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Collisional dynamics of the Milky Way

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Abstract. The effect of gravitational (elastic) encounters between stars and giant molecular clouds on the stability of small-amplitude perturbations of the Milky Way's self-gravitating disk is considered, using the exact Landau (Fokker-Planck type) collision integral, and compared with the results obtained by Griv & Peter (1996), who used the simple phenomenological Bhatnagar-Gross-Krook (Bhatnagar et al. 1954) collisional model. The present analysis is carried out for the case of a spatially inhomogeneous, highly flattened system, i.e., an inhomogeneous system in which the thickness is very small in comparison with the disk's radial extension. According to observations (Grivnev & Fridman 1990), the dynamics of a system with rare, $\kappa^2 \gg \nu_c^2$ (and weak, $\omega^2 \gg \nu_c^2$), interparticle encounters is considered, where κ is the epicyclic frequency, ω is the frequency of excited waves, and $\nu_c \sim 10^{-9}$ yr^{-1} is the effective frequency of star-cloud encounters. The evolution of the stellar distribution is determined primarily by interactions with collective modes of oscillations – gravitational Jeans-type and gradient-dissipative modes – rather than by ordinary ("close") star-cloud encounters. On the basis of a local kinetic theory, it is shown that the Landau integral and the Bhatnagar et al. model give practically identical results in the case of perturbations with the wavelength λ that is comparable to the mean epicyclic radius of stars ρ, that is, in the case of the most dangerous, in the sense of the loss of stability, gravitational Jeans-type perturbations. The models, however, have essentially different qualitative and quantitative behaviors in the extreme limits of long-wavelength perturbations, $(\pi\rho/\lambda)^2 \ll 1$, and of short-wavelength perturbations, $(\pi\rho/\lambda)^2 \gg 1$. Certain observational implications of the present theory are discussed.

Key words: galaxies: interactions – galaxies: kinematics and dynamics – ISM: clouds

1. Introduction

One of the most important recurring problems of stellar dynamics is that of stability of self-gravitating flat galaxies. In principle, a certain quasi-stationary self-consistently found disk state of a galaxy may theoretically be possible, but it might be unstable against collective modes of oscillation and might not survive on the Hubble's time $T \sim 10^{10}$ yr. In this connection, we can learn much about the properties of stellar systems of disk-shaped galaxies analytically by applying mathematical methods developed previously in stability considerations of hot magnetized plasmas (Ichimaru 1973; Mikhailovskii 1974; Alexandrov et al. 1984). In turn, a formal analogy between the collective oscillations in a rotating self-gravitating disk of flat galaxies containing a large number $\sim 10^{11}$ of stars-"particles" and the self-consistent electrostatic oscillations of a hot one-component plasma in a magnetic field has been pointed out (e.g., Fridman & Polyachenko [1984] and Binney & Tremaine [1987]).

Griv & Peter (1996a, 1996b, 1996c) in the series of papers (which will be reffered to as Papers I, II, and III throughout the following) have investigated the stability of the stellar disks in flat, $h \ll R$, galaxies by applying the methods of plasma kinetic theory (h is the typical thickness and R is the characteristic radius of the disk). Almost all the possible spectra of the unstable oscillations of an infinitely thin, collisionless disk of stars were studied in the linear approximation. The instabilities and waves in such a theoretical model are:

(a) Exponentially growing, small-scale $\ll R$, axially symmetric (radial) Jeans-type perturbations in the dynamically cold disk, i.e., in the disk where Toomre's (Toomre 1977) instability parameter $Q = c_r/c_T < 1$ (the notations used in the paper are the same as in Papers I-III). Here c_r is the radial dispersion of residual (random) velocities of stars and c_T is the critical Toomre's (Toomre 1964, 1977) velocity dispersion.

(b) Rapidly growing on the dynamical timescale, $t \sim \Omega^{-1}$, relatively large-scale $\lesssim R$, nonaxisymmetric (spiral) Jeans-type perturbations in the warm model, in which initially Q = 1. Here $\Omega(r)$ is the angular velocity of basic differential rotation at the distance r from a galactic center. In the marginally Jeans-stable fast-rotating disk, the value of Toomre's instability parameter has to be about $Q \approx 2\Omega/\kappa$ in the main part of a system. As a rule, in differentially rotating parts of flat galaxies $2\Omega/\kappa = 1.6 - 1.8$, therefore, the critical $Q = 1.6 - 1.8$ also (see Morozov [1981a, 1981b], Fridman & Polyachenko [1984, Vol. 1, p. 395], and Pa-

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per I for explanations). As usual, $\kappa = 2\Omega[1+(r/2\Omega)(d\Omega/dr)]^{1/2}$ is the epicyclic frequency.

(c) Resonantly excited in a Jeans-stable ($Q \gtrsim 2\Omega/\kappa$; see above) differentially rotating ($2\Omega/\kappa > 1$) disk nonaxisymmetric density waves (cf. the inverse Landau damping effect in plasma physics). The free kinetic energy associated with the differential rotation of the system under study is only one possible source for the growth of the average wave energy. Growth of the oscillations is determined by corotation resonance particles which are capable of exchanging energy with the Jeans-stable spiral perturbations and can thus amplify it. Griv & Yuan (1997, MNRAS, in preparation) studied the Landau-type mechanism of density waves self-excitation at the outer Lindblad resonance (cf. Doppler or Landau cyclotron effect in a magnetized plasma). These spontaneous oscillatory growing waves serve to increase the velocity dispersion of stars (the "temperature") in the galactic plane to values $c_r > (2\Omega/\kappa)c_T$ or $Q > 2\Omega/\kappa$, respectively. It is important to recognize that the resonant-type instabilities, i.e., "Landau" instabilities, can be considered in a consistent fashion only within the framework of kinetic (or microscopic) theory, taking into account all effects of thermal spread of particle random velocities. Clearly, such instabilities were not discovered in theories based on gasdynamical (fluid-like) approach (e.g., Goldreich & Lynden-Bell [1965], Drury [1980], and Lin & Lau [1979]) because the fluid equations do not include any velocity integrals

$$\int \frac{f d\mathbf{v}}{\omega/\mathbf{k} - \mathbf{v}},$$

where $f(\mathbf{v})$ is the particle distribution function, $\omega/\mathbf{k}$ is the phase velocity of excited waves, and $\mathbf{v}$ is the thermal velocities of particles (Paper I).

Summarizing the above results, the unstable oscillations of a collisionless stellar disk arise because either the dispersion of random velocities of stars is too small to suppress the instability of all Jeans hydrodynamic-type perturbations (Paper I, Eq. [28]) or the resonant Landau-type excitation of density waves in the gravitationally (Jeans-) stable disk, which rotates differentially, $d\Omega/dr \neq 0$ (Paper I, Sect. 7.2). As such instabilities grow, it must lead to the production of a more "hot" stellar system with a higher level of particle velocity dispersions. Thus, because the growth times of the instabilities are sufficiently short, $\sim \Omega^{-1} \ll T$, the increase of the stellar velocity dispersions may be achived by the wave-star collective mechanism rather than by ordinary binary collisions (Paper I). In addition, this collisionless collective relaxation (cf. a quasi-linear relaxation in a plasma) will be accompanied by a redistribution of the surface mass density to a more peaked distribution, producing a condensed nucleus of a galaxy and a diffused outer envelope (Griv et al. 1994).

It seems likely that these waves and instabilities explain the observed secular dynamical evolution in spiral galaxies (Courteau et al. 1996), and have already been detected in N-body simulations by Hohl & Hockney (1969) [small-scale, almost axisymmetric Jeans instability of an initially cold computer model of a galaxy, $Q < 1$], Hohl (1971, 1978) [relatively large-scale, spiral Jeans instability of an initially warm, $Q = 1$, differentially rotating model], and Sellwood & Lin (1989) and Donner & Thomasson (1994) [Landau-type instability of a Jeans-stable disk, i.e., resonant self-excitation of density waves in a dynamically hot, differentially rotating disk where $Q \gtrsim 2\Omega/\kappa$; see Griv (1997) for a discussion]. Abbreviated results of the theory have been reported by Griv (1993, 1996); observable consequences have been presented in Papers I-III.

In Paper III an attempt was made to investigate the stability of the collisional (through ordinary pairwise gravitational encounters) Milky Way's disk in the framework of the kinetic theory by investigating collective interactions among stars and giant molecular clouds.[1] It was shown that in addition to instabilities in which collisions play no role there is a gradient-dissipative secular-type instability of star-cloud disks that exists by virtue of the collisional behavior of the system.[2] Earlier, this instability has been investigated in detail by Lynden-Bell & Pringle (1974) and others (Mishurov et al. 1976; Fridman & Polyachenko 1984, Vol. 2, p. 239; Morozov et al. 1985; Willerding 1992; Schmit & Tscharnuter 1995) but using a gasdynamical approach. Recently, by particle N-body simulations, Sterzik et al. (1995) confirmed predictions regarding wavelength and growth time of the instability. The secular instability might arise merely from the dissipation of the energy of regular circular rotation into ever larger amounts of the energy of random motion or "heat" (Mishurov et al. 1976; Morozov et al. 1985; Paper III). To emphasize, Fridman & Polyachenko (1984, Vol. 2) and Sterzik et al. (1995) especially stressed that any dissipative mechanism (e.g., shear-generated turbulence or interparticle inelastic and elastic collisions) can cause this instability in self-gravitating systems, as long as it reflects a hydrodynamic shear viscosity.

