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The maximum rotation of a galactic disc

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Abstract. The observed stellar velocity dispersions of galactic discs show that the maximum rotation of a disc is on average 63% of the observed maximum rotation. This criterion can, however, not be applied to small or low surface brightness (LSB) galaxies because such systems show, in general, a continuously rising rotation curve until the outermost measured radial position. That is why a general relation has been derived, giving the maximum rotation for a disc depending on the luminosity, surface brightness, and colour of the disc. As a physical basis of this relation serves an adopted fixed mass-to-light ratio as a function of colour. That functionality is consistent with results from population synthesis models and its absolute value is determined from the observed stellar velocity dispersions. The derived maximum disc rotation is compared with a number of observed maximum rotations, clearly demonstrating the need for appreciable amounts of dark matter in the disc region and even more so for LSB galaxies. Matters have been illustrated for two examples; the galaxy NGC 6503 and LSB galaxy NGC 1560.

Key words: galaxies: general – galaxies: kinematics and dynamics – galaxies: luminosity function, mass function – galaxies: spiral

1. Introduction

Rotation curves derived from neutral hydrogen observations at the outer regions of spiral galaxies unambiguously show that substantial amounts of dark matter are required (Bosma 1978; Begeman 1987, 1989). Any physically reasonable distribution of this dark matter necessitates the presence of at least some of that in the inner optical disc region, contributing in some degree to the total rotation in that region. Unfortunately, from the observed rotation curve and light distribution one cannot a priori determine the ratio of dark to luminous matter (van Albada et al. 1985). This means that the M/L ratio of the disc cannot be determined from a rotation curve analysis only. There are arguments which might lead to the so called "maximum disc hypothesis" (hereafter: md hypothesis; van Albada & Sancisi 1986; Freeman 1992), favouring a maximum possible rotational contribution of the disc. However, no hard evidence exists proving this hypothesis. To determine not only the amount of dark matter in a galaxy, but also to constrain mass models of dark matter and galaxy formation scenarios (Katz & Gunn 1991), it is of the utmost importance to know the relative rotational contribution of the disc.

Stellar velocity dispersions provide a direct measure of the local surface density of a disc and from that the disc rotation can be calculated. For a sample of 12 disc-dominated galaxies such dispersions have been measured. Results are summarized, discussed and analysed by Bottema (1993, hereafter B93). It appears that the magnitude of the stellar velocity dispersions, both in the radial and vertical direction, is proportional to the square root of the surface density. That can be explained when for a stellar disc, a constant M/L ratio is combined with the observed constancy of the scaleheight as a function of radius (van der Kruit & Searle 1981a,b, 1982). To compare the stellar kinematics of different galactic discs the dispersion was parameterized by fitting a radial relation to the observations and taking the dispersion value at, for instance, one scalelength. Comparison of the inclined and face-on systems showed that the ratio of vertical to radial dispersion is close to 0.6, as is observed in the solar neighbourhood. Moreover, larger and more massive discs have larger velocity dispersions. These matters are illustrated in Fig. 1, for the sample of 12 galaxies with observed dispersions.

For an exponential disc a simple relation can be derived (Freeman 1970; B93) between the maximum rotation of a disc and the vertical velocity dispersion $\langle v_z^2 \rangle^{1/2}$:

$$v_{\rm max}^{\rm disc} = 0.88 \; \langle v_z^2 \rangle_{R=0}^{1/2} \sqrt{\frac{h}{z_0}}, \tag{1}$$

where the maximum is reached at a radius of $2.2h$. This relation involves only the radial scalelength (h) and the vertical sech^2 scale parameter z_0 (van der Kruit & Searle 1981a; see also Eq. 26), which is approximately equal to twice the exponential scaleheight for suitable distances above the plane. The observed dispersions then allow the calculation of the rotation which the disc can supply to the total galactic rotation. For a reasonable h/z_0 behaviour it appears that the maximum disc contribution is 63% ± 10%, roughly independent of the mass of a galaxy. The missing rotation then has to be supplied by the

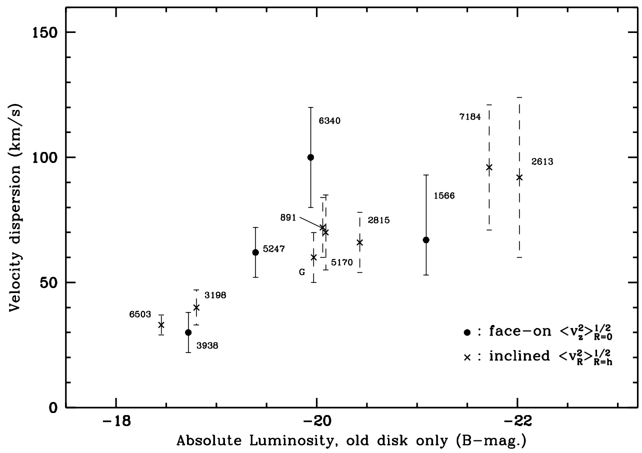

Fig. 1. Observed disc stellar velocity dispersions of a sample of 12 spiral galaxies (B93) as a function of the absolute luminosity of the old disc population (see Sect. 2). The disc dispersion is parameterized in such a way that for $\langle v_z^2\rangle^{1/2}/\langle v_R^2\rangle^{1/2} = 0.6$ data for face-on and inclined systems should fall on the same relation; as appears to be the case. Obviously, the brighter and more massive galaxies have larger velocity dispersions.

dark halo (and the bulge, if present). Note that this 63% criterion does not depend on the colour and surface brightness of the disc. When applying the md hypothesis the disc contributes at 2.2 scalelengths between 85 to 90% of the observed maximum rotation. The value following from the observed velocity dispersion is considerably lower, which leads to disc masses and M/L ratios being a factor of two smaller. Nevertheless, with the 63% contribution, the disc is still dominant in the inner regions.

