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Resonant Alfvén waves in coronal arcades driven by footpoint motions

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Abstract. X-ray spectroscopy performed from different astronomical spacecrafts has shown that the solar corona is structured by magnetic fields having the shape of loops and arcades. These structures are formed by stretching and reconnection of magnetic fields, and remain stable from days to weeks. Also, sporadic or periodic brightenings of such structures have been detected in UV and soft X-ray observations, suggesting the existence of propagating waves and plasma heating within them. In this paper, a mechanism for the deposition of Alfvén wave energy and heating of coronal arcades via resonant absorption is investigated. An analytical solution to the linear viscous, resistive MHD equations that describes the steady state of resonant shear Alfvén oscillations in coronal arcades driven by toroidal footpoint motions is obtained. General expressions for the total amount of dissipated wave energy and for its spatial distribution within the resonant magnetic surface is derived.

Key words: MHD – wave – Sun: corona; oscillations

1. Introduction

More than two decades ago, EUV and soft X-ray observations of the solar corona by Skylab pointed out that the solar atmosphere is structured by the magnetic field in a variety of shapes: loops, arcades, etc. This structuring has been strikingly confirmed by the spectacular soft X-ray pictures taken by the Yohkoh spacecraft during the last few years. Coronal magnetic arcades can be formed by stretching and reconnection of the magnetic field, after the occurrence of a prominence eruption, and may last in a stable way from days to weeks. For example, Acton et al. (1992) show an X-ray image, taken on November 12, 1991, in which a magnetic arcade half a solar radius high, 400,000 km wide and 500,000 km long is present. A large variety of similar structures, with various heights, widths and lengths, can also be seen in many Yohkoh pictures (Watari, Detman, & Joselyn 1996; Weiss et al. 1996). Moreover, the formation and evolution of large coronal arcades can be observed in Yohkoh SXT images (McAllister et al. 1992; Hanaoka et al. 1994) and evidence is growing about the close link of soft X-ray arcades with coronal mass ejections (McAllister et al. 1996).

Observations of coronal structures, such as loops and arcades, using coronal emission lines allow the detection of brightenings in these structures, which can be associated with propagating waves. For example, Vernazza et al. (1975) used coronal EUV lines to search for vertical propagation, such as revealed by time lags between brightenings at different heights, and found evidence for upward propagation of pulses of hydrodynamic or hydromagnetic nature. Antonucci, Gabriel, & Patchett (1984) studied oscillations in the C II, O IV, and Mg X UV emission lines observed by Skylab during a loop brightening and found periodic intensity fluctuations with a period of 141 s. The periodic oscillation was not only localised in the loop region but also extended over a larger area of the same active region, maintaining the same phase. Brightenings in coronal arcades have been also observed by Yohkoh (McAllister et al. 1996).

Coronal arcades are formed by curved magnetic field lines anchored in the relatively dense and highly conductive photospheric plasma. The photospheric footpoints of the magnetic field lines are forced to follow the convective motions of the photospheric plasma. The motions of the footpoints of the magnetic field lines in turn excite MHD waves in the magnetic arcades. Arbitrary footpoint motions generate fast and slow magnetosonic waves and Alfvén waves. Due to the steep density gradients at the photospheric edges these MHD waves are reflected back and forth along the arcade and eventually dissipated by various mechanisms and heat the arcade.

An important property of MHD waves in inhomogeneous plasmas is that individual magnetic surfaces can oscillate with their own Alfvén frequencies. In ideal linear MHD these oscillations occur without interaction with neighbouring magnetic

Send offprint requests to: M. Ruderman surfaces. They are polarized in the magnetic surfaces and perpendicular to the magnetic field lines. Dissipation causes coupling to neighbouring surfaces. However, for large values of the viscous and magnetic Reynolds numbers as in the solar corona, the local Alfvén oscillations are close to those in ideal plasmas. In particular, they are still characterized by large gradients across the magnetic surfaces.

Due to plasma inhomogeneity the Alfvén speed takes different values at different magnetic surfaces. This causes phase mixing of Alfvén waves travelling back and forth along the magnetic surfaces. If the waves are excited long enough then phase mixing leads to the formation of narrow dissipative layers embracing resonant surfaces, which are magnetic surfaces where the frequency of the external driver matches the local Alfvén frequency. In these dissipative layers the amplitudes and gradients of local Alfvén oscillations are very large. As a result, in weakly dissipative plasmas the energy dissipation in the dissipative layers is far more efficient than in homogeneous plasmas. The process of energy dissipation in the dissipative layers is called resonant absorption.

The ability to absorb a large amount of wave energy even in a weakly dissipative plasma, has attracted much attention to resonant absorption. Ionson (1978) proposed resonant absorption of MHD waves as a mechanism for heating magnetic loops in the solar corona. Since this original work, resonant absorption has remained a popular mechanism for explaining the heating of the solar corona (see, e.g., Kuperus, Ionson, & Spicer 1981; Ionson 1985; Davila 1987; Hollweg 1990, 1991; Goossens 1991; Ofman & Davila 1995).

There are two different senarios for the excitation of localised Alfvén oscillations. The first scenario involves lateral driving and an indirect excitation of the localised oscillations. The global waves are supposed to be present either outside the coronal loop and to impinge on the loop or to be present inside the loop. Either way the global wave transfers energy across the magnetic surfaces up to the resonant surface where the frequency of the global wave matches the local Alfvén frequency. In this sense the excitation of the localised oscillations is indirect, since we need magnetosonic waves that propagate across the magnetic surface to excite them.

The second scenario involves driving at the photospheric footpoints of the magnetic field lines. This provides a direct excitation of the localised Alfvén oscillations. Heating of plasmas by dissipation of localised waves driven by footpoint motion was first studied by Heyvaerts & Priest (1983). However these authors concentrated on damping of Alfvén waves due to phase mixing and did not consider resonant absorption. Resonant absorption of directly excited Alfvén waves was first studied numerically in linear MHD by Strauss & Lawson (1989) and, subsequently, by Goedbloed & Halberstadt (1994) and by Halberstadt & Goedbloed (1995a, b). Poedts & Boynton (1996) numerically studied resonant absorption of directly excited Alfvén waves in nonlinear MHD.

Recently, Ruderman et al. (1997) analytically studied resonant absorption of directly excited torsional Alfvén waves in a straight magnetic cylinder, which was used as a model of a coronal loop. These authors assumed that all equilibrium quantities depend on the radial coordinate only, so that they considered a one-dimensional equilibrium state. They obtained a very simple solution to the linear viscous resistive MHD equations that describes the steady state of resonant oscillations.

The aim of the present paper is to extend the analysis by Ruderman et al. (1997) to directly excited resonant Alfvén waves in planar two-dimensional equilibrium magnetic plasma configurations. These configurations can be used as models of coronal arcades. The paper is organised as follows. In the next section we give the basic equations, discuss the main assumptions, and describe the equilibrium state. In Sect. 3 we derive the equation that governs the steady state of resonant Alfvén oscillations driven by footpoint motions. In Sect. 4 we obtain the ideal MHD solution to this equation which is valid far away from the dissipative layer. In Sect. 5 we find the dissipative solution describing wave motions in the dissipative layer embracing the resonant magnetic surface. In Sect. 6 we calculate the total amount of wave energy dissipated in the dissipative layer and the distribution of dissipated energy along the resonant magnetic surface. In Sect. 7 we consider an example of a potential arcade with constant density and, finally, Sect. 8 contains the conclusions.

