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ASTRONOMY AND ASTROPHYSICS

Circumstellar dust shells around long-period variables

VI. An approximative formula for the mass loss rate of C-rich stars

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Abstract. We present an equation which relates the six parameters of dynamical wind models to the resulting mass loss rate. These six parameters are the stellar temperature, the stellar luminosity, the stellar mass, the abundance ratio of carbon to oxygen, the pulsational period and the velocity amplitude of the pulsation. The equation is constructed by applying the maximum-likelihood method to a grid of 48 dynamical models. These models of circumstellar dust shells around carbon-rich long-period variables comprise the calculation of time-dependent hydrodynamics as well as a detailed treatment of dust formation, growth and evaporation.

The main influence on the mass loss rate is given by the stellar temperature, the stellar mass and luminosity. The influence of the carbon abundance, the pulsational period and the amplitude of the piston are almost negligible.

As this mass loss equation is based on physical models it is free of additional arbitrary parameters and could therefore serve as a more reliable description of the mass loss rate e.g. in stellar evolution calculations.

By applying the same method we also constructed two equations which relate the six parameters of dynamical wind models to the averaged outflow velocity and to the averaged dust-to-gas ratio respectively.

Key words: stars: carbon – stars: circumstellar matter – stars: late-type – stars: mass-loss – stars: variables (other)

1. Introduction

Since it became apparent that stars lose considerable amounts of mass in their Asymptotic Giant Branch (AGB) phase (e.g. Deutsch 1956; Gehrz & Woolf 1971; Iben & Renzini 1983; Knapp & Morris 1985; de Jager et al. 1988; Lafon & Berruyer 1991), there were numerous attempts to include the effects of mass loss in models of stellar evolution as well (e.g. Schönberner 1979; Mazzitelli & D'Antona 1986; Iben 1991; Vassiliadis & Wood 1993). Observations of Red Giant Branch (RGB) objects lead Reimers (1975) to an empirical relation for the mass loss rate. This relation introduces a parameter η of order unity. In the early calculations Schönberner (1979) uses a value of η = 1 and concludes that the evolutionary timescale at high luminosities is rather ruled by mass loss than by the growing core mass. However, it turns out that the time scales are very sensitive to the chosen value of η.

Although derived for the RGB, the Reimers relation has also been used for describing the mass loss of AGB objects. However, observations show that AGB stars have very high mass loss rates at the end of their AGB evolution, when the stellar core is stripped from its surrounding material to form a Planetary Nebula. Renzini (1981) coined the term *superwind* for this phase with mass loss rates of order $10^{-5} \ldots 10^{-4} M_\odot \mathrm{yr}^{-1}$. In order to account for these higher AGB mass loss rates a scaling of the Reimers relation with η ranging from 10 to 1000 is introduced (e.g. Iben & Renzini 1983; Mazzitelli & D'Antona 1986).

Bowen & Willson (1991) state that the mass loss rates achieved by the Reimers relation yield a mass loss rate for the entire AGB phase which is increasing too slowly over too long a period of time and reaches too small maximum values of approximately $10^{-6} M_\odot \mathrm{yr}^{-1}$.

The known inadequacy of the Reimers relation leads to alternative relations deduced from observational results to deal with mass loss in stellar evolution calculations (e.g. Volk & Kwok (1988); Vassiliadis & Wood (1993); Blöcker (1995).

Even with the mentioned uncertainties in mind, this kind of description for the mass-losing star nevertheless comprises a first approach for the desired coupling of stellar evolution models with stellar wind models. Substituting the widely used Reimers relation or its descendants (e.g Iben & Renzini 1983; Mazzitelli & D'Antona 1986; Blöcker 1995) by an appropriate equation, that is based on consistent theoretical wind models, would reduce the problems that still obstruct the modeling of the AGB-star evolution.

A main advantage of such an equation is the reduction of the influences introduced by arbitrary parameters like η in the Reimers relation.

The mass loss relation of Dominik et al. (1990) is up to now the only relation that is based on calculated models. This rela-

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tion describes the stationary outflows of carbon-rich low mass stars, but the rather extreme stellar parameters restrict the applicability, if at all, to the very late stages of the AGB evolution.

