expect that fast magnetosonic waves in both directions are equally dominant in the wind of model C. Since the magnetosonic waves are much faster than the unperturbed wind ( $\bar{v}_r = 294 \text{ km s}^{-1}$ ), the inward running waves will not be advected outward. The stable Alfvénic modes (2&5) show no dependence on wavelength

velocity is a lower limit for the velocity of outward running shocks. Rybicki et al. (1990) showed that nonradial perturbations in the stellar wind are damped close to the wind. But the magnetic field of our model C is mostly tangential already at the stellar surface. For the fast magnetosonic modes  $|\delta v_\phi / \delta v_r|$  is 0.5 at the stellar surface and 0.35 at  $r = 2R$ . We expect there-



**Fig. 4a and b.** Amplification timescales **a** and phase velocities (in the frame of the unperturbed wind) **b** versus radius for model C and  $\lambda = 0.1L_{\text{Sobo}}$ . The Alfvénic modes (2&5) missing in **a** are stable. The wave amplification is strongest close to the star, where radiation field and wind acceleration are strong. Fast magnetosonic waves originating here will dominate the wind. They have the shortest amplification timescale for all radii and a high phase velocity even at large radii. This may lead to fast shocks running outward, which have a strong influence on observation

fore, that the effects found by Rybicki et al. will influence but not completely dampen the magnetosonic waves. We emphasize that fast magnetosonic waves propagate fastest perpendicular to the magnetic field with  $v_{\text{ph}} = (\bar{v}_A^2 + c_s^2)^{0.5}$ .

It is very speculative to draw conclusions for nonlinear waves and shocks from a linear stability analysis. But we showed that a magnetic field has a strong influence on the linear analysis. Therefore we expect a strong influence of the magnetic

field on nonlinear perturbations as well. Our speculative scenario for waves in the wind of hot stars is the following: Waves are predominantly generated close to the star where the stellar radiation field is strong and the Sobolev length is short. Waves with a wavelength short compared to the Sobolev length are generated predominantly, because they have the shortest amplification timescale. Inward running waves run into the star and disappear because their phase velocity is higher than the unperturbed wind velocity. Outward running waves, which have, for short wavelengths, the same growth timescale than inward running waves, can run over many stellar radii and grow. For the outward running waves  $\delta\rho$  and  $\delta v_r$  are in phase. Therefore they can steepen into forward shocks with regions of high density at high velocity and influence the measurements for the terminal velocity and mass loss rate.

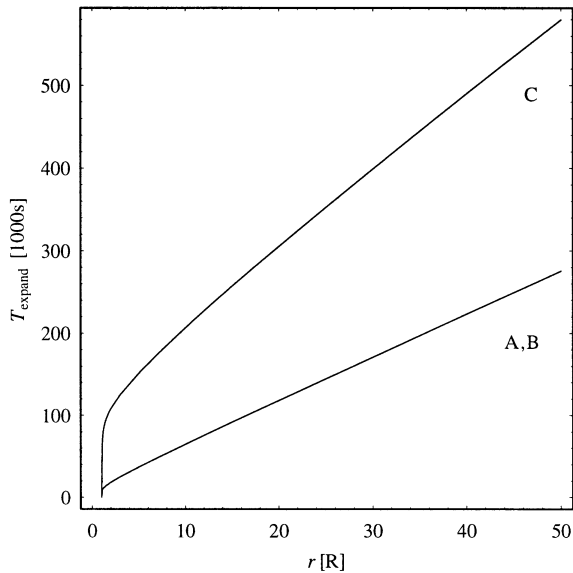
#### 4.2. Nonradial waves

The picture drawn in the last section does not change significantly when nonradial waves are taken into account. There are two new effects compared to the radial case. A third wave mode appears in model A as predicted by Eq. 23. This inward running mode has a long amplification timescale and a relatively low phase velocity. Therefore we do not expect a significant contribution of this mode to the situation in the wind of model A. For model B we find that Alfvén and fast magnetosonic waves have different phase velocities now. The fast magnetosonic waves are now amplified, too. But their amplification timescale is much longer than for slow magnetosonic waves.

### 5. Consequences from our model

In this paper we do a linear stability analysis and show that the waves in the stellar wind can help to understand the observations. In order to calculate quantitative results, which can be compared with observations, it would be necessary to analyze the detailed properties of the shocks resulting from the waves found in this paper. But we can derive some estimates from our calculations.

From Fig. 4b we see that the phase velocity of the outward running waves and possibly shocks in the rest frame of the star is at least twice the terminal velocity of the unperturbed wind. In the outward running waves the oscillations of  $v_r$  and  $\rho$  are in phase. Therefore a significant amount of matter, but presumably not all matter will escape at this or a higher velocity, if the outward running waves steepen into forward shocks. The terminal velocity will then be overestimated at least by a factor of two in the observation. For our model C this would be  $v_{\infty, \text{obs}} \approx 1500 \text{ km s}^{-1}$ . Krolik & Raymond (1985) found that in a nonmagnetic wind shock shells are running much faster than the phase velocity of the waves. In an unperturbed magnetic wind model such a high value for  $v_{\infty}$  combined with a reasonable high value for  $\dot{M}$  can only be obtained with a very high magnetic field and fast rotation, which leads to a spin down problem.