Ideally if one would like to know the disc rotational contribution one should take a spectrum of the galaxy and determine the velocity dispersion of the disc. That is, unfortunately, quite difficult and time consuming. How then to determine quickly the amount of disc rotation and disc mass? For a "normal" galaxy or sample with normal galaxies the 63% criterion can safely be assumed although with an error of 10%. Here normal means comparable to the sample in B93 with $v_{\rm max}^{\rm obs} \gtrsim 100$ km s^{-1}, B-V around 0.7 and central surface brightness (μ_0) close to the value of Freeman's (1970) law; $\mu_0 = \mu_{0,F} = 21.65$ B-mag. arcsec^{-2}. Problems arise for galaxies which are small, faint, or blue. Small galaxies are often faint, meaning having a low surface brightness (LSB) although occasionally also large LSB galaxies are found (de Blok & McGaugh 1996). Such small and/or faint galaxies generally have rotation curves which do not reach a maximum velocity over the observed radial extent (Casertano & van Gorkom 1991). Obviously the 63% criterion then has no meaning, but could the description be generalized in some way?

Most of the light of very blue galaxies originates from a young stellar population which has a negligible mass and consequently this light is not representative for the massive stellar population which determines the velocity dispersion. Therefore a population correction is needed when comparing velocity dispersion with the brightness of a galaxy. This was dealt with in B93 by assigning a so-called "old disc population" absolute magnitude ($M_{\rm od}$) to a disc. An extensive description of this procedure is given in Sect. 2 such that for a galaxy with arbitrary colour the disc rotational contribution can be determined.

The remainder of this paper deals with the construction of a general description for the maximum rotation of a galactic disc. To that aim in Sect. 2 a simple but adequate population correction procedure is discussed. Section 3 analyses the situation for a single colour, fixed central surface brightness disc and in Sect. 4 this is generalized for an arbitrary disc. The inferred mass-to-light ratio is calculated in Sect. 5, and Sect. 6 describes two examples: the normal spiral NGC 6503 and LSB galaxy NGC 1560. Finally in Sect. 7 the method and its applicability is discussed and conclusions are formulated. Throughout a Hubble constant of 75 km s^{-1} Mpc^{-1} is adopted.

2. The old disc population

Velocity dispersions measure the local mass density in the disc. To make any comparison between dynamical quantities derived from the dispersion and the emitted light one would like a reasonable indication of the light emitted by the population that contains nearly all the mass in the disc. In principle, the light of any young, massless population should be subtracted. A mass-to-light ratio $(M/L)_{\rm od}$ can then be assigned to the remaining old disc population and the principal assumption for the remainder of this paper will be that this $(M/L)_{\rm od}$ is the same for all galactic discs. This is equivalent to assuming an equal M/L ratio for galaxies having the same colour. Such an assumption is perfectly reasonable and several arguments for this are given in B93. The basic underlying hypothesis is that for the low mass stars the IMF is the same for all discs and that the range of metallicities is not too broad. Although there is no proof that the low mass end of the IMF is universal, there is certainly no indication for the contrary (Laughlin & Bodenheimer 1993; Wyse 1995). To obtain the luminosity of the old disc, the bulge light, of course, also has to be subtracted from the total light of the galaxy.

In Bottema (1988) a so called "poor man's" population synthesis (pmps) was performed to treat the problem of colour gradients in the disc of NGC 3198. This pmps has been applied to the galaxy sample in B93 and will presently be described and discussed in detail. A galactic disc is assumed to consist of only two stellar populations; an old disc population and a young disc population defined as:

The old disc population:

- contains all the mass in a disc.
- has B-V = 0.97

The young disc population:

- contains no mass.
- has B-V = -0.03.

Using the observed B-V colour the amount of light from each component can be determined in the B or the V band. This is illustrated in Fig. 2, where as a function of B-V colour the ratio

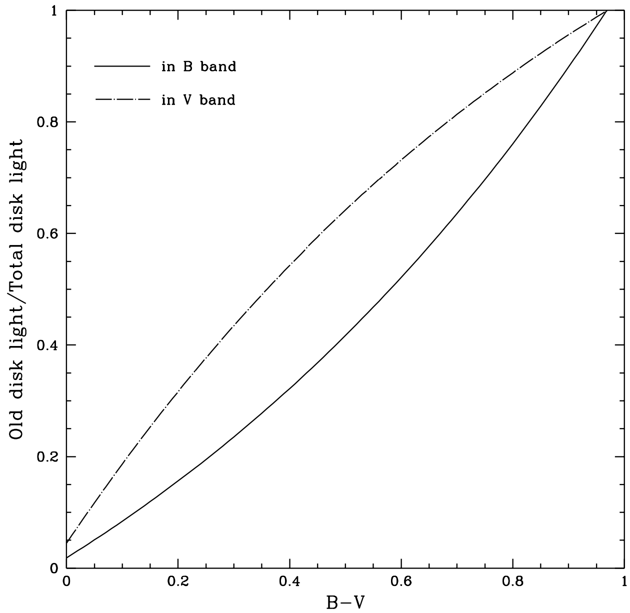

Fig. 2. The proportion of the light of the old stellar, mass containing, population for different B-V colours.