It is the aim of this paper to derive such a mass loss equation based on the models of Fleischer et al. (1992 (henceforth called Paper I)) and Fleischer (1994). These consistent models of circumstellar dust shells around carbon-rich long-period variables (LPV) include time-dependent hydrodynamics, a detailed treatment of the processes of formation, growth and evaporation of dust grains, chemistry, and radiative transfer.

The paper is organized as follows: Sect. 2 briefly outlines the method used to obtain a relation between the fundamental parameters of the model and its mass loss rate or other dependent quantities. The results of our calculations are shown in Sect. 3. The resulting mass loss relation is presented and discussed in Sect. 3.1. Sect. 3.2 presents the relations for the averaged outflow velocity and the averaged dust-to-gas ratio. In Sect. 3.3 we compare the approximative equations with the results obtained by Höfner & Dorfi (1997). The paper ends with some concluding remarks in Sect. 4.

2. Method

The starting point is a series of independent "observations" of data sets, consisting of several independent variables and one dependent variable. In our case each "observation" corresponds to a calculated wind model which is defined by the four independent fundamental stellar parameters: stellar temperature T, stellar luminosity L, stellar mass M and the chemical abundances ε_i. As we also take into account the stellar pulsation two additional independent parameters arise: the pulsational period P and the velocity amplitude Δu of the pulsation. Since the mass loss rate, the outflow velocity and the dust-to-gas ratio are results of the calculation performed for these model parameters they are of course in each case the dependent variable.

The method to obtain a relationship between the model parameters and one of the dependent parameters has to fulfill several requirements which will keep the possible influence of the observers bias to a minimum:

- The aim is a *simple* equation with the six independent or *primary* input quantities which gives an approximation of the dependent or *secondary* quantity.
- The resulting equation should not introduce more coefficients than the number of primary and secondary quantities entering it.
- The error introduced by the approximation of the secondary quantity should be as small as possible, i.e. the correlation between the secondary quantity and its approximation should be maximal.

The above requirements are met by the widely spread multi-dimensional maximum-likelihood method.

The quality of each resulting approximation is checked by calculating the correlation coefficient between the secondary quantity (e.g. $\dot{M}$) and the result of each approximative equation (e.g. $\dot{M}_{\rm fit}$), with the primary quantities used as input data. The correlation coefficient is a measure for the degree of the relation between two quantities. An optimum correlation is reached if the absolute value of the correlation coefficient equals unity. We choose that approximative equation as best which has the highest correlation coefficient.

Table 1. Range of stellar parameters for the dynamical models

T	[K]	2300	...	3000
L	$[L_\odot]$	5000	...	20000
M	$[M_\odot]$	0.6	...	2.0
$\varepsilon_{\rm C}/\varepsilon_{\rm O}$		1.3	...	2.2
P	[d]	325	...	975
Δu	[km s^{-1}]	1.0	...	5.0

3. Results and discussion

3.1. Approximation of the mass loss rate

The dynamical wind models are presented in detail in Paper I and in Fleischer (1994). They describe spherical symmetric circumstellar dust shells around LPVs and include time-dependent hydrodynamics, radiative transfer, chemistry, and a detailed treatment of the processes of formation, growth and evaporation of dust grains. The dust complex is treated by means of a moment method as described in Gail & Sedlmayr (1988) and in Gauger et al. (1990).

The pulsation of the underlying star is simulated by a sinusoidally moving inner boundary (the "piston"). To specify this piston approximation we have to prescribe the (pulsational) period P and the maximum velocity amplitude Δu. For all models in the grid it is assumed that cooling behind the shocks takes place instantaneously, i.e. the isothermal limit case is considered.

Except for carbon we assume solar abundances for all elements. The carbon abundance $\varepsilon_{\rm C}$ is considered as a free model parameter to assure a carbon-rich composition, and thus, relative to the oxygen abundance $\varepsilon_{\rm C}/\varepsilon_{\rm O}$ is always greater than unity.

The models are completely determined by the prescription of the four fundamental stellar parameter (luminosity, mass, temperature, and elemental abundances) and the two parameter arising from the piston approximation.

Table 1 gives the range for the parameter values covered by the dynamical models and Table 2 gives the whole model grid.