To simplify the analysis, in Paper III we found it necessary to abandon the real integral of elastic particle encounters, e.g., the Landau collision integral (Lifshitz & Pitaevskii 1981, p. 168; Binney & Tremaine 1987, p. 506), in favor of the phenomenological integral of collisions suggested by Bhatnagar et al. (1954). Shu & Stewart (1985), Griv & Chiueh (1996), and Griv & Yuan (1996) have used the model Krook and Bhatnagar, Gross, & Krook (BGK) collision operators to study the dynamics of a system of identical spherical-like particles of Saturn's rings with physical (inelastic) interparticle collisions. However, as it has been pointed out to us by the second referee of Paper III, the question that remains is whether the simplified and

1 Collisions can be important if a particulate system has a high density or if it is dynamically cold. Because the typical giant molecular cloud mass in the Milky Way is sufficiently large, star-cloud collisions have a predominant effect (Spitzer & Schwarzschild 1951, 1953; Paper I). In turn, observations have been shown the occurence of gravitational encounters between stars and giant molecular clouds in the disk of the Milky Way (Grivnev & Fridman 1990; Griv 1993; Paper III). In accord with Griv (Grivnev) & Fridman (1990), the time of collisional relaxation in the solar vicinity of the Galaxy $\tau \sim 2 \cdot 10^9$ yr, that is comparable to a time of $\sim$ 10 rotations of the Galaxy in the solar vicinity.

2 See Binney & Tremaine (1987, p. 329) for a discussion of dynamical and secular instabilities in a gravitating medium.

somewhat *ad hoc* BGK model for collision processes is appropriated to observe the essential qualitative effects of the theory. (In plasmas, the BGK description is satisfactory only for interactions between particles of different kind, for instance, it can be applied to weakly ionized plasmas when the scattering of charged particles by neutrals is predominant; Alexandrov et al. [1984]). Also, in plasma physics first Pitaevskii (1963) and then Ivanov & Rukhadze (1964), Bogdankevich & Rukhadze (1967), Rukhadze & Silin (1967, 1969), and Bhadra (1971) have shown that for the investigations of oscillations and stability of a plasma it is necessary to use the exact Boltzmann or Landau collision integrals to observe the essential qualitative effects that appear in extreme short-wavelength and long-wavelength limits. Following them, in the present study the effect of encounters between stars and clouds on the stability of different gravity perturbations of the Galaxy's disk, using the exact Landau collision integral is considered and compared with the results obtained using the BGK model.

In plasma physics, e.g., Rukhadze & Silin (1969) presented several simple approximate methods for solving the kinetic equation with an exact collision integral. It was shown that in some of the solution the integration of the kinetic equation with an exact collision integral does not make the theory more complicated than when the simplified BGK method is used. In the current research, we use the analytical methods developed by Ivanov & Rukhadze (1964), Rukhadze & Silin (1967, 1969), Bogdankevich & Rukhadze (1967), and Alexandrov et al. (1984).

In fact, Julian & Toomre (1966) have already considered the collective supporting response of the stellar disk to the presence of a single, particle-like concentration of interstellar co-orbital material ("cloud"), which may be regarded as a small perturbation.[3] They have obtained the important result that the collective interaction among the stars greatly increases the effective mass of an imposed particle, by typically a factor ~ 10. Incorporating the collective phenomena, Julian (1967) has found the rates of energy input to the stellar random motions, through encounters with massive clouds, to be increased over the standard estimates of Spitzer & Schwarzschild (1951, 1953) roughly by the square of the mass enhancement ratio. (Thorne [1968] considered the dynamical friction experienced by a massive particle traveling within a thin disk of stars. It was found that the collective interactions among stars themselves at most to double that friction.) In the present paper, we extend the analysis of Julian & Toomre (1966) by taking additionally into the consideration the collision operator in the kinetic equation. Aside from this difference, the governing equations Julian & Toomre considered are equivalent to ours. In contrast to their study, however, we investigate additionally the effects of collisions on the perturbed distribution function of stars that comprise a galactic disk. Thus, the evolution of the stellar distribution function is determined by interactions of stars with collective modes of the disk rather than by ordinary binary star-cloud collisions.

[3] Goldreich & Lynden-Bell (1965) have studied the same problem by using a gaseous approach.

It is obvious that the effects of large-angle close star-cloud collisions do not play a significant role in galaxies (Spitzer & Schwarzschild 1951, 1953; Chandrasekhar 1960, p. 48), and the collision processes are due to the long-range gravitational Coulomb-like force that give rise mainly to many small-angle distant collisions (cf. Ichimaru [1973, p. 231]; Lifshitz & Pitaevskii [1981, p. 168]). In plasma physics, it was shown that the appropriate collision operator to use in this case is the Fokker-Planck equation.

Unlike our analysis, Jenkins & Binney (1990) obtained a numerical solution of the Fokker-Planck equation that describes the heating of a stellar disk under the combined influence of stochastic spiral density waves and giant molecular clouds. They did not solve the problem self-consistently, for instance, they neglected the Poisson's equation, and thus the origin of such unstable spiral irregularities in the overall potential is not clear (Jenkins & Binney 1990, p. 245). As has been pointed to us by the referee of this paper, in Jenkins & Binney's calculations collective modes scattered stars entirely independently of any scattering by clouds. In the language of normal modes, they were considering the effects of non-linear mode-mode interactions. Earlier, Barbanis & Woltjer (1967) have studied the heating process through interactions of stars with the gravitational field of growing spiral waves on the basis of exact numerical integration of the equations of motion. Similarly to Jenkins & Binney (1990), they did not solve the problem self-consistently. Also, Binney & Lacey (1988) attempted to solve this problem quantitatively by considering the scattering both clouds and spiral waves (see Binney & Tremaine [1987, p. 470] and Jenkins & Binney [1990] for review of the problem). To repeat, in the present paper we study the stability and collective oscillations of a fast-rotating disk of stars and giant molecular clouds, treating the problem by means of a *self-consistent* solution of the kinetic Boltzmann and Poisson's equations.

This paper is constructed as follows. First, in §2 the basic equations of the linear kinetic theory are given. Additionally, the relevant Landau collision integral is determined. In §3 the general dispersion relation in a star-cloud collisional galactic disk is obtained. We study the influence of gravitational encounters between stars and molecular clouds on the growth/damping rate of the Jeans-type and gradient-type waves in §3.1 and §3.2, respectively. §4 contains a summary and a discussion of the results of the work. A brief first report has been published by Griv (1996).

2. Basic equations

2.1. Boltzmann and Poisson's equations

The starting equation to describe the collisional motion of an ensemble of particles in the plane of the system is the Boltzmann kinetic equation for the change of the particle distribution function (Fridman & Polyachenko 1984; Paper III),

$$\frac{df_\alpha}{dt} \equiv \frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f_\alpha}{\partial \mathbf{v}} = \sum_\beta \left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha\beta}, \tag{1}$$

where d/dt is the total derivative along the trajectory of the particle, $f_\alpha(\mathbf{r}, \mathbf{v}, t)$ is the number of particles of type α per unit volume in the phase space of configuration and velocity, and Φ is the average gravitational potential determined self-consistently from the Poisson's equation. The quantity $(\partial f_\alpha/\partial t)_{\alpha\beta}$ is called the collision integral which defines the change of f_α due to particle close gravitational encounters with particles of type β (where β, in particular, can coincide with α). Further, we shall consider only star-cloud encounters (Paper III; cf. the problem of a Lorentzian plasma studied, e.g., by Lifshitz & Pitaevskii [1981]).

Below an important special case of wave propagation in an infinitely thin galactic disk will be considered. The latter means that we investigate gravity perturbations with a radial wavelength that is much greater than the characteristic disk half-thickness (Shu 1970; Ginzburg et al. 1972; Paper I). Note that N-body computer simulations have been shown that in general the effects of three-dimensional motion in thin galactic disk models are small (Hohl 1978). Also, we are interested in the evolution of the distribution function of stars only regarding the clouds as at galactic rotation with given distribution. This is because the cloud velocity is only slightly changed in each star-cloud collision because of the disparity in mass. (In most giant galaxies, including our own Galaxy, the stars contribute the bulk of the luminous mass, probably $\sim 90\%$, and, therefore, control the overall dynamics. According to observations, the amount of gas in flat galaxies is probably between 5 and 10% of the total mass of the system only.) Thus, even though in reality the stellar system and the giant molecular clouds are coupled to each other by their common gravitational field, in the lowest-order approximation of the theory we interpret in Eq. (1) of f_α as the distribution function of stars (and the index β corresponds to clouds).

In addition, in our model there is no systematic movement of stars relative to clouds, and the clouds are considered immobile compared with the stars. In other words, since the stars have small masses, their mean peculiar velocities are large compared with those of the clouds. In the following, in accordance with observations we assume that clouds move on almost circular orbits with a Maxwellian distribution of residual velocities being distributed almost isotropically (Blitz 1993; Lada 1995; Myers 1995).