2. Basic equations and assumptions

We consider the motions of a viscous, resistive plasma, described by the dissipative MHD equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \,, \tag{1}$$

$$\begin{aligned} \rho \left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} \right] &= -\nabla p + \frac{1}{\mu}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \rho \mathbf{g} \\ &- \nabla \times (\rho\nu\nabla \times \boldsymbol{v}) + \nabla(\rho\nu\nabla \cdot \boldsymbol{v}) \,, \end{aligned} \tag{2}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times (\eta \nabla \times \boldsymbol{B}) \,, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) + \boldsymbol{v} \cdot \nabla \left(\frac{p}{\rho^\gamma}\right) = \mathscr{L} \,, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0 \,. \tag{5}$$

Here ρ is the density, p the pressure, $\boldsymbol{v}$ the velocity, and $\boldsymbol{B}$ the magnetic field. The gravitational acceleration $\mathbf{g}$ and the adiabatic index γ are constant. The spatial dependences of the coefficients of viscosity ν and magnetic field diffusion η will be specified later on the basis of mathematical simplicity. The quantity $\mathscr{L}$ in the entropy equation (4) represents the effect of dissipation due to viscosity and finite resistivity. We do not give the explicit expression for $\mathscr{L}$ because we shall not use the entropy equation in what follows.

We adopt the Cartesian coordinates x, y, and z with the z-axis antiparallel to the gravitational acceleration $\mathbf{g}$ and consider an equilibrium magnetic plasma configuration that is independent of y ($\partial/\partial y = 0$). In addition, the equilibrium magnetic

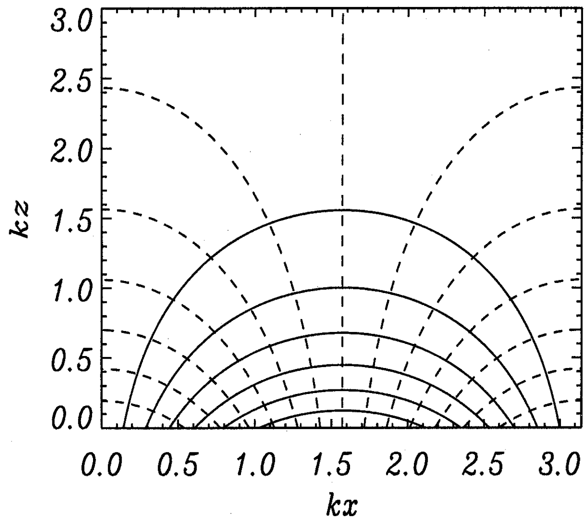

Fig. 1. Sketch of the unperturbed state. Solid curves are magnetic field lines, determined by the condition ψ = const, while dashed curves are determined by the condition ϕ = const (cf. Eq. [17]).

field $\boldsymbol{B}_0$ is assumed to be poloidal, i.e. $B_{0y} = 0$. The equilibrium quantities satisfy the magnetostatic equations

$$\nabla p_0 = \frac{1}{\mu}(\nabla \times \mathbf{B}_0) \times \mathbf{B}_0 + \rho_0 \mathbf{g} , \quad (6)$$

$$\nabla \cdot \mathbf{B}_0 = 0 , \quad (7)$$

where the subscript '0' indicates an equilibrium quantity. The solenoidality equation (7) enables us to write $\mathbf{B}_0$ in terms of a flux function $\psi(x, z)$,

$$B_{0x} = \frac{\partial \psi}{\partial z} , \quad B_{0z} = -\frac{\partial \psi}{\partial x} . \quad (8)$$

The magnetic surfaces are given by the equation $\psi(x, z)$ = const. The surface $z = 0$ is the boundary between the hot, rarefied corona ($z > 0$) and the dense photosphere ($z < 0$). We assume that the magnetic surfaces have the form of arcades with the feet of magnetic field lines frozen into the dense photosphere. Priest (1988) considered a special case of equilibrium poloidal magnetic field where

$$\nabla^2 \psi = \alpha \psi , \quad (9)$$

with α a constant. One of the simplest solutions to Eq. (9) is

$$\psi = -A \exp(-lz) \sin(kx) , \quad (10)$$

where A, l, and k are positive constants, $k \leq l$, and α, l, and k are related by $\alpha = l^2 - k^2$. A particular magnetic configuration described by Eq. (10) is shown in Fig. 1. Substitution of Eq. (9) into Eq. (6) results in

$$\nabla \left(p_0 + \frac{\alpha}{2\mu}\psi^2 \right) = \rho_0 \mathbf{g} . \quad (11)$$

It immediately follows from this expression that $\rho_0 = \rho_0(z)$. Then integration of Eq. (11) gives

$$p_0 + \frac{\alpha}{2\mu}\psi^2 + g \int_0^z \rho_0(\bar{z})\, d\bar{z} = \text{const} , \quad (12)$$

where the function $\rho_0(z)$ can be chosen arbitrarily, and so Eq. (12) gives the spatial dependence of the equilibrium pressure p_0. The spatial dependence of the equilibrium temperature T_0 is determined by the Clapeiron equation for a fully ionised plasma,

$$p_0 = \frac{2k_B}{m_p}\rho_0 T_0 , \quad (13)$$

with k_B the Boltzmann constant and m_p the proton mass. Note that the magnetic plasma configuration described by Eqs. (10), (12), and (13) was used by Oliver, Hood, & Priest (1996) with $g = 0$ when studying magnetosonic waves in coronal arcades. The particular case of this configuration with $\alpha = 0$ ($l = k$), corresponding to a potential arcade, was used by Čadež & Ballester (1996), Čadež, Oliver, & Ballester (1995, 1996a, b), and Oliver, Murawski, & Ballester (1997) when studying wave propagation in coronal arcades.

3. Derivation of the Alfvén mode governing equation

In what follows we consider Alfvén oscillations only, so that perturbations of all quantities but the y-component of the velocity, v, and the magnetic field, b, are zero. Consequently we use the linearised y-components of Eqs. (2) and (3) only,

$$\rho_0 \frac{\partial v}{\partial t} = \frac{1}{\mu}(\mathbf{B}_0 \cdot \nabla) b + \nabla \cdot (\rho_0 \nu \nabla v) , \quad (14)$$

$$\frac{\partial b}{\partial t} = (\mathbf{B}_0 \cdot \nabla) v + \nabla \cdot (\eta \nabla b) . \quad (15)$$

Note that gravity does not appear in these equations, so that it affects wave motions only through the equilibrium state.