To normalize the model parameters, we take the standard model given by: $L = 10^4\ L_\odot, T = 2600$ K, $M = 1 M_\odot, \varepsilon_{\rm C}/\varepsilon_{\rm O} = 1.8$, $P = 650$ d and $\Delta u = 2.0$ km s^{-1}.

Due to the relatively high computational effort necessary for the dynamical models the model grid up to now consists of only 48 models. These were chosen such that all extreme cases, i.e. those in which the mass loss barely starts, were included. With increasing luminosity, decreasing stellar mass or stellar temperature the mass loss becomes increasingly easier to produce. Insofar a successful high temperature, high mass and low luminosity model (with respect to Table 1) represents a kind of lower limit for the mass loss rate.

Table 2. Grid of models used for determining the approximative equations. Model parameters: T, L, M, $\varepsilon_C/\varepsilon_O$, P and Δu. Model results: mass loss rate $\dot{M}$, averaged outflow velocity v_∞ and dust-to-gas ratio ρ_d/ρ_g. Model number 23 is the standard model.

No.	T [K]	L [$L_\odot$]	M [$M_\odot$]	$\varepsilon_C/\varepsilon_O$	P [d]	Δu [km/s]	$\dot{M}$ [$M_\odot$/yr]	ρ_d/ρ_g [10^{-3}]	v_∞ [km/s]
1	2300	$5.0 \cdot 10^3$	1.0	1.8	650	2.0	$1.2 \cdot 10^{-5}$	4.1	17.5
2	2300	$1.0 \cdot 10^4$	1.0	1.8	650	2.0	$4.2 \cdot 10^{-5}$	4.1	21.4
3	2300	$2.0 \cdot 10^4$	1.0	1.8	650	2.0	$6.6 \cdot 10^{-5}$	3.6	23.6
4	2400	$1.0 \cdot 10^4$	0.8	1.8	650	2.0	$4.5 \cdot 10^{-5}$	3.5	20.2
5	2400	$1.5 \cdot 10^4$	1.2	1.8	650	2.0	$3.1 \cdot 10^{-5}$	4.0	26.5
6	2500	$9.0 \cdot 10^3$	0.9	1.4	388	3.0	$2.3 \cdot 10^{-5}$	1.6	11.7
7	2500	$9.0 \cdot 10^3$	0.9	1.45	388	3.0	$3.2 \cdot 10^{-5}$	2.0	13.9
8	2500	$9.0 \cdot 10^3$	0.9	1.5	388	3.0	$3.8 \cdot 10^{-5}$	2.5	17.9
9	2500	$1.0 \cdot 10^4$	1.0	1.8	650	2.0	$1.5 \cdot 10^{-5}$	3.7	23.7
10	2500	$2.0 \cdot 10^4$	1.2	1.8	650	2.0	$4.2 \cdot 10^{-5}$	3.5	25.5
11	2600	$5.0 \cdot 10^3$	0.8	1.8	650	2.0	$8.2 \cdot 10^{-6}$	3.3	19.0
12	2600	$5.0 \cdot 10^3$	1.0	1.8	650	2.0	$3.2 \cdot 10^{-6}$	4.0	19.8
13	2600	$1.0 \cdot 10^4$	0.6	1.8	650	2.0	$3.6 \cdot 10^{-5}$	4.3	23.2
14	2600	$1.0 \cdot 10^4$	1.0	1.3	650	2.0	$7.4 \cdot 10^{-6}$	1.3	8.8
15	2600	$1.0 \cdot 10^4$	1.0	1.4	650	2.0	$8.5 \cdot 10^{-6}$	1.8	12.3
16	2600	$1.0 \cdot 10^4$	1.0	1.5	650	2.0	$8.9 \cdot 10^{-6}$	2.6	16.5
