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Hipparcos subdwarfs and globular cluster ages: the distance and age of M 92*

F. Pont[1], M. Mayor[1], C. Turon[2], and D.A. VandenBerg[3]

[1] Observatoire de Genève, CH-1290 Sauverny, Switzerland
[2] DASGAL/URA CNRS 335, Observatoire de Paris-Meudon, France
[3] Department of Physics and Astronomy, University of Victoria, Canada

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Abstract. A new determination of the distance and age of the old globular cluster M 92 (NGC 6341) is obtained from a consideration of a set of more than 500 subdwarf candidates with Hipparcos parallaxes. Precise [Fe/H] values are derived for most stars using the equivalent width of the Coravel cross-correlation function.

We examine at some length the biases affecting the determination of the mean luminosity of a set of subdwarfs selected by metallicity and $\sigma_\pi/\pi < lim$ criteria, by means of Monte Carlo simulations of the whole selection and analysis procedure. Effects other than the classic Lutz-Kelker bias are found to be significant, so that in most cases considered, the resulting bias acts as a slight shift in a direction opposite to the Lutz-Kelker correction. The effect of the presence of detected (and suspected) binaries is also examined and taken into account.

Our best estimate of the distance of M 92 is $(m-M)_V$ = 14.67 ± 0.08 mag, from a fit of the cluster main sequence to the 17 subdwarfs in our set with [Fe/H]< -1.8 and $\sigma_\pi/\pi < 15\%$. Bias and binarity corrections are included. The adoption of an alternative [Fe/H] scale causes only minor differences in this result. A more classic treatment of binaries (i.e., simply excluding the detected binaries from the sample) leads to $(m-M)_V$ = 14.74 mag. The location of the evolved field subdwarfs (along the subgiant branch) provides a strong indication that M 92 and the most metal-poor subdwarfs are coeval.

The M 92 C-M diagram is confronted with up-to-date stellar evolutionary models (recent opacities and nuclear reaction rates, non-ideal equation of state, α-element enhancement). The agreement between theory and observations is excellent, including the position of the horizontal branch, if the models are shifted by $\delta(B-V)$ = +0.012 mag. An age of 14 Gyr is derived from the luminosities of the cluster turnoff and subgiant branch stars. A similar colour shift is found from an examination of an independent set of subdwarfs having $-1.8 <$ [Fe/H]< -1.1 and very accurate parallaxes ($\sigma_\pi/\pi < 5\%$). Because the inferred adjustment to the synthetic colours seems to be nearly constant over a fairly wide range in metallicity, we conclude that current theoretical models predict the systematic variation in the location of the lower main sequence as a function of [Fe/H] reasonably well. As discussed herein, a greater distance (and younger age) for M 92 would pose considerable difficulties for stellar structure theory.

Thanks to more precise globular cluster (GC) distances, the dominant source of uncertainty in GC ages now becomes associated with the model stellar interiors and atmospheres. In particular, helium diffusion, which was not treated in the present models, is expected to lead to about a 10% age reduction, while the use of an alternative bolometric correction scale would imply higher ages by $\sim$ 1.5 Gyr. If GCs formed about 1 Gyr after the Big Bang, then we obtain a minimum age for the Universe of at least 14 Gyr, confirming the necessity of rather low values of H_0 in the context of standard cosmological models.

Key words: globular clusters: M 92 – stars: kinematics – HR-diagram – stars: abundances – Galaxy: kinematics and dynamics

1. Introduction

In the current picture of the birth and evolution of the Universe, the gaseous medium from which the stars and galaxies formed was progressively enriched from primordial chemical abundances by successive generations of supernovae, so that the most metal-deficient stars are also the oldest. In particular, extreme subdwarfs and very metal-poor globular clusters (GCs) were probably among the first objects to form after the Big Bang (Peebles & Dicke 1968). Consequently, their ages give the best available constraint on the minimum age of the Universe.

In this regard, and for stellar evolution studies in general, GCs have been invaluable because each one contains typically 10^5 to 10^6 stars that all formed at about the same time and with

Send offprint requests to: F. Pont

* Based on data from the ESA Hipparcos Astrometry Satellite, and measurements made at the European Southern Observatory, Chile and Observatoire de Haute-Provence, France.

essentially the same chemical composition (e.g., see Stetson 1993). There are a few well-known exceptions to this rule — notably ω Cen and M22 — but most GCs have exceedingly tight photometric sequences that can be readily compared with computed isochrones to yield estimates of their ages. Unfortunately, it has not been possible to determine *absolute* cluster ages to within $\sim$ 20% (cf. Renzini 1991, Chaboyer 1995), although *relative* age studies have advanced to the point where we can say with some confidence that all (or nearly all) GCs having [Fe/H] < -2 are very close to being coeval (see the review by Stetson et al. 1996), independent of their location in the Galaxy (Harris et al. 1997). The main problem insofar as absolute GC ages are concerned has been our inability to derive sufficiently precise distances to these systems. The scarcity of parallax data for local subdwarfs and, to some extent, the practical difficulties of measuring GC main sequences free of photometric biases have hindered the determination of accurate distances by the most direct method; namely, the main-sequence fitting of cluster colour-magnitude diagrams (CMDs) to nearby field subdwarfs of known distance.

In recent years, very well-defined fiducials down to $M_V > 8$ have been obtained for several GCs using large ground-based telescopes (e.g., Mandushev et al. 1996) or the *HST* (e.g., Piotto et al. 1997). Furthermore, the astrometric satellite Hipparcos has measured precise parallaxes for a large number of subdwarf candidates, enabling a much improved definition of metal-deficient main-sequence loci on the $[(B-V)_0, M_V]$-plane and a new determination of GC distances via the main-sequence fitting technique.

However, precise metal abundances for the subdwarfs are just as important as well-determined distances to these stars since the local subdwarf sequence to which a cluster CMD is fitted must correspond to the metallicity of the cluster stars themselves. Several large surveys of subdwarf metallicities have been completed in recent years (Schuster & Nissen 1989; Ryan & Norris 1992; Carney et al. 1994). In order to measure metallicities for newly-discovered subdwarfs and to ensure homogeneity of the [Fe/H] values for the whole sample, we derive new metal abundances for most Hipparcos subdwarfs with the Coravel radial velocity spectrometer (Baranne et al. 1979), which correlates an object spectrum with a physical template of more than a thousand weak metallic lines. The cross-correlation function can be used for a precise calibration of the metallicity of F and G dwarfs, as already shown by Mayor (1980).

The globular cluster M 92 (NGC 6341) is known to be one of the oldest and most metal-poor such systems in the Galaxy (see the compilation of GC properties by Harris 1996). There is general agreement as to its metallicity and reddening, and its main sequence has been measured precisely down to very faint magnitudes by Stetson & Harris (1988). For these reasons, we will focus on M 92 here; but, given the apparent uniformity in age among [Fe/H] < -2 GCs (as noted above), the conclusions drawn from this case should be valid for the entire old globular cluster system.

At present, the best estimate for the age of the oldest GC is in the range 15–16 Gyr, with a total span of 12–20 Gyr once all of the observational and theoretical uncertainties are taken into account (Chaboyer et al. 1996; VandenBerg et al. 1996). As is widely appreciated, these values conflict with the predictions of current inflationary cosmological models if the Hubble Constant has a value near 80 km s^{-1}Mpc^{-1} (e.g., Ferrarese et al. 1996, Tonry et al. 1997), and they are only marginally consistent with $H_0 \sim$ 60–65 km s^{-1}Mpc^{-1} (Hamuy et al. 1996, Riess et al. 1996) or the somewhat lower estimates favoured by, e.g., Sandage et al. (1996). For instance, an age for the universe of 15 Gyr would require $H_0 \simeq 44$ km s^{-1}Mpc^{-1} if Ω_{matter} = 1, or $H_0 \simeq 58$ km s^{-1}Mpc^{-1} if Ω_{matter} = 0.1 (assuming that the cosmological constant, Λ, is zero). GC ages therefore provide an exceedingly important constraint on cosmology, and obtaining precise ages for the globulars brings us nearer to the resolution of this conflict, be it by reducing the ages or by amending the cosmological models.

Sect. 2 below presents the Hipparcos parallax subdwarf data and the Coravel metallicity determinations. In Sect. 3, the resultant location of the subdwarf sequence for different values of [Fe/H] is considered. In Sect. 4, we examine at some length how biases and the presence of binaries affect GC distance estimates, while Sect. 5 presents our distance determination for M 92 as obtained by main-sequence fitting to the most metal-deficient subdwarfs. Finally, Sect. 6 discusses the implications of our results for the age of M 92 and of the subdwarfs themselves by comparison with stellar evolution models, and compares the model predictions with other available constraints.

2. The data

2.1. Sample description

The Hipparcos astrometric satellite collected precise parallax, proper motion and magnitude data for thousands of stars in its three years of operation.

The subsample of the Hipparcos Catalogue used in this study is made up of two parts, corresponding to two Hipparcos proposals submitted by CT and MM:

- (1) 330 stars identified as subdwarf candidates from various sources at the time of the 1982 Hipparcos proposals;
- (2) 216 stars that were markedly (more than 0.6–0.75 mag, depending on spectral type) below the Hyades sequence in the HR diagram according to Hipparcos data, and with $\sigma_\pi/\pi < 20\%$[1]. Objects of known or suspected binarity, or with large reddenings, were excluded[2].

[1] The parallax and its uncertainty are denoted as π and σ_π in this article.