It is convenient to have an orthogonal curvilinear system of coordinates with the magnetic field lines as one family of coordinate lines. In what follows we shall see that the equations describing Alfvén waves in arcades take especially simple forms in this coordinate system. In particular, the variable ψ is present in the ideal MHD equations as a parameter only. The second curvilinear coordinate is denoted by $\phi(x, z)$. Then the orthogonality condition is

$$\frac{\partial \phi}{\partial x}\frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial z}\frac{\partial \psi}{\partial z} = 0 . \quad (16)$$

As an example, when the flux function ψ is given by Eq. (10), the function ϕ can be taken in the form

$$\phi = -\exp(-k^2 z/l) \cos(kx) . \quad (17)$$

The coordinate ψ increases with the height z at a fixed x. On the other hand, the sign of the function ϕ is chosen so that ϕ increases along a magnetic field line from the left to the right.

The functions ϕ and ψ satisfying Eq. (16) constitute an orthogonal curvilinear coordinate system in the xz-plane, shown in Fig. 1 for ψ and ϕ given by (10) and (17). The coordinate lines $\psi = \text{const}$ coincide with magnetic field lines, while the coordinate lines $\phi = \text{const}$ are perpendicular to magnetic field lines.

It is straightforward to check that the Jacobian of the coordinate transform

$$J = \frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial z} - \frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial x}$$

is positive: $J > 0$. The following formula then holds,

$$(\mathbf{B}_0 \cdot \nabla) = J\frac{\partial}{\partial\phi} . \tag{18}$$

The theory developed in the following Sects. 3 to 6 applies to any pair of curvilinear coordinates ϕ and ψ. An application to a potential arcade is made in Sect. 7, where Eqs. (10) and (17) with $k = l$ are considered in the region $0 < x < L$, $z > 0$, with $L = \pi/k$.

In what follows we consider resonant Alfvén oscillations in a weakly dissipative plasma. For these oscillations dissipation is only important in the narrow dissipative layer that embraces the resonant magnetic surface, while it can be neglected outside the dissipative layer. In the dissipative layer the gradients of the perturbations in the ψ-direction (across magnetic surfaces) are far larger than the gradients in the ϕ-direction (along magnetic field lines). On the other hand, the gradients of ρ_0, ν, and η in the ψ-direction are of the same order as the gradients in the ϕ-direction. Hence we can neglect all terms on the right-hand sides of Eqs. (14) and (15), except those containing $\partial^2 v/\partial\psi^2$ and $\partial^2 b/\partial\psi^2$. Then, taking Eq. (18) into account, we rewrite Eqs. (14) and (15) as

$$\frac{\partial v}{\partial t} = \frac{J}{\mu\rho_0}\frac{\partial b}{\partial\phi} + \frac{\nu}{h^2}\frac{\partial^2 v}{\partial\psi^2} , \tag{19}$$

$$\frac{\partial b}{\partial t} = J\frac{\partial v}{\partial\phi} + \frac{\eta}{h^2}\frac{\partial^2 b}{\partial\psi^2} , \tag{20}$$

where

$$\frac{1}{h^2} = \left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2 = B_0^2 . \tag{21}$$

Now, for the sake of mathematical simplicity we assume that the quantities ν/h^2 and η/h^2 are constant. We differentiate Eq. (19) with respect to t and substitute $\partial b/\partial t$ from Eq. (20) in order to get

$$\frac{\partial^2 v}{\partial t^2} = \frac{J}{\mu\rho_0}\frac{\partial}{\partial\phi}J\frac{\partial v}{\partial\phi} + \frac{\nu}{h^2}\frac{\partial^3 v}{\partial t\partial\psi^2} + \frac{J\eta}{\mu\rho_0 h^2}\frac{\partial^3 b}{\partial\phi\partial\psi^2} . \tag{22}$$

The assumption that η/h^2 is constant makes it possible to avoid the appearance of a term proportional to $\partial\eta/\partial\phi$ in the right-hand side of this equation, which would otherwise complicate the analysis.

We next use Eq. (19) to eliminate $\partial b/\partial\phi$, neglect small terms of the order of $\nu\eta$, and once again retain only terms containing $\partial^2 v/\partial\psi^2$ when calculating the term proportional to η. As a result, we obtain the equation

$$\frac{\partial^2 v}{\partial t^2} = \frac{J}{\mu\rho_0}\frac{\partial}{\partial\phi}J\frac{\partial v}{\partial\phi} + \frac{\nu+\eta}{h^2}\frac{\partial^3 v}{\partial t\partial\psi^2} . \tag{23}$$

This is the basic equation for the study of Alfvén waves in magnetic arcades.

We assume that the arcade is shaken at its left end by photospheric motions, while its right end remains immobile. This means that we impose the boundary conditions

$$v = f(t,\psi) \;\text{ at }\; \phi = \phi_1(\psi) , \quad v = 0 \;\text{ at }\; \phi = \phi_2(\psi) , \tag{24}$$

where $\phi_1(\psi)$ and $\phi_2(\psi)$ are the coordinates of the left and right footpoints of the magnetic field line determined by the equation $\psi = \text{const}$. Note that this choice of photospheric forcing is rather general as it allows different field lines being displaced with different amplitudes.

In what follows we assume that the driver is harmonic and consider only the steady state of driven oscillations. This allows us to take v and f proportional to $\exp(-i\omega t)$ with ω real. Then Eq. (23) takes the form

$$\omega^2 v + \frac{J}{\mu\rho_0}\frac{\partial}{\partial\phi}J\frac{\partial v}{\partial\phi} - \frac{i\omega(\nu+\eta)}{h^2}\frac{\partial^2 v}{\partial\psi^2} = 0 . \tag{25}$$

In order to obtain homogeneous boundary conditions we introduce a new variable

$$V = v + \frac{\phi - \phi_2(\psi)}{\phi_2(\psi) - \phi_1(\psi)}f(\psi) . \tag{26}$$

Substituting Eq. (26) into Eq. (25) we obtain

$$\frac{\partial}{\partial\phi}J\frac{\partial V}{\partial\phi} + \frac{\mu\rho_0\omega^2}{J}V - \frac{i\mu\rho_0\omega(\nu+\eta)}{h^2 J}\frac{\partial^2 V}{\partial\psi^2} = \frac{\mu\rho_0}{J}G(\phi,\psi) , \tag{27}$$

where

$$G(\phi,\psi) = \left[\frac{J}{\mu\rho_0}\frac{\partial J}{\partial\phi} + (\phi - \phi_2)\omega^2\right]\frac{f}{\phi_2 - \phi_1} . \tag{28}$$

In deriving Eqs. (27) and (28) it has been assumed that the characteristic scale of variation of the driver amplitude is much larger than the thickness of the dissipative layer. This assumption has enabled us to neglect terms proportional to ν and η when obtaining expression (28).

The function V satisfies the homogeneous boundary conditions

$$V = 0 \;\text{ at }\; \phi = \phi_1(\psi),\ \phi_2(\psi) . \tag{29}$$

Equation (27) with boundary conditions (29) will be used in the following to study the steady state of driven resonant Alfvén oscillations in coronal arcades.