17	2600	$1.0 \cdot 10^4$	1.0	1.55	650	2.0	$7.6 \cdot 10^{-6}$	1.7	17.0
18	2600	$1.0 \cdot 10^4$	1.0	1.65	650	2.0	$7.5 \cdot 10^{-6}$	3.1	20.5
19	2600	$1.0 \cdot 10^4$	1.0	1.75	650	2.0	$9.3 \cdot 10^{-6}$	3.9	23.1
20	2600	$1.0 \cdot 10^4$	1.0	1.8	325	2.0	$9.0 \cdot 10^{-6}$	3.9	25.1
21	2600	$1.0 \cdot 10^4$	1.0	1.8	450	2.0	$1.7 \cdot 10^{-5}$	3.6	24.9
22	2600	$1.0 \cdot 10^4$	1.0	1.8	650	1.0	$7.3 \cdot 10^{-6}$	3.9	24.6
23	2600	$1.0 \cdot 10^4$	1.0	1.8	650	2.0	$9.4 \cdot 10^{-6}$	3.7	23.5
24	2600	$1.0 \cdot 10^4$	1.0	1.8	650	3.0	$2.1 \cdot 10^{-5}$	3.6	24.5
25	2600	$1.0 \cdot 10^4$	1.0	1.8	975	2.0	$1.5 \cdot 10^{-5}$	4.0	23.9
26	2600	$1.0 \cdot 10^4$	1.0	2.0	650	2.0	$9.6 \cdot 10^{-6}$	4.7	27.6
27	2600	$1.0 \cdot 10^4$	1.0	2.2	650	2.0	$1.1 \cdot 10^{-5}$	6.0	33.0
28	2600	$1.0 \cdot 10^4$	1.5	1.8	650	2.0	$4.4 \cdot 10^{-6}$	3.4	21.7
29	2600	$1.0 \cdot 10^4$	2.0	1.8	650	2.0	$1.2 \cdot 10^{-6}$	3.6	21.3
30	2600	$1.5 \cdot 10^4$	0.8	1.8	650	2.0	$4.5 \cdot 10^{-5}$	3.4	23.5
31	2600	$1.5 \cdot 10^4$	1.0	1.8	650	2.0	$2.9 \cdot 10^{-5}$	3.0	23.7
32	2600	$2.0 \cdot 10^4$	1.0	1.4	650	2.0	$4.9 \cdot 10^{-5}$	2.4	18.2
33	2600	$2.0 \cdot 10^4$	1.0	1.8	650	2.0	$3.0 \cdot 10^{-5}$	4.0	29.7
34	2750	$5.0 \cdot 10^3$	1.0	1.5	350	5.0	$3.1 \cdot 10^{-6}$	1.4	9.2
35	2800	$8.5 \cdot 10^3$	1.0	1.4	400	1.0	$5.3 \cdot 10^{-6}$	1.7	12.3
36	2800	$1.0 \cdot 10^4$	0.8	1.8	650	2.0	$1.0 \cdot 10^{-5}$	3.3	25.3
37	2800	$1.0 \cdot 10^4$	1.0	1.8	650	2.0	$5.2 \cdot 10^{-6}$	3.5	25.8
38	2800	$1.5 \cdot 10^4$	1.2	1.8	650	2.0	$7.5 \cdot 10^{-6}$	2.6	22.3
39	2800	$2.0 \cdot 10^4$	1.0	1.8	650	2.0	$2.0 \cdot 10^{-5}$	2.9	28.0
40	2900	$5.0 \cdot 10^3$	1.0	1.6	400	2.0	$6.2 \cdot 10^{-7}$	2.8	13.7
41	3000	$5.0 \cdot 10^3$	0.6	1.5	350	5.0	$7.7 \cdot 10^{-6}$	2.3	11.7
42	3000	$5.0 \cdot 10^3$	1.0	1.5	350	5.0	$9.5 \cdot 10^{-7}$	1.4	9.8
43	3000	$5.0 \cdot 10^3$	1.0	1.7	350	5.0	$2.2 \cdot 10^{-6}$	3.0	16.4
44	3000	$5.5 \cdot 10^3$	1.3	1.7	350	5.0	$1.5 \cdot 10^{-6}$	2.3	17.8
45	3000	$7.5 \cdot 10^3$	1.0	1.5	350	5.0	$3.6 \cdot 10^{-6}$	1.8	14.5
46	3000	$1.0 \cdot 10^4$	1.0	1.5	350	5.0	$1.7 \cdot 10^{-6}$	1.2	13.3
47	3000	$1.0 \cdot 10^4$	1.0	1.8	650	2.0	$5.0 \cdot 10^{-6}$	2.6	21.8
48	3000	$1.5 \cdot 10^4$	1.0	1.5	350	5.0	$6.7 \cdot 10^{-6}$	1.8	18.5

