Astron. Astrophys. 328, 253–260 (1997)

ASTRONOMY AND ASTROPHYSICS

Do T Tauri stars rotate differentially?

M. Küker[1] **and G. Rüdiger**[1]

Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany

Received 14 April 1997 / Accepted 29 July 1997

Abstract. We compute the rotation profile of a T Tauri star from the balance between the different components of the Reynolds stress and the meridional circulation. We first use a simple model to study the influence of the meridional flow on the differential rotation and vice versa. A more realistic model is then presented, which includes the stratification of the turbulence derived from a stellar model. We find that for a strictly spherical star without any latitudinal gradients in temperature, density and pressure the rotation is very close to the rigid-body state. We conclude that the stellar magnetic field must be generated by an α^2- rather than an $\alpha\Omega$-dynamo. It must thus have non-axisymmetric geometry and will not show cyclic behaviour.

Key words: hydrodynamics – turbulence – stars: magnetic fields – stars: pre-main sequence – stars: rotation

1. Introduction

One of the key problems in the theory of star formation is how the star can be formed from a rotating cloud without accreting the angular momentum together with the matter, since otherwise the star would have to spin up more and more, finally reaching the breakup velocity and stopping the process of star formation. There must, therefore, be a mechanism that carries angular momentum away from the star without significantly reducing the stellar mass.

It is now widely accepted, that magnetic fields play an important role in extracting angular momentum from the star and thus braking its rotation (cf. Bodenheimer 1995). Two mechanisms are known. First, a wind coupled to the star by the stellar magnetic field can exert a strong torque due to the large radius of the magnetosphere. Second, the magnetic field can thread a circumstellar accretion disk, disrupting the disk inside the corotation radius and leaving those parts of the disk that rotate slower than the star. It will then accelerate the rotation of the disk and brake the star, fixing its rotation rate at certain equilibrium value (Ghosh & Lamb 1979a,b, Camenzind 1990, Königl 1991, Cameron & Campbell 1993, Yi 1994, Cameron et al. 1995, Ghosh 1995, Lovelace et al. 1995, Armitage & Clarke 1996).

Send offprint requests to: M. Küker

There is some evidence that the latter braking mechanism is indeed at work in classical T Tauri stars (CTTS). Stars of this type are surrounded by by disks, while weak line T Tauri stars (WTTS) are not (cf. Bertout 1989). The rotation rates of CTTS are found to be considerably smaller than those of WTTS, in agreement with the assumption that disks brake the stellar rotation (Bouvier et al. 1993, Edwards et al. 1993). The present model of the pre-main sequence (PMS) evolution of stellar rotation includes both braking by disks and winds. During the T Tauri phase, the star is prevented from spinning up as long as the disk exists. The star begins to spin up when the disk disappears, finally being spun down again by the wind (Bouvier 1994).

In any case the geometry as well as the strength of the magnetic field are decisive for the efficiency of the braking process. Since pre-main sequence stars are fully convective, the decay time of a fossil field would be some years only and thus a dynamo process is necessary to generate the stellar magnetic field. The field structure depends on the type of dynamo, especially the relative magnitude of the two field generating processes, namely the rotational shear and the mean helicity (α-effect) of the convective motions. In the solar dynamo the shear dominates and the resulting magnetic field is axisymmetric and has dipole geometry. It is, however, not clear at all if stellar magnetic fields are always generated by $\alpha\Omega$-dynamos. Observations show that for lower main sequence stars the normalized surface differential rotation,

$$\delta\Omega = \frac{\Omega_{\text{equator}} - \Omega_{\text{pole}}}{\Omega_{\text{equator}}}, \tag{1}$$

decreases with increasing rotation rate (Hall 1991, Donahue et al. 1996), a behaviour also found from theoretical work by Rüdiger et al. (1997). Johns-Krull (1996) derived the differential rotation of two CTTS and one WTTS from the profiles of photospheric absorption lines. He found that the data are consistent with polar acceleration or rigid rotation but not with equatorial acceleration. Doppler imaging of the WTTS V410 Tauri shows a non-axisymmetric distribution of stellar spots and that differential rotation, if present at all, must be very small and solar-type (Rice & Strassmeier 1996). We must therefore expect a rotational shear much smaller than in the Sun, probably with opposite sign.

2. The model

In a stellar convection zone or a fully convective star the turbulent convective motions cause an additional stress on the mean (global) motion known as Reynolds stress. While this stress can be described as an additional viscosity in case of a non-rotating convection zone this is no longer correct as soon as the rotation period becomes comparable with or smaller than the convective turnover time. In that case a non-viscous contribution, the Λ-effect, arises that forces differential rotation.

In Küker et al. (1993), Reynolds stress was considered as the only transporter of angular momentum in the solar convection zone. This model yields a rotation pattern that agrees almost perfectly with the observations from helioseismology and thus confirms the theory of the Reynolds stress as well as the assumption, that the meridional flow is of minor importance for the problem of solar differential rotation. The latter assumption can, however, not be correct for stars in general since a model based on Reynolds stress alone always yields a normalized differential rotation that increases with increasing rotation rate, in contradiction to the observations mentioned above.

We therefore present a model of a rapidly rotating fully convective pre-main sequence star in which Reynolds stress and meridional circulation are treated consistently, i.e. we solve the full Reynolds equation,

$$\rho \left[\frac{\partial \bar{\boldsymbol{u}}}{\partial t} + (\bar{\boldsymbol{u}} \cdot \nabla)\bar{\boldsymbol{u}}\right] = -\nabla \cdot (\rho Q) - \nabla \bar{p} + \rho \boldsymbol{g} + \nabla \cdot \pi, \tag{2}$$

rather than only its azimuthal component. In (2), $Q_{ij} = \langle u'_i u'_j \rangle$ is the correlation tensor of the fluctuating part $\boldsymbol{u}'$ of the velocity field while $\bar{\boldsymbol{u}}$ denotes the mean velocity. The molecular stress tensor π can be neglected since it is many orders of magnitude smaller than the Reynolds stress.