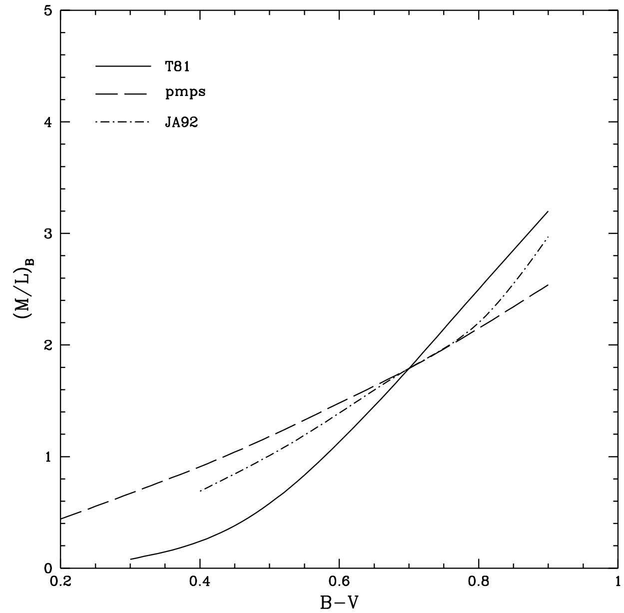

Fig. 3. Total mass-to-light ratio (in B) versus B-V colour according to Tinsley (T81), the described poor man's population synthesis (pmps), and according to Jablonka & Arimoto (JA92). The curves have been scaled to coincide at $(M/L)_B$ = 1.79 for B-V = 0.7. Despite its simplicity, the predicted mass-to-light ratio for the pmps is similar to that of the sophisticated population synthesis models.

of old disc to total disc light is presented. For example, for B-V = 0.6, 52% of the light in the B-band originates from the old disc population. The absolute magnitude of the old disc population in the B-band ($M^B_{\rm od}$) is related to the total magnitude in B ($M^B_{\rm tot}$) as

$$M^B_{\rm od} = M^B_{\rm tot} - ct, \tag{2}$$

with the correction term ct given by

$$ct = 2.5 \log_{10} \left[\frac{1 - 0.973W}{1.470W} \right], \tag{3}$$

and

$$W = 10^{-0.4(B-V)}. \tag{4}$$

This treatment has its shortcomings. A preferable complete population synthesis, however, is much more complicated (Larson & Tinsley 1978; Bruzual & Charlot 1993; Worthey 1994), both in handling such models and in applying it to the present specific situation. Even then effects of dust and metallicity are not or only partially included. In Sect. 7 the influence of particularly these two parameters on the employed pmps is investigated. It appears that a good assessment of the effects can be made; which counteract one another and are individually always below a 15% level for the galaxies of interest.

The pmps gives a total mass-to-light ratio $(M/L)_B$ proportional to $10^{0.4ct}$, and after stellar velocity dispersions are compared with the luminosity of galaxies in the following sections, the absolute scale of the mass-to-light ratio is fixed at

$$(M/L)_B = 2.84\ 10^{0.4ct} = 1.93\ 10^{0.4(B-V)} - 1.88, \tag{5}$$

in solar units. This can be compared with predictions of stellar population models. Unfortunately such models cannot predict the absolute scale of the mass-to-light ratio, only the functionality with colour. This is caused by the uncertainty of the IMF at the low mass end. For instance, $M/L \propto m_l^{1-x}$, proportional to the low mass cutoff (m_l) to the power $1-x$, where x = 1.35 for a Salpeter (1955) IMF. This means that mass-to-light ratios can be increased simply by adding more low mass stars. The uncertainty in absolute M/L ratio is even more increased because M/L ratios derived from observations are proportional to the adopted Hubble constant. Hence M/L ratios can only be compared differentially and presently values will be fixed at $(M/L)_B$ = 1.79 for B-V = 0.7 as given by Eq. (5). Results of the pmps are compared with population synthesis models of Tinsley (1981, hereafter T81) and those of Jablonka & Arimoto (1992, hereafter JA92). Presented in Fig. 3 is a comparison of the B-band mass-to-light ratio as a function of B-V, for the pmps, T81 and JA92. In all cases $(M/L)_B$ is increasing towards redder colours, though for the T81 models more steeply than the others. It is striking that, despite its simplicity, the pmps shows a nearly identical trend as the other more sophisticated models. This supports the applicability of the method.

The present paper deals with the amount of mass in a galactic disc, which is given by the observed velocity dispersions. Therefore it is possible to fix the absolute scale of the mass-to-light ratio when comparing dispersions with luminosities and colours. To that aim a few relations will be derived for an exponential disc, leading to a relation which can eventually be compared with the observed dispersions in Fig. 1. For an exponential old disc

$$L_{\rm od} = 2\pi(\mu_0)_{\rm od} h^2, \tag{6}$$

$$\sigma_0 = (\mu_0)_{\rm od} \left(\frac{M}{L}\right)_{\rm od}, \tag{7}$$

and

$$v_{\rm max,od} = v_{\rm max}^{\rm disc} = 0.88\ \sqrt{\pi G \sigma_0 h}, \tag{8}$$

(Freeman, 1970) such that

$$v_{\rm max}^4 = 0.3\pi G^2 (\mu_0)_{\rm od} \left(\frac{M}{L}\right)^2_{\rm od} L_{\rm od}, \tag{9}$$

where $L_{\rm od}$ is the total luminosity of the old disc, $(\mu_0)_{\rm od}$ the central surface brightness of the old disc in linear units e.g. $L_\odot {\rm pc}^{-2}$, σ_0 the central surface density and h the scalelength. Equation (9) holds exactly and as noted above, $(M/L)_{\rm od}$ is considered a universal constant.

3. All discs have the same colour, B-V = 0.7, and obey Freeman's law

For such a situation Eq. (9) can be written as

$$M_{\rm od} = -10\ \log_{10}(v_{\rm max}^{\rm disc}) + P, \tag{10}$$

which is a kind of Tully-Fisher relation. It appears that for an exponential old disc this TF relation holds with a coefficient of exactly 10. Or,

$$v_{\rm max}^{\rm disc} = 10^{0.1P} \cdot 10^{-0.1 M_{\rm od}}. \tag{11}$$

For an exponential disc the maximum rotation (at $2.2h$) is related to the vertical velocity dispersion through Eq. (1), which, when combined with Eq. (11) gives

$$\langle v_z^2 \rangle^{1/2}_{R=0} = A^{-1} \sqrt{\frac{z_0}{h}}\ 10^{-0.1 M_{\rm od}}. \tag{12}$$