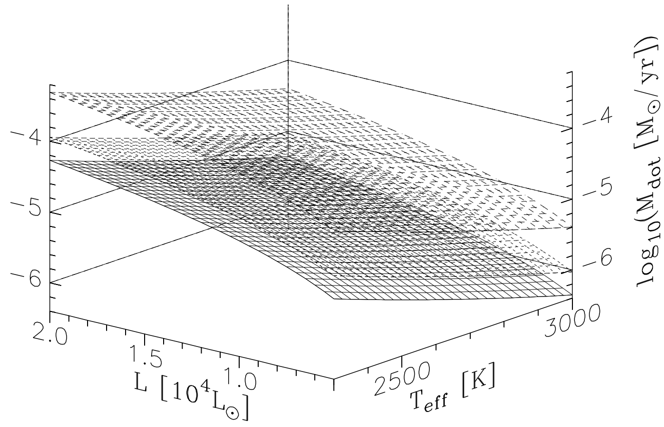

Fig. 1. Representation of Eq. (1) for the LPV mass loss rate as a function of L and T with stellar mass isosurfaces for M = 0.6, 1.0, 1.3 $M_\odot$ (top to bottom). Fixed parameters: $\varepsilon_C/\varepsilon_O = 1.8, P = 650$ d and $\Delta u = 2.0$ km s^{-1}

The models calculated in Paper I can not be a part of our model grid since they do not deal with self-gravitation of the circumstellar material. This can be seen in the different outcome of model E from Paper I, which has the same set of parameters as our standard model (No. 23 in Table 2). We also have to exclude the models specifically designed for IRC +10216 (Winters et al. 1997) and AFGL 3068 (Fleischer et al. 1997), since they are calculated with a constant flux at the inner boundary, i.e. a variable stellar temperature.

The resulting approximative equation for the mass loss rate of dynamical wind models is given in Eq. (1).

$$\begin{aligned}\log \dot{M}_{\rm fit} \;=\; & -4.95 - 9.45 \cdot \log\left(\frac{T[K]}{2600}\right) \\ & +1.65 \cdot \log\left(\frac{L[L_\odot]}{1 \cdot 10^4}\right) - 2.86 \cdot \log\left(M[M_\odot]\right) \\ & +0.470 \cdot \log\left(\frac{\varepsilon_C/\varepsilon_O}{1.8}\right) - 0.146 \cdot \log\left(\frac{P\ [\rm d]}{650}\right) \\ & +0.449 \cdot \log\left(\frac{\Delta u\ [\rm km\ s^{-1}]}{2.0}\right) \end{aligned} \qquad (1)$$

This equation does have a nearly optimal correlation coefficient of 0.954 to the calculated mass loss rate and the mean relative error for $\log \dot{M}_{\rm fit}$ amounts to $\pm 2.4\%$.

The relatively large coefficient for the temperature results from the drop of the mass loss rate of slightly more than one order of magnitude in the small temperature range between 2300 K and 3000 K. This gives the stellar temperature the strongest influence on the mass loss rate.

Fig. 1 depicts the three-dimensional representation of Eq. (1) in the (T, L, M)-range. The topmost surface in Fig. 1 corresponds to the smallest stellar mass ($0.6 M_\odot$). With increasing stellar mass the mass loss rate is reduced.

It should be noted that Fig. 1 depicts a partially idealized view: for one solar mass the surface shown is completely based on calculated models. For masses below $1 M_\odot$ areas with temperatures over 2600 K and luminosities below $1.5 \cdot 10^4 L_\odot$ are covered by calculated models. Since a pulsationally enhanced dust driven wind becomes more effective for lower stellar masses (as well as with increasing luminosity and decreasing temperature) it is allowed to extend the description by Eq. (1) over the whole area of the (T, L)-range shown in Fig. 1 for masses below one solar mass. For a mass of $1.3 M_\odot$ we were able to obtain a model with T = 3000K and $L = 5 \cdot 10^3 L_\odot$ which can be seen as a lower limiting case. From there on it gets easier to obtain models, losing mass by a pulsationally enhanced dust driven wind, with lower stellar temperatures and higher luminosities. Therefore it is allowed to extend the lowest isosurface in Fig. 1 from its bottom corner over the whole (T, L)-range in Fig. 1.