4. Ideal solution

In this section we consider the ideal MHD solution to Eq. (27). To obtain this solution we take $\nu = \eta = 0$ in this equation and rewrite it as

$$\frac{\partial}{\partial\phi} J \frac{\partial V}{\partial\phi} + \frac{\mu\rho_0\omega^2}{J} V = \frac{\mu\rho_0}{J} G(\phi, \psi) , \quad (30)$$

where the function $V(\phi, \psi)$ must satisfy boundary conditions (29). Equation (30) is an ordinary differential equation with respect to ϕ, where the variable ψ is a parameter. This means that we can solve this equation on each magnetic surface independently. There is no coupling between different magnetic surfaces. Taking $G = 0$ we obtain an eigenvalue problem for the function V for each particular magnetic surface, with ω^2 as an eigenvalue. This eigenvalue problem is the Sturm-Liouville problem, so that there is an infinite set of eigenfunctions, $\{u_n(\phi, \psi)\}$, for any fixed ψ with corresponding eigenvalues $\{\omega_n^2(\psi)\}$. For every fixed ψ these eigenfunctions constitute a complete set of functions in the interval $\phi_1(\psi) \le \phi \le \phi_2(\psi)$. Quantities $\{\omega_n(\psi)\}$ are eigenfrequencies of Alfvén oscillations of particular magnetic surfaces. When one varies ψ the eigenfrequency $\omega_n(\psi)$ is smeared out in the Alfvén continuum

$$[\min\, \omega_n(\psi)\, , \max\, \omega_n(\psi)] \quad (31)$$

for each fixed n. Hence we have an infinite number of Alfvén continua.

Let us expand the functions V and G in Fourier series with respect to the set of functions $\{u_n\}$,

$$V = \sum_{n=1}^{\infty} V_n(\psi) u_n(\phi, \psi) , \quad G = \sum_{n=1}^{\infty} G_n(\psi) u_n(\phi, \psi) , \quad (32)$$

with $V_n(\psi)$ and $G_n(\psi)$ the Fourier coefficients. The function V satisfies boundary conditions (29), so that the Fourier series converges to V for any value of ϕ in the interval $[\phi_1, \phi_2]$. The function G, however, does not satisfy boundary conditions (29), so the Fourier series for G converges to G for every value of ϕ but $\phi = \phi_1$ and $\phi = \phi_2$.

In addition, the eigenfunctions $u_n(\phi, \psi)$ satisfy the orthogonality condition

$$\int_{\phi_1}^{\phi_2} \frac{\rho_0}{J} u_n u_m \, d\phi = 0 , \quad m \neq n . \quad (33)$$

With the use of this orthogonality condition, the equation for $u_n(\phi, \psi)$, and expression (28) for $G(\phi, \psi)$, we get

$$G_n(\psi) \int_{\phi_1}^{\phi_2} \frac{\rho_0}{J} u_n^2 \, d\phi = \frac{f(\omega^2 - \omega_n^2)}{\mu\omega_n^2(\phi_2 - \phi_1)} \int_{\phi_1}^{\phi_2} J \frac{\partial u_n}{\partial\phi} \, d\phi - \frac{f\omega^2}{\mu\omega_n^2} \left(J \frac{\partial u_n}{\partial\phi} \right) \Bigg|_{\phi=\phi_1} . \quad (34)$$

We substitute expansions (32) into Eq. (30) and obtain

$$V_n(\psi) = \frac{G_n(\psi)}{\omega^2 - \omega_n^2(\psi)} . \quad (35)$$

Let us assume that $G_n(\psi) \neq 0$ only for $\psi_m < \psi < \psi_M$. In what follows we use the term 'essential part of Alfvén continuum' to denote a part of the Alfvén continuum that is given by (31) where maximum and minimum are calculated in the interval $[\psi_m, \psi_M]$. Solution (35), with G_n given by Eq. (34), is mathematically and physically acceptable only if the frequency of the driver lies out of the essential part of all Alfvén continua, i.e.

$$\omega_n^2(\psi) \neq \omega^2 \quad (36)$$

for $\psi_m < \psi < \psi_M$ and for any positive integer n. In what follows we assume that there is exactly one value of n, $n = n_0$, such that inequality (36) is violated for exactly one value of ψ, $\psi = \psi_A$ ($\psi_m < \psi_A < \psi_M$), with $\psi(x, z) = \psi_A$ the resonant magnetic surface. The ideal solution (35) is valid everywhere for $n \neq n_0$, while it is valid only far enough from the resonant magnetic surface for $n = n_0$. In the next section we derive the solution for the resonant harmonic $V_{n_0}(\phi)$ in the vicinity of the resonant magnetic surface.

5. Dissipative solution in the vicinity of the resonant magnetic surface

One can infer from Eq. (35) that in the vicinity of the resonant magnetic surface $V_{n_0} = \mathscr{O}[(\psi - \psi_A)^{-1}]$, so that

$$\frac{1}{V_{n_0}} \frac{dV_{n_0}}{d\psi} = \mathscr{O}[(\psi - \psi_A)^{-1}] . \quad (37)$$

This estimate enables us to use the approximate formula

$$\frac{\partial^2}{\partial\psi^2}(V_{n_0} u_{n_0}) \approx u_{n_0} \frac{d^2 V_{n_0}}{d\psi^2} . \quad (38)$$

We substitute expansions (32) into Eq. (27) and use approximation (38) to get

$$[\omega^2 - \omega_{n_0}^2(\psi)] V_{n_0} - \frac{i\omega(\nu + \eta)}{h^2} \frac{d^2 V_{n_0}}{d\psi^2} = G_{n_0} . \quad (39)$$

We now follow Sakurai, Goossens, & Hollweg (1991), Goossens, Ruderman, & Hollweg (1995), and Ruderman et al. (1997), and assume that there is a quantity s_A such that in the interval $[\psi_A - s_A, \psi_A + s_A]$ the function $\omega^2 - \omega_{n_0}^2(\psi)$ can be represented by the first term of its Taylor expansion,

$$\omega^2 - \omega_{n_0}^2 = \Delta(\psi - \psi_A) , \quad (40)$$

with

$$\Delta = \frac{d(\omega^2 - \omega_{n_0}^2)}{d\psi} \Bigg|_{\psi=\psi_A} . \quad (41)$$

Dissipation is only important in a thin resonant layer embracing the resonant magnetic surface where the first term on the left-hand side of Eq. (39) is of the order of the second term. Comparison of these two terms results in the thickness of the dissipative layer, δ_A, given by

$$\delta_A = \left| \frac{\omega(\nu + \eta)}{h^2 \Delta} \right|^{1/3} . \quad (42)$$

Note that δ_A is not the spatial thickness of the dissipative layer but its thickness in the plane of the curvilinear coordinates ψ, ϕ.