As usual in the theory of self-consistent disk oscillations (Fridman & Polyachenko 1984; Paper I), Eq. (1) must be solved jointly with the Poisson's equation connecting the total gravitational potential $\Phi(\mathbf{r}, t)$ with the surface density $\sigma(\mathbf{r}, t)$:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G\sigma\delta(z), \tag{2}$$

where G is the constant of gravitation, $\delta(z)$ is the Dirac delta-function, $\sigma = \int f d\mathbf{v}$, and r, φ and z are the galactocentric cylindrical coordinates and the axis of galactic rotation is taken oriented along the z-axis. Such a solution describes the ordered behavior (collective oscillations) of a medium near its equilibrium or quasi-equilibrium state.

In plasma physics, in the case of the exact integral, e.g., Landau collision integral (the collision integral in the Fokker-Planck approximation), equations in two limiting cases of weak collisions $\omega^2 \gg \nu^2$ and strong collisions $\omega^2 \ll \nu^2$ have been obtained by entirely different methods, with ω and ν being the frequency of excited waves and the frequency of interparticle collisions, respectively (Rukhadze & Silin 1969; Mikhailovskii 1974, Vol. 2). By replacing the exact collisional integral in the kinetic equation by an approximate term in the form of BGK model the problem can be solved with a single method (Mikhailovskii & Pogutse 1964, 1966; Mikhailovskii 1974, Vol. 2, p. 71). In Paper III, it has been pointed out, however, that the greatest defect of the BGK model is that the frequency of collisions does not fall off with increasing velocity, as do the diffusion coefficients given by Fokker-Planck equation. It has also been pointed that sometimes the use the BGK model can produce misleading results; the correct procedure for handling the collisions is the use of an exact collision operator. The simple phenomenological BGK model has been used in Paper III. In the present paper we consider the disk dynamics in the case of weak collisions, by using the exact Landau collision integral. In plasma physics, the form of such the Fokker-Planck operator was originally derived by Landau (Lifshitz & Pitaevskii 1981, p. 168; see also Rosenbluth et al. [1957]). Binney & Tremaine (1987, p. 506) modified this operator in the case of a gravitating galactic disk.

Let us consider a disk with the property that the epicyclic frequency as well as the angular velocity of regular circular rotation exceeds the frequency ν_c of binary collisions between stars and clouds. Then, the resulting Boltzmann kinetic equation for the perturbed distribution function of stars can be solved by successive approximations, neglecting the influence of star-cloud encounters on the equilibrium distribution of stars in the zero-order approximation (Mikhailovskii & Pogutse 1964, 1966). In the linear approximation, the kinetic equation (1) can be rewritten in the form of the set of equations (Lin et al. 1969; Shu 1970; Paper III)

$$v_r \frac{\partial f_0}{\partial r} + \left(2\Omega v_\varphi + \frac{v_\varphi^2}{r}\right)\frac{\partial f_0}{\partial v_r} - \left(\frac{\kappa^2}{2\Omega}v_r + \frac{v_r v_\varphi}{r}\right)\frac{\partial f_0}{\partial v_\varphi} = \left(\frac{\partial f_0}{\partial t}\right)_{\alpha\beta} \tag{3}$$

and

$$\frac{df_1}{dt} = \frac{\partial \Phi_1}{\partial r}\frac{\partial f_0}{\partial v_r} + \frac{\partial \Phi_1}{r\partial \varphi}\frac{\partial f_0}{\partial v_\varphi} + \left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta}, \tag{4}$$

where $f_1(\mathbf{r}, \mathbf{v}, t)$ is the small perturbation of the equilibrium time-independent distribution function of stars $f_0(r, \mathbf{v})$ with $|f_1/f_0| \ll 1$ for all $\mathbf{r}$ and t. In Eqs. (3) and (4) the potential was divided into a smooth part $\Phi_0(r)$ satisfying the equilibrium condition

$$\partial \Phi_0/\partial r = \Omega^2 r,$$

and a rapidly fluctuating part $\Phi_1(\mathbf{r}, \mathbf{v}, t)$ with $|\Phi_1/\Phi_0| \ll 1$. The collision integral has been linearized by virtue of condition

$\Omega^2 \gg \nu_c^2$: the collisions begin to influence f_0 only after a time of $\tau \sim \nu_c^{-1} \gg \Omega^{-1}$ rotations of the Galaxy, when the perturbations of the distribution function are large already as a result of the instability.

In order to find the perturbed distribution f_1, it is convenient to integrate Eq. (4) along the ordinary unperturbed Lindblad's elliptic-epicyclic trajectories of particles (Spitzer & Schwarzschild 1953; Paper I):

$$r = r_0 - \frac{v_\perp}{\kappa}[\sin(\phi_0 - \kappa t) - \sin\phi_0], v_r = v_\perp \cos(\phi_0 - \kappa t), \quad (5)$$

$$\varphi = \Omega t + \frac{2\Omega}{\kappa}\frac{v_\perp}{r_0\kappa}[\cos(\phi_0 - \kappa t) - \cos\phi_0],$$

$$v_\varphi = \frac{\kappa}{2\Omega} v_\perp \sin(\phi_0 - \kappa), \quad (6)$$

where r_0 is the radius of the chosen circular orbit in the (r, φ) plane, $v_\perp$ and ϕ_0 are constants of integration, $v_\perp/r_0\kappa \simeq \rho/r_0 \ll 1$, ρ is the mean epicyclic radius, v_r and v_φ are the components of the particle's random purely oscillatory velocity at each point relative to the local rotating standard of rest, and $\Omega = \Omega(r_0)$, $\kappa = \kappa(r_0)$. In Eqs. (5)-(6), the total azimuthal velocity of the stars was represented as a sum of v_φ and the circular velocity $r\Omega$. In addition, we ignored the small drift contributions to the star velocity (Paper I). This is because we consider the main, nonresonant part of the system only (Paper I).

According to observations (Grivnev & Fridman 1990), the case is considered with rare collisions, when

$$\kappa^2 \sim \Omega^2 \gg \nu_c^2. \quad (7)$$

In other words, we shall assume that $\Omega\tau > 1$, where $\tau \sim 1/\nu_c$ is the relaxation time for star-cloud collisions, then the single particle analysis is still valid.

As a solution of Eq. (3), the equilibrium distribution function f_0 can be constructed with the use of the constants I_1, I_2, . . ., of the equilibrium star motion $f_0 = F_0(I_1, I_2, \ldots)$, where F_0 is, generally speaking, an arbitrary function. One can choose as the steady state distribution of stars, the Maxwellian-type distribution. The collision correction for f_0 is of order $\nu_c/\kappa \ll 1$, and can be neglected (Mikhailovskii & Pogutse 1964, 1966). The anisotropic Schwarzschild distribution (Maxwellian distribution with respect to $v_\perp$) for the unperturbed system is particularly important because it corresponds to observations. Therefore, the equilibrium distribution function $f_0 = f_0(v_\perp^2/2, r_0)$ in Eq. (4) may be choosen as (Lin et al. 1969; Shu 1970; Paper I)

$$f_0 = \frac{\sigma_0(r_0)}{2\pi c_r(r_0) c_\varphi(r_0)} \exp\left[-\frac{v_r^2}{2c_r^2(r_0)} - \frac{v_\varphi^2}{2c_\varphi^2(r_0)}\right]$$

$$= \frac{2\Omega(r_0)}{\kappa(r_0)} \frac{\sigma_0(r_0)}{2\pi c_r^2(r_0)} \exp\left[-\frac{v_\perp^2}{2c_r^2(r_0)}\right], \quad (8)$$

where $\sigma_0(r)$ is the equilibrium surface density and r_0 represents approximately the r-coordinate of the star guiding center. In Eq. (8), the fact was used that the radial and azimuthal dispersions are not independent, but according to Eqs. (5)-(6) they are related in the rotating system through $c_r \simeq (2\Omega/\kappa)c_\varphi$.

The asymptotic solution of Poisson's equation (2), connecting the perturbed local surface density $\sigma_1(\mathbf{r}, t)$ and gravitational potential $\Phi_1(\mathbf{r}, t)$, is

$$\sigma_1(r) = -\frac{|k|\Phi_1(r)}{2\pi G}\left\{1 - \frac{i}{k_r r}\frac{d\ln}{d\ln r}\left[r^{1/2}\delta\Phi(r)\right]\right\}, \quad (9)$$

where $k^2 = k_r^2 + k_\varphi^2$, $k_r = k\cos\psi$ and $k_\varphi = k\sin\psi$ are the radial and azimuthal components of the wavevector $\mathbf{k}$, $k_r \gtrsim k_\varphi$, ψ is the perturbation pitch angle, $|\sigma_1| \ll \sigma_0$, and $\delta\Phi$ is the perturbation amplitude (Lin & Lau 1979). Solution (9) determines the perturbed surface density required to support the perturbed gravitational potential up to second order in the so-called Lin-Shu asymptotic approximation of moderately tightly wound perturbations (Bertin 1980).