As the model grid does not represent a good basis for masses over $1.5 M_\odot$, we restrict the valid range for Eq. (1) to masses below $\approx 1.8 M_\odot$. We are aware that there may exist carbon stars on the AGB with masses above the upper mass limit considered in our computations of carbon-rich Mira-stars, as suggested by the initial masses given by Barnbaum et al. (1991).

In the scope of the current models the influence of the parameters $\varepsilon_C/\varepsilon_O$, P and Δu proves to be negligible compared to that of T, L and M. Of course the abundance ratio still has to exceed $\varepsilon_C/\varepsilon_O \approx 1.2$ to assure a sufficient amount of condensible carbon-rich material.

At least the pulsational amplitude Δu has a small influence which takes the fourth place behind T, L and M. The mass loss rate increases together with the amplitude. The abundance ratio has only a minor influence on the mass loss rate. An increasing abundance ratio leads to an insignificant increase of the mass loss rate. From all parameters the pulsational period has the smallest influence on the mass loss rate. An increasing period reduces the mass loss rate but does so only in an insignificant way.

Considering the weak dependence on $\Delta u, \varepsilon_C/\varepsilon_O$ and P a reduced mass loss equation can be derived which is based only on temperature, luminosity and mass, and which might be more convenient for practical purposes. The reduced equation has a correlation coefficient of 0.943 and the mean relative error for the reduced equation amounts to a factor of $\pm 2.6\%$ in $\log \dot{M}_{\rm fit}$.

$$\begin{aligned}\log \dot{M}_{\rm fit} \;=\; & -4.93 - 8.26 \cdot \log\left(\frac{T[K]}{2600}\right) \\ & +1.53 \cdot \log\left(\frac{L[L_\odot]}{1 \cdot 10^4}\right) - 2.88 \cdot \log\left(M[M_\odot]\right) \end{aligned} \qquad (2)$$

3.2. Approximative equations for ρ_d/ρ_g and v_∞

Applying the maximum-likelihood method to the dynamical models within the range of Table 1 also allows to give approximative equations for the averaged dust-to-gas ratio ρ_d/ρ_g as well as the averaged outflow velocity v_∞ as a function of the fundamental stellar parameters.

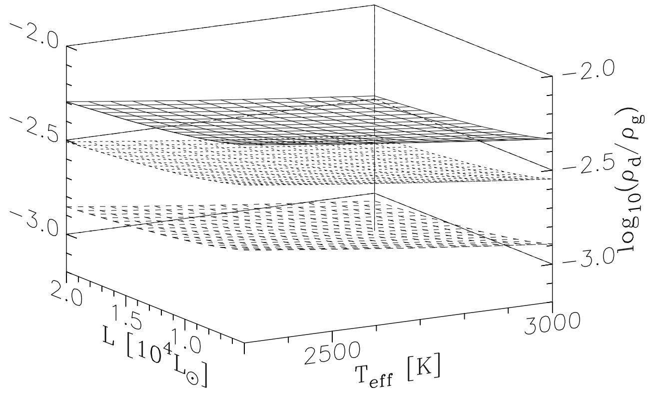

Fig. 2. Representation of Eq. (3) for the dust-to-gas ratio as a function of L and T with abundance ratio isosurfaces for $\varepsilon_C/\varepsilon_O$ = 2.2, 1.8, 1.3 (top to bottom) Fixed parameters: M = 1.0 $M_\odot$, P = 650 d and Δu = 2.0 km s^{-1}