[2] the actual selection criteria were:
$3 > M_V - [1.0 + 5.7(B-V) - 1.26((B-V) - 0.75)^2] > 1.05 - 0.3(B-V)$
$7 - 22((B-V) - 1.05) > M_V > 5 - 3((B-V) - 0.5)$
which defines a quadrilateral below the solar-abundance main sequence; no Hipparcos binarity, variability or high data rejection rate (i.e., fields H10, H43, and H59 in the Hipparcos Catalogue had to be empty; H29 had to be smaller than 6%, H52 had to be empty or contain "C", and H61 must not contain "S"); there must not have been any pre-

The first set contains mostly F stars at or near the turnoff stage, though a few are subgiants, residing between the turnoff and the bottom of the red-giant branch in the CMD. The second set samples unevolved subdwarfs of later spectral type. The Hipparcos Catalogue provides parallaxes, V magnitudes and $B-V$ colours for all of these stars. The V magnitudes are derived from the Hipparcos Hp magnitudes, while the $B-V$ indices are compiled from the Tycho mission measurements and ground-based values.

2.2. Coravel measurement campaign

Coravel is a radial velocity scanner that computes the cross-correlation function (CCF) of a star's spectrum with a K0 spectrum template (Baranne et al. 1979). Spectral cross-correlation allows a precise evaluation of the average metallic line blocking in the spectrum. Therefore, the CCF equivalent width, denoted "W", is a sensitive indicator of the metallicity, even for low signal-to-noise spectra in which individual lines are not resolved. Using Coravel, homogeneous, precise values of [Fe/H] can be obtained for faint stars with a short observing time. The calibration of W in terms of [Fe/H] is presented in the next section.

A measurement campaign, which was started in the summer of 1996 for the samples defined above, is now completed for set (1) and about half of set (2) ($21h < \alpha < 12h$). The measurements for the whole sample should be finished by the end of 1997. Coravel data for the target stars of other Geneva observation programmes have also been included in the analysis.

Altogether, we now have at our disposal 503 subdwarf candidates from both sets with [Fe/H] determinations either from Coravel or from other subdwarf studies (Carney et al. 1994, hereafter CLL94; Ryan & Norris 1991; Axer et al. 1994; Schuster & Nissen 1989), with σ_π/π <20%.

2.3. The Coravel metallicity determination

The primary use of the Coravel CCF is the measurement of the radial velocity, given by the position of the minimum of the CCF. However the CCF equivalent width, "W", is also related to the temperature, [Fe/H], gravity, etc., in the way in which the individual lines forming the correlation template depend on these quantities. It was realised early on that, via a calibration of the temperature dependence, W was a sensitive indicator of metallicity for F to K dwarfs. The preliminary calibration of Mayor (1980) shows that a precision better than 0.16 dex in [Fe/H] is easily attainable.

More than 60,000 stars have now been measured with Coravel, and the existence of this extensive database permits a more detailed calibration (Pont 1997). In the F-dwarf domain, for [Fe/H] > -0.8, the [Fe/H] determination is calibrated using the sample of high-precision spectroscopic metallicities for 189 F-dwarfs by Edvardsson et al. (1993). A dispersion of 0.06 dex is obtained between Edvardsson et al. and Coravel determinations when the temperature is known independently, or 0.09 dex if the $B-V$ colour is used as the temperature indicator. These values show that the accuracy of the metallicities from the Coravel CCF surface are comparable to the best spectroscopic determinations, with the additional advantages of homogeneity, long term stability and cheapness in telescope time.

vious detection of radial velocity variations by Coravel or by Carney et al. (1994); and $E(B-V) < 0.05$ mag and $|b| > 10^o$.

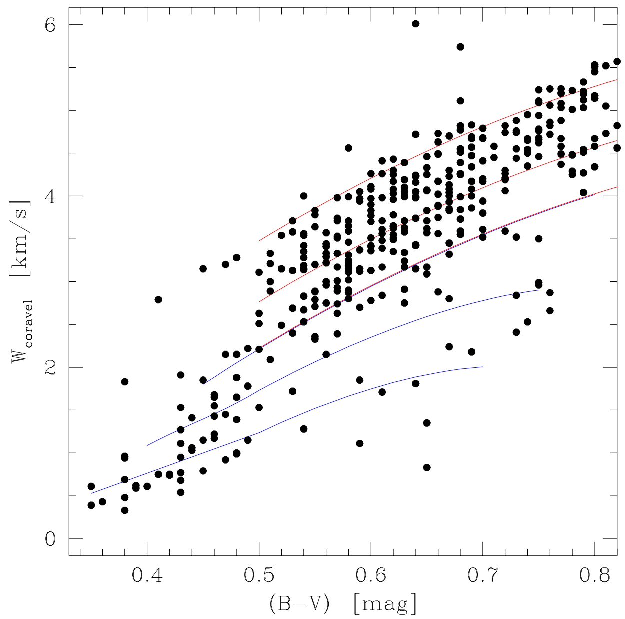

Fig. 1. The [Fe/H] calibration of the Coravel CCF equivalent width "W". Lines of constant [Fe/H] are shown from [Fe/H] = 0 to [Fe/H] = -2 at 0.5 dex intervals, together with the location of the CLL94 calibrators.

For G dwarfs and subdwarfs, the calibration is based on the extensive CLL94 study: 325 stars from their investigation have also been measured with Coravel. Fig. 1 shows the lines of constant [Fe/H] in the W vs. $B-V$ plane according to the calibration, as well as the position of the calibrating objects. The calibrators cover the whole subdwarf domain down to [Fe/H] $\simeq -2.2$ (Fig. 2).

The drawback of the Coravel metallicity scale is that it is not directly connected to stellar atmosphere models and has to rely on another scale for calibration. There are, in some cases, significant differences between the metallicity scales reported in different studies, especially for extreme subdwarfs. The Schuster & Nissen (1989) Strömgren photometric scale, for instance, as well as the high dispersion spectroscopic study of Axer et al. (1994), give a significantly more compressed (i.e., more metal rich) scale than the one by CLL94. We estimate from Fig. 8 in Axer et al.:

$$[\mathrm{Fe/H}]_{CLL94} \simeq 1.12 \times [\mathrm{Fe/H}]_{Axer}. \quad (1)$$

The results of the present study will be examined using both the CLL94 and Axer et al. scales. The conversion will be made using Eq. 1.

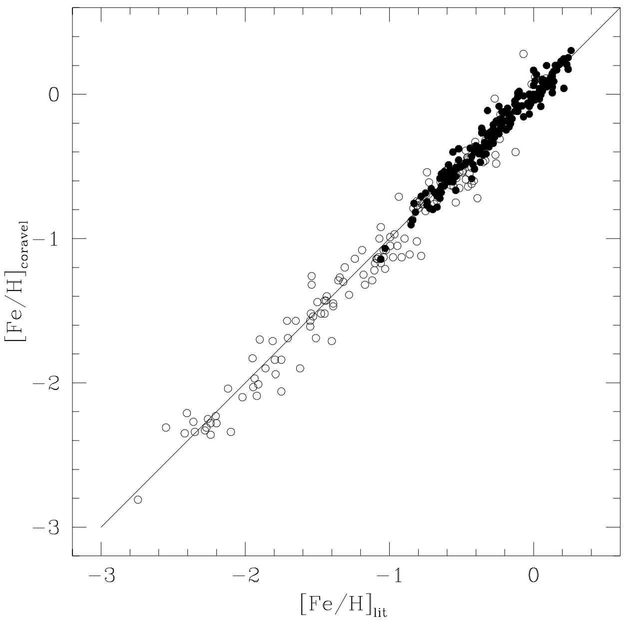

Fig. 2. $[Fe/H]_{Coravel}$ vs. $[Fe/H]_{literature}$ for the calibrators. The dispersion is 0.09 dex for the F-dwarfs from Edvardsson et al. (1993, filled circles), and 0.17 dex for the G dwarfs and subdwarfs from Carney et al. (1994, open circles).

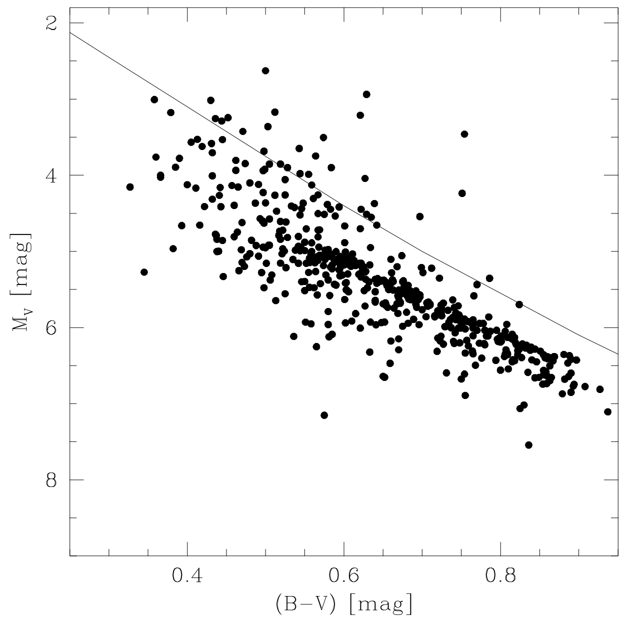

Fig. 3. Position of the sample objects of known [Fe/H] and $\sigma_\pi/\pi < 20\%$ in the HR diagram using Hipparcos parallaxes, magnitudes and colours. The solid line is the Hyades main sequence.

