Astron. Astrophys. 327, 428–431 (1997)

ASTRONOMY AND ASTROPHYSICS

On the stochastic gravitational radiation background produced by an ensemble of single neutron stars

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Received 13 January 1997 / Accepted 12 May 1997

Abstract. The possible stochastic background produced by single galactic neutron star is studied. The upper limit to this background independent of the NS ellipticities is derived. If ~ 0.1 of old neutron star population have a low surface magnetic field ($< 10^7$ G; from the beginning or due to field decay), this background may be detected by the advanced LIGO/VIRGO interferometers with a sensitivity of $\sim 10^{-25}$ at 100 Hz during 1-year integration.

Key words: gravitational waves – stars: neutron

1. Introduction

Among possible sources of gravitational radiation (GR), neutron stars (NS), both single and entering binary systems, are considered as most promising (see Thorne 1987; Abramovici et al. 1992; Schutz 1997 for full review). NS are the end product of evolution of massive ($\geq$ 8–10 $M_{\odot}$) stars, so their number in the Galaxy should amount to $\sim 10^9$ over the galactic lifetime of $\sim 10^{10}$ years. The number of binary NS is more controversial. Simple estimate based upon the binary pulsar statistics (Phinney 1991; Curran & Lorimer 1995; van den Heuvel & Lorimer 1996) yield the coalescence rate of binary NS in the Galaxy of order 1 per 10^5 years, so to have an acceptable detection rate of these sources one must have a detector sensitivity of at least 10^{-21}–10^{-22} at the frequency 100 Hz, which the initial laser inetrferometers currently under construction are aimed at. Theoretical estimates of the binary NS coalescence rate are typically an order of magnitude higher (1 per 10^4 years), and the possible solution of the discrepancy may be connected with the NS not being observed as a radiopulsar in a binary NS+NS system (see Lipunov, Postnov & Prokhorov (1997) for more detail).

The situation with single NS, however, differs from binary NS in that the latter are the most relaible sources of GR, which is confirmed by observations of the binary pulsar PSR 1913+16 orbit decay (Taylor 1992), whereas to be a noticable source of GR, the form of an isolating rotating NS must deviate from axisymmetry. This deviation is usually described in terms of the relative difference of moment inertia along the different axis of the non-spherical body of the star, $\epsilon = 1 - a_2/a_1 \approx \Delta I/I$, where a_1 is the semimajor axis of the equatorial section, and a_2 the semiminor axis.

In the last years, different mechanisms of symmetry bracking have been proposed for young NS (see, e.g., Lai & Shapiro 1995; Bonazzola, Frieben & Gourgoulhon 1996). It has also been suggested (Zimmermann 1978; Gal'tsov, Tsvetkov & Tsirulev 1984; Bonazzola & Gourgoulhon 1996) that an internal strong magnetic field ($B \sim 10^{15}$–10^{16} G) may cause the asymmetrical shape of the NS. It has been shown by Blair (1996) that the asymmetry of young NS may lead to appearance of a stochastic GR background at frequencies 1–300 Hz, and provided that the supernova explosions in the entire Universe are frequent enough, it can be marginally detected with the advanced LIGO/VIRGO interferometer. Recently, Giazotto, Bonazzola & Gourgoulhon (1997) studied the possibility of the detection of the GR background generated by all old NS in the Galaxy with only one GW interferometer using a quadratic detection technique. We are highly ignorant about old NS distribution in the Galaxy. However, old NS may populate an extensive halo around the galaxy ($\sim$100–300 kpc even without significant kick velocity; see e.g. calculations by Gurevich et al. (1993); Prokhorov & Postnov (1994)), so the modulation of the signal by Earth rotation considered by Giazotto et al. (1997) could be smeared out, making quadratic detection by one GW interferometer questionable.

In this paper, we consider the stochastic GW background produced by the old NS population in the Galaxy and beyond taking into account spin-down evolution of old NS. We show that in the limiting case of angular mometum loss only due to GR, the upper limit on the GR background formed by old NS is determined by NS production rate only. We also briefly discuss the possibility of its detection in future GR experiments.

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2. Stochastic GR background produced by rotating single NS

Let us assume a stationary situation, i.e. that the number of NS in the Galaxy is determined by a constant formation rate $\mathscr{R}$. Assuming the present star formation rate in the Galaxy 1 $M_\odot$/yr, Salpeter mass function $f(M) \propto M^{-2.35}$ (Salpeter 1955), and the minimal mass of the star $M_{\rm min} = 0.1\ M_\odot$ (such a choice yields the total stellar mass in the Galaxy $10^{11}\ M_\odot$ during 1.5×10^{10} years), we find that the mean formation rate of massive stars (>10 $M_\odot$ to produce NS) are about 1 per 30 years. Below we shall normalize all calcualtions to this rate, $\mathscr{R}_{30} \equiv 1/30\,{\rm yr}^{-1} \approx 10^{-9}\ {\rm s}^{-1}$.