**Fig. 5.** Expansion time versus radius. Model C has a slower acceleration close to the star and a lower terminal velocity. A fast magnetosonic wave with an amplification timescale of about 2500s (cf. Fig 4a and b.a) grows by a factor of  $\approx e^{40}$ , while the wind expands from  $r = 1R$  to  $r = 1.5R$ . The electron scattering opacity is  $\tau \approx 2/3$  at  $r = 1.3R$ . Therefore we can expect shocks already at this radius, where line observation starts. The objections of Lucy (1984) do not change this situation qualitatively

The influence of our model on  $\dot{M}$  is more difficult to estimate. We gain a real factor on 28 in  $\dot{M}$  between our model A&B and model C even in the unperturbed wind due to the driving force of the rotating magnetic field. Furthermore the observation of  $\dot{M}$  is influenced by the clumping of the wind matter. But to calculate the clumping factor  $\langle \rho^2 \rangle / \langle \rho \rangle^2$  at large radii, where radio observations are made, it would be necessary to do a nonlinear calculation including large radii, because the waves steepen into shocks very rapidly due to the short amplification timescales. This is beyond the linear model presented here.

Magnetic rotator models for hot stars are often criticized for their short spin down times. We reduced this problem by using the equations of Biermann & Cassinelli (1993) for our unperturbed wind model C. These equations do not assume that the magnetic field is radial close to the star. Therefore we find a smaller value for the Alfvénic radius  $R_A$ , which controls as a lever arm the angular momentum loss. The magnetic field increases the angular momentum loss in the equatorial plane compared to a nonmagnetic star with the same rotation rate by  $(R_A/R)^2$ , which is 3.2 for our model C.

In order to decide, whether the increased angular momentum loss would rule out a rotating magnetic field as an important component of the wind of these stars, one would need a detailed stellar evolution model. Such a model must e.g. consider the angular momentum structure and evolution inside the star, which is yet unknown in the case of a strong magnetic field. It is also not clear, whether the magnetic field is present in the wind from the beginning of the stellar evolution, or whether it is generated

later, as was suggested by Biermann & Cassinelli (1993) and Biermann (1997) even for stars with convective cores and radiative envelopes. The angular momentum loss from the wind outside the equatorial plane is also not calculated yet. Due to these serious uncertainties in this issue most people (including the authors) use the very simple spin down formula of Friend & MacGregor (1984, Eq. 38)

$$\tau_J = \frac{J}{\dot{J}} \approx \frac{3}{5} \frac{R^2}{R_A^2} \frac{M}{\dot{M}} = \frac{3}{5} \frac{R^2}{R_A^2} \tau_M \quad (24)$$

to roughly guess the spin down timescale of the star. This formula guesses the angular momentum of the star by assuming, that the star is a *rigidly* rotating *sphere* of *constant* density. All these three assumptions are of course wrong. The angular momentum loss from outside the equatorial plane is just guessed from the value in the equatorial plane using a factor of 2/3. Even if we could accurately calculate the current spin down  $J/\dot{J}$  and mass loss  $M/\dot{M}$  timescales,  $\tau_J < \tau_M$  does not automatically mean that post main sequence star does not rotate significantly. E.g. an artificial nonmagnetic star ( $R = R_A$ ), which fullfills the assumptions of Eq. 24 mentioned above, has no internal angular momentum transport and a constant density over this lifetime, has  $\tau_J = 3/5 \tau_M < \tau_M$ , but the rotation rate  $\Omega$  remains constant over the whole lifetime of such a star. The reason is that the outer layers of the star have, due their larger average distance from the rotation axis, more angular momentum per unit mass than inner layers in spite of the rigid rotation. This example shows that Eq. 24 is just not accurate enough to verify or rule out a model with  $\tau_J(\text{Eq. 24}) \approx \tau_M$  within an order of magnitude. For our model C we find using Eq. 24  $\tau_J = 2.5 \times 10^5 \text{yr}$  and  $\tau_M = 1.4 \times 10^6 \text{yr}$ .

## 6. Conclusions

In this paper we analyzed the interaction between a magnetic field and linear waves induced by the radiative instability. We found both models complement each other. The magnetic field suppresses the inward running waves, which dominate in non-magnetic winds. This may allow the outward running waves to support the unperturbed wind as described by Koninx (1992) and to form high density shock shells running out at a high speed. These shock shells may explain the high terminal velocities measured in winds of massive stars. The outward running waves will also lead to an overestimation of  $\dot{M}$  due to wind clumping and the overestimated  $v_\infty$ . Wind clumping also occurs without a magnetic field. But in this case the resulting shock shells will run inward; and the argument about the overestimated  $v_\infty$  would not apply. The overestimated  $\dot{M}$  and  $v_\infty$  put unnecessarily strong restrictions on fast magnetic rotator wind models. We argued that even for a magnetic field with  $B_{r0} = 500\text{G}$  the spin down time is consistent with the lifetime of the star inferred from the mass loss rate considering the uncertainties in the stellar structure. From the observation of nonthermal radio emission in many OB and Wolf-Rayet stars we know that these stars have a nonnegligible magnetic field. Further direct observations are necessary to infer the actual strength of these fields.

We showed that a luminous fast magnetic rotator model plus wind perturbations by waves or shocks can help to explain the observed high values for  $\dot{M}$  and  $v_\infty$  without being ruled out by the spin down problem. Further observation of the magnetic field and further theoretical work on the evolution of stellar rotation are necessary to evaluate to role of magnetic fields in winds of massive hot stars.

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