In what follows we assume that $s_A \gg \delta_A$. It is convenient to introduce a dimensionless quantity $\tau = (\psi - \psi_A)/\delta_A$, so that in the dissipative layer $\tau \sim 1$, while $\psi \to \psi_A \pm s_A$ corresponds to $\tau \to \pm\infty$. In terms of the new variable τ Eq. (39) can be cast as

$$\frac{d^2 V_{n_0}}{d\tau^2} + i\tau V_{n_0} \mathrm{sign}\Delta = \frac{iG_{n_0}}{\delta_A|\Delta|} . \quad (43)$$

Since we have assumed that the characteristic scale of variation of the driver is much larger than δ_A, we can take $G_{n_0} \approx G_{n_0}(\psi_A)$, so the solution to Eq. (43) bounded for $|\tau| \to \infty$ is given by (see e.g. Goossens et al. 1995)

$$V_{n_0} = -\frac{iG_{n_0}(\psi_A)}{\delta_A|\Delta|} F(\tau) , \quad (44)$$

where $F(\tau)$ is the universal function, first introduced by Boris (1968),

$$F(\tau) = \int_0^\infty \exp(ik\tau \,\mathrm{sign}\Delta - k^3/3)\, dk . \quad (45)$$

It can be shown that the dissipative solution given by Eq. (44) matches the ideal solution for V_{n_0} given by Eq. (35) in the two overlap regions determined by the conditions $\delta_A \ll |\psi - \psi_A| \lesssim s_A$.

6. Energetics

6.1. Total dissipated Alfvén wave energy

In this section we calculate the energy dissipated in the coronal arcade. As the z-component of the velocity at the base of the arcade is zero, the energy flux through the left foot of the arcade is given by the z-component of the Poynting vector $\mathbf{E} \times \mathbf{B}/\mu$ integrated over the part of the surface $z = 0$ where the driver amplitude is non-zero. With the use of Ohm's and Ampere's laws we get

$$\frac{1}{\mu}\mathbf{E} \times \mathbf{B} = \frac{1}{\mu}[\mathbf{v}B^2 - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})] + \frac{\eta}{\mu}(\nabla \times \mathbf{B}) \times \mathbf{B} . \quad (46)$$

Taking into account that only the y-components of the velocity and the magnetic field perturbations differ from zero, we obtain

$$\frac{1}{\mu}(\mathbf{E} \times \mathbf{B})_z = -\frac{\eta}{\mu}\frac{\partial\psi}{\partial z}\nabla^2\psi + \frac{1}{\mu}\frac{\partial\psi}{\partial x}vb - \frac{\eta}{\mu}b\frac{\partial b}{\partial z}. \quad (47)$$

The first term on the right-hand side of this expression is due to the electric current present in the equilibrium state and is equal to zero in a potential arcade. The second and third terms are due to wave motion. In the following we are only interested in the energy flux arising from wave motion.

Let us now introduce the magnetic Reynolds number $R_m = v_A l_{\mathrm{eq}}/\eta$, where l_{eq} is a characteristic scale of the equilibrium state and v_A and η are typical values of the Alfvén velocity and the magnetic diffusion coefficient. The ratio of the third to the second term on the right-hand side of Eq. (47) is of the order of R_m^{-1}, which together with $R_m \gg 1$, allows us to neglect the third term. The z-component of the instantaneous Poynting flux per unit length in the y-direction related to wave motion, $S(t)$, can then be written as

$$S(t) = \frac{1}{\mu}\int_{x_1}^{x_2} \frac{\partial\psi}{\partial x} vb\, dx , \quad (48)$$

where $[x_1,\, x_2]$ is the interval in the x-axis where the driver amplitude is non-zero ($f(t,x) \neq 0$) and all quantities are calculated at $z = 0$ ($\phi = \phi_1(\psi)$), so that, in particular, $v = f(t,x)$.

Let us now use the analytical solution obtained in the previous section for calculating $S(t)$. We first use Eq. (20) in order to find b. An order-of-magnitude calculation indicates that the ratio of the second to the first term on the right-hand side of Eq. (20) is of the order R_m^{-1} far away from the dissipative layer, while this ratio is of the order $R_m^{-1/3}$ within the dissipative layer. These estimates enable us to neglect the second term on the right-hand side of Eq. (20). With the use of Eqs. (26) and (32), which determine the solution for v, we obtain from Eq. (20)

$$b = \frac{iJ}{\omega}\left(\sum_{n=1}^{\infty} V_n \frac{\partial u_n}{\partial\phi} - \frac{f}{\phi_2 - \phi_1}\right) . \quad (49)$$

When looking for a solution proportional to $e^{-i\omega t}$, we implicitly assume that the solution is of the form $q(t,x,z) = Re[q(x,z)e^{-i\omega t}]$, where q is any perturbed quantity. Therefore, when calculating $S(t)$, we need to evaluate expression (49) at $z = 0$, multiply it by $e^{-i\omega t}$, and substitute the real part of the result into Eq. (48). Similarly we have to substitute $v = Re[f(x)e^{-i\omega t}]$. In what follows we take $f(x)$ to be real, which implies that at the footpoints ($z = 0$) all magnetic surfaces ψ = const oscillate synchronically, i.e. there are no phase shifts between the oscillations of different magnetic surfaces. The choice of a complex function $f(x)$ would allow us to introduce such a phase shift.

In what follows we are only interested in the Poynting flux averaged over one period, $\bar{S}$. It is straightforward to obtain

$$\bar{S} = -\frac{1}{\mu}\int_{x_1}^{x_2} \frac{Jf}{2\omega}\frac{\partial\psi}{\partial x}\sum_{n=1}^{\infty} Im\left(V_n\frac{\partial u_n}{\partial\phi}\right) dx , \quad (50)$$

where all quantities are calculated at $z = 0$. Changing the integration variable, we rewrite expression (50) as

$$\bar{S} = Im\int_{\psi_m}^{\psi_M} \frac{J^2 f}{2\mu\omega}\frac{\partial z}{\partial\phi}\left|\frac{\partial x}{\partial\psi} + \frac{\partial x}{\partial\phi}\frac{d\phi_1}{d\psi}\right|\sum_{n=1}^{\infty} V_n\frac{\partial u_n}{\partial\phi} d\psi , \quad (51)$$

where $\psi_m = \psi(x_2, z = 0)$ and $\psi_M = \psi(x_1, z = 0)$, and all quantities in this expression are calculated at $\phi = \phi_1(\psi)$. All quantities in the integral in Eq. (51) are real, except for V_{n_0} whose imaginary part is different from zero in the dissipative layer. This consideration enables us to reduce Eq. (51) to

$$\bar{S} = \frac{\delta_A J f_A}{2\mu\omega}\frac{\partial u_{n_0}}{\partial\phi}\int_{-\infty}^{\infty} Im[V_{n_0}(\tau)]\, d\tau , \quad (52)$$

where $f_A = f(\psi_A)$ and all quantities in expression (52) are calculated at $\psi = \psi_A$ and $\phi = \phi_1(\psi_A)$. When deriving Eq. (52) we have used the identity

$$\frac{\partial z}{\partial \phi}\left|\frac{\partial x}{\partial \psi} + \frac{\partial x}{\partial \phi}\frac{d\phi_1}{d\psi}\right| = J^{-1} , \tag{53}$$

which can be obtained with the aid of Eq. (16). Substitution of the imaginary part of V_{n_0} from expression (44) into Eq. (52) results in

$$\bar{S} = -\frac{\pi J f_A G_{n_0}(\psi_A)}{2\mu\omega|\Delta|}\frac{\partial u_{n_0}}{\partial \phi} . \tag{54}$$