A rotating non-axisymmetric NS with the ellipticity ϵ loses energy in the form of GR at a rate (Shapiro & Teukolsky 1983)

$$\dot{E}_{GR} = \frac{32G}{5c^5} I^2 \epsilon^2 \omega^6 \tag{1}$$

where G and c are the Newtonian gravity constant and speed of light, $\omega = 2\pi\nu$ is the NS rotation frequency, I is the moment of inertia.

If the NS were emitted GR at strictly twice rotational frequency and this frequency were not changed, in principle each star may be distinguished by an ideal detector provided its frequency band is sufficiently narrow. However, the rotating NS radiates both at ω and 2ω (and possibly at other higher harmonics if its form is more complex) and its rotational frequency are constantly changing by the energy conservation law

$$I\omega\dot{\omega} = \dot{E}_{GR} + \dot{E}_{em} + \dot{E}_{...} \tag{2}$$

where we have explicitly written down possible rotational energy losses – electromagnetic ($\dot{E}_{em}$) and others. In the ideal case of GR being the only source of energy loss we would retain only first term in the expression above. We should note that the spin evolution of a magnetized NS becomes much more complicated when the NS is in a binary system (e.g. Lipunov 1992); however, their fraction among the total number of NS is hardly higher than 10%, and we will not consider them here.

As laser interferometers are broad-band detectors ($\Delta\omega \sim \omega$), a large number of sources within the sensitivity band would produce a stochastic background. Long-term continuous observations, however, allow to make the sensitivity band efficiently narrower provided that the faweform of the signal is known (in fact, as $\Delta\nu \sim 1/T$, where T is the integration time; this permits to increase signal-to-noise ratio for continous source observations as $\sqrt{T}$ using match filtering thechnique). For old NS, however, match filtering technique of data analysis would be excessively demanding for modern computers (Schutz 1997) (a priori we do not know the form of the signal, its frequency, source location on the sky, etc.), so for old NS the interferometer always works as a broad-band detector.

Clearly, the condition that a stochastic signal appears within the detector band reads

$$\Delta t\,\mathscr{R} \geq 1 \tag{3}$$

where Δt is the time for a typical source to pass through the detector band $\Delta\omega$. This time is determined by a particular mechanism of energy losses, and we examine it separately for GR and electromagnetic losses.

1. Elecromagnetic losses. They are described by the law

$$\dot{\omega} = A\omega^3; \qquad A = \frac{2\mu^2}{3c^3 I} \tag{4}$$

Here μ is the dipole magnetic moment of the NS. The solution to the equaiton (4) reads

$$\Delta t_{em} = \frac{1}{2} A\omega^{-2}((\frac{\omega}{\omega - \Delta\omega})^2 - 1) \tag{5}$$

Assuming $\Delta\omega = \frac{1}{2}\omega$ we find the upper frequency of the stochastic background

$$\nu_0^{em} \approx 10^3 ({\rm Hz}) \mathscr{R}_{30}^{1/2} I_{45}^{1/2} \mu_{30}^{-1} \tag{6}$$

with $\mu_{30} = \mu/(10^{30}{\rm G\ cm}^3)$.

2. GR losses. These are

$$\dot{\omega} = B\omega^5; \qquad B = \frac{32G}{5c^5} I^2 \epsilon^2 \omega^6\,. \tag{7}$$

Then

$$\Delta t_{GR} = \frac{1}{4} B\omega^{-4}((\frac{\omega}{\omega - \Delta\omega})^4 - 1) \tag{8}$$

and under the same assumption about $\Delta\omega$ we find

$$\nu_0^{GR} \approx 1.4 \times 10^4 ({\rm Hz}) \mathscr{R}_{30}^{1/4} I_{45}^{-1/4} \epsilon_{-6}^{-1/2} \tag{9}$$

with $\epsilon_{-6} = \epsilon/10^6$.

Therefore, for plausible values of the NS magnetic fields ($\mu_{30} = 10^{-4}$–10^2) and ellipticities ($\epsilon_{-6} = 10^{-3}$–10^2), at any frequency $<10^3$ Hz we deal with stochastic backgrounds from galactic NS. Physically, this is due to the inability of old NS to leave frequency interval $\Delta\omega \sim \omega$ during the typical time between consecutive supernova explosions. This is not so for young NS (see, e.g., Lai & Shapiro 1995).