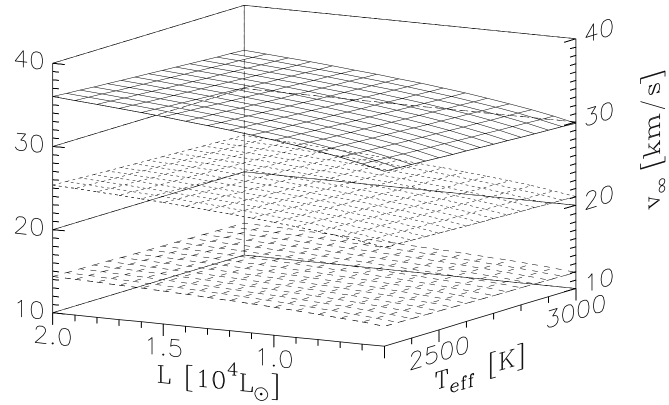

Fig. 3. Representation of Eq. (5) for the averaged outflow velocity as a function of L and T with abundance ratio isosurfaces for $\varepsilon_C/\varepsilon_O$ = 2.2, 1.8, 1.3 (top to bottom) Fixed parameters: M = 1.0 $M_\odot$, P = 650 d and Δu = 2.0 km s^{-1}

The resulting approximative equation for the averaged dust-to-gas ratio is given by Eq. (3), which exhibits a very good degree of correlation that amounts to 0.972 and a mean relative error of $\pm1.7\%$ in $\log(\rho_d/\rho_g)_{\rm fit}$.

$$\begin{aligned}\log\left(\frac{\rho_d}{\rho_g}\right)_{\rm fit} &= 2.74 - 1.62 \cdot \log T[K] \\ &-0.248 \cdot \log L[L_\odot] - 0.658 \cdot \log M[M_\odot] \\ &+2.45 \cdot \log\left(\varepsilon_C/\varepsilon_O\right) + 0.230 \cdot \log P \text{ [d]} \\ &-4.93\,10^{-3} \cdot \log \Delta u \text{ [km s}^{-1}\text{]}\end{aligned} \quad (3)$$

As shown in Fig. 2 the abundance ratio has the strongest influence in Eq. (3). An increasing carbon abundance (relative to oxygen) leads to a strong increase in the dust-to-gas ratio. The abundance ratio is closely followed by the mass and the temperature and to a lesser extent by the luminosity. Increasing values in M, T and L lead to a decrease of the dust-to-gas ratio. It is possible to obtain a reduced approximation since both, the pulsational period and the piston amplitude have only a minor influence.

$$\begin{aligned}\log\left(\frac{\rho_d}{\rho_g}\right)_{\rm fit} &= 3.56 - 1.69 \cdot \log T[K] \\ &-0.242 \cdot \log L[L_\odot] - 0.669 \cdot \log M[M_\odot] \\ &+2.46 \cdot \log\left(\varepsilon_C/\varepsilon_O\right)\end{aligned} \quad (4)$$

The reduced Eq. (4) results in a smaller degree of correlation of 0.968, but the mean relative error in $\log(\rho_d/\rho_g)_{\rm fit}$ remains unchanged at $\pm1.7\%$.

In the case of the outflow velocity of the dynamical models the approximation is given by:

$$\begin{aligned}\log(v_{\infty,\rm fit}) &= 0.730 - 0.148 \cdot \log T[K] \\ &+0.107 \cdot \log L[L_\odot] - 6.40\,10^{-2} \cdot \log M[M_\odot] \\ &+1.74 \cdot \log\left(\varepsilon_C/\varepsilon_O\right) + 9.56\,10^{-2} \cdot \log P \text{ [d]} \\ &-7.07\,10^{-4} \cdot \log \Delta u \text{ [km s}^{-1}\text{]}\end{aligned} \quad (5)$$

The correlation coefficient of this equation to the calculated outflow velocity is 0.956 and the mean relative error for $\log(v_{\infty,\rm fit})$ amounts to $\pm2.3\%$.

Additionally, Eq. (5) is limited to values above $v_{\infty,\rm fit} \approx$ 10 km s^{-1}, because only one model with a velocity significantly below this limit is part of the model grid.