2.4. Colours and reddenings

Intrinsic colour, in our case $(B–V)_0$, must be known in addition to the parallax and magnitude in order to locate a star in the HR diagram. Moreover, as $(B–V)_0$ is used as a temperature indicator for the Coravel [Fe/H] determination, colours and reddenings also have an indirect effect on the derived metallicities.

$(B-V)$ colours were taken from the Hipparcos Catalogue, except those for HIP 8130 and HIP 19797, for which Strömgren photometry shows another ground-based value to be more likely. Some estimates are unfortunately not very precise ($\sigma_{(B–V)} \simeq 0.03$ mag), and some important objects deserve additional photometry.

Reddenings were determined in two different ways[3], so as to permit an evaluation *a posteriori* of the influence of the reddening determination on the final results:

1. from a spatial model developed by Arenou et al. (1992), specifically designed for the analysis of Hipparcos data.
2. from a model based on the Burstein & Heiles (1982) H I absorption maps, on the hypothesis of a gaussian distribution of the absorbing matter around the Galactic disc with a scale height σ = 150 pc — a procedure similar to that adopted by Ryan & Norris (1991). The reddening is then expressed as a function of position and distance as $E(B-V) = E(B-V)_\infty \cdot \mathrm{erf}(\cos b \cdot dist/\sqrt{2}\sigma)$, where $E(B-V)_\infty$ is taken from Figs. 4 and 5 of Burstein & Heiles.

[3] It is important that reddenings be computed from the spatial position of the objects and not from some assumed intrinsic photometric property, as these properties are precisely what is sought from the Hipparcos data.

2.5. Detection of binaries

The accuracy of Coravel radial velocities is sufficient to detect velocity variations down to $\sim 1\ \mathrm{km\,s^{-1}}$. Most stars in the sample have been measured more than once, and many spectroscopic binaries have been detected (by the $P(\chi^2) < 0.01$ criterion — indicating a false detection rate of 1%). The CLL94 study had previously revealed a number of binaries, and the Hipparcos mission also isolated some binary suspects (as indicated by flag "S" in field 61 of the Hipparcos Catalogue, or field 58 indicating the number of resolved components). It is obviously important that proper account be taken of the binaries, since their unrecognized presence can significantly increase the mean luminosity of a given sample (Sect. 4.2).

3. Absolute luminosity and metallicity of subdwarfs

The Hipparcos, Coravel and general data from the previous section form a catalogue from which the position of each individual object in the $[(B-V)_0, M_V]$ plane can be extracted. This catalogue will be published in a later article, once the Coravel measurement campaign is completed.

The absolute magnitude is computed from parallax, visual magnitude and colour data via

$$M_V = m_v + 5\log(\pi) + 5 - 3.1E(B-V)$$

where π is the parallax in arcsec/yr. The resulting HR diagram is displayed in Fig. 3.

[Fe/H] values are taken from the Coravel determination or from the following sources: CLL94, Ryan & Norris (1991), Axer et al. (1994). The metallicity scales are equivalent except the one by Axer et al., which was transformed into our system using Eq. 1. When more than one determination is available an average is made. The Coravel value is ignored when $\sigma_{[\rm Fe/H]} > 0.2$ dex (if another value is available). In three cases, HIC 21609, 89215 and 108592, the extreme metal deficiency obtained by Ryan & Norris (1991) was found to be incompatible with Coravel data: the former values were ignored for these three objects.

Metallicity data from other sources, such as the Cayrel de Strobel et al. (1992) catalogue, are not used, given the often very large variation in the results from different sources.

The size of our sample permits two important improvements over previous studies:

1. The sample can be separated into several [Fe/H] intervals between [Fe/H] = -1 and [Fe/H] = -3 — thus reducing the need to shift the subdwarf colours using theoretical isochrones to define a "mono-metallicity" sequence (cf. VandenBerg et al. 1996) — and compared directly to the lower main sequences of globular clusters.
2. The sample reveals, for the first time, the position of the turnoff and subgiant branch *in the field subdwarfs*. By direct comparison with theoretical isochrones it is possible to determine their mean age.

Fig. 4 shows the location of the subdwarfs in the HR diagram for two [Fe/H] intervals. The colours have been shifted to a central metallicity by means of the standard method using the D'Antona et al. (1997) isochrones [Bertelli et al. (1994) or VandenBerg et al. (1997) isochrones give very similar offsets (cf. Mandushev et al. 1996)]. As the metallicity slices are narrow, the colour corrections are quite small (less than 0.02 mag, except for subgiant-branch stars).

The clarity of the sequences in these diagrams up to the subgiant branch is remarkable, especially for the lowest metallicity interval. The low dispersion around a mean locus indicates that the age spread of the field subdwarfs is small. In fact, the turnoff region is sufficiently well-defined for a mean age to be directly determined for the subdwarfs themselves, without the mediation of GCs (see Sect. 6).

The higher dispersion at higher [Fe/H] in the turnoff region may be due to a higher age spread among these objects and/or to contamination by disc objects of higher [Fe/H] with large errors. Some additional scatter is introduced by the uncertainties in the reddening.

4. Treatment of biases – binaries

The aim of this study is, as pointed out in the Introduction, to derive distances to globular clusters by direct fitting of their main sequences to local subdwarfs. This classical method has been widely used before (Sandage 1970, VandenBerg et al. 1996), although with considerably fewer objects: the reader is referred to these references for a description of the method.

For the first time, we possess precise parallaxes not only for unevolved subdwarfs, but also for turnoff and subgiant-branch subdwarfs. Under the assumption that field subdwarfs and GCs have similar ages, the *evolved* subdwarfs provide a very strong constraint on globular cluster distances and ages.

It is well known that the calculation of the mean luminosity of a sample of stars selected by parallax is strongly biased (see the pioneering study by Lutz & Kelker 1973, or the elaboration by Smith 1987). A proper compensation of biases is necessary before deriving GC distance moduli from field subdwarf parallaxes. Furthermore, the presence of binaries can have the effect of increasing the apparent luminosity of objects at a given colour and should also be properly taken into account.

4.1. Biases affecting main-sequence fitting of globular clusters to field subdwarfs

We contend that the specificity of Hipparcos parallaxes and the way the sample is selected mean that pure Lutz-Kelker-type (LK) biases are not dominant in the procedure, and that other significant biases affect the results in different ways. Indeed, the simulations presented below outline an effective bias in a sense *opposite* to LK. While it is true that individual absolute magnitudes must be corrected towards brighter values, a given sample will indicate an absolute magnitude too bright for its colour, because some fainter objects would have been left out by the sample selection.

The biases affecting the fit of a globular cluster sequence to the Hipparcos subdwarf data can be seen as consisting of three dominant parts:

1. The "pure" LK bias. In our case, this bias is moderated by the existence of strong constraints on m_V (Smith 1987, for instance, shows how they reduce the effect of the LK bias). The accuracy of the Hipparcos parallaxes and the large number of objects allow for rigorous selection limits (e.g. $\sigma_\pi/\pi < 15\%$), which also limits the LK bias.
2. The bias caused by the fact that the Hipparcos parallax uncertainties sharply increase with magnitude. For the 330 stars of our first set, we fit
$$\sigma_\pi \simeq 0.6 + (m_V/10)^{5.5} \qquad (2)$$
where m_V is the apparent visual magnitude. Therefore, at a given colour and [Fe/H], brighter objects are strongly favoured by a selection criteria of the type $\sigma_\pi/\pi < limit$.
3. The strongly asymmetric distribution of [Fe/H] in the field stars (less metal-poor stars are much more numerous than more metal-poor stars at any given subdwarf metallicity) interacts with the observational uncertainties to cause an "LK-type" average underestimation of [Fe/H].

The interplay between biases "2" and "3", and the fact that stars at a given colour become fainter with decreasing metallicity, means that at a given $(B-V)_0$, the inclusion in a sample defined by [Fe/H]$_{measured}$ = F, of objects with [Fe/H]$_{real}$ = $F+\delta$ (brighter and more numerous) will be favoured over the detection of objects at [Fe/H]$_{real}$ = $F-\delta$. This leads to an *overestimation* of the absolute magnitude corresponding to a given $(B-V)_0$ and [Fe/H]$_{meas}$, contrary to the classic LK bias.