This relation is equal to Eq. (19) in B93 for

$$A = 0.88\ 10^{-0.1P}. \tag{13}$$

For Eq. (12) a fit can be made to the observed velocity dispersions as a function of $M_{\rm od}$ by chosing a certain h/z_0 behaviour. In B93 three choices of this behaviour are presented: one where h/z_0 is constant at five, secondly a functionality such that the dispersion versus luminosity relation (Fig. 1) becomes linear, and thirdly, an intermediate situation where $h/z_0 = 0.6M_{\rm od} + 17.5$. All three give a satisfactory fit to the dispersion data yielding a disc TF relation (Eq. 10) with almost the same constant P. Still, the last behaviour is preferred. This is because then the h/z_0 value is somewhat larger for the smaller galaxies as might have been observed (Bottinelli et al. 1983; Heidmann et al. 1972). In addition, the fit to the observed dispersions is marginally better than for the $h/z_0 = 5$ (van der Kruit & Searle 1981a, b, 1982) case (see Fig. 8 in B93). A linear dispersion versus $M_{\rm od}$ relation leads to the undesired property that for the least massive galaxies the velocity dispersion in the disc becomes negative.

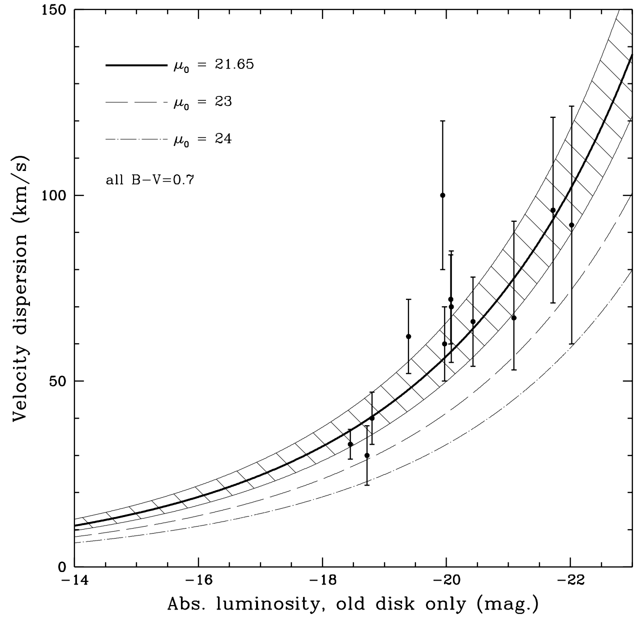

Fig. 4. The observed velocity dispersion values of Fig. 1. Given by the solid line and shaded area is a fit to these data of Eq. (12) for the adopted h/z_0 behaviour. The data are for galactic discs with on average $\mu_0 \sim 21.65$ B-mag. arcsec^{-2} and B-V = 0.7. Also indicated are the expected dispersion functionalities for lower surface brightness galaxies with μ_0 = 23 and 24 B-mag. arcsec^{-2}.

Observations of h/z_0 values are scarce and it is not a priori predictable if and how h/z_0 is related to galaxy size. Therefore, at the moment, with the limited information available, the best suited linear functionality is adopted. Individual deviations in h/z_0 values will certainly be the largest source of scatter in any diagram of velocity dispersion versus galaxy size.

Taking $h/z_0 = 0.6M_{\rm od}+17.5$, Eq. (12) can now be compared with the observed dispersions. This has been done in Fig. 8b in B93 and presently in Fig. 4. The best fit is achieved for $A = 0.75$. By eye an error estimate has been made which is shown in Fig. 4 as the shaded area around the best fit for $A = 0.75 \pm 0.1$. From this a $(M/L)_B$ ratio was derived for B-V equal to 0.7 of 1.79 ± 0.48. Substituting the value found for A into Eq. (13) and Eq. (11) one gets

$$v_{\rm max}^{\rm disc} = (1.17 \pm 0.16)\ 10^{-0.1 M_{\rm od}}, \tag{14}$$

providing the maximum rotation of a single colour (B-V = 0.7) galactic disc where $M_{\rm od}$ is given by Eq. (2) as $M_{\rm od} = M_{\rm tot} + 0.5$. In Fig. 5 this maximum rotation of the disc only is compared with the observed maximum rotation for the total galaxy. The observations are for optical emission line rotation curves of a sample of Sc and Sb galaxies by Rubin et al. (1985) and of the galaxies by Mathewson et al. (1992). For the latter absolute B magnitudes were obtained from the ESO-LV catalogue (Lauberts & Valentijn 1989) by Rhee (1996). The shaded area shown in Fig. 5 corresponds to the error given by the shading in Fig. 4. It is obvious, comparing the disc-only TF relation (Eq. 14) with the data in Fig. 5 that the maximum rotation of the disc is considerably lower than the observed maximum rotation. In fact, this is the 63% criterion cast into a Tully-Fisher representation. This now provides the possibility to investigate the consequences for a galactic disc with less restricted parameters.

4. Discs with different colours and different central surface brightnesses

To investigate this general case Eq. (9) is rewritten to

$$v^4_{\rm max} = 0.3\pi G^2 \mu_{0,F} \frac{(\mu_0)_{\rm od}}{\mu_{0,F}} \left(\frac{M}{L}\right)^2_{\rm od} L^{\rm od}, \qquad (15)$$

where $\mu_{0,F}$ is Freeman's value which is constant (by definition). Use

$$\frac{\mu_0^{\rm od}}{\mu_0} = \frac{L_{\rm od}}{L_{\rm tot}} = 10^{0.4ct}, \qquad (16)$$

and one finds

$$v^{\rm disc}_{\rm max} = {\rm const} * \left(\frac{\mu_0}{\mu_{0,F}}\right)^{1/4} 10^{0.1ct}\ 10^{-0.1M_{\rm od}}. \qquad (17)$$