Now we ask the question: how many sources with changing frequency are to be simultaneously observed within a frequency interval $\Delta\omega \sim \omega$? The answer is immediate: Under stationary conditions the continuity equation implies

$$N(\omega) \equiv \frac{dN}{d\ln\omega} = \frac{dN}{dt}\frac{\omega}{\dot{\omega}} = \mathscr{R}\frac{\omega}{\dot{\omega}} \tag{10}$$

Here we assumed that all sources come into the interval through its upper boundary. This assumption is correct if the upper boundary of the interval lies sufficiently far from the initial frequency of NS (i.e. less than about 100 Hz). Now, if the number of sources within a frequency interval $\Delta\omega \sim \omega$ interval is more than unity, the resulting GR signal at the frequency ω would read

$$h_\Sigma(\omega)^2 = \sum_{i=1}^{N(\omega)} h_i^2(\omega) \tag{11}$$

where dimensionless strain amplitude from one source relates to the energy flux $F(\omega)$ at the frequency ω as

$$h(\omega)^2 = \frac{16\pi G}{c^3\omega^2}F(\omega) = \frac{4G}{c^3\omega^2}\frac{\dot{E}_{GR}(\omega)}{r^2} \quad (12)$$

where r is the distance to the source.

If we perform a long-term search for the background ($T \gg 1$ day) and thus not interested in the possible modulation of the signal by Earth rotation or if old NS populate an extended halo, Eq. (11) may be rewritten in the form

$$h_\Sigma(\omega) = \sqrt{\tilde{h}^2(\omega)N(\omega)} \quad (13)$$

where $\tilde{h}^2(\omega) \propto (1/\tilde{r}^2)\omega^4$ and $\tilde{r}$ is the inverse-square average distance to the typical source. Using Eq. (2) and (10), we obtain to within an unimportant trigonometrical coefficient of order unity (see Giazotto et al. (1997) for more detailed calculations of this factor)

$$h_\Sigma(\omega) \approx \frac{1}{\tilde{r}}\sqrt{\frac{4GI}{c^3}\frac{\mathscr{R}}{1+\dot{E}_{em}/\dot{E}_{GR}}} \quad (14)$$

where we omitted all but electromagnetic and GR loss terms. For purely GR-driven NS spin-down the resulting spectrum is independent of the unknown value of ϵ in the NS population. (The independency of the resulting signal on the ellipticity when the pulsar spin-down is governed by GR losses only was noted by Thorne (1987) with the reference to private communication from R. Blandford in 1984). Note that any additional braking mechanism always lowers the resulting signal. For example, taking typical values $I = 10^{45}$ g cm^2, $\mathscr{R} = 1/30$ yr^{-1} we obtain

$$h_{lim} = \frac{1}{\tilde{r}}\sqrt{\frac{4GI\mathscr{R}}{c^3}} \approx 3\times10^{-24}\left(\frac{10\text{kpc}}{\tilde{r}}\right)\mathscr{R}_{30}^{1/2}I_{45}^{1/2} \quad (15)$$

(here we assumed the characteristic distance to NS population of order 10 kpc). The GR background of such strength could be detected by the advanced LIGO/VIRGO interferometers in one year integration (Thorne 1987; Giazotto 1997).

What kind of losses governs the NS spin evolution for realistic NS parameters? The ratio of electromagnetic losses to GR losses $x = \dot{E}_{em}/\dot{E}_{GR}$ is

$$x = \frac{A}{B}\omega^{-2} \quad (16)$$

with A and B determined as above, and for typical parameters μ and ϵ we find

$$x \approx 4000\mu_{30}^2\epsilon_{-6}^{-2}\left(\frac{100\text{Hz}}{\nu}\right)^2 \quad (17)$$

that is electormagnetic losses becomes insignificant only at frequencies

$$\nu > \nu_{cr} \approx 6.3(\text{kHz})\frac{\mu_{30}}{\epsilon_{-6}} \quad (18)$$

i.e. they dominate practically always. If we would take $\epsilon_{-6} = 10^{-3}$ and $\mu_{30} = 10^{-4}$ as in millisecond pulsars, we would obtain $\nu_{cr} \approx 630$ Hz, however millisecond pulsars are spun up by accretion in binary systems and are not considered here.