Equations (4) and (9) with appropriate boundary conditions give a complete description of the problem for collisional disk oscillations in the linear approximation of a self-consistent field problem. In the next section, we shall simplify the system of Eqs. (4) and (9) by applying the local approximation of the classical Wentzel-Kramers-Brillouim (WKB) method, to derive asymptotic Lin-Shu type dispersion relation modified by collisions. The dispersion relation connects the frequency of excited oscillations with the wavenumber for every $\mathbf{r}$ and describes the ordered behavior of a medium near its equilibrium or quasi-equilibrium state.

2.2. Landau collision integral

In the spirit of the Chandrasekhar's "molecular-kinetic" theory (Chandrasekhar 1960, p. 48), for observed parameters of giant molecular clouds, $m_{cl} = (2-5)\cdot 10^5\ M_\odot$ and $N_{cl} \sim 4000$ (Binney & Tremaine 1987; Blitz 1993; Lada 1995; Myers 1995), only star-cloud encounters may be important in the problem of dynamical relaxation of the Galaxy, and for such parameters of molecular clouds the collision frequency $\nu_c \simeq 1/\tau \sim 10^{-9}$ yr^{-1} (Spitzer & Schwarzschild 1951, 1953; Grivnev & Fridman 1990; Paper III). Here m_{cl} is the typical mass of a cloud and N_{cl} is the total number of clouds. Star-star encounters may be neglected because the relaxation time of the Galaxy due to such interactions considerably exceeds the lifetime of the universe (Chandrasekhar 1960).

For star-cloud encounters the Landau integral of collisions reads (Lifshitz & Pitaevskii 1981, p. 168; Alexandrov et al. 1984, p. 54)

$$\left(\frac{\partial f}{\partial t}\right)_{\alpha\beta} = 2\pi G^2 m_s^2 m_{cl}^2 \ln\Lambda_s \frac{\partial}{\partial p_i}\int d\mathbf{p}'$$

$$\times\left\{f'(\mathbf{p}')\frac{\partial f(\mathbf{p})}{\partial p_j} - f(\mathbf{p})\frac{\partial f'(\mathbf{p}')}{\partial p_j'}\right\}\frac{w^2\delta_{ij} - w_i w_j}{w^3}, \quad (10)$$

where $\ln\Lambda_s$ is the so-called Coulomb (or Newton) logarithm, by means of which the long-range nature of the Coulomb (gravitational) force is taken into account (Lifshitz & Pitaevskii

1981; Alexandrov et al. 1984), $\Lambda_s \simeq b_{max} v_s^2/Gm_{cl}$, and $b_{max} \approx R$ is the maximum impact parameter considered (Binney & Tremaine 1987, p. 510). The collision term was derived by Landau by introducing appropriate cut-offs of the integration divergence at a minimum and maximum distance. A classical justification of the cut-off at small distances $r < b_{min} = Gm_{cl}/v_s^2$ can be given by a consideration of many particle collisions. Clearly, at large distances $r > R$ the interaction may be neglected (in a completely ionized plasma, where only collisions of charged particles are essential, the cut-off at large distances is necessary since the interaction is screened due to the plasma polarization). In addition, in equation (10) $f'(\mathbf{r}, \mathbf{v_{cl}}, t)$ is the distribution function of clouds, $p = m_s v_s$, $p' = m_{cl} v_{cl}$, v_s and v_{cl} are the random velocities of stars and clouds, $m_s \sim M_\odot$ is the typical mass of a star, and $\mathbf{w} = \mathbf{v} - \mathbf{v}'$ is the relative velocity of the colliding particles. It may be shown by integrating over velocities that the collisional term in such a form instantaneously conserves the number of particles, the linear momentum, and the particle's energy. The Landau form for the collision integral is based on an assumption that the duration of a collision is much less than the time between collisions; it vanishes if we substitute in it a one-dimensional Maxwellian distribution.

For encounters between stars and giant molecular clouds, the linearized Landau integral (10) becomes:

$$\left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta} = 2\pi G^2 m_s^2 m_{cl}^2 \ln \Lambda_s \frac{\partial}{\partial p_i} \int d\mathbf{p}' \frac{w^2\delta_{ij} - w_i w_j}{w^3} \times \left\{ f_0' \frac{\partial f_1}{\partial p_j} + f_1' \frac{\partial f_0}{\partial p_j} - f_0 \frac{\partial f_1'}{\partial p_j'} - f_1 \frac{\partial f_0'}{\partial p_j'} \right\}, \qquad (11)$$

where f_1 and f_1' are the perturbed distribution function of stars and clouds, f_0 and f_0' are the equilibrium distribution functions, and $|f_1/f_0| \ll 1$ and $|f_1'/f_0'| \ll 1$. Further, we shall assume $f_1' = 0$. For $c_{cl}^2 \ll c_s^2$, where c_{cl} and c_s are the velocity dispersions of clouds and stars, one can expand the integrand in powers of v_{cl}/v_s which yields (according to observations, $c_s \sim 20$ km s^{-1} and $c_{cl} \sim 6$ km s^{-1}; Crovisier [1978])

$$\frac{w^2\delta_{ij} - w_i w_j}{w^3} \approx \frac{v^2\delta_{ij} - v_i v_j}{v^3} - v_\gamma' \frac{\partial}{\partial v_\gamma} \frac{v^2\delta_{ij} - v_i v_j}{v^3}.$$

Using the expansion above, up to terms $m_s/m_{cl} \ll 1$ the linearized collision integral (11) may be rewritten as

$$\left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta} = \frac{2\pi G^2 m_{cl}^2 N_{cl}}{v} \ln \Lambda_s \times \frac{\partial}{\partial v_i} \left\{ \delta_{ij} \frac{\partial f_1}{\partial v_j} - \frac{v_i v_j}{v^2} \frac{\partial f_1}{\partial v_j} \right\}. \qquad (12)$$

By choosing the polar coordinates in velocity space $v_\perp$ and $\theta = \arctan(v_\varphi/v_r)$, from Eq. (12) the following simplified expression results:

$$\left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta} \approx \nu_c \left[(1 - 2\sin^2\theta \cos^2\theta) \frac{\partial^2 f_1}{\partial \theta^2} - 2\cos^3\theta \sin\theta \frac{\partial f_1}{\partial \theta} - v_\perp \frac{\partial f_1}{\partial v_\perp} \right], \qquad (13)$$

where $\nu_c = 2\pi G^2 m_{cl}^2 N_{cl} \ln \Lambda_s / v_\perp^3$ is the frequency of star-cloud encounters, and $v_\perp \sim c_r$. Here a disk with a weak nonuniform rotation was considered, i.e., $2\Omega/\kappa \approx 1$. As one can see, the collision frequency ν_c is approximately $1/\tau_{ch}$, where τ_{ch} is the well-known Chandrasekhar's relaxation time for the establishment of two-body equilibrium. It is easy to show that the relaxation time for the establishment of star-cloud equilibrium is $\sim (m_{cl}/m_s)\nu_c^{-1} \gg T$.

In the present subsection only, we neglected the effects of galactic rotation (magnetic field in a plasma, respectively) on the collision process. That is, following Chandrasekhar (1960) and Landau (Lifshitz & Pitaevskii [1981]), we treat interactions in the very dilute system as sums of independent interactions between pairs of particles whose orbits at great separations $\sim \rho \approx c_r/\kappa$,

$$\frac{\rho c_r^2}{2Gm_{cl}} \gg 1,$$

(in the vicinity of the Sun this ratio is about 10^2) become practically straight lines. (In addition, it is assumed that the total exchange of energy occurs at the moment of minimum separation: in this sense the collisions are instantaneous.) As it has been shown in plasma physics, a new "anomalous" temperature-relaxation process arises from the presence of the strong magnetic field (epicyclic, non-rectilinear particulate orbits) that is as effective as the ordinary two-body relaxation process (Ichimaru 1973, p. 246). Thus, from an analogy between the oscillations in a rotating stellar disk and the oscillations of a hot plasma in a magnetic field, one can expect that the frequency ν_c discussed above has to be increased by a factor ~ 2. Therefore, one can replace

$$\nu_c \to 2 \cdot \nu_c.$$

The collision integral discussed so far does not take into account the detailed mechanism of the star-cloud interaction such as the finite size of clouds and their distribution in the Milky Way. It seems that the Landau integral can give qualitatively correct results in considered rarefied galactic disks where the detailed effects of elastic collisions may be ignored.

3. Dispersion relation

In accordance with Eqs. (5)-(6) and (8), the partial derivatives in Eq. (4) transform as follows:

$$\frac{\partial}{\partial v_r} = v_r \frac{\partial}{\partial \mathscr{E}}; \quad \frac{\partial}{\partial v_\varphi} = \left(\frac{2\Omega}{\kappa}\right)^2 v_\varphi \frac{\partial}{\partial \mathscr{E}} + \frac{2\Omega}{\kappa^2} \frac{\partial}{\partial r_0},$$

where $\mathscr{E} = v_\perp^2/2$ (Paper I). Then, a subsystem of stars of an infinitely thin collisional star-cloud disk is described by the linearized Boltzmann equation

$$\frac{df_1}{dt} = \mathbf{v}_\perp \frac{\partial \Phi_1}{\partial \mathbf{r}} \frac{\partial f_0}{\partial \mathscr{E}} + \frac{2\Omega}{\kappa^2} \frac{\partial \Phi_1}{r\partial \varphi} \frac{\partial f_0}{\partial r_0} + \left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta}, \qquad (14)$$

and by the asymptotic solution of Poisson's equation in the form of Eq. (9). In Eq. (14), $v_\perp^2 = v_r^2 + (2\Omega/\kappa)^2 v_\varphi^2$.