Finally, "2" also means that undetected binary stars with $\mathscr{M}_1/\mathscr{M}_2 \sim 1$ are strongly favoured for inclusion in the sam-

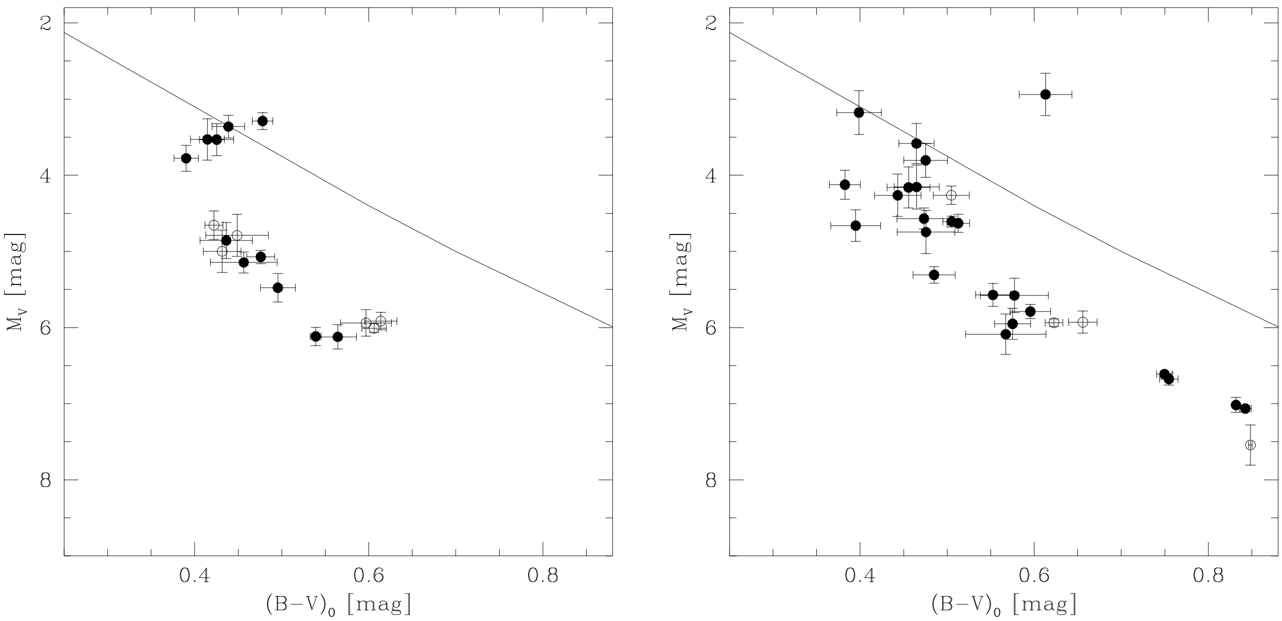

Fig. 4a and b. Position of the subdwarfs with $\sigma_\pi/\pi < 15\%$ grouped into two [Fe/H] intervals: [Fe/H] < -1.8 colour-shifted to [Fe/H] = -2.2 (*left*), $-1.8 <$ [Fe/H] < -1.2 colour-shifted to [Fe/H] = -1.5 (*right*). Detected or suspected binaries are indicated as open circles. The solid line is the Hyades main sequence.

ple, leading to an additional overestimation of the magnitude at a given colour.

Therefore, the different biases affecting the final results interact in a complex way that would be very arduous to treat analytically. We resorted to building a Monte Carlo simulation of the whole procedure. This simulation assumes a uniform density of stars, and approximates the Hipparcos input selection by a progressive magnitude cut. For each realisation, the Monte Carlo simulation selects a star at a random spatial position within a sphere of 200 pc radius centred on the Sun. The $B-V$ colour is chosen randomly between 0.4 and 0.8 mag. The [Fe/H] value is selected with a probability distribution:

$$\mathscr{P}([\mathrm{Fe/H}]) = 1.4\exp([\mathrm{Fe/H}]+3) - 1, \quad (3)$$

which approximately reproduces the observed [Fe/H] distribution in the sample.

Random errors are then added to produce $[\mathrm{Fe/H}]_{meas}$:

$$[\mathrm{Fe/H}]_{meas} = [\mathrm{Fe/H}] + \sigma_{[\mathrm{Fe/H}]} \cdot \mathrm{gauss}(0,1)$$

where gauss(0,1) is a random value following a normal distribution. The absolute magnitude is determined by assuming a linear shift of the main sequence with metallicity:

$$M_V = 1.7 + 5.7(B-V) - 0.6[\mathrm{Fe/H}] + \Delta MS \cdot \mathrm{gauss}(0,1) \quad (4)$$

where ΔMS is an intrinsic dispersion of the stars on the main sequence, expressing the fact that the main sequence has a finite width for a given metallicity. The colour dependence is approximated by the Hyades main sequence slope, while the [Fe/H] dependence is taken from stellar evolution models.

Separate simulations were also carried out for stars evolved past the turnoff on the subgiant branch. In that case the selection sphere was increased to 800 pc, and the absolute magnitude was approximated from the models in the following way:

$$M_V = 4 + 0.5[\mathrm{Fe/H}] + \Delta MS \cdot \mathrm{gauss}(0,1). \quad (5)$$

To reproduce the Hipparcos catalogue characteristics, we considered the sample to be complete up to magnitude $m_V = 9$, and then increasingly incomplete with a probability for inclusion:

$$\mathscr{P}(m_V) = 10^{-0.6(m_V-9)}. \quad (6)$$

The recovered parallax π is computed from the distance by

$$\pi_{meas} = 1/dist_{true} + \sigma_\pi \cdot \mathrm{gauss}(0,1)$$

where σ_π is the parallax error calculated from Eq. 2. The simulated data are then accepted if

- $|[\mathrm{Fe/H}]_{meas} - [\mathrm{Fe/H}]_{target}| < 0.2$ dex
- $\sigma_\pi/\pi < (\sigma_\pi/\pi)_{lim}$
- "detected" by magnitude completeness with the probability of Eq. 6.

Finally, relative colour corrections are applied to compensate for the difference between the measured and target [Fe/H], as would be done with the real data.

There are 4 parameters in the simulation: the centre of the metallicity interval "$[\mathrm{Fe/H}]_{\mathrm{target}}$", the uncertainty in [Fe/H] "$\sigma_{[\mathrm{Fe/H}]}$", the main sequence intrinsic width "ΔMS", and the limiting relative parallax error "$(\sigma_\pi/\pi)_{lim}$". The key parameter is the observational uncertainty in [Fe/H]. Comparing the

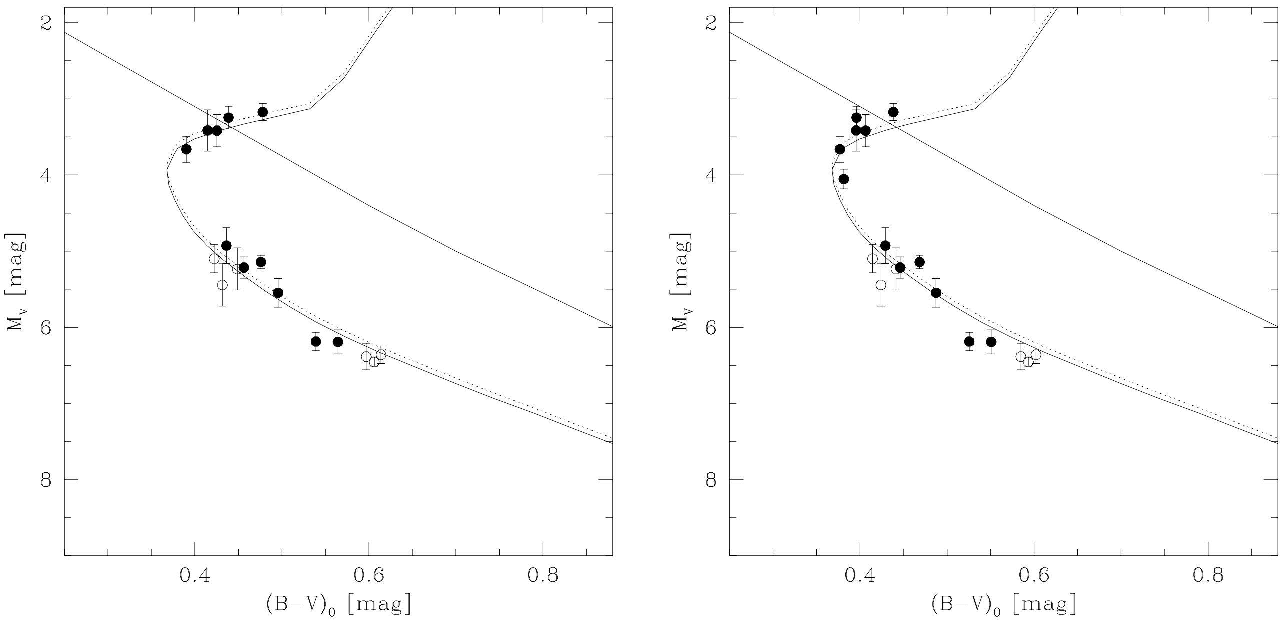

Fig. 5a and b. The fiducial sequence of M 92 fitted to the $-2.6 < [Fe/H] < -1.8$ subdwarfs, using the Coravel/CLL94 metallicity scale (left), and the scale of Axer et al. (right). Bias compensations and average binarity corrections to the absolute magnitude have been applied. The M 92 sequence is plotted with μ_0 = 14.61 mag (solid line, fit with binaries corrected), and μ_0 = 14.68 mag (dotted line, fit without including binaries). Suspected or detected binaries are shown as open circles. Colour-shifts $\delta(B-V)$ have been applied towards a central [Fe/H] of -2.2 dex. The upper line is the Hyades main sequence.

subdwarf [Fe/H] values from different studies (CLL94, Ryan & & Norris 1991, Axer et al. 1994, this study), we estimate $\sigma_{[Fe/H]}$ = 0.15 dex for [Fe/H] = -1, and $\sigma_{[Fe/H]}$ = 0.2 dex[4] Mean results of the Monte Carlo simulation for some interesting values of the parameters are given in Table 1.

The conclusion of this simulation is that the position of the main sequence at a given metallicity, when fitted to Hipparcos subdwarfs, must be shifted downwards (towards fainter luminosities) by about 0.06 mag at [Fe/H] = -2 and 0.01 mag at [Fe/H] = -1 — a correction opposite to the LK correction. Another interesting result is that the objects that have evolved past the turnoff are subject to a bias in the opposite direction (lines 8 and 9 of Table 1) because the magnitude dependence on metallicity is reversed after the turnoff.

4.2. Binaries

The high incidence of binaries among field stars (possibly as high as 60%) is a well-established fact. When both components have similar luminosities, an unresolved binarity causes an apparent increase in luminosity of up to 0.75 magnitudes. For reason "2" in the preceding section (the sharp dependence of the Hipparcos parallax uncertainty with magnitude), a binary star of a given colour also has a higher likelihood of being included in a sample selected by σ_π/π limits than a single star.