We only treat the axisymmetric case. The velocity field can then be separated into a global rotation and the meridional flow:

$$\bar{\boldsymbol{u}} = r \sin\theta\, \Omega\, \hat{\boldsymbol{\phi}} + \bar{\boldsymbol{u}}^{\rm m}, \tag{3}$$

where $\hat{\boldsymbol{\phi}}$ is the unit vector in the azimuthal direction. The azimuthal component of the Reynolds equation reads,

$$\frac{\partial \rho r^2 \sin^2\theta\, \Omega}{\partial t} + \nabla \cdot \boldsymbol{t} = 0, \tag{4}$$

where

$$\boldsymbol{t} = r \sin\theta \left[\rho r \sin\theta \Omega \bar{\boldsymbol{u}}^{\rm m} + \rho \langle u'_\phi \boldsymbol{u}' \rangle\right]. \tag{5}$$

The meridional circulation can be expressed by a stream function A,

$$\bar{\boldsymbol{u}}^{\rm m} = \left(\frac{1}{\rho r^2 \sin\theta}\frac{\partial A}{\partial \theta}, \frac{1}{\rho r \sin\theta}\frac{\partial A}{\partial r}, 0\right). \tag{6}$$

An equation for the stream function is obtained by taking the azimuthal component of the *curl* of (2):

$$\frac{\partial \omega}{\partial t} = - \left[\nabla \times \frac{1}{\rho}\nabla(\rho Q)\right]_\phi + r \sin\theta \frac{\partial \Omega^2}{\partial z} + \frac{1}{\rho^2}(\nabla\rho \times \nabla p)_\phi, \tag{7}$$

where $\omega = (\nabla \times \bar{\boldsymbol{u}})_\phi$ is the curl of the meridional flow velocity and $\partial/\partial z = \cos\theta \cdot \partial/\partial r - \sin\theta/r \cdot \partial/\partial\theta$ is the gradient along the axis of rotation. In (7), we have omitted all nonlinear terms except the one including Ω^2. Since the code described below allows the inclusion of nonlinear terms we made some test calculations including the full advective term, but the results did not change significantly. We use the anelastic approximation,

$$\nabla \cdot (\rho \bar{\boldsymbol{u}}) = 0, \tag{8}$$

i.e. the density is constant with time but varies with depth. The stream function and the vorticity ω are related via the equation

$$DA - \frac{1}{\rho}\frac{\partial \rho}{\partial r}\frac{\partial A}{\partial r} - \frac{1}{\rho r}\frac{\partial \rho}{\partial r}A = -\rho\omega, \tag{9}$$

where $D = \Delta - 1/(r^2 \sin^2\theta)$. Eq. (9) can be reduced to a set of ordinary differential equations by an expansion in terms of Legendre functions $P_l^1(\cos\theta)$, which are eigenfuntions of the latitudinal part of D.

In the correlation tensor Q, a viscous and a non-viscous part can be distinguished:

$$Q_{ij} = -\mathcal{N}_{ijkl}\frac{\partial \bar{u}_k}{\partial x_l} + \Lambda_{ijk}\Omega_k, \tag{10}$$

where $\boldsymbol{\Omega} = \Omega\, \hat{\boldsymbol{z}}$. The viscous part,

$$Q^\nu_{ij} = -\mathcal{N}_{ijkl}\frac{\partial \bar{u}_k}{\partial x_l}, \tag{11}$$

is given by

$$\begin{aligned} \mathcal{N}_{ijkl} &= \nu_1(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) \\ &+ \nu_2\left(\delta_{il}\frac{\Omega_j\Omega_k}{\Omega^2} + \delta_{jl}\frac{\Omega_i\Omega_k}{\Omega^2} + \delta_{ik}\frac{\Omega_j\Omega_l}{\Omega^2}\right. \\ &\left. + \delta_{jk}\frac{\Omega_i\Omega_l}{\Omega^2} + \delta_{kl}\frac{\Omega_i\Omega_j}{\Omega^2}\right) \\ &+ \nu_3\delta_{ij}\delta_{kl} - \nu_4\delta_{ij}\frac{\Omega_k\Omega_l}{\Omega^2} + \nu_5\frac{\Omega_i\Omega_j\Omega_k\Omega_l}{\Omega^4} \end{aligned} \tag{12}$$

(Kitchatinov et al. 1994). The viscosity coefficients,

$$\nu_n = c_\nu \tau_{\rm corr}\langle \boldsymbol{u}'^2\rangle \phi_n(\Omega^*), \qquad n = 1, \dots, 5 \tag{13}$$

depend on the angular velocity as well as on the convective turnover time, $\tau_{\rm corr}$, via the Coriolis number, $\Omega^* = 2\tau_{\rm corr}\Omega$. In the limiting case of very slow rotation, $\Omega^* \ll 1$, the viscous stress becomes isotropic and reduces to the well known expression,

$$Q^\nu_{ij} = -\nu_{\rm T}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) + \zeta_T \frac{\partial \bar{u}_k}{\partial x_k}\delta_{ij}. \tag{14}$$

The second contribution, the Λ-effect, is the source of differential rotation. In spherical polar coordinates, it is only present in the components $Q_{r\phi}$ and $Q_{\theta\phi}$:

$$Q^\Lambda_{r\phi} = \nu_{\rm T}(V^{(0)} + V^{(1)}\sin^2\theta)\sin\theta\Omega, \tag{15}$$

$$Q^\Lambda_{\theta\phi} = \nu_{\rm T} H^{(1)}\sin^2\theta\cos\theta\Omega. \tag{16}$$

The functions ϕ_n have been derived in Kitchatinov et al. (1994) and those for $V^{(0)}$, $V^{(1)}$, and $H^{(1)}$ can be found in Küker et al. (1993).