Using the local version of the WKB approximation, the perturbations will be taken to be of the form $f_1, \Phi_1, \sigma_1 \sim \delta f, \delta\Phi, \delta\sigma \cdot \exp[-i\omega_* t + im\varphi + ik_r r]$, with $\delta f, \delta\Phi, \delta\sigma$ being constant amplitudes and $|k_r| r \gg 1$. The quantity $\omega_* = \omega - m\Omega$ is the Doppler-shifted complex frequency of excited waves, m (= number of spiral arms) is the positive azimuthal mode number, and the term $m\Omega > 0$ takes into account the possibility of different harmonics (many-armed waves) in the rotating reference frame. It is convenient, to write the eigenfrequency in a form of the sum of the real part and the imaginary part, $\omega_* \to \mathrm{Re}\,\omega_* + i\mathrm{Im}\,\omega_*$; the existence of solutions with Im ω_* greater than zero implies instability. Note that in contrast to our analysis Lin & Shu (1966), Lin et al. (1969), and Shu (1970) as well as Toomre (1964, 1977), Julian & Toomre (1966), Lynden-Bell & Pringle (1974), and Mishurov et al. (1976) were interested only in almost radial, axisymmetric perturbations. Therefore, they excluded in the above expressions for perturbations all nonaxisymmetric terms except that in the expression for $\omega_* = \omega - m\Omega$.

In the local WKB approximation the solution of Eq. (14) with collision integral in the form (13) can be written as

$$f_1 = e^{-\nu_{eff} t} \int_{-\infty}^{t} dt' e^{\nu_{eff} t'}$$

$$\times \left\{ \left[\frac{d\Phi_1}{dt} + i\omega_* \Phi_1 \right] \frac{\partial f_0}{\partial \mathscr{E}} + \frac{2\Omega}{\kappa^2} \frac{\partial \Phi_1}{r\partial\varphi} \frac{\partial f_0}{\partial r_0} \right\}, \qquad (15)$$

where in the lowest-order approximation the integration is performed, as before, along the unperturbed epicyclic stellar orbits given by Eqs. (5)-(6). In Eq. (15), we defined an effective collision frequency as follows:

$$\nu_{eff} = 2\nu_c \left[\frac{k_*^2 v_\perp^2}{\kappa^2} (1 - 2\sin^2\theta\cos^2\theta)\cos^2\kappa t \right.$$

$$\left. -i2\frac{k_* v_\perp}{\kappa}\cos^4\theta\sin\theta - i\frac{k_* v_\perp}{\kappa}\sin\kappa t \right] \to \nu_c \frac{k_*^2\rho^2}{2},$$

where the value $\phi_0 - \zeta = 0$ was chosen (see below for the definition of ζ) and a long-time behavior has been singled out in the last term. The effective collision frequency ν_{eff} describes properly the more rapid collisional smoothing of the small-scale inhomogeneities of the stellar distribution function, and $\nu_{eff} \to 0$ for long-wavelength perturbations, $k_*^2\rho^2 \to 0$.

After integrations of expression (15) along unperturbed stellar trajectories, we obtain the following equation for the perturbed distribution function

$$f_1 = \Phi_1 \frac{\partial f_0}{\partial \mathscr{E}} \left[1 - \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \omega_* \frac{e^{i(n-l)(\phi_0-\zeta)} J_l(\chi) J_n(\chi)}{\omega_* - l\kappa + i\nu_{eff}} \right]$$

$$-\Phi_1 \frac{2\Omega}{\kappa^2} \frac{m}{r_0} \frac{\partial f_0}{\partial r_0} \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{i(n-l)(\phi_0-\zeta)} J_l(\chi) J_n(\chi)}{\omega_* - l\kappa + i\nu_{eff}}, \qquad (16)$$

where the argument of the Bessel function J_l of order l is $\chi = k_* v_\perp/\kappa$ and $k_*^2 = k^2\{1 + [(2\Omega/\kappa)^2 - 1]\}\sin^2\psi$ is the squared generalized wavenumber. In obtaining equation (16), using the definitions $\tan\psi = k_r/k_\varphi$ and $\tan\zeta = (2\Omega/\kappa)\tan\psi$, the exponential factor occurring in the orbit integral (15) was expressed as

$$\exp[i\mathbf{k}\cdot(\mathbf{r} - \mathbf{r_0}) - i\omega_* t + \nu_{eff} t] = \exp[-i(\omega_* + i\nu_{eff})t]$$

$$\times \exp\left\{ i\frac{k_* v_\perp}{\kappa} [\sin(\phi_0 - \zeta) - \sin(\phi_0 - \zeta - \kappa t)] \right\}.$$

The method of integration has been described in Papers I-III in detail. Equation (16) corresponds to that obtained in Paper III (Eq. [9]), by using the BGK model, but the principal difference is that here the collision frequency ν_c is replaced by $\nu_{eff} \approx \nu_c(k_*^2\rho^2/2)$. Clearly, the latter will lead to essentially different results obtained using the Landau and BGK models in the limiting cases of long-wavelength perturbations, $k_*^2\rho^2/2 \ll 1$, and short-wavelength ones, $k_*^2\rho^2/2 \gg 1$.

Note also that in such a form Eq. (16) for the perturbed distribution function of stars may be derived using the widely used in plasma physics Lenard-Bernstein collision operator (Lenard & Bernstein 1958). Indeed, if we are interested in those problems associated with a change in the distribution function arising from relatively small momentum transfer, relaxation processes may be described by a simple collision integral of Fokker-Planck type, proposed by Lenard & Bernstein

$$\left(\frac{\partial f}{\partial t}\right)_{\alpha\beta} = \nu\frac{\partial}{\partial v}\left[vf + c^2\frac{\partial f}{\partial v} \right], \qquad (17)$$

where v is the velocity component in the direction of wave propagation, ν is a characteristic velocity-independent collision frequency, which may be evaluated, for example, by using the simple "molecular-kinetic" theory (e.g., Chandrasekhar [1960]) and c^2 is the squared velocity dispersion ("temperature"). The linearized Eq. (17) becomes

$$\left(\frac{\partial f_1}{\partial t}\right)_{\alpha\beta} \simeq \nu\left[1 + i\frac{k_* v_\perp}{\kappa}\sin\kappa t - \frac{k_*^2 v_\perp^2}{\kappa^2}\sin^2\kappa t \right] f_1$$

$$\to -\nu\frac{k_*^2 c^2}{2\kappa^2} f_1,$$

where again a long-time behavior has been singled out in the last term (Ichimaru 1973, p. 123). The similarity between the equation above and Eq. (13) can easily be found.

Integrating expression (16) over velocity space,

$$\int\int f_1 dv_r dv_\varphi = \frac{\kappa}{2\Omega}\int_0^\infty f_1 v_\perp dv_\perp \int_0^{2\pi} d\phi_0 \equiv \sigma_1,$$

and applying the relation between perturbed gravitational potential Φ_1 and surface density σ_1 from the asymptotic solution of the Poisson's equation (9), the following general dispersion relation is obtained (the method of solution is given in Papers I-III)

$$\frac{k^2 c_r^2}{2\pi G\sigma_0|k|} = 1 - \sum_{l=-1}^{1} \omega_* \frac{e^{-x} I_l(x)}{\omega_* - l\kappa + i\nu_{eff}}$$

$$+ 2\Omega \frac{m\rho^2}{r_0 L} \sum_{l=-1}^{1} \frac{e^{-x} I_l(x)}{\omega_* - l\kappa + i\nu_{eff}}. \qquad (18)$$

In Eq. (18), $0 < |\omega_*| < \kappa$, I_l is the modified Bessel function, $x = k_*^2 c_r^2/\kappa^2 \simeq k_*^2\rho^2$, $|L| = |\partial \ln(2\Omega\sigma_0/\kappa c_r^2)/\partial r_0|^{-1}$ is the radial scale of stellar disk spatial inhomogeneity, and only in-phase terms are included (Paper I).

Only the main part of the disk between the inner $l = -1$ and outer $l = 1$ Lindblad's resonances is considered, all resonant effects are neglected and resonances of a higher order ($|l| > 1$) are dynamically of less importance (Paper I). The dispersion relation (18) describes the ordinary Jeans and gradient, $\partial f_0/\partial r_0 \neq 0$, branches of oscillations modified by collisions, $\nu_{eff} \neq 0$.

3.1. Jeans-type perturbations

Using the dispersion relation (18), let us first determine the dispersion law for ordinary Jeans perturbations. To repeat, with the Landau collision integral only the case of rare, $\kappa^2 \gg \nu_c^2$ (more correct, $\kappa^2 \gg \nu_{eff}^2$), and weak, $\omega_*^2 \gg \nu_{eff}^2$, collisions can be considered. Strong (and rare) star-cloud collisions have been studied in Paper III by using the model BGK integral.