All objects used in this study have been extensively measured in radial velocity, either with Coravel or by CLL94, and several spectroscopic binaries were detected. It is clear from Fig. 4 that at least some binaries are above the single-star sequence corresponding to their metallicity, and that failing to recognize and correct for their presence would cause significant errors in the estimation of that sequence.

From the Praesepe open cluster data of Bolte (1991), we estimate that the effect of binaries can be modeled by assuming 25% of the stars to be 0.75 mag above the main sequence (average of the 21 binary suspects in Bolte's Fig. 1), the other binaries having too small a mass ratio to be separated from the single-star sequence. Assuming a total binary rate of 50%, we then arrive at an average shift of 0.375 mag above the sequence for binaries. We therefore correct the absolute magnitudes of the detected binaries in the sample by +0.375 mag. It is clear that this is an average correction, that will be either too small or too large in individual cases depending on the real mass ratio.

Fig. 5 displays the subdwarf data for the lowest metallicities, with binarity and bias corrections applied. A posteriori, the direction and scale of the correction is confirmed, especially by the better agreement of the M 92 sequence with the subgiant branch.

The binaries (at least the bluest ones) possibly seem a little too faint after correction, but this is to be expected since the

[4] While this may appear high, the difficulty of determining very low metallicities precisely should be appreciated. An error of 0.022 on $(B-V)_0$, or 7% on W, for instance, implies an error of 0.1 dex in the metal abundance at [Fe/H] = -2. The average uncertainties for the faint, metal-deficient subdwarfs used on Table 2 are 6% on W and 0.03 on $(B-V)_0$ (including the reddening uncertainty). The quadratic sum of these contributions with the dispersion of the Coravel calibration give about 0.2 dex at [Fe/H] = -2.

Table 1. Some results of the Monte Carlo simulation for different parameter values of interest. The parameters are: the intrinsic width of the main sequence ΔMS, the limit on the accepted relative parallax uncertainty $(\sigma_\pi/\pi)_{lim}$, the target [Fe/H], the measurement uncertainties in [Fe/H], $\sigma_{\rm [Fe/H]}$. The mean bias is defined for all of the "accepted" objects (see the text) as the "true" magnitude at a given colour defined by Eq. 4 for the first 9 lines, and by Eq. 5 for the last two, minus the mean "measured" magnitude.

Number of realizations	ΔMS [mag]	$(\sigma_\pi/\pi)_{lim}$	[Fe/H] [dex]	$\sigma_{\rm [Fe/H]}$ [dex]	Mean bias [mag]	Note
25×10^5	.10	.15	−2.0	.20	**+0.064**	**Adopted [Fe/H]=−2**
15×10^5	.10	.20	−2.0	.20	+0.053	Higher σ_π/π limits
25×10^5	.10	.15	−2.0	.15	+0.024	Lower $\sigma_{\rm [Fe/H]}$
25×10^5	.10	.15	−2.0	.25	+0.083	Higher $\sigma_{\rm [Fe/H]}$
25×10^5	.0	.15	−2.0	.20	+0.036	Without MS width
25×10^5	.25	.15	−2.0	.20	+0.109	Higher MS width
25×10^5	.10	.15	−1.0	.15	**+0.011**	**Adopted [Fe/H]=−1**
25×10^5	.25	.15	−1.0	.15	+0.055	Higher MS width
5×10^5	.0	.15	−1.0	.0	−0.025	No MS or [Fe/H] dispersion
25×10^5	.10	.15	−2.0	.20	**−0.115**	**Subgiants [Fe/H]=−2**
25×10^5	.10	.15	−1.0	.15	**−0.071**	**Subgiants [Fe/H]=−1**

way in which the correction is applied means that it is valid not in individual cases but as an average of the whole sample. Some low mass ratio spectroscopic binaries would have been corrected although the secondary does not contribute very much to the total luminosity, and some high mass ratio binaries would have been undetected because the velocity variations are too small or too slow (long-period binaries).

5. Distance of M 92

5.1. Main-sequence fit and distance

As one of the Galaxy's most metal-deficient globular clusters with a low and uncontroversial value of reddening, and with a very well-measured sequence in the CMD down to the lower main-sequence (Stetson & Harris 1988), M 92 is an ideal standard beacon to establish the age of the oldest GC. CMD comparisons show that it is probably as old as other clusters of similar metallicity (Stetson et al. 1996). We fit the M 92 sequence of Stetson & Harris to the subdwarf data, corrected for biases and binaries according to the procedure developed in Sect. 4. Unevolved and evolved stars are shifted in magnitude according to lines 1 and 10 of Table 1, respectively, and 0.375 mag is then added to the absolute magnitude of suspected or detected binaries. The tight sequence defined by the evolved subdwarfs on Fig. 4 indicates that they all share a similar age, and the fact that they outline a turnoff shape at least as red as the M 92 turnoff (Fig. 5) indicates that they are not significantly younger.

We adopt for M 92, $E(B-V)$ = 0.02 and [Fe/H] = -2.2 (cf. Caretta & Gratton 1997). A least-square fit is done for the apparent distance modulus $(m-M)_V$ on the sample subdwarfs with $-2.6 <$ [Fe/H] < -1.8 dex and $\sigma_\pi/\pi < (\sigma_\pi/\pi)_{lim}$. The results of the fit are shown in Table 3 for different cases: using all or only unevolved stars, accepting or rejecting binaries, or with a wide or narrow σ_π/π limit. (When subgiants are used in the fits, we are implicitly assuming that the very metal-deficient field subdwarfs — those with [Fe/H] < -1.8 — have a similar age to that of M 92.). The results based on all objects with $\sigma_\pi/\pi < 15\%$ are displayed in Fig. 5. The data for these objects is given in Table 2.

The agreement of the overall shape is impressive. The subdwarfs evolved past the turnoff provide a particularly strong constraint on the distance of M 92, and an equally strong indication that M 92 and the oldest subdwarfs all formed at the same epoch. The binary corrections seem globally too large in Fig. 5, although the binaries are indeed overluminous for their colours. With the binary corrections included, the M 92 subgiant branch becomes fainter than the corresponding field subdwarfs, and a few binaries appear to be somewhat below the sequence (although, as explained in Sect. 4.2, that does not necessarily mean that the corrections are too large). On the other hand, globular clusters do contain binaries, even if the binarity rate appears to be smaller than in the field (Hut et al. 1992), so that the measured sequence of M 92 may be slightly brighter than the single-star sequence. Stetson & Harris (1998) tentatively identify a line of stars to the right of their main-sequence fiducial as being the binary star sequence, and exclude it from their determination of the M 92 main sequence locus. Only low mass-ratio binaries with a small magnitude shift from the single-star sequence are then expected to influence their determination. These binaries may still be sufficient to make the sequence slightly too bright.

We adopt line 2 of Table 3, $(m-M)_V$ = 14.67 mag (μ_0 = 14.61 mag) as our best value, noting that it is also representative of the average results using slightly different procedures (other lines of Table 3), with the provision that a slightly higher modulus (up to $(m-M)_V$ = 14.74 mag) is possible if the binary corrections are too large.

Note that if (i) only unevolved stars are considered, (ii) no correction is applied for binaries, and (iii) only the classical Lutz-Kelker correction for biases is adopted, then the "standard method" yields an apparent distance modulus of 14.91 mag, or higher, for M 92, implying a much younger age. This value is, however, highly incompatible with the subgiant data and the location of the detected binaries.

Table 2. Table of data used for Fig. 5. $\delta(B-V)$ is the colour shift used to bring the data to [Fe/H] = -2.2.

HIC	Name	$(B-V)_0$	E(B-V)	M_V	σ_{M_V}	[Fe/H]	$\delta(B-V)$	Binarity Note
14594	HD 19445	0.48	0.01	5.07	0.09	−2.20	0.000	C
16404	BD +66 268	0.60	0.06	5.94	0.17	−2.10	−0.005	C SH
21609	HD 29907	0.61	0.03	5.92	0.12	−2.28	0.004	SB
24316	HD 34328	0.47	0.03	5.15	0.14	−1.88	−0.014	C
38541	HD 64090	0.62	0.00	6.01	0.06	−1.92	−0.015	SV
44124	BD −03 2525	0.44	0.04	5.00	0.28	−1.94	−0.008	SB2
46120	CPD−80 349	0.54	0.02	6.12	0.12	−2.26	0.003	C
48152	HD 84937	0.39	0.01	3.78	0.17	−2.21	0.000	C
60632	HD 108177	0.44	0.00	4.86	0.24	−1.91	−0.009	C
65201	HD 116064	0.42	0.03	4.66	0.19	−2.39	0.006	H2
68464	HD 122196	0.41	0.05	3.53	0.27	−2.22	0.002	C
72461	BD +26 2606	0.44	0.00	4.79	0.28	−2.58	0.012	SB
73385	HD 132475	0.44	0.06	3.53	0.21	−1.95	−0.020	C
76976	HD 140283	0.44	0.04	3.29	0.11	−2.41	0.034	C
98532	HD 189558	0.50	0.07	3.36	0.15	−1.84	−0.064	C
99267	BD +42 3607	0.50	0.01	5.48	0.19	−2.13	−0.002	C
106924	BD +59 2407	0.58	0.05	6.12	0.16	−1.91	−0.017	C

Binarity notes – C: constant radial velocity; SB: spectroscopic binary; SB2: double-line spectroscopic binary; SV: possible small radial velocity variations; SH: suspected non-single from Hipparcos catalogue; H2: two components resolved by Hipparcos.