Define the average colour correction term $\langle ct \rangle$ for the sample for which dispersions have been measured. The sample has galaxies with colours all close to B-V = 0.7 so that $\langle ct \rangle \sim -0.5$. Insert this and Eq. (1) into Eq. (17) to get

$$\langle v^2_z \rangle^{1/2}_{R=0} = \frac{\rm const}{0.88} 10^{0.1\langle ct \rangle} \sqrt{\frac{z_0}{h}} \frac{10^{0.1ct}}{10^{0.1\langle ct \rangle}} \cdot \left(\frac{\mu_0}{\mu_{0,F}}\right)^{1/4} 10^{-0.1M_{\rm od}}, \qquad (18)$$

which can in principle again be fitted to Bottema's sample of galactic disc dispersion measurements. For these $\mu_0 \sim \mu_{0,F}$ and $ct \sim \langle ct \rangle = -0.5$. Adopt again $h/z_0 = 0.6M_{\rm od} + 17.5$, and one finds for the same fit to the same data that:

$${\rm const} * 10^{0.1\langle ct \rangle} = 1.17. \qquad (19)$$

Substitute back into Eq. (17) to find:

$$v^{\rm disc}_{\rm max} = 1.17 \left(\frac{\mu_0}{\mu_{0,F}}\right)^{1/4} 10^{0.1(ct-\langle ct \rangle)}\ 10^{-0.1M_{\rm od}}. \qquad (20)$$

Once more $M_{\rm od}$ can be converted to observed absolute magnitudes

$$v^{\rm disc}_{\rm max} = 1.17 \left(\frac{\mu_0}{\mu_{0,F}}\right)^{1/4} 10^{-0.1M^B_{\rm tot}} \cdot 10^{0.2(ct+0.25)}, \qquad (21)$$

which is the most general expression for the maximum rotational velocity of a disc and hence the principal result of this paper.

The stellar velocity dispersion of a disc is found by substituting Eq. (19) into Eq. (18) to get

$$\langle v^2_R \rangle^{1/2}_{R=0} = 1.33 \sqrt{\frac{z_0}{h}} \left(\frac{\mu_0}{\mu_{0,F}}\right)^{1/4} 10^{0.1(ct-\langle ct \rangle)} \cdot 10^{-0.1M_{\rm od}}. \qquad (22)$$

This relation is shown in Fig. 4 for B-V = 0.7 and μ_0 = 23 and 24 mag. arcsec^{-2}. Different B-V values have not been plotted, to avoid confusion, but the result of any preferred colour - surface brightness combination can be inferred from Eq. (22). Fig. 4 shows that the LSB discs have lower stellar velocity dispersions than normal discs with the same luminosity. However, this is only valid for an isolated stellar disc. For small and/or LSB discs, for example, there may be large quantities of gas available, which will increase the dispersion. Also a dark halo will increase the stellar velocity dispersion in the outer parts of the disc (B93). The extrapolation to $M_{\rm od} > -18$ in Fig. 4 is for the adopted behaviour of h/z_0 as a function of luminosity. For a different behaviour the result will, of course, be different.

At this stage the implications of the general disc-only TF relation (Eq. 21), will be investigated. For $\mu_0 = \mu_{0,F}$ and B-V = 0.7, Eq. (14) is retrieved as a special case which is already depicted in Fig. 5 where disc-only rotations are compared with total observed rotations of a galaxy. In addition in Fig. 5 Eq. (21) is plotted for B-V = 0.7 and μ_0 = 23 and 24 mag. arcsec^{-2} and for B-V = 0.4 with μ_0 = 24; the regime of low surface brightness galaxies. For such objects the disc-only rotation is at lower velocities than that of the normal surface brightness discs and at even lower velocities than given by the observations. Since LSB galaxies seem to follow the same observed TF relation as non LSB galaxies (Zwaan et al. 1995) it can thus be concluded that LSB discs contain a larger fraction of dark matter than normal discs.

5. The mass-to-light ratio

For an exponential disc:

$$\langle v^2_z \rangle^{1/2}_{R=0} = \sqrt{\pi G \mu_0 \left(\frac{M}{L}\right)_{\rm tot} z_0}. \qquad (23)$$

If we substitute Eq. (22) into Eq. (23) and eliminate the dispersion one gets after some algebra and unit conversions:

$$\left(\frac{M}{L}\right)_B = 28.1\ \ 10^{-0.2M^{\odot}_B}\ 10^{0.4ct} 10^{-0.2\langle ct \rangle}. \qquad (24)$$

Where $M^{\odot}_B$ is the absolute magnitude of the sun. For $M^{\odot}_B$ = 5.48 (Allen 1973) Eq. (24) changes to

$$\begin{aligned} \left(\tfrac{M}{L}\right)_B &= 1.79\ \ 10^{0.4ct}\ 10^{0.2(0.5-\langle ct \rangle)} \\ &= 1.93\ \ 10^{0.4(B-V)} - 1.88, \end{aligned} \qquad (25)$$

being equal to the result already given in Eq. 5. This shows that the observed dispersions actually fix the mass-to-light ratio of the stellar population in an absolute sense. There is a small