Taking into account that $\omega_{n_0}(\psi_A) = \omega$, Eq. (34) yields

$$G_{n_0}(\psi_A)\int_{\phi_1}^{\phi_2}\frac{\rho_0}{J}u_{n_0}^2\,d\phi = -\frac{f_A}{\mu}\left(J\frac{\partial u_{n_0}}{\partial \phi}\right)\Big|_{\phi=\phi_1} . \tag{55}$$

Upon substituting expression (55) into (54) we finally obtain

$$\bar{S} = \frac{\pi J^2 f_A^2}{2\mu^2\omega|\Delta|}\left(\frac{\partial u_{n_0}}{\partial \phi}\right)^2\left(\int_{\phi_1}^{\phi_2}\frac{\rho_0}{J}u_{n_0}^2\,d\phi\right)^{-1} . \tag{56}$$

All quantities in this expression are taken at $\phi = \phi_1$ and $\psi = \psi_A$. This formula clearly shows that the quantity $\bar{S}$ is independent of the dissipation coefficients ν and η, which means that the amount of dissipated energy is independent of the actual dissipation mechanism. It is fairly simple to prove $\bar{S} > 0$ from expression (56), as can be expected from a physical point of view.

6.2. Spatial profile of energy dissipation

It is also instructive to calculate the amount of energy dissipated in a part of the dissipative layer between positions s and $s+\Delta s$, averaged over one period. Here s is the length along the resonant magnetic field line in the xz-plane from $z = 0$ to a point (x,z), and is given by

$$s = \int_{\phi_1(\psi_A)}^{\phi}\left[\left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial z}{\partial \phi}\right)^2\right]^{1/2} d\phi . \tag{57}$$

The integrand in this equation is calculated at $\psi = \psi_A$. We assume that $\Delta s \ll l_{\rm eq}$, so that the energy dissipated between s and $s+\Delta s$ is equal to $Q(\phi)\Delta\phi$, where the density of dissipated energy $Q(\phi)$ is given by

$$Q(\phi) = \frac{\omega\delta_A}{2\pi}\left[\left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial z}{\partial \psi}\right)^2\right]^{1/2} \times \int_0^{\frac{2\pi}{\omega}} dt\int_{-\infty}^{\infty}\left[\frac{j^2}{\sigma} + \rho_0\frac{\nu}{h^2}\left(\frac{\partial v}{\partial \psi}\right)^2\right] d\tau , \tag{58}$$

where all quantities in this equation are calculated at $\psi = \psi_A$. Here $\mathbf{j}$ is the electric current density and σ is the electric conductivity,

$$\mathbf{j} = \frac{1}{\mu}\nabla\times\mathbf{B} , \quad \sigma = \frac{1}{\mu\eta} . \tag{59}$$

When deriving Eq. (58) we have taken into account that in the dissipative layer the derivative of v with respect to ψ is much larger than the derivative with respect to ϕ. We now express the electric current $\mathbf{j}$ as

$$\mathbf{j} = \frac{1}{\mu}\mathbf{e}_y\nabla^2\psi - \frac{1}{\mu}\mathbf{e}_y\times\nabla b , \tag{60}$$

with $\mathbf{e}_y$ the unit vector along the y-axis. The first term in this equation represents the electric current flowing in the equilibrium state, while the second term describes the current generated by footpoint motions. Substituting expression (60) into Eq. (58) we obtain $Q = Q_0 + Q'$, with Q_0 the energy density dissipated by the equilibrium current and Q' the energy density dissipated by footpoint motions. In what follows we are only interested in the quantity Q'. Once again we neglect derivatives with respect to ϕ in comparison with derivatives with respect to ψ and obtain

$$Q' = \frac{\omega\delta_A}{2\pi h^2}\left[\left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial z}{\partial \psi}\right)^2\right]^{1/2} \times \int_0^{\frac{2\pi}{\omega}} dt\int_{-\infty}^{\infty}\left[\rho_0\nu\left(\frac{\partial v}{\partial \psi}\right)^2 + \frac{\eta}{\mu}\left(\frac{\partial b}{\partial \psi}\right)^2\right] d\tau . \tag{61}$$

Next we take $v(t,\phi,\psi) = Re[v(\phi,\psi)e^{-i\omega t}]$, $b(t,\phi,\psi) = Re[b(\phi,\psi)e^{-i\omega t}]$, and neglect the second term in Eq. (20) in order to arrive at

$$b(\phi,\psi) = \frac{iJ}{\omega}\frac{\partial v}{\partial \phi} . \tag{62}$$

Substituting Eq. (62) into Eq. (61) we get

$$Q' = \frac{\delta_A}{2h^2}\left[\left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial z}{\partial \psi}\right)^2\right]^{1/2} \times \int_{-\infty}^{\infty}\left(\rho_0\nu\left|\frac{\partial v}{\partial \psi}\right|^2 + \frac{\eta J^2}{\mu\omega^2}\left|\frac{\partial^2 v}{\partial\phi\partial\psi}\right|^2\right) d\tau . \tag{63}$$

Now we use the approximate formula $v \approx V_{n_0}(\psi)u_{n_0}(\phi,\psi)$, valid in the dissipative layer, and also take into account that the characteristic scale of $V_{n_0}(\psi)$ is δ_A, while the characteristic scale of $u_{n_0}(\phi,\psi)$ with respect to ψ is $l_{\rm eq}$. As a result we have

$$Q' = \frac{\delta_A}{2h^2}\left[\left(\frac{\partial x}{\partial \psi}\right)^2 + \left(\frac{\partial z}{\partial \psi}\right)^2\right]^{1/2} \times \left(\rho_0\nu\,|u_{n_0}|^2 + \frac{\eta J^2}{\mu\omega^2}\left|\frac{\partial u_{n_0}}{\partial \phi}\right|^2\right)\int_{-\infty}^{\infty}\left|\frac{dV_{n_0}}{d\psi}\right|^2 d\tau . \tag{64}$$

Next we make use of Eqs. (44) and (45) to calculate the last integral,

$$\int_{-\infty}^{\infty}\left|\frac{dV_{n_0}}{d\psi}\right|^2 d\tau = \frac{\pi[G_{n_0}(\psi_A)]^2}{\delta_A^4\Delta^2} . \tag{65}$$

Substituting Eq. (65) into Eq. (64) and using Eq. (55) we finally obtain

$$Q' = \frac{\pi f_A^2}{2h^2\delta_A^3\Delta^2\mu^2}\left[\left(\frac{\partial x}{\partial\psi}\right)^2 + \left(\frac{\partial z}{\partial\psi}\right)^2\right]^{1/2} \times \left(\rho_0\nu\,|u_{n_0}|^2 + \frac{\eta J^2}{\mu\omega^2}\left|\frac{\partial u_{n_0}}{\partial\phi}\right|^2\right) \times \left[\left(J\frac{\partial u_{n_0}}{\partial\phi}\right)\Big|_{\phi=\phi_1}\right]^2 \left(\int_{\phi_1}^{\phi_2}\frac{\rho_0}{J}u_{n_0}^2\,d\phi\right)^{-2}. \quad (66)$$

With the help of the equation for u_{n_0} (cf. Eq. [30] with zero right-hand side) it is easy to derive the following formula

$$\bar{S} = \int_0^{s_0} Q'(s)\,ds\,, \quad (67)$$

where $s_0 = s[\phi_2(\psi_A)]$ is the length of the resonant magnetic field line. This equation tells us that the total amount of energy deposited in the dissipative layer equals the total Poynting flux through the photosphere at the arcade base, as can be expected from the physical point of view.