Table 3. Apparent distance modulus for M 92 using different subdwarf sets. The M 92 fiducial sequence from Stetson & Harris (1988) is fitted to the corrected subdwarf data. Unevolved stars are defined to be those with $M_V > 4.5$ mag.

Selection	[Fe/H] scale	$\sigma(\pi)/\pi$ limit	[Fe/H] interval [dex]	Fitted $(m-M)_V$ [mag] With binaries	Without binaries
Unevolved	I	0.15	$-2.6 - -1.8$	14.64 ± 0.06	14.71 ± 0.07
All	I	0.15	$-2.6 - -1.8$	$\mathbf{14.67 \pm 0.04}$	$\mathbf{14.74 \pm 0.05}$
All	I	0.20	$-2.6 - -1.8$	14.70 ± 0.04	14.76 ± 0.05
All	II	0.15	$-2.6 - -1.8$	14.64 ± 0.04	14.73 ± 0.05

5.2. Alternative metallicity scale

The possibility of a more compressed metallicity scale than the one we use, namely the scale advocated by Axer et al. (1994), has been described in Sect. 2.3. The procedure was repeated with the [Fe/H] values for the subdwarfs adjusted to the scale of Axer et al. using Eq. 1. The result is shown in Table 3 and displayed on the right panel of Fig. 5. The derived distance modulus in this case, $(m-M)_V$ = 14.64 mag, differs only slightly from the result discussed above.

5.3. Uncertainty in the distance modulus

The formal uncertainty of the subdwarf fit is 0.05 mag. To this must be added an uncertainty for the adopted corrections. As some ingredients in the corrections are poorly known, we take the error in the corrections to be 50%, so that the uncertainty in the distance modulus is half the difference between the uncorrected and corrected moduli, or 0.06 mag. The quadratic addition of these two components gives 0.08 mag for the final uncertainty: $(m-M)_V^{\rm M\ 92}$ = 14.67 $\pm$ 0.08 mag.

5.4. Modified assumptions

The effect of adopting the colour excess determination of Arenou et al. (1992) is examined by repeating the procedure with the second colour excess scale described in Sect. 2.4. The differences in the recovered GC distance moduli are small, at most a few hundredths of a magnitude. The second E(B-V) values lead, for instance, to a distance modulus 0.02 mag smaller for M 92[5].

There is some (small) lingering uncertainty associated with the reddening and metallicity of M 92. Adopting 0.01 mag for the reddening of M 92 decreases its recovered μ_0 by 0.02 mag (0.05 mag in $(m-M)_V$). The dependence of the recovered distance modulus on the [Fe/H] attributed to M 92, $\delta\mu/\delta[Fe/H]$, is about 0.3 mag/dex, so that, for instance, adopting [Fe/H] = -2.1 for M 92 increases the distance modulus by 0.03 mag. Given the present accuracy of the reddening and metallicity for M 92, we conclude from the estimations given in this paragraph that the remaining small uncertainties in these parameters are not a dominant source of error in the distance determination.

6. The age of M 92 and related discussion

Based on the eight classical Population II subdwarfs with good parallax measurements (see Sandage 1970, Carney 1979) that have been widely used in globular cluster studies over the past

[5] Both reddening scales give slightly higher E(B-V) values for well-known subdwarfs than CLL94 — the average difference is 0.012 mag.

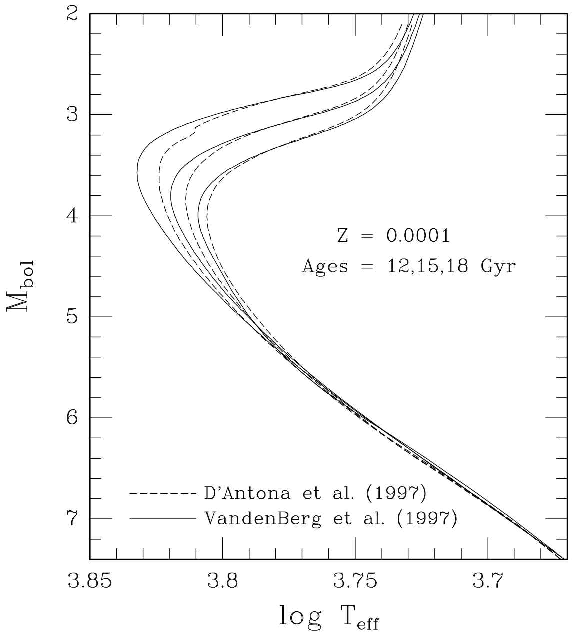

Fig. 6. Comparison of D'Antona et al. (1997) and VandenBerg et al. (1997) isochrones on the theoretical plane.

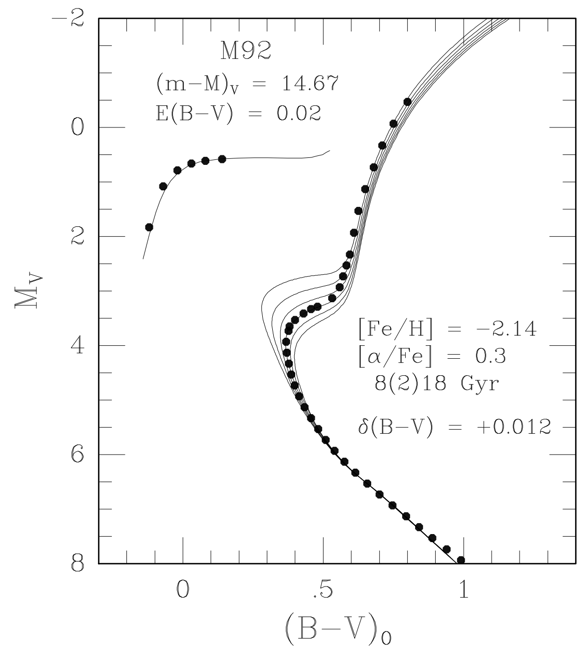

Fig. 7. Comparison of the M 92 sequence with VandenBerg et al. (1997) model isochrones, for the parameters shown.

two decades, Stetson & Harris (1988) derived an apparent distance modulus $\approx$ 14.6 and a corresponding age of 16–17 $\pm$3 Gyr for M 92. These estimates were revised to $(m - M)_V$ = 14.65 and 15^{-3}_{+5} Gyr by VandenBerg et al. (1996; hereafter VBS96) using a slightly enlarged sample of such stars, with updated properties, and employing isochrones that allowed for enhancements in the α-elements (O, Ne, Mg, Si, etc.) as well as Coulomb interactions and other non-ideal effects in the equation of state. [VBS96 actually derived an age close to 16 Gyr from their isochrone fits to the M 92 photometry, but as the model calculations that they used did not allow for the diffusion of helium, they argued that 15 Gyr was a more realistic estimate since the gravitational settling of helium is believed to cause a $\sim$ 1 Gyr age reduction (cf. Proffitt & VandenBerg 1991).] Since our best estimate of the M 92 distance from Hipparcos observations is nearly the same as that determined by VBS96 from ground-based subdwarf data, which is itself a very encouraging development, so must our estimate of the cluster age be quite similar.

However, VandenBerg (1997a) has noted that recent improvements in the construction of model atmospheres has resulted in a significant revision to the bolometric corrections appropriate to very metal-poor stars (also see the pertinent discussion in Harris et al. 1997). To be specific, if exactly the same isochrones used by VBS96 were fitted to the M 92 CMD on the assumption of exactly the same distance and reddening, then the predicted age would decrease by $\approx$ 1.5 Gyr just from the change in the BC_V's that are used to transpose the models from the theoretical to the observed plane. Although it is not yet clear that the revised BC_V scale is necessarily to be preferred over, for instance, the ones predicted by VandenBerg & Bell (1985) or Buser & Kurucz (1992), the latest calculations of this quantity by both Kurucz (1992) and Bell (1996) appear to agree extremely well with one another (Unfortunately, their predictions for most colour indices do not yet show the same level of agreement). It is truly quite an unexpected and disconcerting development that the uncertainty in the BC_V scale may well be the largest uncertainty in the determination of GC ages, now that Hipparcos has dramatically reduced the errors in their distances as estimated from main-sequence fits to local subdwarfs. Further work to better establish the bolometric correction scale is strongly encouraged.

Were we to fit D'Antona et al. (1997) isochrones for Z = 0.0001 to the M 92 CMD, an age near 16 Gyr would be obtained. This is very similar to the age that would be derived from VandenBerg et al. (1997) isochrones for [Fe/H] = -2.3 and [α/Fe] = 0.0. Indeed, on the theoretical plane, the D'Antona et al. isochrones agree rather well with those computed by VandenBerg et al. — as illustrated in Fig. 6. Both the location of the lower main sequence and the subgiant branch (for a given age) are nearly coincident. There are some differences in $T_{\rm eff}$ in the vicinity of the turnoff and small variations in the turnoff luminosity versus age relations, which arise as a consequence of different treatments of convection. D'Antona et al. adopt the Canuto & Mazzitelli (1992) theory of turbulent convection whereas VandenBerg et al. employ the usual mixing-length theory. As both sets of isochrones utilize the same transformations to the observed plane (aside from slight differences in the choice of the solar $M_{\rm bol}$ and M_V values), both must yield very nearly the same age, as judged from the match of the predicted and observed location of subgiant-branch stars. The main cause of the $\approx$ 2 Gyr difference in age noted above is, therefore, the assumption of higher [Fe/H] and [α/Fe] values in the VandenBerg et al. isochrones. This exemplifies the considerable importance of the adopted chemistry to the determination of GC ages and the insensitivity of the results to the particular set of isochrones that is used in the analysis.