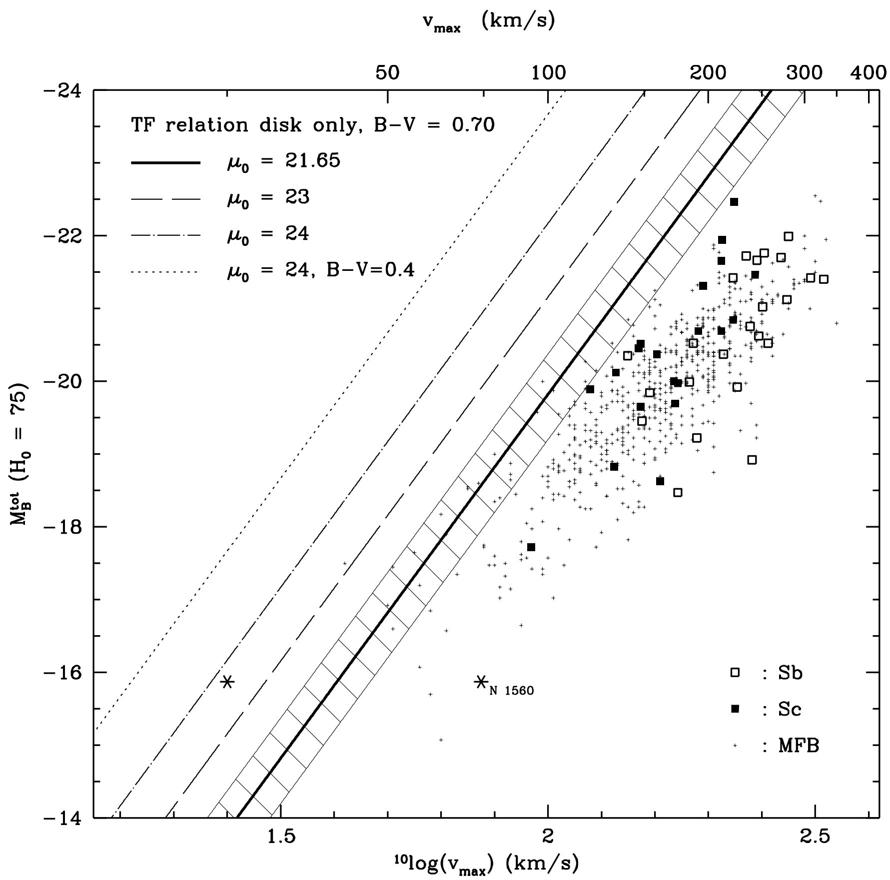

Fig. 5. Representation of the TF relation for observed maximum rotations given by the squares and plusses, and for the disc only maximum rotation, as implied by the measured disc velocity dispersions, given by the lines. Observational rotations are derived from optical emission lines for Sb and Sc galaxies by Rubin et al. (1985) and for a sample of late type galaxies by Mathewson et al. (1992, MFB). The disc-only maximum rotation follows from Eq. (21) and is shown for a few central surface brightness and colour combinations; the shaded area corresponds to the shading in Fig. 4. The disc-only has considerably lower rotational velocities than observed for the total galaxy, a difference becoming even more pronounced for the lower surface brightness systems. As an example, for NGC 1560, the disc-only and total rotation is indicated by the left and right asterisk respectively.

discrepancy with B93 because in that paper $M_B^{\odot}$ = 5.41 was used (Allen 1963) leading to a coefficient of 1.85 instead of 1.79 in Eq. (25). Therefore now for $B - V$ = 0.7, the value used for the one colour, one brightness disc in B93, one finds $ct = -0.5$ and $\langle ct \rangle = -0.5 \Rightarrow (M/L)_B = 1.79$. Note that the mass-to-light ratio of the general disc does not depend on the central surface brightness of the disc. This is not surprising since one of the assumptions was that the old disc population has the same mass-to-light ratio for all discs. Hence discs with equal colours also have equal total mass-to-light ratios irrespective of the brightness.

6. Two examples: NGC 6503 and NGC 1560

To get a feeling for the implications of the results derived, two specific examples will be discussed. For two galaxies for which detailed and well resolved rotation curves have been measured, a decomposition will be made of these curves into the contributions of the galactic constituents. This is done for the maximum disc hypothesis situation and for the disc contribution determined by velocity dispersions and colour as given in Eq. (21).

The galaxy is supposed to consist of three components. First, a disc with density distribution $\rho(z)$ as

$$\rho(z) = \rho(z = 0)\ \mathrm{sech}^2 \left(\frac{z}{z_0} \right), \tag{26}$$

with thickness parameter z_0 being equal to 1/6 radial scalelength. The radial density distribution was proportional to the observed radial photometric profile. A rotation curve was calculated according to Casertano (1983). Secondly, a thin gas layer with surface density proportional to the observed radial H I density profile, multiplied with a factor 1.4 to account for Helium. Thirdly, disc and gaslayer are embedded in a spherical pseudo isothermal dark halo (Carignan & Freeman 1985) with rotation curve

$$v_{\rm halo}(R) = v_{\rm max}^{\rm halo} \sqrt{1 - \frac{R_{\rm core}}{R} \arctan \left(\frac{R}{R_{\rm core}} \right)}. \tag{27}$$

A least-squares fit is made of the sum of the individual contributions to the observed rotation and best fitting parameters $v_{\rm max}^{\rm halo}$ and $R_{\rm core}$ are determined.

The examples are NGC 6503 and NGC 1560; the first being a normal surface brightness galaxy of moderate size and the second a typical example of an LSB galaxy with μ_0^B = 23.23 mag. arcsec^{-2}. A number of relevant galaxy parameters and results of the fit are given in Table 1. Figs. 6 and 7 show the results for NGC 6503 and NGC 1560 respectively. For the md hypothesis case and velocity dispersion implied case the fits are equally valid. But there are appreciable differences in the parameters of the individual components. In the case where the disc mass is based on dispersions, the disc is less massive and core radius and asymptotic halo velocity are smaller than for the