We take the stratifications of density, temperature and luminosity from a model of a fully convective PMS star by Palla & Stahler (1993) and calculate the rms value of the turbulent convective velocity from mixing-length theory. The star has 1.5 solar masses and is at the beginning of his contraction phase, with a radius of 4.6 solar radii. Both density and pressure are assumed to be functions of the fractional stellar radius only and their gradients are thus aligned. As a consequence, the last term in (7) vanishes and the rotation pattern is determined by the balance between the meridional flow and the Reynolds stress.

Standard mixing-length theory does not include the rotational influence on the turbulent heat transport. We therefore use the eddy heat flux derived by Kitchatinov et al. (1994),

$$F_i = C_p \rho \chi_{ij} \nabla_j \Delta T, \tag{17}$$

where

$$\chi_{ij} = \chi_{\rm T} \left(\Phi(\Omega^*)\delta_{ij} + \Phi_{\|}(\Omega^*) \frac{\Omega_i \Omega_j}{\Omega^2} \right) \tag{18}$$

is the turbulent heat conductivity tensor, $\nabla \Delta T = \nabla T - \nabla T_{\rm ad}$ the deviation from adiabatic stratification, and $\Omega^* = 2\tau_{\rm corr}\Omega$ denotes the Coriolis number. For slow rotation, i.e. $\Omega^* \ll 1$, the heat conductivity reduces to the scalar quantity

$$\chi_{\rm T} = \frac{1}{3} \tau_{\rm corr} u_{\rm T}^2. \tag{19}$$

For arbitrary rotation rates, the functions Φ and $\Phi_{\|}$ give the rotational influence on the isotropic and anisotropic part of the heat conductivity, respectively. $\tau_{\rm corr}$ is the convective turnover time. We are only interested in the magnitude of the heat transport and do not regard any anisotropy. We further replace Φ with its limit for large Coriolis numbers,

$$\Phi(\Omega^*) \to \frac{3\pi}{8\Omega^*}. \tag{20}$$

With these approximations the radial convective heat flux is

$$\boldsymbol{F} = C_p \rho \chi_{\rm T} \frac{3\pi}{8\Omega^*} \nabla \Delta T. \tag{21}$$

For fully ionized matter and the composition of a standard solar model the specific heat capacity C_p is approximately given by

$$C_p = 3.4 \cdot 10^8 \frac{\rm cm^2}{\rm s^2 K}. \tag{22}$$

The superadiabatic temperature gradient $\nabla \Delta T$ is eliminated using the standard mixing-length relation

$$u_{\rm T}^2 = \langle u'^2 \rangle = \frac{l_{\rm corr}^2 g}{4T} |\nabla \Delta T|. \tag{23}$$

We equate the convective heat flux and the stellar luminosity,

$$F = \frac{L}{4\pi r^2} = C_p \rho \chi_{\rm T} u_T^2 \frac{4T}{g l_{\rm corr}^2} \frac{3\pi}{8\Omega^*}, \tag{24}$$

yielding

$$u_{\rm T}^4 = \frac{L g l_{\rm corr}^2 \Omega}{\pi^2 r^2 \rho C_p T}. \tag{25}$$

We choose $\alpha_{\rm MLT} = \frac{5}{3} \Rightarrow l_{\rm corr} = H_\rho$ for a stratification very close to the adiabatic case, and use the relation

$$\tau_{\rm corr} = \frac{l_{\rm corr}}{u_{\rm T}} \tag{26}$$

for the convective turnover time.

To solve Eqs. (4) and (7), we use a time-dependent code based on a finite difference method in both space and time. Every time step includes the solution of a second order partial differential equation to compute the stream function A from ω. The domain of the computation should principally be the whole star, but we must exclude a small volume around the center for numerical reasons. We take a value of 0.1 for the fractional radius of this artificial inner boundary. As boundary conditions, we require that both the radial component of the meridional motion and $Q_{r\phi}$ vanish, i.e. neither matter nor angular momentum may be transported across the boundaries.

The numerical scheme used to solve Eqs. (4) and (7) is an explicit finite difference scheme with a staggered grid. The main advantage of this scheme is that it is relatively easy to implement a complicated tensor structure for Q and to ensure the conservation of angular momentum. The ordinary differential equations resulting from the expansion of Eq. (9) in terms of Legendre functions are solved by applying a standard second order finite difference scheme and solving the resulting tridiagonal linear equations. The results for the T Tauri model were tested for sensitivity to changes in the resolution of the grid. The necessary number of grid points turned out to be determined by the requirement of stability rather than accuracy, especially in case of large Taylor numbers. Once the resolution is sufficient for stability, further increase does not change the result significantly. We use a non-uniform distribution of the radial grid points with enhanced resolution close to the boundaries and a uniform distribution of latitudinal grid points. A grid of (100×80) points is appropriate for Ta = 10^8.

3. Results

We use rigid-body rotation and vanishing meridional flow as initial values. Due to the Λ-effect, it is not a possible steady state. Consequently, the Reynolds stress starts immediately to redistribute the angular momentum, thus forcing differential rotation which in turn gives rise to the meridional flow. The meridional flow counteracts the generation of the differential rotation and finally the whole system approaches an equilibrium. Note that as long as there is no further force, the meridional flow is driven by the differential rotation and not vice versa. It thus always weakens the differential rotation, driving the whole system towards rigid-body rotation.