Fig. 7 compares VandenBerg et al. (1997) isochrones for [Fe/H] = -2.14, [α/Fe] = 0.3, and ages from 8 to 18 Gyr (in 2 Gyr increments) with the Stetson & Harris (1988) fiducial for M 92, assuming our best estimate of the cluster distance and the generally-accepted foreground reddening, $E(B-V)$ = 0.02 mag (these models assume a value of 1.89 for the usual mixing-length parameter, based on the requirements of a standard solar model). A zero-point offset amounting to 0.012 mag had to be added to the model $B-V$ colours in order to achieve a coincidence of the predicted and observed lower main sequences. Such a small shift is well within the uncertainties in the synthetic colour-$T_{\rm eff}$ relations that were used (see VandenBerg 1997b), the predicted effective temperatures themselves, and such cluster properties as the chemical composition and the reddening. But once this adjustment is made, the agreement between theory and observation is rather good, indicating an age near 14 Gyr [There remains a slight discrepancy along the giant branch, which could *easily* arise from a variety of factors (e.g., the colour-$T_{\rm eff}$ relations, the treatment of convection, the low-temperature opacities), and therefore does not represent a serious concern].

It is worth stating that the VandenBerg et al. (1997) isochrones explicitly treat the adopted α-element enhancement, which is favoured by most spectroscopic studies (e.g., see Ryan, Norris, & Beers 1996; the review by Carney 1996). That is, opacities analogous to those published by Rogers & Iglesias (1992) for temperatures greater than 8000 K, and to those reported by Alexander & Ferguson (1994) for lower temperatures, were calculated for the specific heavy-element mixtures that were assumed. Furthermore, in most respects, the physics incorporated in the models is up-to-date. Non-ideal equation-of-state effects, of which the Coulomb interaction is the most important, were taken into account, and the improved rates for H-burning nuclear reactions described by Bahcall & Pinsonneault (1992) were adopted. The only potentially important physical process that was not considered (besides rotation, perhaps) is helium diffusion. As discussed by Proffitt & VandenBerg (1991), uninhibited He diffusion causes considerable problems for the shapes of synthetic CMDs; which suggests that, at least in the surface layers, something (circulation currents, turbulence at the base of the convection zone?) acts to prevent the gravitational settling of helium. However, diffusion may still have important effects in the cores of globular cluster stars and, if this is the case, then our estimate of 14 Gyr for M 92 should probably be reduced by $\sim$ 1 Gyr.

Within the past few months, Reid (1997) and Gratton et al. (1997) have reported the results of their analyses of selected Hipparcos observations of nearby subdwarfs, emphasizing the implications of their data for globular cluster distances and ages. In the case of M 92, they determined $(m-M)_V$ = 14.99 and 14.83, respectively, from main-sequence fits to local subdwarf sequences. Their findings are quite different from ours — which can be attributed to several factors. First, our subdwarf sample will necessarily provide tighter constraints on the M 92 distance and age simply by virtue of the fact that it contains many more stars, including a few (very valuable) subgiants. Second, as we had complementary radial velocity observations, we were able to identify binaries and to make suitable corrections for them. And third, we found that a bias arising mainly from the very skewed nature of the metallicity distribution towards solar abundances acts in the opposite direction to the classical Lutz-Kelker correction and amounts to several hundredths of a magnitude at [Fe/H] = -2.2.

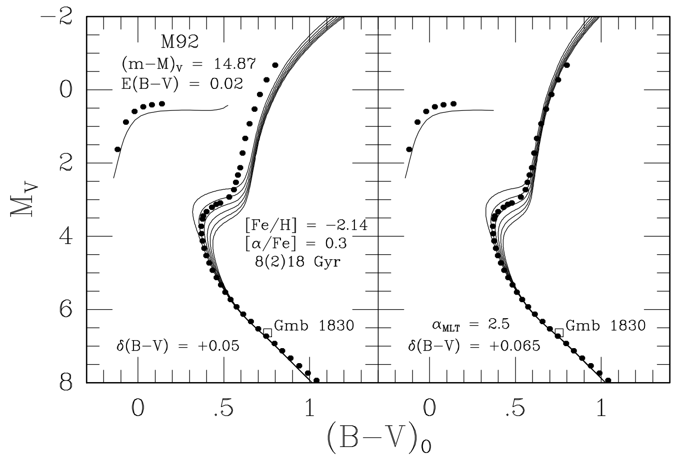

Fig. 8. Comparison of the M 92 sequence with VandenBerg (1997) model isochrones, for a higher value of the distance to M 92.

Some comments are in order concerning the considerable difficulties that would be posed for stellar evolutionary theory if the distances derived by Reid and by Gratton et al. are correct. The left-hand panel of Fig. 8 illustrates a comparison of the M 92 CMD with the same isochrones plotted in Fig. 7 if the adopted distance modulus is taken to be 0.2 mag larger than our best estimate. In order to obtain a coincidence of the predicted and observed loci for the unevolved, lower-main-sequence stars, the model $B-V$'s had to be adjusted redward by 0.05 mag. This is a very large correction for relatively warm, metal-poor turnoff stars whose colours are not an especially strong function of $T_{\rm eff}$. And, given that VandenBerg (1977b) has found that the turnoffs of metal-rich open clusters like M67 can be matched by suitable isochrones without requiring much, if any, correction to the synthetic $B-V$ colours predicted by Bell & Gustafsson (1978; also see VandenBerg 1992), it would seem highly unlikely that the colour-$T_{\rm eff}$ relations appropriate to M 92 turnoff stars can be incorrect by as much as 0.05 mag.

Nor is it very probable that the predicted effective temperatures are wrong by an amount which could explain such a colour shift. VandenBerg (1997b) has shown, for instance, that the predicted $T_{\rm eff}$'s for the giant branches in globular clusters agree very well with those inferred from infrared photometry (e.g., see Frogel et al. 1981). Because the giant branch rises so steeply, uncertainties in the assumed cluster distances do not significantly affect this result. Nonetheless, it is possible to produce

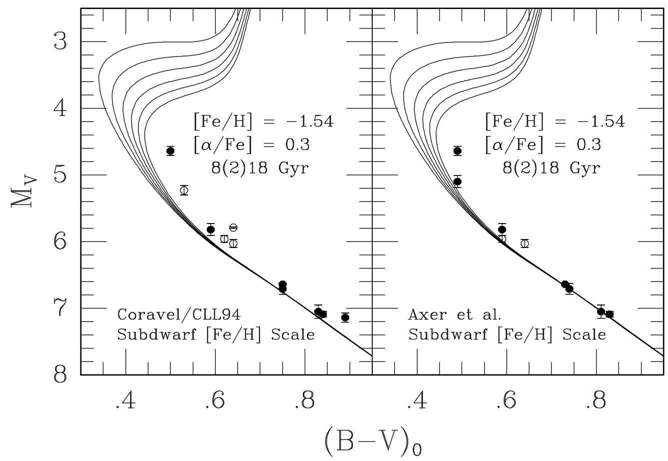

Fig. 9. Hipparcos subdwarfs with $-1.8 <$[Fe/H]< -1.1 dex, and $\sigma_\pi/\pi < 5\%$, colour shifted to [Fe/H] = -1.54, compared with theoretical isochones from Vandenberg et al. (1997), for the Coravel/CLL94 (left) and Axer et al. (right) metallicity scales. Detected or suspected binaries are shown as open circles.

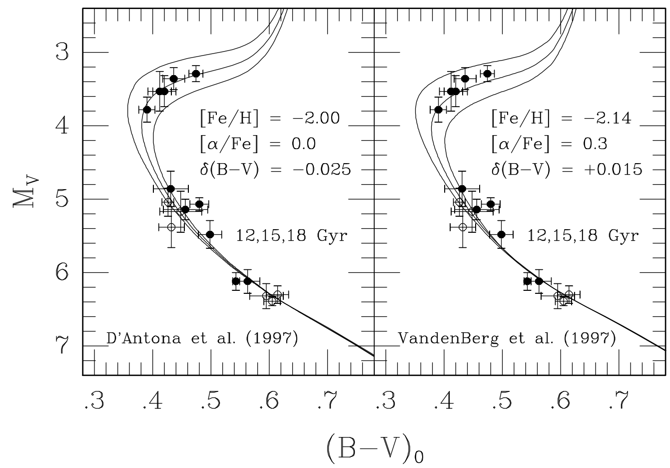

Fig. 10. Subdwarfs with $-2.6 <$[Fe/H]< -1.8 dex, shifted in colour to [Fe/H] = -2.2, and isochrones for 12, 15, 18 Gyr from D'Antona et al. (1997) and VandenBerg et al. (1997). As noted, the isochrone colours were slightly adjusted in order to achieve the best possible fits to the unevolved subdwarfs. As the D'Antona et al. isochrones are for a scaled-solar mix of the heavy elements, isochrones for Z = 0.0002 ([Fe/H] = -2.00) have been plotted to partially compensate for the neglect of an α-element enhancement in these models. The mass-fraction abundance of all elements heavier than helium in the VandenBerg et al. computations is Z = 0.000254.

a synthetic CMD that does a much better job, in a systematic sense, of matching the observed M 92 fiducial — simply by choosing a larger value for the mixing-length parameter (see the right-hand panel of Fig. 8). But, in this case, an even larger zero-point adjustment to the predicted colours is needed (0.065 mag).