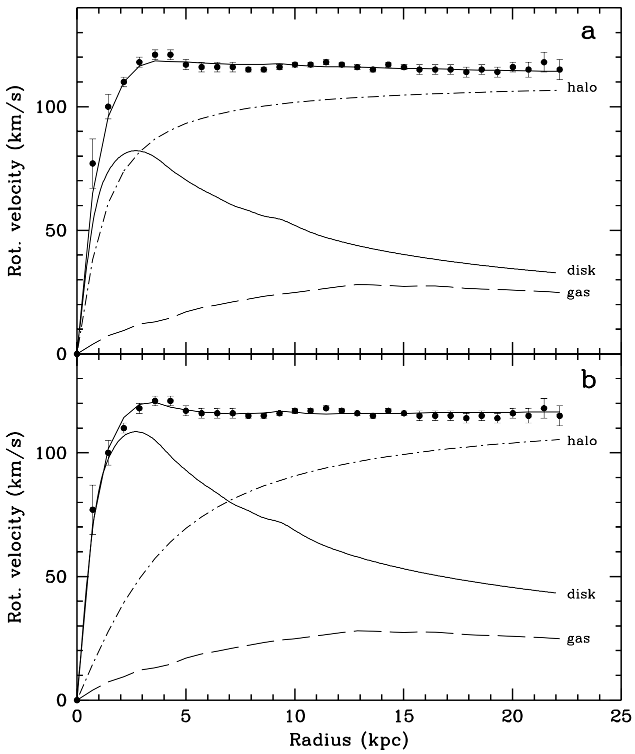

Fig. 6a and b. Rotation curve decomposition for NGC 6503. The dots are the observed values and upper solid line is the fit to these for: **a** disc mass and rotation implied by stellar velocity dispersions (Eq. 21) and **b** The maximum disc hypothesis.

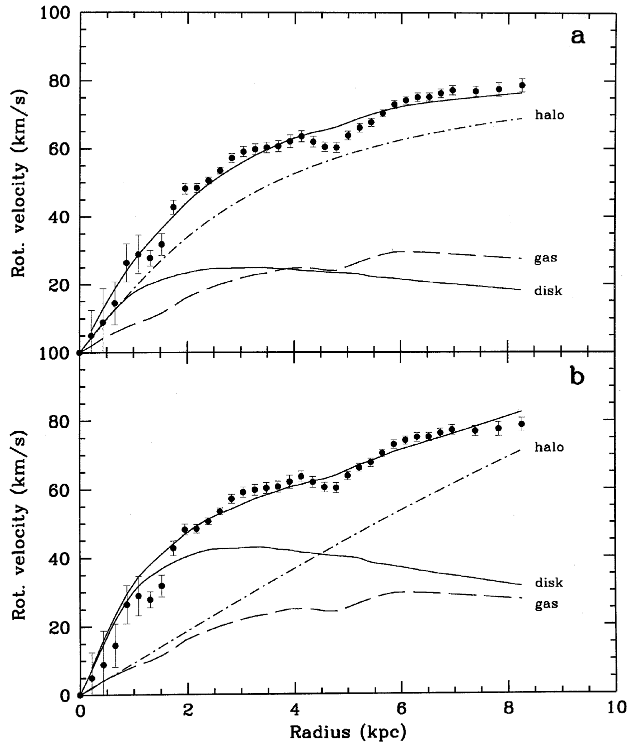

Fig. 7a and b. As Fig. 6, but now for the LSB galaxy NGC 1560.

md hypothesis situation. This implies that there is approximately a factor of two more dark matter present in the disc region, a conclusion which also applies for galaxies other than the two discussed here.

NGC 6503 offers the unique opportunity to compare maximum velocities of a disc derived in three different ways because it is in the sample for which dispersions have been measured. Based on Eq. (21) a $v_{\rm max}^{\rm disc}$ of 82 ± 11 km s^{-1} is found. According to the 63 % criterion ($\pm$ 10%) one finds $v_{\rm max}^{\rm disc} = 76 \pm 8$ km s^{-1} and the maximum velocity of the disc can be calculated directly from the observed dispersions using Eq. (1) with $h/z_0 = 6$ and $\langle v_R^2 \rangle_{R=h}^{1/2} = \langle v_z^2 \rangle_{R=0}^{1/2} = 33 \pm 4$ km s^{-1} such that $v_{\rm max}^{\rm disc} = 71 \pm 9$ km s^{-1}. The three results agree and show which errors can be expected.

For NGC 1560, $v_{\rm max}^{\rm disc}$ based on dispersion is 25 km s^{-1} which is plotted in the TF relation in Fig. 5. The observed maximum H I rotation is 78 km s^{-1} and optically one would obtain at $\sim 4\frac{1}{2}$ h a maximum velocity of 72 km s^{-1}. The average of these two is also given in Fig. 5 representing the observed maximum rotation. The distance between disc only and observed data point then gives a graphical representation of the amount of dark matter. It illustrates nicely the dominance of that component in LSB galaxies.

7. Discussion and conclusions

The developed description for maximum disc rotational velocities (Eq. 21) can be applied for all discs, no matter the form of the observed rotation curve. Furthermore, an additional value of the method is that it has a physical basis. Namely the equal mass-to-light ratio for stellar populations having the same colour. This mass-to-light ratio is gauged by the observed velocity dispersion of the sample of galactic discs. The present description is unlike that of the md hypothesis, which is purely ad-hoc, or the 63% criterion which is established observationally.

However, there are some disadvantages of the method. To calculate the maximum velocity of the disc according to Eq. (21), one needs to know the absolute magnitude, central surface brightness, and the colour of the disc. The absolute magnitude depends on the distance and galactic and internal absorption correction. If these are ill determined appreciable errors will result. To obtain the central surface brightness a correction to face-on is needed, also a source of errors. In addition the colour can never be determined with infinite accuracy. Fortunately none of the parameters enters into Eq. (21) in a dominant manner so that some error will not directly generate a huge error in the maximum rotational velocity of the disc.