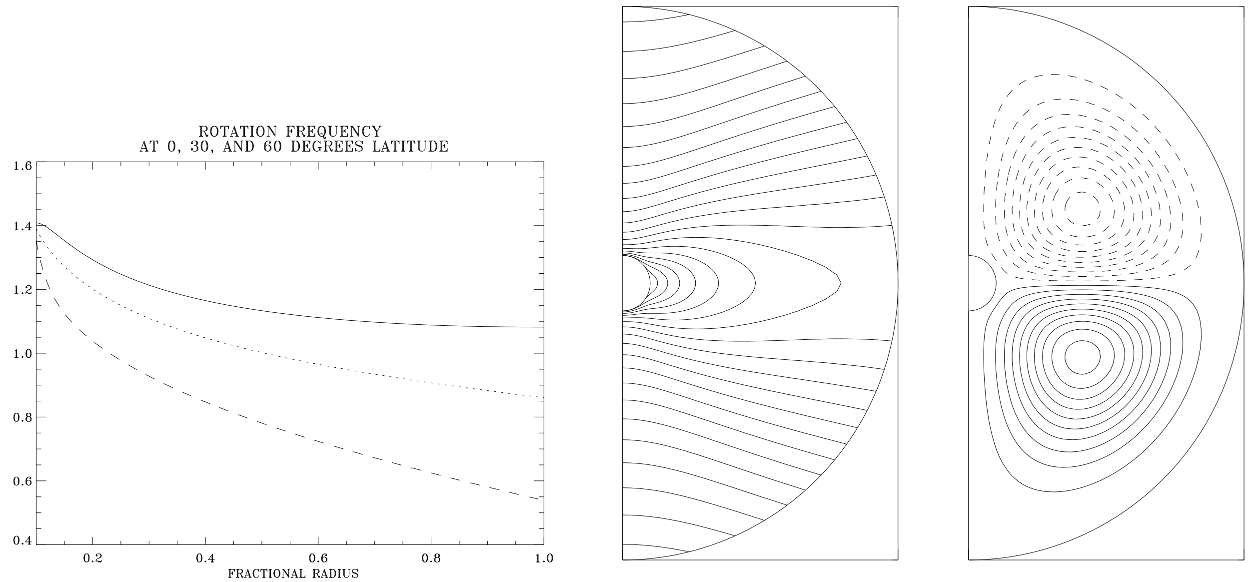

Fig. 1. The rotation pattern and meridional motion of a T Tauri star with Ta = 10 from the simplified model. Left: The radial variation of the normalized angular velocity for the equator (solid line), 30° latitude (dotted line) and 60° latitude (dashed line). Middle: Isocontour plot of the angular velocity. Right: Isocontour plot of the stream function. The dashed lines refer to negative values, i.e. counterclockwise circulation.

3.1. Simplified model

To illustrate this mechanism, we first study a simplified model. The viscosity coefficient $\nu_{\rm T}$ as well as the Λ- effect are constant throughout the whole star. The simple form (14) is assumed for the viscous part of the stress tensor. We choose $V^{(0)} = -1.0$ and $V^{(1)} = H^{(1)} = 1.0$, which is a good approximation for the case of rapid rotation, $\Omega^* \gg 1$, when $V^{(0)} + V^{(1)} = 0$. We measure the speed of the rotation by its Taylor number,

$$\mathrm{Ta} = \frac{4\Omega^2 R_*^4}{\nu_{\rm T}^2}, \qquad (27)$$

which gives the strength of the centrifugal force relative to the Reynolds stress.

Fig. 1 gives the result for Ta=10, a case where the rotation pattern is determined by the Λ-effect alone and not significantly influenced by the meridional flow. The equator rotates twice as fast as the high latitude and the radial shear is negative throughout the whole star. There is one cell of meridional circulation per hemisphere, the surface flow being directed towards the poles.

In Fig. 2, the result for an intermediate Taylor number of 10^4 is given. Although the Λ-effect still dominates, the pattern has changed significantly and the total latitudinal shear is only 20 percent of the equatorial angular velocity rather than 50 percent. The radial variation of Ω is essentially unchanged close to the boundaries, but at a depth of half the stellar radius there is no radial shear any more. The meridional flow pattern has not changed.

For the large Taylor number of 10^7, as shown in Fig. 3, the total horizontal shear at the stellar surface has been further reduced to about 2 percent. There is no radial shear in the interval [0.4,0.9] of the fractional stellar radius x. Outside a cylindrical surface touching the (artificial) inner core at the equator, the shear is now restricted to a layer of thickness 0.1 close to the upper and lower boundary, respectively. The deviation from rigid-body rotation is less than 4 percent close to the lower and up to two percent at the upper boundary.

For fixed angular velocity increasing the Taylor number means decreasing the viscosity. Fig. 4 shows the maximum value of the latitudinal velocity component as a function of the Taylor number for this case. It increases with Ta for small Taylor numbers, reaches a maximum between Ta = 10^3 and Ta = 10^4, and *decreases* as Ta is increased for Ta $> 10^4$. The shape of the flow pattern, represented by the stream function, changes, however, only slightly. There is always only one flow cell per hemisphere and the surface flow is always directed towards the poles.

3.2. Full stratification

The complete model includes the stratification of the turbulence. The convective turnover time as well as the convective velocity vary strongly with the fractional stellar radius, leading to a depth dependence of the Coriolis number. In Fig. 5 the Coriolis number is plotted vs. the fractional stellar radius for an angular velocity $\Omega = 10^{-5}\mathrm{s}^{-1}$ and three different values of the mixing-length parameter $\alpha_{\rm MLT}$. In our calculations we always assumed $\alpha_{\rm MLT} = 5/3$, which, for fully ionized gas and a stratification that deviates only slightly from the adiabatic case, means that the mixing length is equal to the density scale height. The cases $\alpha_{\rm MLT} = 1$ and $\alpha_{\rm MLT} = 2.4$ are shown for comparison. Fig. 5 shows that, although the exact value of the Coriolis number increases by a factor of 1.6 as $\alpha_{\rm MLT}$ is increased from 1.0 to 2.4, it