Fig. 3 proves the major role of the abundance ratio in Eq. (5). The small variation given for $\varepsilon_C/\varepsilon_O$ by Table 1 dominates the approximation of the outflow velocity, which increases with increasing abundance ratio or luminosity and decreasing temperature. The mass, the pulsational period and the piston amplitude prove to be of minor significance for Eq. (5). This leads to the reduced Eq. (6) which yields a nearly equally good correlation coefficient of 0.942 and a mean relative error for $\log(v_{\infty,\rm fit})$ of $\pm2.8\%$.

$$\begin{aligned}\log(v_{\infty,\rm fit}) &= 1.36 - 0.293 \cdot \log T[K] \\ &+0.132 \cdot \log L[L_\odot] + 1.84 \cdot \log\left(\varepsilon_C/\varepsilon_O\right)\end{aligned} \quad (6)$$

We stress that Eqs. (3) to (6) are only valid within the parameter range given by Table 1.

3.3. Comparison with other authors

Höfner & Dorfi (1997) present a set of 22 carbon-rich LPV models. This sample allows to check at least the applicability if not even the accuracy of the presented approximative equations.

Their set of models has to be reduced by excluding the models R5, R5C, R5P and R7C18 because they do not fall inside the limits of Table 1. Additionally, model R10C15 and R13 have to be excluded because of an outflow velocity below the limit of Eq. (5) or Eq. (6), respectively.

Table 3. Comparison of the approximative equations with the results of Höfner & Dorfi (1997)

Param.	Eq.	r	$\bar{e}$ [%]	e_+ [%]	e_- [%]
$\log(\dot{M})$	(1)	0.970	5.0	+11.6	-2.8
	(2)	0.917	5.2	+14.3	-3.4
$\log(\rho_d/\rho_g)$	(3)	0.952	2.4	+4.5	-3.0
	(4)	0.943	2.7	+5.3	-3.7
$\log(v_\infty)$	(5)	0.901	3.3	+9.4	-4.1
	(6)	0.897	3.8	+8.7	-5.5

Table 3 shows the results of our comparison. The first column gives the approximated parameter and the equation it results from. r is the correlation coefficient between the sample of Höfner & Dorfi (1997) and our approximation thereof. In this context $\bar{e}$ denotes the mean relative error made by the approximation and e_+ is the maximal over- and e_- the maximal underestimation the approximation yields.

The results of Höfner & Dorfi (1997) prove to be well approximated by the whole set of our equations. The mass loss rate and the averaged dust-to-gas ratio can be described with high confidence by the full equations. Even the worst case still yields a degree of correlation around ≈ 0.9. Of course the degree of correlation decreases and the mean error increases with the use of the reduced Eqs. (2), (4) and (6) compared to the full Eqs. (1), (3) and (5).

Eq. (1) proves capable of describing the mass loss rate of wind models obtained by a different modeling method (for a comparison see Höfner et al. 1996) within a factor of two, on average $\dot{M}_{\rm fit}$ underestimates $\dot{M}$ by a factor of 1.84.

Since the Reimers relation even fails to give the correct order of magnitude for the mass loss rate, we consider our approximative equation to be a major improvement for describing the mass loss rate of AGB stars in stellar evolution calculations.

4. Conclusions

An equation is derived which relates the six parameters of dynamical wind models to the resulting mass loss rate. These six parameters are the stellar temperature, the stellar luminosity, the stellar mass, the abundance ratio of carbon to oxygen, the pulsational period and the amplitude of the pulsation. This mass loss equation is constructed by applying the maximum-likelihood method to a grid of 48 dynamical models.

It turns out that the main influence on the mass loss rate is given by the stellar temperature followed by the stellar mass and luminosity whereas the influence of the the amplitude of the piston, the carbon overabundance and the pulsational period are almost negligible.

Similar approximative equations are derived for the averaged outflow velocity and the averaged dust-to-gas ratio. With an extended model grid we would expect minor changes in the values of the coefficients of the approximative equations but the overall picture and the general dependences should not change significantly.

As the deduced equations are based on physical models they are free of arbitrary parameters and could therefore serve as a more reliable description of e.g. the mass loss rate in stellar evolution calculations. A first application to evolutionary calculations and the comparison of the mass loss equation presented here with mass loss rates widely used in this field of research can be found in Wagenhuber (1996).

As soon as comparable oxygen-rich models become available it would of course be desirable to obtain similar equations for the oxygen-rich case as well in order to cover a broader range of the evolutionary cycle of the stars.

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