The poor man's population synthesis as described in section 2, applies for a metallicity regime not too far from solar abundances. For the majority of the nearby high surface brightness (hereafter HSB; meaning $\mu_0 \sim \mu_{0,F}$) galactic discs the metallicity can indeed be assumed to ly close to solar. However, for the LSB galactic discs abundance determinations

Table 1. Parameters of the galaxies

	NGC 6503	NGC 1560	Ref.
$(B-V)^o_{\rm T}$	0.57	0.57	RC3/RC3, but corrected
Distance (Mpc)	6.0	3.0	Bottema (1989)/ Broeils (1992)
$M_B^{o,i}$ (mag)	-18.76	-15.87	Sandage & Tammann (1981)/ Broeils (1992)
μ_0^B (mag. arcsec^{-2})	20.9	23.23	Wevers et al. (1986) + Bottema (1989)/ Broeils (1992)
ct	-0.78	-0.78	Eq. (3)
$v_{\rm max}^{\rm disc}$ (km s^{-1})	82	25	Eq. (21)
$M_{\rm disc}$ (10^9 $M_\odot$)	5.42	0.52	calculated
$R_{\rm core}^{\rm halo}$ (kpc)	1.08	2.63	lsq fit
$v_{\rm max}^{\rm halo}$ (km s^{-1})	111	89	lsq fit
$v_{\rm max}^{\rm disc}$ md hyp. (km s^{-1})	108	43	lsq fit
$M_{\rm disc}$ md hyp. (10^9 $M_\odot$)	9.46	1.53	calculated
$R_{\rm core}^{\rm halo}$ md hyp. (kpc)	3.37	15	lsq fit
$v_{\rm max}^{\rm halo}$ md hyp. (km s^{-1})	119	243	lsq fit

(Mc Gaugh 1994) indicate a metallicity content of typically 0.3 to 0.1 times the solar values. This could pose a problem and therefore the effect of a lower metallicity on the pmps will be investigated. Less metals in the same stellar population with the same age produce a bluer B-V colour. For a solar abundance a B-V of 0.97 for the old disc population (5 to 10 Gyrs) was assumed in section 2. For a metal poorer od population the colour can be determined from Worthey (1994, his fig 34). Assuming a worst case scenario with $Z = 0.1Z_\odot$ the od B-V colour has to be decreased to $\sim$ 0.8. The young disc colour is assumed to be roughly independent of metallicity remaining at B-V = -0.03; indicated by the observed small range in colours of young star forming regions. The pmps was repeated for $(\text{B-V})_{\rm yd}$ = -0.03 and $(\text{B-V})_{\rm od}$ = 0.8 giving a new colour correction term ct' = 2.5 $\log_{10}[(1-0.973W)/1.116W]$. Comparison with the solar abundance colour correction term ct (Eq. 3) shows that $ct' - ct$ = 0.30 for all colours. This means that for Z = $0.1Z_\odot$ the old disc population is 0.3 magnitudes brighter for the same observed colour; and hence the stellar disc contains more mass generating a higher maximum rotation. The latter can now be calculated simply from Eq. (21) when the metal poor ct' is substituted resulting in a maximum disc velocity for Z = $0.1Z_\odot$ being 15% higher than that for a solar abundance. Applied to the LSB example NGC 1560 this means that if Z(N1560) = $0.1Z_\odot$ the maximum disc velocity should be increased from 25 km s^{-1} to 29 km s^{-1}. This still falls substantially below the md hypothesis value of 43 km s^{-1}. It should be kept in mind that this calculation is for a case expected to be a limiting situation. The 15% increase is therefore a maximum.

In relation to LSBs there is another matter that has to be discussed. The central surface brightness is an observed quantity; only a correction to face-on is made but no absorption correction. HSB discs are semi transparent with most of the extinction concentrated in the inner regions (Huizinga & van Albada 1992; Giovanelli et al. 1994). On the other hand, low luminosity spiral discs and LSB discs appear to be by and large transparent over the whole extent of the disc (Bosma et al. 1992; Mc Gaugh 1994). Thus the Freeman's law value derived for HSB discs of 21.65 B mag. arcsec^{-2} is compromised by extinction with respect to LSB discs. In order to correctly compare central surface densities of HSB and LSB systems a correction should be applied to the observed central surface brightness. Or, such a correction has to make the surface brightness of HSBs larger, or, that of LSBs smaller. Either way, in calculating maximum disc rotations the central surface brightness quotient $\mu_0/\mu_{0,F}$ has to be lowered for LSB systems with the typical amount of extinction in HSB discs. Various studies indicate that this typical extinction ranges between 0.5 to 1 mag. in B (Keel 1983; Andredakis & van der Kruit 1992; White & Keel 1992; Knapen & van der Kruit 1991; Byun et al. 1994; Huizinga 1994). Inserting this correction in Eq. (21) results in a lower maximum disc rotation for LSB systems of the order of 10 to 20%.

According to the scheme developed in this paper a maximum disc rotational velocity can be calculated for a galactic disc akin to that of the solar neighbourhood. The lower metallicity in LSBs would lead to a higher rotational velocity of at most 15%. On the other hand a different dust content of LSBs would lead to a lower rotational velocity of 10 to 20%. There is no direct evidence that lower metals in discs is accompanied by less dust, but it is more than likely. Therefore it is expected that in LSBs both effects approximately cancel such that the original description also applies for these systems. Moreover, effects of up to 15% will leave all conclusions essentially unchanged.

The maximum disc velocity following from Eq. (21) is for a strict exponential disc. In that sense it can be applied very well in a statistical way comparing galaxies with one another as for example in Fig. 5. For individual cases, when the photometric profile deviates strongly from exponential, the method can in principle be extended to be used in a radially differential way.

Finally, a compilation of the main conclusions:

– A general relation has been derived giving the maximum rotation of a galactic disc as a function of its absolute magnitude, central surface brightness, and colour. As a physical

basis for this relation serves an adopted universal M/L ratio as a function of colour.
- This relation and the involved M/L ratios are fixed in an absolute sense by the observed stellar velocity dispersions.
- Comparing derived maximum rotations of a disc with the observed total rotation shows that even in the disc region, normal galaxies contain an appreciable amount of dark matter. Disc only rotations are lower by a factor of 0.7 compared to the rotation implied by the maximum disc hypothesis.
- Low surface brightness galaxies contain an even larger amount of dark matter.
- Derived disc M/L ratios are 1.79 ± 0.48 in the B-band for a B-V of 0.7. This M/L value is around a factor or two lower than the md hypothesis values.

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