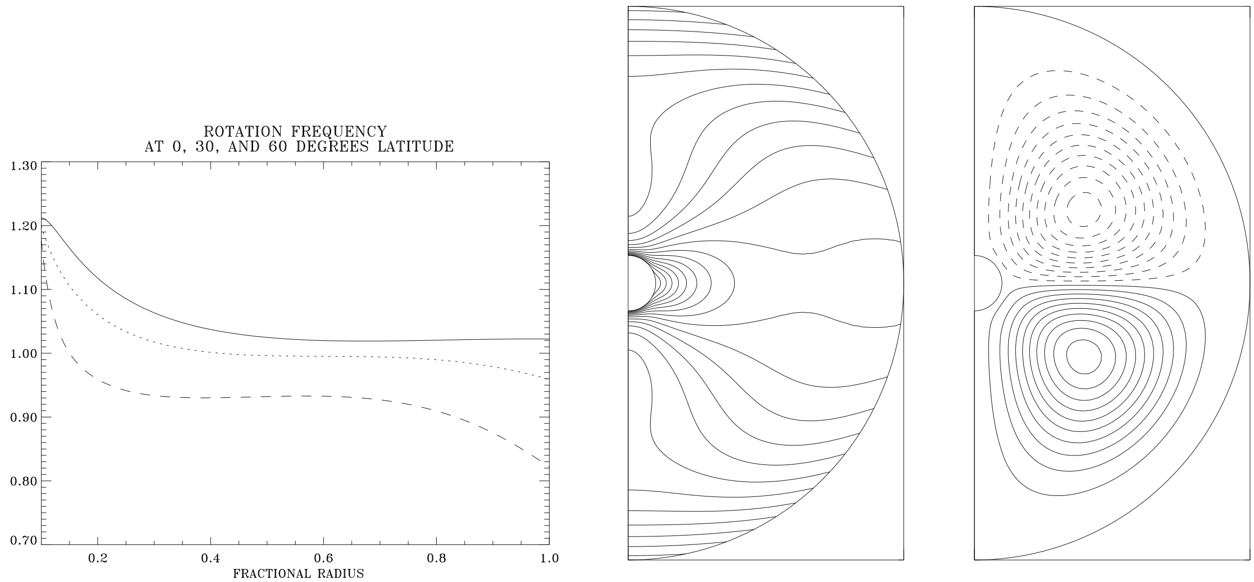

Fig. 2. Same as Fig. 1, but Ta = 10^4

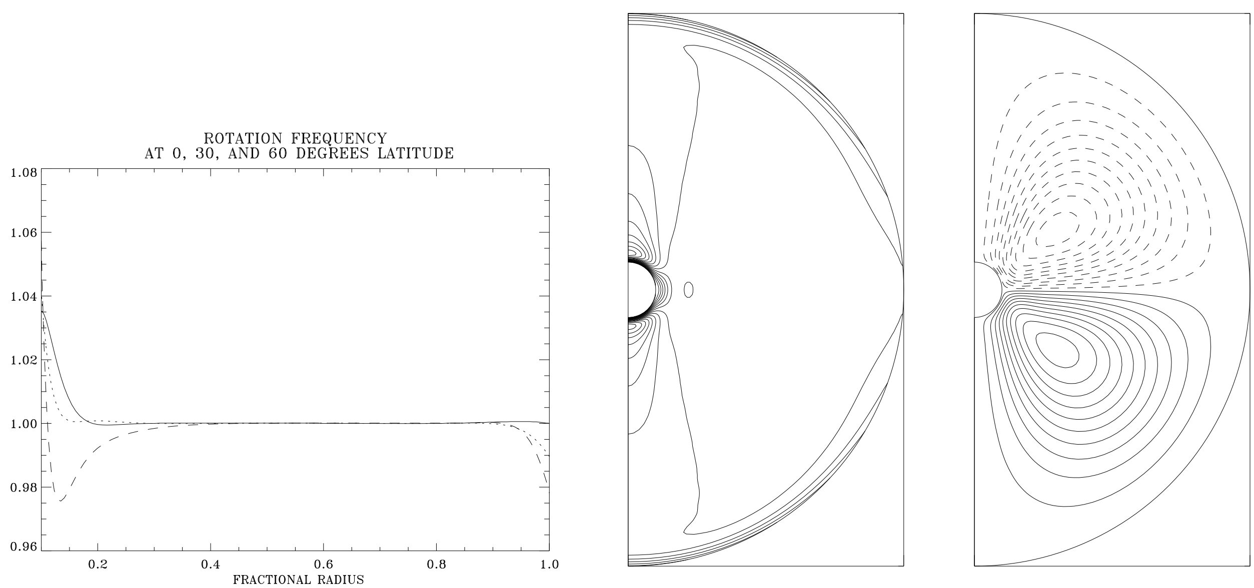

Fig. 3. Same as Fig. 1, but Ta = 10^7

is in all cases a number between 10 and 100 almost throughout the whole stellar volume.

The Reynolds stress is thus strongly reduced, especially in the inner part of the star. In a model without meridional circulation this would, however, not affect the resulting rotation law, since the diffusive and non-diffusive part vary in the same way in the case of large Coriolis numbers . In the case considered here, the balance between the centrifugal force and the turbulent viscosity is strongly affected. The turbulent viscosity of the non-rotating star,

$$\nu_{\rm T} = c_\nu \tau_{\rm corr} \langle u'^2 \rangle, \qquad (28)$$

is a moderately varying function of the density with a magnitude of $10^{15}{\rm cm}^2/{\rm s}$ for $c_\nu = 1/3$. Rotational quenching makes it strongly depth-dependent and reduces it by up to two orders of magnitude. The resulting value of $10^{13}{\rm cm}^2/{\rm s}$ together with the stellar radius of 4.6 $R_\odot$ yields a Taylor number of $4.3 \cdot 10^{10}$, too large for our code to produce stable results. We therefore use $c_\nu = 3$, providing a Taylor number two orders of magnitude smaller but still very large.