7. Example: potential arcade with uniform density

In this Section we apply the general theory developed in the previous sections to Alfvén oscillations in a coronal arcade described with a flux function given by Eq. (10). In order to further simplify the analysis we consider only the particular case where $l = k$, i.e. a potential arcade. Then the variables ϕ and ψ are related to x and z by

$$\phi = -e^{-kz}\cos(kx)\,, \quad \psi = -Ae^{-kz}\sin(kx)\,. \quad (68)$$

The region of variation of these variables is determined by the inequalities

$$-A < \psi < 0, \quad -\frac{1}{A}(A^2-\psi^2)^{1/2} \le \phi \le \frac{1}{A}(A^2-\psi^2)^{1/2}, \quad (69)$$

and corresponds to half an ellipse (the shaded region in Fig. 2).

The inverse transform to Cartesian variables is given by

$$x = \frac{1}{k}\left[\pi - \arccos\frac{\psi}{(A^2\phi^2+\psi^2)^{1/2}}\right], \quad (70)$$

$$z = -\frac{1}{2k}\ln\left(\phi^2 + \frac{\psi^2}{A^2}\right). \quad (71)$$

The Jacobian of this coordinate transform is

$$J = Ak^2\left(\phi^2 + \frac{\psi^2}{A^2}\right). \quad (72)$$

The function $u_n(\phi,\psi)$ has to satisfy Eq. (30) with zero right-hand side, that can be now written as

$$A^2k^4\left(\phi^2 + \frac{\psi^2}{A^2}\right)\frac{\partial}{\partial\phi}\left(\phi^2 + \frac{\psi^2}{A^2}\right)\frac{\partial u_n}{\partial\phi} + \mu\omega^2\rho_0(z)u_n = 0\,, \quad (73)$$

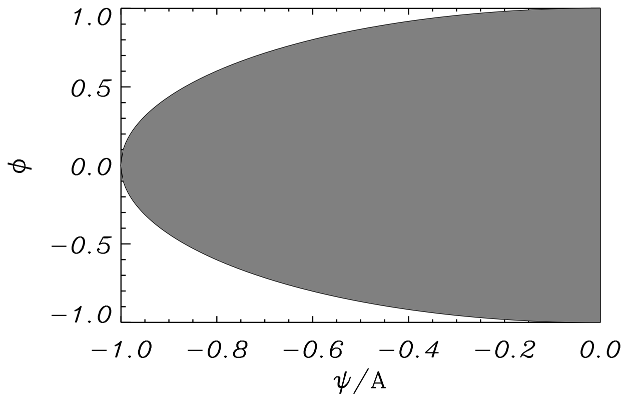

Fig. 2. The shaded region shows the range of variation of the curvilinear coordinates ϕ and ψ, determined by inequalities (69).

together with the boundary conditions

$$u_n = 0 \ \text{ at } \ \phi = \pm\frac{1}{A}(A^2-\psi^2)^{1/2}. \quad (74)$$

Let us consider ρ_0 = const, a particular choice relevant for low magnetic arcades whose height is smaller than the scale height of the solar corona, typically of the order of 10^5 km. Then the substitution

$$\phi = \frac{\psi}{A}\tan\frac{\xi\psi}{A} \quad (75)$$

reduces Eq. (73) to an equation with constant coefficients,

$$\frac{\partial^2 u_n}{\partial\xi^2} + \frac{\omega^2}{\omega_A^2}u_n = 0\,, \quad (76)$$

where

$$\omega_A^2 = \frac{A^2k^4}{\mu\rho_0}. \quad (77)$$

Boundary conditions (74) now take the form

$$u_n = 0 \ \text{ at } \ \xi = \pm\xi_0\,, \quad (78)$$

with

$$\xi_0 = \frac{A}{|\psi|}\arccos\frac{|\psi|}{A}. \quad (79)$$

Now it is straightforward to obtain the eigenvalues and eigenvectors,

$$\omega_n = \frac{\pi n\omega_A}{2\xi_0(\psi)}, \quad u_n = \sin\frac{\omega_n[\xi+\xi_0(\psi)]}{\omega_A}. \quad (80)$$

As a particular example, let us assume that the resonant condition $\omega = \omega_n(\psi)$ is satisfied for the fundamental mode ($n = 1$) at the magnetic surface $\psi = -A/2$. This condition leads to $\xi_0 = 2\pi/3$ and $\omega = 3\omega_A/4$. It is easy to show that $\xi_0(\psi)$ is a monotonically growing function, with $\xi_0(-A) = 0$ and $\xi_0(\psi) \to \infty$ as $\psi \to 0$, so that for each n the Alfvén continuum consists of the semi-infinite interval $]0,\infty[$. This and expression (80) for ω_n imply that there is a monotonically growing sequence $\{\psi_n\}_{n=1}^{\infty}$ ($\psi_1 = -A/2$) such that the resonant condition for the nth harmonic is satisfied at $\psi = \psi_n$, i.e. $\omega_n(\psi_n) = \omega$. In what follows we assume that $0 < \psi_m < -A/2 < \psi_M < \psi_2$ (let us recall that $f(\psi) = 0$ for $\psi < \psi_m$ and $\psi > \psi_M$). Then the essential part of the nth Alfvén continuum consists of the interval

$$\left[\frac{\pi n \omega_A}{2\xi_0(\psi_M)}, \frac{\pi n \omega_A}{2\xi_0(\psi_m)}\right]. \tag{81}$$

It is straightforward to see that ω is out of the essential part of all Alfvén continua but the first one, and only the first harmonic is in resonance with the driver at the resonant magnetic surface. Hence, the only eigenfunction needed for the calculation of the energy dissipated is u_1, which takes the following form at the resonant magnetic surface

$$u_1 = \cos\frac{3\xi}{4}. \tag{82}$$

It is straightforward to obtain

$$\frac{1}{h^2} = k^2(A^2\phi^2 + \psi^2) = \frac{\psi^2 k^2}{\cos^2(\xi\psi/A)}. \tag{83}$$

In accordance with the assumption that ν and η are proportional to h^2 we take

$$\nu = \nu_0 \cos^2\left(\frac{\xi\psi}{A}\right), \quad \eta = \eta_0 \cos^2\left(\frac{\xi\psi}{A}\right). \tag{84}$$

with ν_0 and η_0 constant. Then, with the use of the result

$$\Delta = \frac{4\omega^2(\pi + \sqrt{3})}{\pi A}, \tag{85}$$

we obtain

$$\delta_A = \left[\frac{\pi k^2 A(\nu_0 + \eta_0)}{4\omega(\pi + \sqrt{3})}\right]^{1/3}. \tag{86}$$

Note that δ_A is independent of ϕ. In physical space the thickness of the dissipative layer is given by

$$\bar{\delta}_A = \delta_A h = \frac{\delta_A}{k\psi}\cos\frac{\xi}{2}. \tag{87}$$

Hence the spatial thickness of the dissipative layer varies within the resonant magnetic surface, taking its maximum value δ_A/k at the top and its minimum value $(\delta_A/k)\cos(\xi_0/2) = \delta_A/2k$ at the photospheric level.