Be that as it may, the main problem with such a large (or larger) distance modulus is that the M 92 lower main sequence would then nearly overlay that defined by stars which are ~ 0.7–0.8 dex more metal rich, of which Gmb 1830 is a well-known, representative example[6]. It is a fundamental prediction of stellar evolutionary theory that the main sequence for [Fe/H] = -2.2 stars (like those in M 92) is separated by at least 0.2–0.3 mag, at a given colour, from the one for [Fe/H] ~ -1.5 stars. Therefore, either a high distance modulus for M 92 cannot be considered credible or the subdwarf sequence for [Fe/H] ~ -1.5 also has to be correspondingly brighter.

There are in our dataset a number of intermediate-metallicity subdwarfs ($-1.8 <$ [Fe/H] < -1.1) with very precise Hipparcos parallaxes ($\sigma_\pi/\pi < 5\%$), that can be used to define the main sequence at [Fe/H] = -1.5 dex, as shown in Fig. 9. Again, binary candidates revealed themselves by showing radial velocity variations. The single stars follow a very tight sequence, which the models are able to fit precisely if their colours are adjusted by $\delta(B-V) \simeq 0.015$ mag[7]. Incidentally, this colour shift is almost the same as the one adopted in the case of M 92, on the basis of an independent subdwarf dataset. This indicates that the predicted dependence of the main-sequence luminosity on metallicity is essentially the same as the observed dependence. We therefore conclude that an M 92 modulus as large as $(m-M)_V \sim 14.9$ would bring its lower main sequence so close to the locus of [Fe/H] = -1.5 stars as to contradict a fundamental theoretical prediction. This provides a strong argument that M 92 cannot be as young as 11 Gyr (the age indicated by the isochrone fit in Fig. 8).

Another point in favour of intermediate ages, independent of the M 92 distance, comes from the direct age dating of the local subdwarfs that have evolved onto the subgiant branch. As illustrated in Fig. 10, their location in the CMD is not consistent with ages as young as 11 Gyr or as high as 17 Gyr [if VandenBerg et al. (1997) isochrones are used; the models computed by D'Antona et al. (1997) constrain their ages to the range 12–18 Gyr]. Hence, a large distance for M 92 would imply that the extreme field subdwarfs are older than the cluster itself, in which case their ages would have to be taken as the primary

[6] Poul Nissen has pointed out to us that the detection of a flaring M-dwarf companion to Gmb 1830 had been claimed by van de Kamp (1969). However, a clear confirmation has never been obtained, and the orbit subsequently proposed by Beardsley et al. (1974) is not compatible with later radial velocity observations by Duquennoy & Mayor (1991) or Mazeh et al. (1996). In any case, a possible companion would be at least 5 mag fainter than Gmb 1830 (Carney 1983), thus of negligible photometric influence.

[7] HIC 104659 (the bluest point) is far off the sequence in a position that cannot be explained by evolution alone. Comparing its location with that of detected binaries, the most plausible explanation is that it is an undetected $\mathcal{M}_1/\mathcal{M}_2 \sim 1$ binary. HIC 57939 (Gmb 1830), which is the star at [(B-V)$_0$=0.75, M_V=6.61], has frequently been used in the past to define the subdwarf sequence, and it is reassuring that is appears to lie on the locus defined by stars of similar metallicity.

constraint on the age of the oldest stars in the Galaxy. From our distance determination, however, and from the close similarity of the morphology of the M 92 CMD and the subdwarf locus, we infer that the extremely metal-deficient subdwarfs probably have the same age as M 92.

Fig. 7 indicates that excellent agreement is obtained between the zero-age horizontal-branch (HB) models computed by VandenBerg et al. (1997) with their observed counterparts in M 92 if our best estimate of the cluster distance modulus is adopted. This agreement may be partly fortuitous as those models did not take into account recent revisions to the rate of the plasma neutrino cooling process (Haft et al. 1994, 1995). This is expected to increase the helium core mass at the tip of the giant branch by about $0.005\mathcal{M}_\odot$ (cf. Catelan et al. 1994), and thereby to increase horizontal-branch luminosities by a few hundredths of a magnitude. However, the HB models produced by some workers (e.g., Caloi et al. 1997) appear to be considerably brighter than this, for reasons which are not entirely clear (though differences in the assumed He core masses are certainly part of the explanation). Thus a consensus has not yet been reached among the model builders concerning the luminosities of core He-burning stars, and it remains something of an open question as to whether or not the uncertainty in the M 92 modulus can accommodate the predictions of canonical models for the core He-burning phase. Such models cannot be too bright, as otherwise, they would cause the same problem discussed above concerning the relative location of Gmb 1830 and the M 92 lower main sequence in the CMD (if they are used to infer the cluster distance). The Hipparcos subdwarf data clearly provides interesting constraints on the luminosities of real HB stars and the models used to represent them.

7. Conclusions

The Hipparcos data considerably expands the sample of field subdwarfs with precise trigonometric parallaxes. They make possible a direct determination of the distance to very metal-deficient GCs by main-sequence fitting to the local subdwarfs. Moreover, the amount and accuracy of subdwarf data are now sufficient for the uncertainties due to parallax errors to become smaller than the uncertainties caused by systematic biases or reddening/metallicity scale questions.

Extremely metal-deficient Hipparcos subdwarfs ([Fe/H] < -1.8) define a very tight sequence in the HR diagram, including objects in the turnoff region or evolved towards the subgiant branch. These evolved field subdwarfs provide an especially strict constraint on subdwarf ages and globular cluster distances.

We have developed in Sect. 4 a detailed treatment of the systematic biases specific to our analysis. The proper account of binaries is especially important, but can only be considered in a statistical way because complete binarity and mass ratio information cannot be determined for all of the objects. As the number of stars is not high enough to completely damp small-number statistics, the presence of binaries and the values of their mass ratio may still introduce an uncertainty of the order of 0.03 mag (Poissonian noise of the binarity correction on the 17 objects of Table 2) in the distance modulus results.

Removing the detected binaries from the sample and using the remaining set to fit the single-star sequence is not a completely sound way to proceed, because undetected binaries are also expected in any sample, and as the mass ratio is not a priori related to the period or angular separation, high mass ratio binaries, 0.75 mag above the single star sequence, can remain undetectable by present techniques at distances of the order of 100 pc. Indeed it can be argued that an average correction for binaries, with its associated uncertainties, is preferable to the exclusion of detected binaries from the sample.

The M 92 sequence closely matches the Hipparcos subdwarf sequence for the most metal-deficient objects, including the evolved ones, justifying the assumption of a common age. A distance modulus of μ_0 = 14.61$\pm$0.08 mag ($(m-M)_V$=14.67) is derived for M 92, using all objects, corrected for biases and binarity. Similar results are obtained using slightly different assumptions. The uncertainty includes the contribution of reasonable changes in the parameters of the correction procedure. The resultant cluster age, as derived from VandenBerg et al. (1997) isochrones for [Fe/H] = -2.14 and [α/Fe] = 0.3 is 14 $\pm$ 1.2 Gyr. By comparison, an age of 16 $\pm$ 1.5 Gyr is obtained using D'Antona et al. (1997) isochrones for [Fe/H] = -2.30 and [α/Fe] = 0.0, showing how the results are affected by differences in the assumed chemistry. Although both VandenBerg et al. and D'Antona et al. adopt the latest BC_V's predicted by Kurucz (1992) (and Bell 1996), the uncertainties in these quantities remain sufficiently large (according to Bell) that one cannot say with certainty that the lastest predictions are a definite improvement over previous values. Hence higher ages by 1–2 Gyr cannot be precluded.

The Hipparcos subdwarf data at intermediate metallicities ($-1.8 <$ [Fe/H] < -1.1) confirm the M 92 results in the sense that a similar colour shift ($\approx$ 0.015 mag in $B-V$) must be applied to the relevant models in both cases in order to achieve agreement between theory and observations (Note that no colour adjustment whatsoever is required, for either dataset, if the metallicity scale of Axer et al. (1994) is used in preference to that of CLL94). This indicates that the observed shift in the main-sequence locus due to a change in the metal abundance is accurately predicted by current stellar evolutionary models.

The previous dominant uncertainty in globular cluster ages — the cluster distances — has been greatly decreased by the Hipparcos subdwarf data. Indeed, this investigation confirms the intermediate ages (14–16 Gyr) that have been widely thought to be appropriate for these systems, while sensibly reducing their error bars. Based on models that employ up-to-date physics and the latest available colour-T_{eff} relations and bolometric corrections, but which do not treat diffusion, an age of 14 ± 1.2 Gyr was obtained for M 92 (and presumably the majority, if not all, of the extremely metal-deficient GCs). Unfortunately, the uncertainty in the BC_V scale (which was not included in the age uncertainty) is such that the age of the oldest GCs could be higher than our estimate by up to $\sim$ 2 Gyr. Our 14 Gyr age estimate (without He diffusion) or 13 Gyr (with He diffusion) should, therefore, probably be regarded as a lower bound. To this age one should add perhaps 1 Gyr to account for the elapsed time between the Big Bang and the GC formation epoch (see the relevant discussion in Sandage 1993), in which case the minimum age of the universe is at least 14 Gyr. If $\Omega = 1$ and $\Lambda = 0$, then H_0 must be ≤ 48 km s^{-1}Mpc^{-1}. Alternatively, if the Hubble constant is larger than this value, then one of the other assumptions is incorrect.

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