The resulting rotation law and the stream function are shown in Fig. 6. It resembles very much that for Ta = 10^7 from the simple model. The deviation from rigid rotation is less than 0.5 percent except close to the (artificial) lower boundary. The re-

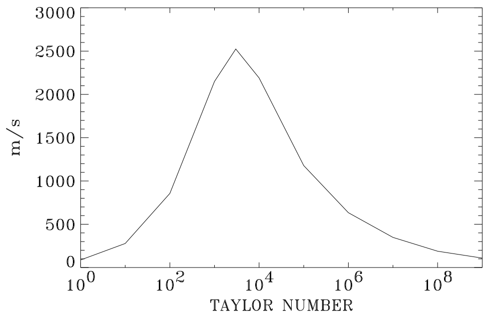

Fig. 4. The maximum value of the latitudinal velocity component in m/s as a function of the Taylor number for fixed angular velocity

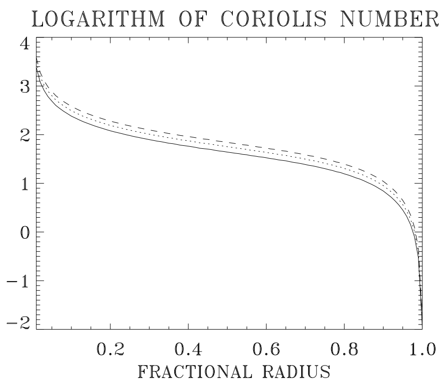

Fig. 5. The radial variation of the Coriolis number in a PMS star with 1.5 solar masses at the beginning of the contraction phase for different values of the mixing-length parameter. Solid line: $\alpha_{\rm MLT}$ = 1, dotted line: $\alpha_{\rm MLT}$ = 5/3, dashed line: $\alpha_{\rm MLT}$ = 2.4

maining small shear follows the Taylor-Proudman theorem in the bulk of the stellar volume. Close to the upper and lower boundary, respectively, there is a layer where the rotation pattern is determined by the boundary conditions, which do not match the Taylor-Proudman state. At the outer boundary, the requirement that the surface be stress-free together with small values of the Coriolis number leads to positive radial shear and a strong reduction of the latitudinal shear within the outer 5 percent of stellar radius. Due to the rapidly decreasing density, the meridional flow velocity is maximal at the surface, although the stream function has very small values there. The speed of the latitudinal flow reaches a maximum value of about 70 m/s close to the stellar surface.

4. Discussion

In our model of the rotation and meridional circulation of a fully convective pre-main sequence star the Reynolds stress is the source of differential rotation. The meridional circulation is a consequence of differential rotation and not the source, as is often assumed. Eq. (7) has a steady solution only if the Reynolds stress due to the meridional motion exactly balances the gradient of the rotation frequency along the z-axis, $\partial\Omega/\partial z$. In the limiting case of vanishing viscosity this gradient must therefore vanish, as required by the Taylor-Proudman theorem. For finite viscosity, any deviation from the Taylor-Proudman state forces a meridional flow which not only produces the shear necessary for a steady state but also transports angular momentum and therefore changes the rotation pattern.

It is known from the work of Kippenhahn (1963) and Köhler (1969) on the solar convection zone that positive radial shear produces a meridional flow which is directed towards the equator at the surface while in case of negative shear the direction of the flow is opposite. In that model the horizontal shear is produced by the meridional motion and hence positive in case of positive vertical shear and negative for negative vertical shear. In our model, the radial Λ-effect is negative everywhere except at the equator where the contributions of $V^{(0)}$ and $V^{(1)}$ cancel. The resulting circulation decelerates the rotation at the equator and thus counteracts the horizontal Λ-effect. One could therefore expect a reversal of the surface differential rotation for sufficiently fast meridional flow. For very large Taylor numbers this is indeed the case, but the latitudinal shear is very small and the rotation is almost rigid.

As the variation of the meridional flow velocity as a function of the Taylor number in Fig. 4 shows, there are two asymptotic regimes. For small Taylor numbers, the differential rotation is completely determined by the Reynolds stress and the meridional motion is just a consequence of rotational shear. Hence, its velocity grows with Ta since as $\partial\Omega/\partial z$ is not yet affected by the circulation and a decrease of the viscosity must be compensated for by an increase in the flow velocity. For very large Taylor numbers, the meridional flow drives the system towards the Taylor-Proudman state and its velocity decreases with increasing Taylor number. The maximum value of the meridional flow velocity is reached at intermediate values of Ta, i.e. between 10^3 and 10^4, when on the one hand large velocities are needed to produce enough stress to compensate for $\partial\Omega/\partial z$, while on the other hand the Λ-effect is still strong enough to maintain differential rotation.

According to the Taylor-Proudman theorem, for high Taylor numbers the rotation rate becomes a function of the distance from the axis only, $\Omega = f(r \sin\theta)$. The function f is, however, not prescribed by the theorem. As we know that the Λ-effect is the only source of differential rotation, we solved the Reynolds equation for several artificially chosen combinations of $V^{(0)}$ and $V^{(1)}$. The radial variation of Ω at the equator turned out to be mainly determined by the Λ-effect even for large Taylor numbers. But since in the latter case the rotation rate does not depend on z any more, it is completely determined by its radial variation at the equator. The essentially rigid rotation found for the T Tauri star is thus due to the cancelation of $V^{(0)}$ and $V^{(1)}$ at the equator.

We have modeled the rotation of a T Tauri star under the assumption that the star is barotropic, which should be true in a fully convective star. The angular velocity turns out to be almost constant throughout the whole star and the surface differential rotation is smaller than one percent. This result is in agreement with Johns-Krull (1996) as well as with Rice & Strassmeier (1996) and clearly contradicts the assumptions made by Smith