Analyzing the dispersion relation (18), it is important to distinguish the cases of the long-wavelength perturbations, $x \equiv k_*^2\rho^2 \lesssim 1$ and short-wavelength perturbations $x \gg 1$. Also, one can see a decrease in the effective collision frequency in long-wavelength perturbations approximately by the square of the ratio of the mean epicyclic radius of the stars to the wavelength of the oscillations in the plane of the disk, $\nu_{eff} = \nu_c(k_*^2\rho^2/2) \sim \nu_c(\pi\rho/\lambda)^2$, and an increase in the effective collision frequency in the short-wavelength perturbations. Of course, the short-wavelength perturbations are not so dangerous in the problem of galactic disk stability as perturbations with $x \lesssim 1$, since they lead only to very small-scale $\ll 2\pi\rho$ oscillations of the density (Paper III).

In the limit $x < 1$, the asymptotic expansion of the modified Bessel function of imaginary argument:

$$I_l(x) = \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^{l+2n} \frac{1}{n!(n+l)!}.$$

Hence, up to terms $\propto x^3$, one can use the approximations $I_0(x) \simeq 1 + x^2/4$, $I_1(x) \simeq x/2$, and $\exp(-x) \simeq 1 - x + x^2/2$. As a result, Eq. (18) can be greatly simplified:

$$\omega_*^3 - \omega_*\omega_J^2 + \frac{\kappa^2}{k_*^2\rho^2}\left(i\nu_{eff} + \Omega\frac{m\rho^2}{r_0 L}\right) = 0, \qquad (19)$$

where the square of the Jeans frequency ω_J^2 is (Paper I)

$$\omega_J^2 \simeq \kappa^2 - 2\pi G\sigma_0 \frac{k_*^2}{|k|}[1 - (3/4)k_*^2\rho^2]. \qquad (20)$$

Since according to observations in the Milky Way $\nu_{eff}/\Omega \ll 1$ and $\rho^2/r_0 L \ll 1$ to simplify the analysis, in Eq. (19) we retain only $l = 0$ terms in a Bessel expansion, proportional to these small parameters. From Eq. (20) it is easy to find the generalized local stability criterion to suppress the instability of arbitrary but not only axisymmetric Jeans-type perturbations, $\omega_J^2 > 0$, including the most dangerous, in the sense of the loss of stability, nonaxisymmetric perturbations (see below).

From relation (19) in the frequency range

$$|\omega_*|^3 \sim |\omega_J|^3 \gg 2\Omega\frac{\kappa^2}{k_*^2}\frac{m}{r_0|L|} \quad \text{and} \quad |\text{Re } \omega_*| \gg |\text{Im } \omega_*|, \qquad (21)$$

(we have made use of the fact that this is typically for spiral galaxies, where usually $|\omega_J| \sim \Omega$ and $m \sim 1$) one determines the dispersion law for the Jeans branch of oscillations (Paper III)

$$\omega_{*1,2} = \pm p|\omega_J| - \frac{\kappa^2}{2\omega_J^2 k_*^2\rho^2}\left(i\nu_{eff} + 2\Omega\frac{m\rho^2}{r_0 L}\right), \qquad (22)$$

where $p = 1$ for perturbations with $\omega_J^2 > 0$ and $p = i$ for perturbations with $\omega_J^2 < 0$. The second term on the right-hand side is the small correction to the Jeans frequency ω_J. From Eq. (22), we conclude that Jeans-stable perturbations ($\omega_J^2 > 0$) will decay (Im $\omega_* < 0$) in the presence of collisions, and Jeans-unstable ($\omega_J^2 < 0$) perturbations will undergo additional weak destabilization (Im $\omega_* > 0$). Fridman & Polyachenko (1984, Vol. 2, p. 243) and Morozov et al. (1985) have explained such an influence of the dissipation on Jeans perturbations by considering the dissipation of the perturbation energy.[4] Because we consider oscillations in the range (21), the collisional correction $\sim \nu_{eff}\kappa^2/4k_*^2\rho^2\omega_J^2 = \nu_c\kappa^2/4\omega_J^2$ (where $\kappa^2 > |\omega_J^2|$ is always true) to the Jeans frequency $|\omega_J| \sim \Omega$ is not great, $\nu_c\kappa^2/4\omega_J^2 \sim \nu_c < \Omega$.

In the opposite case of short-wavelength perturbations, $x \gg 1$, the asymptotic expansions of the modified Bessel functions are

$$I_0(x) \simeq I_1(x) \simeq \frac{e^x}{\sqrt{2\pi x}}\left[1 + O\left(\frac{1}{x}\right)\right].$$

Using the expansions above, in this extreme limit the general dispersion relation (18) can be rewritten in the form

$$\omega_*^3 - \omega_*\omega_J'^2 + \frac{\kappa^2}{2}\left(i\nu_{eff} + 2\Omega\frac{m\rho^2}{r_0 L}\right) = 0, \qquad (23)$$

where now the square of the Jeans frequency is (Paper I)

$$\omega_J'^2 \simeq \kappa^2 - 2\pi G\sigma_0|k|\frac{1}{k^2\rho^2}\left[1 - \sqrt{\frac{1}{2\pi}}\frac{1}{k_*\rho}\right]. \qquad (24)$$

[4] It was shown that the sign of the energy density of Jeans perturbations sign (E_J) = sign (ω_J^2). Hence Jeans perturbations in a gravitationally stable disk ($\omega_J^2 > 0$) will decay if dissipative effects are taken into account, while in a gravitationally unstable dissipative system the negative energy perturbations ($E_J < 0$) will be even more unstable (Morozov et al. 1985). Note that in plasma physics the negative energy waves are well known, e.g., the negative energy loss-cone mode (Ichimaru 1973; Alexandrov et al. 1984).

In the frequency range

$$|\omega_*|^3 \sim |\omega'_J|^3 \gg 2\Omega \frac{\kappa}{2} \frac{m\rho^2}{r_0|L|} \quad \text{and} \quad |\text{Re}\;\omega_*| \gg |\text{Im}\;\omega_*|, \tag{25}$$

the Jeans-roots of Eq. (23) are

$$\omega_{*1,2} \simeq \pm p|\omega'_J| - \frac{\kappa^2}{4\omega_J'^2} \left(i\nu_{eff} + 2\Omega \frac{m\rho^2}{r_0 L} \right), \tag{26}$$

where $p = 1$ if $\omega_J'^2 > 0$ and $p = i$ if $\omega_J'^2 < 0$. Similar to the case of $x \lesssim 1$ (Eq. [22]), here also Jeans-stable perturbations ($\omega_J'^2 > 0$) will decay and Jeans-unstable perturbations ($\omega_J'^2 < 0$) will suffer weak $\text{Im}\;\omega_* \simeq \nu_{eff} < |\omega'_J|$ (and $|\omega'_J| \sim \Omega$) desctabilization because of star-cloud encounters.

It is important to recognize from Eqs. (22) and (26) that the collisional correction to the Jeans frequency in the limit of short wavelengths, $k_*^2\rho^2 \gg 1$, is more important than in the opposite case of long wavelength, $k_*^2\rho^2 < 1$ (this is because for short-wavelength perturbations $\nu_{eff} \gg \nu_c$). The collisionless approximation (Eq. [7]) for short-wavelength Jeans perturbations must be rewritten in the limit $k_*^2\rho^2 \gg 1$ as

$$\Omega^2, \kappa^2, \kappa^2 - \omega_*^2 \gg v_{eff}^2.$$

It means that the collisionless approximation for short-wavelength Jeans perturbations of a star-cloud disk is valid when encounters are much more infrequent than in the event of long-wavelength perturbations. (The similar conclusion has been obtained in the theory of a collisional plasma by Pitaevskii [1963] and Rukhadze & Silin [1969].) This result is quite obvious: as the energy associated with the motion is fed into smaller and smaller scale sizes the role of viscous effects becomes more important because of the higher spatial gradients.

Thus, as a result of collisions the Jeans-stable wave will be damped both in the case of long-wavelength $x < 1$ and in the opposite case of short-wavelength perturbations, $x \gg 1$. The Jeans-unstable perturbations will undergo further weak destabilization. Although the damping rate of the Jeans-stable oscillations is not great (the time necessary for the Jeans-stable wave amplitude to fall to $1/e$ of its initial value is about the collision time, $t \sim \nu_c^{-1}$), it is sufficient to damp the standard Lin-Shu density waves on the Hubble's time T. Here, we adopted $|\omega_J| \sim \Omega$ and $x \sim 1$. Thus, in the disk of the Milky Way the damping time is about $t \sim 10^9 < T$ yr.