Therefore, for realistic NS we must consider the case $x \gg 1$. Using Eqs. (14) and (17) we derive that the stochastic background from old NS is

$$h_\Sigma(\nu) \approx 5\times10^{-28}\left(\frac{10\text{kpc}}{\tilde{r}}\right)\mathscr{R}_{30}^{1/2}I_{45}^{1/2}\epsilon_{-6}\mu_{30}^{-1}\nu \quad (19)$$

Note the *frequency dependence* appeared in this expression. If we take the estimate of magnetic field from observations of pulsar periods P and their change rate $\dot{P}: \mu_{30} \approx \left(P\dot{P}_{-15}\right)^{1/2}$ (here $\dot{P}_{-15} \equiv \dot{P}/10^{-15}$) and assume maximum possible ellipticity allowed by P–$\dot{P}$ observations: $\epsilon_{max} \approx 5.7\times10^{-3}\left(P^3\dot{P}_{-15}\right)^{1/2}$, Eq. (19) immediately yields the same Eq. (15) for h_{lim} as above.

3. Discussion

Now consider the contribution of old NS population from other galaxies. For distances described by Euclidean geometry (<100 Mpc) we may do a crude estimate as follows. For specific events, the rate within the volume V (Mpc^{-3}) relates to the galactic event rate $\mathscr{R}_G$ (e.g., Phinney 1991) as $\mathscr{R}_V \approx 0.01\times\mathscr{R}_G h_{100}$, where $h_{100} = H_0/100$ km s^{-1} Mpc^{-1} is the Hubble constant. Therefore, for a population of old NS within $\sim$ 100 Mpc we obtain

$$h_{lim} \sim 10^{-25}\left(\frac{100\text{Mpc}}{\tilde{r}}\right)\left(\frac{\mathscr{R}_G}{1/30\text{yr}}\right)^{1/2}I_{45}^{1/2} \quad (20)$$

ten times smaller than from galactic NS.

Going further away, however, cosmological effects become significant. Old NS population from other galaxies may fairly well be considered isotropic and of probably not strongly varying comoving density. Then we should use the mean photometric distance in (15) which is in the standard flat Friedman Universe is $\langle r_{ph}\rangle = \sqrt{20/3}(c/H_0)$ (if $z_{lim} \gg 1$). For limiting redshifts $z_{lim} = 5$ we find $\langle r_{ph}\rangle \approx 10h_{75}^{-1}$ Gpc. The supernova rate even with strong evolutionary effets is $< 10^9$ per year (for baryonic content in stars $\Omega_b \approx 0.005$; see Jørgensen et al. (1997) for more detail), so we obtain $h_{lim} < 10^{-25}$.

We have shown that if the NS form ellipticity is present, the stochastic GR background produced by old NS population is naturally formed due to NS rotation braking. In the limiting case when only GR angular momentum loss causes NS spin-down, this background is *independent* on both exact value of the NS form ellipticity ϵ and frequency and can be detected by advanced LIGO/VIRGO interferometers. In reality, the magnetic field of NS causes more effective electromagnetic energy loss: to be insignificant, the magnetic field of a NS should be less than (see Eq. (17))

$$\mu < 1.5\times10^{26}(\text{G cm}^3)\epsilon_{-6}\,\nu \quad (21)$$

According to Urpin & Muslimov (1992), the magnetic field can decay very fastly provided that the field was initially concentrated in the outer crust layers with the density $< 10^{10} - 10^{11}$ g cm^{-3}, and such very low magnetic field for old NS may be possible. In the limiting case that the NS magnetic field does not decay at all (for example, if only accretion-induced field decay is possible in binary systems (Bisnovatyi–Kogan & Komberg 1974)), old NS should lose their energy through electromagnetic losses and be very slow rotators with periods of about a few seconds. Then the initial magnetic field distribution becomes crucial. If it is centered at $\sim 10^{12}$ G (as implied by radipulsar P–$\dot{P}$ measurements), we have little chances to detect the old NS population at 10–100 Hz frequency band unless close mean distances ($<$10 kpc) are assumed (Giazotto et al. 1997). However, if nature prefers a scale-free law (like $f(\mu) \propto 1/\mu$), the fraction of low-field NS could amount to a few 10% and they can contribute to the frequency-independent GR background. Then Eq. (15) implies that such a background can be detected by the advanced LIGO/VIRGO interferometer in the frequency band 10–1000 Hz in one-year integration even if the formation rate of such NS is as small as 1 per 300 years and the characteristic distance to them is 100 kpc.

Acknowledgements. The work was partially supported by Russian Fund for Basic Research through Grant No 95-02-06053-a and by INTAS Grant No 93-3364. KAP acknowledge the staff of Cosmic Radiation Laboratory of RIKEN for hospitality and JISTEC through STA Fellowship Grant No 496057 for financial support. We acknowledge the referee, Dr. E. Gourgoulhon, for useful notes.

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