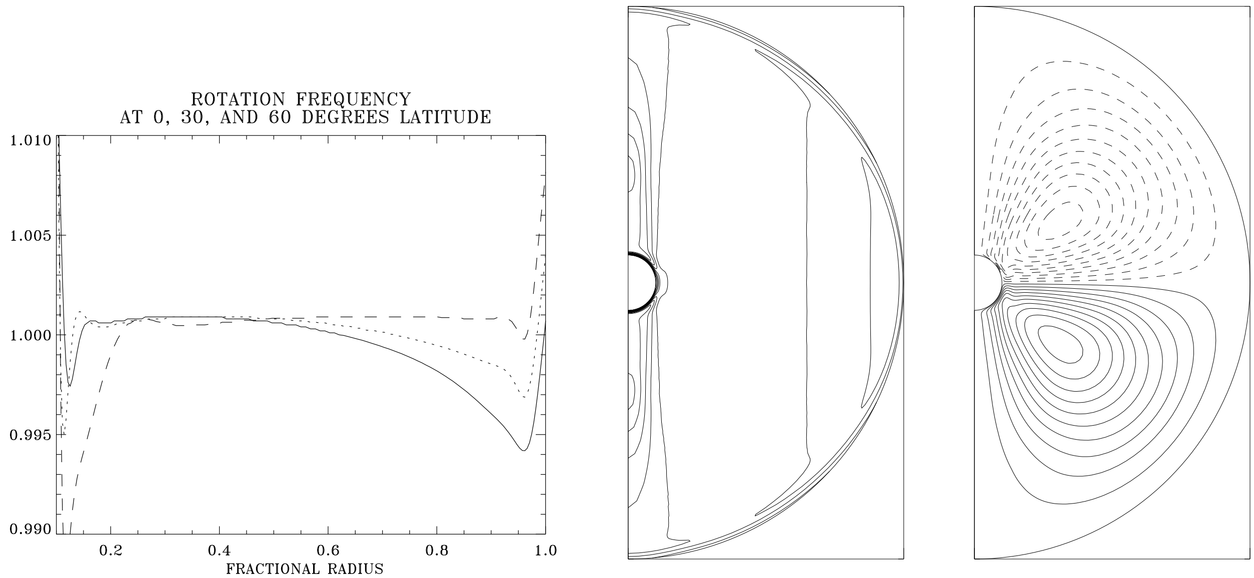

Fig. 6. Same as Fig. 1, but for the more realistic model including both anisotropy and stratification of the stress tensor.

(1994), who proposed an alternative explanation of the higher rotation rates of WTTS compared with CTTS in terms of equatorial acceleration and different latitudes of spot occurrence.

We have assumed a stress-free stellar surface. This means that we neglect the torques exerted on the stellar surface by a wind or an accretion disk via the stellar magnetic field. Since the time scale of the redistribution of angular momentum by Reynolds stress and meridional motion is only a few decades and hence much smaller than that of wind braking, the neglect of the latter is surely justified. In a CTTS the torques by accretion and magnetic field can be much larger than those of the wind and a slight change of the rotation pattern in case of strong coupling between disk and star is therefore possible. This will, however, only affect the surface layer to which differential rotation is restricted and not change our finding of essentially rigid rotation.

In our model, the effects of Reynolds stress and meridional motion cancel out each other. For the Sun, the rotation pattern resulting from a similar model are much to close to rigid body rotation. The situation changes if the stratification is not strictly barotropic. In that case, an additional Term $-(g/rT)\partial\Delta T/\partial\theta$ arises on the RHS of (7) which drives an additional meridional flow. Kitchatinov & Rüdiger (1995) solved the Reynolds equation together with the equation for the turbulent heat transport for the solar convection zone and the outer part of the radiative core. They found a polar surface temperature 5 K higher than the surface temperature at the equator, which was sufficient to balance the meridional flow and cancel out its effect on the differential rotation. We therefore checked our model for its sensitivity to horizontal temperature gradients by adding a latitude-dependent perturbation, $\delta T = T_0 \cos(2\theta)$, to the temperature T from the stellar model. We found that T_0 = 10K does not affect the rotation law while 100 K cause an equatorial acceleration of about 4 percent and 1000 K cancel out the effect of the meridional flow. Negative values of T_0, i.e. cool poles, cause a polar acceleration of 3 and 25 percent for 100K and 1000K, respectively. As for T Tauri stars the eddy heat conductivity tensor is the same as for the Sun, it seems unlikely that a latitudinal temperature gradient of several hundred degrees could be produced by anisotropic heat conduction. Spotted regions on the stellar surface are more than 1000 K cooler than the remainder of the surface, but it is questionable whether there is a strong effect on the rotation since their depth is probably small and their distribution strongly non-axisymmetric. We thus conclude that WTTS indeed rotate rigidly.

5. Consequences for the stellar dynamo

We do not find any remarkable gradient of the rotation frequency either in the radial or in the latitudinal direction. This raises the question about the nature of the dynamo process in this type of star, which might be quite different from the solar dynamo. The latter is believed to be of $\alpha\Omega$-type, producing an axisymmetric oscillating dipole field with a toroidal field component much stronger than the poloidal. This mechanism of field generation does not work in stars with weakly differential or uniform rotation. On the other hand, a fossil field can survive over a time scale

$$\tau_{\rm diff} = \frac{R_\star^2}{\eta_{\rm T}}, \tag{29}$$

where $\eta_{\rm T} \simeq \nu_{\rm T}$ is the turbulent magnetic diffusivity and $R_\star$ the stellar radius. Even if the efficiency of turbulent diffusion is reduced by two orders of magnitude due to the global rotation, the diffusive time scale is only about 300 years and thus much too small for a large-scale field to survive. The magnetic field of the TTS (and of the main sequence star that evolves from it) must therefore be dynamo-generated.

Of the variety of known dynamo mechanisms (cf. Krause & Rädler 1980), the α^2-dynamo seems to be the most likely one in case of uniform rotation, since for large Coriolis numbers the magnetic diffusivity decreases as $1/\Omega^*$ while the α-effect has a finite limit (Rüdiger & Kitchatinov 1993) and the dynamo must therefore necessarily become supercritical for sufficiently fast rotation. This case has been considered by Moss & Brandenburg (1995), who found that the S1 mode was the first to become supercritical, at a Coriolis number of 27.9. Since in our model $\Omega^* > 27.9$ for the inner 75 percent of the stellar radius, we can indeed expect the action of an α^2-dynamo.