By neglecting the weak effect of spatial inhomogeneity (Morozov 1981a), from Eq. (19) one can see that at the limit of gravitational stability, the two conditions

$$\frac{\partial\omega_*^2}{\partial k} = 0, \quad \omega_*^2 \geq 0 \tag{27}$$

are fulfilled. The first condition determines the most unstable wavelength (the modified Jeans-Toomre wavelength), that is, the smallest wavenumber which could be Jeans-unstable:

$$k_{crit} \simeq \left\{ 1 + \left[(2\Omega/\kappa)^2 - 1 \right] \sin^2\psi \right\}^{-1/2} \frac{\kappa}{c_r}.$$

Accordingly, the velocity dispersion shifts the limit of stability to a smaller wavenumber (a longer wavelength). It follows from the equation above, by including the pitch angle dependent effects (m and $\psi \neq 0$), the limit of gravitational instability of differentially rotating disk ($2\Omega/\kappa > 1$) is shifted toward a smaller wavenumber (or a longer wavelength) than it follows from the ordinary Toomre's critical wavenumber $k_T \simeq \kappa/c_r$. In galaxies $1 + \left[(2\Omega/\kappa)^2 - 1\right] \sin^2\psi \sim 1$, therefore

$$k_{crit} \sim \frac{\kappa}{c_r} \sim \frac{1}{\rho} \quad \text{or} \quad k_{crit} \cdot \rho \sim 1.$$

To order of magnitude, the critical wavelength is

$$\lambda_{crit} \equiv 2\pi/k_{crit} = \frac{2\Omega}{\kappa} 2\pi\rho.$$

As a rule, in galaxies $2\Omega/\kappa$ = 1.4–1.7, thus, $\lambda_{crit} \sim (5 - 10) \cdot \rho$.

Use of the second condition (27) determines the critical radial velocity dispersion for the stability of arbitrary Jeans perturbations (modified Toomre's critical velocity dispersion):

$$c_r \geq c_G \simeq c_T \left\{ 1 + \left[(2\Omega/\kappa)^2 - 1 \right] \sin^2\psi \right\}^{1/2}, \tag{28}$$

where $c_T \simeq 3.4G\sigma_0/\kappa$ is the ordinary Toomre's critical radial velocity dispersion to suppress the instability of axisymmetric perturbations only (Toomre 1964, 1977). The criterion above takes into account the destabilizing effect of finite inclination of spiral arms, $\psi \neq 0$ (Griv & Peter 1996). To emphasize, it is follows from Eq. (28) that stability of the nonaxisymmetric perturbations (m and $\psi \neq 0$) in a nonuniformly rotating disk ($2\Omega/\kappa > 1$) requires a larger dispersion of random velocities than the ordinary Toomre's critical value c_T. Note that a relationship exists between the proposed generalized criterion c_G for nonaxisymmetric gravitational instability and what Toomre (Toomre 1977) called "swing amplification." Earlier, effects of finite but not large inclination of spiral arms have been studied by Bertin & Mark (1978), Lin & Lau (1979), Bertin (1980), Polyachenko (1989), Fridman et al. (1991), and Griv & Peter (1996) using both a gaseous and a kinetic approaches. More accurate expressions for the critical velocity dispersion have been obtained by Morozov (1981a, 1981b) by including the weak effects of the finite thickness and spatial inhomogeneity of the disk.

As follows from Eq. (28), in order to suppress the most unstable gravitational perturbations in the form of a bar ($\psi \to 90^o$), c_r should obey the generalized local stability criterion $c_r \geq (2\Omega/\kappa)c_T$ (or $Q \geq 2\Omega/\kappa$, respectively). Hence, the critical value of c_r in disk-shaped galaxies is $c_G \approx (1.5 - 1.7) \cdot c_T$, since according to observations $2\Omega/\kappa = 1.5 - 1.7$ in galaxies. Note that in Paper I we have shown that the Jeans-stable waves ($Q > 2\Omega/\kappa$) in a differentially rotating disk may be a subject to the oscillating Landau-type instability (Cherenkov or Doppler radiation effects in plasmas).

It is of interest to find out the difference in results in the cases of the Landau and BGK models. As opposed to the results of Eqs. (22) and (26) obtained in the case of the exact collision integral, in Paper III (Paper III, Eqs. [17] and [22]) using the BGK model we found that the collisional corrections to the Jeans frequency are Im $\omega_* \sim \nu_c/(k_*^2\rho^2)$ and Im $\omega_* \sim \nu_c$ in the cases of long-wavelength and short-wavelength perturbations, respectively. Thus, in the limits of long-wavelength and short-wavelength oscillations the exact Landau and the BGK model collision integrals lead to results that differ qualitatively approximately by the square of the ratio of the mean epicyclic radius of stars to the wavelength of the oscillation, $x = (k_*\rho)^2 \sim (2\pi\rho/\lambda)^2$. In the range of values $x \sim 1$ the results from two models have roughly similar quantative and qualitative behavior. In Paper I and above, we have shown that in a disk the most dangerous, in the sense of the loss of stability, are the Jeans-type perturbations with $k_*\rho \sim 1$. We conclude, therefore, that the Landau and the BGK integrals lead to almost identical results only in the case of the most unstable Jeans-type perturbations. The above means that in the problem of the gravitational Jeans-type stability of the collisional star-cloud disk of our Galaxy and other spiral galaxies instead of the complicated exact collision integral one can use the simple BGK model operator (or the simplest Krook collision term; Shu & Stewart [1985]).

Also, it is obvious that rare elastic collisions between particles (which is the approximation being used) do not affect the generalized local stability criterion (28).

3.2. Gradient-dissipative perturbations

By neglecting the effect of collisions, $\nu_c = 0$, in the frequency ranges (21) and (25), the dispersion relations (19) and (23), respectively, have also other roots equal to

$$\omega_{*3} \simeq \Omega \frac{\kappa^2}{\omega_0^2} \frac{m\rho^2}{r_0 L} \frac{I_0(x)}{I_1(x)}, \tag{29}$$

where $\omega_0 \equiv \omega_J$ in the case of the long-wavelength or $\omega_0 \equiv \omega'_J$ in the case of the short-wavelength oscillations, respectively. In contrast to Eqs. (22) and (26), the roots (29) describe the gradient branch of oscillations of a stellar disk. This branch of oscillations can exist only when there is spatial inhomogeneity of the disk, $\partial f_0/\partial r_0 \neq 0$ or $L^{-1} \neq 0$, respectively. Apparently, the gradient perturbations are stable (natural oscillations, Re $\omega_* \neq 0$ and Im $\omega_* = 0$) and are independent of stability of Jeans perturbations. We shall show now that these perturbations become oscillatory unstable in the presence of rare collisions.

Let us investigate the influence of collisions on the oscillation spectrum of gradient perturbations. Apart from the Jeans roots (22) and (26) in the frequency ranges (21) and (25), the simplified dispersion relations (19) and (23) have other roots equal to

$$\omega_{*3} \simeq \left(2\Omega\frac{m\rho^2}{r_0 L} + i\nu_{eff}\right)\frac{\kappa^2}{\omega_0^2}\frac{I_0(x)}{I_1(x)}. \tag{30}$$

Accordingly, the gradient perturbations are now unstable (Im $\omega_{*3} > 0$) in the presence of interparticle encounters in the Jeans-stable ($\omega_0^2 > 0$) disk and are slowly fade (Im $\omega_{*3} < 0$) in the Jeans-unstable ($\omega_0^2 < 0$) disk. Lynden-Bell & Pringle (1974), Fridman & Polyachenko (1984, Vol. 2, p. 243), and Morozov et al. (1985) have already explained the cause of this dissipative secular-type instability: the secular instability might arise merely from the dissipation of the energy of ordered rotation into even larger amounts of heat. Obviously, this oscillating (Re $\omega_* \neq 0$ and Im $\omega_* > 0$) dissipative instability leads to a lower free energy state of the system (Lynden-Bell & Pringle 1974; Fridman & Polyachenko 1984, Vol. 2, p. 243). The instability has an essential dependence both on the self-gravitation of the galactic matter and the collisions between particles.[5] To order of magnitude the instability growth rate is

$$\text{Im}\ \omega_{*3} \approx \nu_c \frac{\kappa^2}{|\omega_0|^2} \sim \nu_c, \tag{31}$$

where we have considered the case $|\omega_0| \simeq \Omega \sim \kappa$, and $x \lesssim 1$, then $I_0(x)/I_1(x) \simeq 2/x$. Hence, the instability growth rate is not large, $< \Omega$, and the instability will develop on the timescale of many galactic rotations. According to Eq. (30), in the most important limit $x \lesssim 1$ the wavelength of unstable oscillations is arbitrary. To emphasize, the instability develops only in a disk which is stable to all Jeans-type perturbations, $\omega_0^2 > 0$, that is, $\omega_J'^2 > 0$ and $\omega_J^2 > 0$. In other words, if $Q > 2\Omega/\kappa$ (or $c_r > (2\Omega/\kappa)c_T$) [Paper I; §3.1 of the present paper].

Comparing Eq. (30) with the corresponding Eq. (26) in Paper III, we find again that the result obtained with the aid of the Landau integral differs from that obtained with the aid of the BGK model collision integral by the ratio $(k_*\rho/2)^2 \sim (\pi\rho/\lambda)^2$.