The dissipated energy density related to oscillations, $Q'(\phi)$, is given by

$$Q' = \frac{A^2 k^3 f_A^2}{\mu\omega}\bar{Q}(\xi), \tag{88}$$

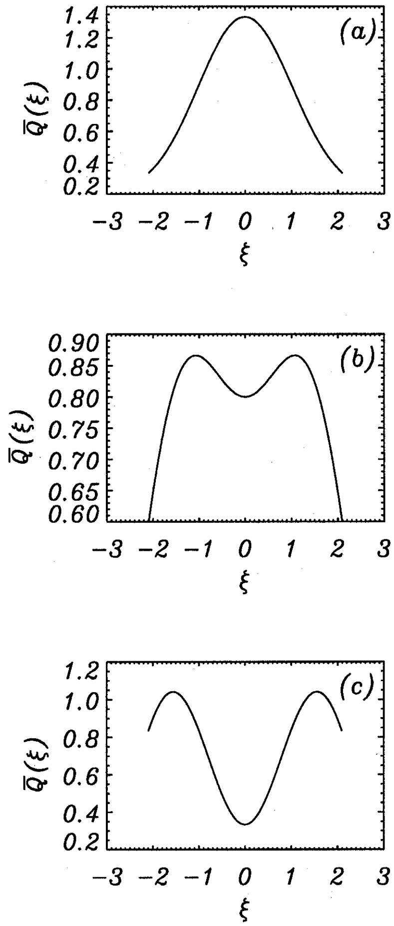

Fig. 3a–c. The function $\bar{Q}(\xi)$ for different values of η_0/ν_0. **a** $\eta_0 = \nu_0/2$, **b** $\eta_0 = 3\nu_0/2$, and **c** $\eta_0 = 5\nu_0$ respectively.

where

$$\bar{Q}(\xi) = \frac{9\cos(\xi/2)}{32(\pi + \sqrt{3})}\left(1 + \frac{\nu_0 - \eta_0}{\nu_0 + \eta_0}\cos\frac{3\xi}{2}\right) \approx 0.0577\left(1 + \frac{\nu_0 - \eta_0}{\nu_0 + \eta_0}\cos\frac{3\xi}{2}\right)\cos\frac{\xi}{2}. \tag{89}$$

The dimensionless function $\bar{Q}(\xi)$ has a different qualitative behaviour for different values of ν_0 and η_0.

i) $\eta_0 < 11\nu_0/9$. In this case the function $\bar{Q}(\xi)$ takes its minimum value $0.0577\eta_0/(\nu_0 + \eta_0)$ at $\xi = \pm\xi_0 = \pm 2\pi/3$ and its maximum value $0.115\nu_0/(\nu_0 + \eta_0)$ at $\xi = 0$. This function monotonically grows in the interval $[-2\pi/3, 0]$ and monotonically decreases in the interval $[0, 2\pi/3]$. The graph of function $\bar{Q}(\xi)$ for $\eta_0 = \nu_0/2$ is shown in Fig. 3a.

ii) $11\nu_0/9 < \eta_0 < 2\nu_0$. In this case the function $\bar{Q}(\xi)$ takes its minimum value $0.0577\eta_0/(\nu_0 + \eta_0)$ at $\xi = \pm 2\pi/3$ and its maximum value

$$\bar{Q}_M = 0.0577\chi\left[1 + \frac{\chi(3 - 4\chi^2)(\eta_0 - \nu_0)}{\nu_0 + \eta_0}\right]$$

at $\xi = \pm\xi_M$, where $\xi_M = 2\arccos\chi$ and the quantity χ is the only positive root of the equation

$$16\chi^3 - 6\chi - \frac{\nu_0 + \eta_0}{\eta_0 - \nu_0} = 0 \,. \tag{90}$$

The function $\bar{Q}(\xi)$ monotonically grows in the intervals $[-2\pi/3, -\xi_M]$ and $[0, \xi_M]$, and monotonically decreases in the intervals $[-\xi_M, 0]$ and $[\xi_M, 2\pi/3]$. The graph of this function for $\eta_0 = 3\nu_0/2$ is shown in Fig. 3b.

iii) $\eta_0 > 2\nu_0$. In this case the function $\bar{Q}(\xi)$ takes its minimum value $0.115\nu_0/(\nu_0 + \eta_0)$ at $\xi = 0$ and its maximum value $\bar{Q}_M$ at $\xi = \pm\xi_M$. This function increases monotonically in the intervals $[-2\pi/3, -\xi_M]$ and $[0, \xi_M]$, and monotonically decreases in the intervals $[-\xi_M, 0]$ and $[\xi_M, 2\pi/3]$. The graph of the function $\bar{Q}(\xi)$ for $\eta_0 = 5\nu_0$ is shown in Fig. 3c.

We see that the distribution of the dissipated energy along the dissipative layer strongly depends on the ratio η_0/ν_0, which may have important implications for the heating of the solar corona by Alfvén wave energy dissipation.

Finally, the total amount of energy deposited in the dissipative layer is

$$\bar{S} = \frac{3\pi A^2 k^2 f_A^2}{16\mu\omega(\pi + \sqrt{3})} \,. \tag{91}$$

8. Conclusions

In the present paper we have considered resonant shear Alfvén waves in coronal arcades. Coronal arcades have been modelled by planar, two-dimensional magnetic plasma configurations with a purely poloidal magnetic field. Solutions to the viscous, resistive MHD equations that describe the steady state of resonant Alfvén oscillations driven by toroidal footpoint motions have been obtained. In order to derive these solutions we have assumed that the viscous and magnetic Reynolds numbers are very large, as is the case in the solar corona, and have then used the method of matched asymptotic expansions, by which viscosity and resistivity are only taken into account in a narrow dissipative layer embracing the ideal resonant magnetic surface, while the ideal MHD equations are used far away from the resonant magnetic surface. General expressions for the total amount of dissipated wave energy and for its spatial distribution within the resonant magnetic surface have been derived. It turns out that the latter quantity depends on the ratio of the viscosity coefficient to the magnetic diffusion coefficient. In contrast, the total amount of dissipated energy is independent of this parameter.

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