Linear dynamo theory has shown that in case of an α^2-dynamo in a sphere with isotropic α-effect the most easily excited mode is always axisymmetric (Rädler 1986) while anisotropic α leads to non-axisymmetric fields unless there is strong rotational shear (Rüdiger 1980, Brandenburg et al. 1989, Rädler et al. 1990, Rüdiger & Elstner 1994). Since for Coriolis numbers as large as in case of T Tauri stars the α-tensor is indeed strongly anisotropic, the finding of Moss & Brandenburg is in agreement with the results of previous work. The actual field geometry is, however, not determined by one single mode since the back reaction of the magnetic field on the turbulent electromotive force makes the dynamo equation nonlinear and therefore there will be a coupling of different modes. Moss & Brandenburg (1995) found that in this case the field geometry always resembled the S1 mode although contributions of higher modes were present and that the field in the surrounding space was mainly that of a dipole with the axis lying perpendicular to the axis of rotation.

Although the spatial structure of stellar magnetic fields is generally not observable, Doppler imaging provides some hints since it tells us about the distribution of spots on the stellar surface. Unlike the sunspots, spots on T Tauri stars appear to have lifetimes of several years and cover a significant fraction of the stellar surface. Joncour et al. (1994a,b) found large spots close to the visible poles of V 410 Tauri and HDE 283572 and hence no restriction of stellar activity to low latitudes. Moreover, the distribution of spots on the surface of V 410 Tauri is strongly non-axisymmetric and covers all latitudes. This result, also found by Rice & Strassmeier (1996), indicates that the large-scale magnetic field may indeed be non-axisymmetric. We may therefore conclude that the absence of surface differential rotation and the non-axisymmetric geometry of the magnetic field support our finding that there should not be any significant differential rotation in weak line T Tauri stars.

The symmetry of the magnetic field has important consequences for stellar activity. The cyclic behaviour of solar activity is quite normal for $\alpha\Omega$-dynamos, which produce dynamo waves that propagate along the lines of constant rotation rate (Parker 1955, Yoshimura 1975). Non-axisymmetric field modes, however, can only propagate in longitudinal direction without any cyclic variation of the total field energy (cf. Rädler 1986). We can therefore not expect activity cycles for uniformly rotating stars. This provides a strong observational test for dynamo theory.

Acknowledgements. This work was supported by Deutsche Forschungsgemeinschaft.

References

Armitage, P.J., Clarke, C.J., 1996, MNRAS 280, 458
Bertout, C., 1989, Ann. Rev. Astron. Astroph. 1989, 351
Bodenheimer, P., 1995, Ann. Rev. Astron. Astroph. 1995, 199
Bouvier, J., Cabrit, S., Fernandez, M., Martin, E.L., Matthews J.M., 1993, A&A 272, 176
Bouvier, J., 1994, in Caillault, J.P. (ed.), Cool Stars, Stellar Systems and the Sun, Eighth Cambridge Workshop, ASP Conference Series Vol. 64
Brandenburg, A., Tuominen, I., Rädler, K.-H., 1989, Geophys. Astrophys. Fluid Dyn. 49, 45
Camenzind, M., 1990, in Klare, G. (ed.), Reviews in Modern Astronomy 3, Springer-Verlag, Berlin
Cameron, A.C., Campbell, C.G., 1993, A&A 274, 309
Cameron, A.C., Campbell, C.G., Quaintrell, H., 1995, A&A 298, 133
Donahue, R.A., Saar, S.H., Baliunas, S.L., 1996, ApJ 466, 384
Edwards, S., Strom, S.E., Hartigan, P., Strom, K.M., Hillenbrandt, L.A., Herbst, W., Attridge, J., Merrill, K.M., Probst, R., Gatley, I., A.J., 106, 372
Ghosh, P., Lamb, F.K., 1979a, ApJ 232, 259
Ghosh, P., Lamb, F.K., 1979b, ApJ 234, 296
Ghosh, P., 1995, MNRAS 272, 763
Hall, D., 1991, in Tuominen, I., Moss, D., Rüdiger, G., The Sun and Cool Stars: activity, magnetism, dynamos, Springer-Verlag, Berlin
Johns-Krull, C.M., 1996, A&A 306, 803
Joncour, I., Bertout, C., Ménard, F., 1994a, A&A 285, L25
Joncour, I., Bertout, C., Bouvier, J., 1994b, A&A 291, L19
Kippenhahn, R., 1963, ApJ 137, 664
Kitchatinov, L.L., Rüdiger, G., 1993, A&A 276, 96
Kitchatinov, L.L., Pipin V.V., Rüdiger, G., 1994, Astron. Nachr. 315, 157
Kitchatinov, L.L., Rüdiger, G., 1995, A&A 299, 446
Köhler, H., 1969, Ph D thesis, Göttingen
Königl, A., 1991, ApJ 370, L39
Krause, F., Rädler, K.H., 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory, Akademie-Verlag Berlin
Küker M., Rüdiger, G., Kitchatinov, L.L., 1993, A&A 279, L1
Lovelace, R.V.E., Romanova, M.M., Bisnovatyi-Kogan, G.S., 1995, MNRAS 275, 244
Moss, D., Brandenburg, A., 1995, Geophys. Astrophys. Fluid Dyn. 80, 229
Palla, F., Stahler, S.W., 1993, ApJ **418**, 414
Parker, E.N., 1955, ApJ 122, 293
Rädler, K.-H., 1986, Astron. Nachr. 307, 89
Rädler, K.-H., Wiedemann, E., Brandenburg, A., Meinel, R., Tuominen, I., 1990, A&A 239, 413
Rice, J.B., Strassmeier, K.G., 1996, A&A 316, 164
Rüdiger, G., 1980, Astron. Nachr. 301, 181
Rüdiger, G., Elstner, D., 1994, A&A 281, 46
Rüdiger, G., von Rekowski, B., Donahue, R.A., Baliunas, S.L., 1997, ApJ , in press
Smith, M.D., 1994, A&A 287, 523
Yi, I., 1994, ApJ 428, 760
Yoshimura, H., 1975, ApJ 201, 740

This article was processed by the author using Springer-Verlag LaTeX A&A style file *L-AA* version 3.