In closing of the section, let us show that the value of the growth rate of the dissipative instability becomes large in parts of the disk that are marginally Jeans stable gravitationally ($\omega_0^2 \to 0$). It is apparent from Eq. (30) that here the dissipative instability growth rate is a maximum. In the marginally Jeans stable disk $k_*^2\rho^2 \simeq 1$, $|k| \simeq \pi G\sigma_0/c_r^2$, $c_r \simeq \pi G\sigma_0/\kappa$, $\exp(-x)I_0(x) \simeq 0.5$, and $\exp(-x)I_1(x) \simeq 0.2$, and the conditions (21) and (25) are violated (Papers I and III). Now Eqs. (19) and (23) in the frequency range $\omega_*^3 \sim \nu_c\kappa^2 \gg (2\Omega\kappa^2)(m\rho^2/r_0|L|)$, i.e., in the case of nearly homogeneous disk, has roots

$$\omega_{*\ 1} \simeq \omega_d\left[i + \frac{\Omega}{\nu_c}\frac{m\rho^2}{r_0 L}\right], \tag{32}$$

$$\omega_{*\ 2,3} \simeq \omega_d\left[\frac{\pm 3}{2} - \frac{\Omega}{2\nu_c}\frac{m\rho^2}{r_0 L} - i\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}\frac{\Omega}{\nu_c}\frac{m\rho^2}{r_0 L}\right)\right], \tag{33}$$

where

$$\omega_d = \left(\nu_c\kappa^2\right)^{1/3}.$$

The result of Eq. (32) indicates that a marginally Jeans-stable gravitationally disk with encounters between particles is violently unstable, Im $\omega_* \sim \left(\nu_c\kappa^2\right)^{1/3} \sim (0.3{-}0.4)\cdot\Omega > \nu_c \ll T$,

[5] Lynden-Bell & Pringle (1974) claimed this instability to be analogous to the well-known viscous mechanism that converts Maclaurin spheroids to Jacobi ellipsoids.

with respect to dissipative perturbations (assuming with accordance to observations that in large portions of a stellar disk $\Omega \simeq \kappa$). The characteristic wavelength of these unstable perturbations is $\lambda = 2\pi/|k_*| \simeq 2\pi\rho$; the modes are oscillatory growing, with the small real part of the eigenfrequency $\sim \kappa m\rho^2/r_0|L| \ll \kappa$.

As has been noted in Paper I, according to observations the disk of the Milky Way is near the boundary of gravitational stability, $Q \sim 2$ (Toomre 1977; Fridman et al. 1991; Paper I). Also, $\nu_c \approx 10^{-9}$ yr^{-1}, $\kappa \approx 10^{-8}$ yr^{-1}, and the dissipative instability growth time determined from Eq. (32) for the nearly homogeneous disk is about $1/\omega_{*\,1} \sim \left(\nu_c\kappa^2\right)^{-1/3} \approx 5 \cdot 10^8$ yr. This is smaller than the ages of the bulk of stars and comparable to a time of a single revolution of the Galaxy in the solar vicinity.

Thus, in marginally Jeans-stable galactic disks with $\nu_c \gtrsim 10^{-9}$ yr^{-1}, dissipative instabilities may develop effectively on the time span of a galaxy's age $T \sim 10^{10}$ yr. In this sense, in disk-shaped spiral galaxies similar to our Galaxy this local instability may be considered both as a generating mechanism for the rapidly evolving, short-scale spiral filaments with characteristic radial length $\lambda_r = 2\pi\rho \sim 3$ kpc and as a mechanism for the effective dynamical relaxation of the system (Papers I and III).

4. Conclusions and discussion

In concluding we wish to summarize the basic features of the present work. We extend previous works by Lin, Shu, Julian, Toomre, and others to investigate how the instability range of the Milky Way disk is modified in the presence of gravitational elastic encounters between stars and giant molecular clouds. The kinetic theory of the stability of the Milky Way's self-gravitating disk with gravitational encounters between stars and giant molecular clouds was based on the Boltzmann kinetic equation with an exact Landau collision integral and the Poisson's equation. The evolution of the stellar distribution is determined primarily by the interaction with collective modes – gravitational Jeans-type and gradient modes – modified by collisions.

We have shown that rare, $\kappa^2 \sim \Omega^2 \gg \nu_c^2$, and weak, $\omega_*^2 \gg \nu_c^2$, binary collisions between stars and clouds in the Milky Way's disk lead to the damping of Jeans-stable modes and the further weak destabilization of Jeans-unstable modes. Generally, the effect is not great: if the dissipative damping of Jeans-stable modes occurs in the Galaxy, the damping time is $t \gtrsim 1/\nu_c \sim 10^9$ yr. Although this time is longer than the characteristic time of a single revolution of the Galaxy in the solar vicinity, it is sufficient to damp the standard Lin-Shu quasi-stationary density waves on the Hubble's time T. Typically, collisions will disrupt the organized wave motion on a timescale of the order of the mean time τ between collisions. According to Grivnev & Fridman (1990), in the Milky Way $\tau \simeq 2 \cdot 10^9$ yr.

In turn, gradient Jeans-stable modes that are stable in the collisionless case now become unstable in the presence of star-cloud encounters. Here also the effect is not strong: the dissipative secular-type instability, which grows only in a Jeans-stable disk ($Q > 2\Omega/\kappa$ and $c_r > (2\Omega/\kappa)c_T$), will develop on the timescale of many galactic rotations, $\sim \nu_c^{-1} > \Omega^{-1}$.

From the physical point of view, the results described above – weak influence of collisions on a disk dynamics – are quite obvious. Indeed, Lifshitz & Pitaevskii (1981, p. 115) have discussed plasma phenomena in which interparticle collisions are unimportant, and such a plasma is said to be collisionless (and in the lowest-order approximation of the theory one can neglect the collision integral in the kinetic equation). It was shown that a necessary condition is that $\nu \ll \omega$: then the collision operator in the kinetic equation is small in comparison with $\partial f/\partial t$. The same case that is under consideration in the present research by investigating only the limit of weak collisions. (Lifshitz & Pitaevskii [1981] have pointed out that collisions may be neglected also if the particle mean free path is large compared with the wavelength of collective oscillations. Then the collision integral in Eq. (1) is small in comparison with the term $\mathbf{v} \cdot (\partial f/\partial \mathbf{r})$.)

In §3.2 of the paper we found, however, "fast" dissipative secular-type instabilities in parts of the star-cloud galactic disk those are marginally Jeans-stable, i.e., for which $c_r \to (2\Omega/\kappa)c_T$, $Q \to 2\Omega/\kappa$, and ω_J^2 or $\omega_J'^2 \to 0$, correspondingly. Similar result has been obtained Morozov et al. (1985), using a gasdynamical model. We have shown that in this case, ω_J^2 and $\omega_J'^2 \ll \kappa^2$, the growth rate of dissipative instability becomes sufficiently large, Im $\omega_{*3} > \nu_c \sim \Omega$. Under conditions thought to be realized in the Galaxy, this instability may develop effectively on the timescale of a few Milky Way's rotations only. Hence, this fast secular-type instability may be considered as a generating mechanism for unstable density waves, thereby leading to a small-scale ~ 3 kpc structuring of the disk of the Galaxy and a rapid $\sim 3 \cdot \Omega^{-1} \sim 5 \cdot 10^8$ yr collective dynamical relaxation of the system (see Paper III for a discussion).

It is found that the results obtained with the aid of the exact Landau (Fokker-Planck type) collision integral differ from that obtained with the aid of the BGK model collision integral approximately by the square of the ratio of the mean epicyclic radius of stars to the wavelength of the oscillation. Therefore, the BGK model integral leads to results that differ qualitatively from the true ones in the limiting cases of short-wavelength, $(\pi\rho/\lambda)^2 \gg 1$, and long-wavelength, $(\pi\rho/\lambda)^2 \ll 1$, oscillations. In the range of values $(\pi\rho/\lambda)^2 \sim 1$ the results from these two different models have roughly similar quantitative and qualitative behavior. In Paper I and §3.1 of the present paper, it was demonstrated that the most dangerous, in the sense of the loss of stability, Jeans-type perturbations are ones with $2\pi\rho/\lambda \sim 1$. Therefore, in the problem of the stability of small oscillations of the star-cloud disk of the Galaxy it is a good approximation to use the simple BGK or Krook phenomenological models instead of the complicated exact collision integral, e.g., the Landau collision integral.

Occasionally doubts have been raised about the validity of two-dimensional N-body simulations of stellar disks of flat galaxies. For example, White (1988) found a few computer models in the exactly planar simulations which are affected by noise and two-body relaxation (see, however, Hohl [1973]). The physical effect of relaxation in the N-body simulations is to generate viscosity and heat conduction. One obvious effect of short-term relaxation is a heating of the disk, and, therefore, some of the two-dimensional N-body simulations cannot be trusted (White 1988). The present work, however, shows that in general the effect of such rare elastic collisions is very small, and may be important only on a timescale of the order of the mean time of many galactic rotations, typically $\sim$ 100 rotations (see also Sterzik et al. [1995]). This is greater than (or comparable to) the Hubble's time. Thus, relaxation effects in such N-body models probably do not yield any interesting physics on a timescale of a few first tens of rotations. Further discussion of the problem is clearly needed.

Lastly, we add a brief comment on our work. Here equations in the limiting case of weak collisions $\omega_*^2 \gg \nu_{eff}^2$ in a rarefied star-cloud galactic disk $\kappa^2 \sim \Omega^2 \gg \nu_{eff}^2$ have been considered using the exact Landau integral formulation. The other question that remains is the stability of the collisional disk in the extreme case of strong collisions $\omega_*^2 \ll \nu_{eff}^2$. It is shown in plasma physics to extend the theory to this limit, equations should be solved by an entirely different method (see Mikhailovskii [1974, Vol. 2] for explanations). We intend to consider the problem in the following publication of the present series.

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