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Towards a hydrodynamical model predicting the observed solar rotation profile

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Abstract. We present here the results of a numerical study performed to investigate the role of gravity waves in the evolution of the solar rotation profile. We show that, together with meridional circulation and shear turbulence, they may enforce solid body rotation in the solar interior. The transient phase is characterized by an outer portion of the radiative zone rotating at the velocity of the convection zone and an inner portion in strong differential rotation with depth. The gravity waves slow down the outer portion, whereas it is the meridional circulation which dominates the transport of momentum in the interior. With our treatment of the excitation and damping of these waves, solid body rotation is achieved on a timescale of a few giga-years, depending on the initial velocity of the model.

Key words: hydrodynamics – turbulence – Sun: interior – Sun: rotation – stars: interiors – stars: rotation

1. Introduction

It is now a well known result of helioseismology that the radiative part of the Sun rotates almost as a solid body, at least down to $r \simeq 0.4 R_\odot$ (*cf. e.g.* Brown et al. 1989). Until recently, the only way to achieve that result was to advocate a magnetic field. Indeed, both rotation-induced turbulent diffusion (Endal & Sofia 1978; Pinsonneault et al. 1989) and wind-driven meridian circulation (Zahn 1992) fail to extract sufficient angular momentum from the radiative interior (Chaboyer et al. 1995; Matias & Zahn 1997). However, there is a problem with the magnetic solution: if the field lines are anchored in the convection zone, they should enforce differential rotation in the radiative interior, unless the link occurs in narrow region at mid latitude.

A step forward was taken recently by Zahn et al. (1997) (which will be referred to as ZTM) and by Kumar & Quataert (1997) when they examined the role of the gravity waves generated at the base of the convective zone. They found that such waves transport momentum "non-locally" on a rather short timescale, and that they tend to flatten the rotation profile.

Let us recall briefly the somewhat different approaches used by the two groups. Gravity waves conserve their momentum as long as they are not damped. Thus when assuming that both prograde (+m) and retrograde ($-m$) waves are excited with the same intensity at the base of the convective region, differential damping is required in order to get a net deposit of momentum. This differential damping will be provided by the Doppler shift due to differential rotation, since radiative damping varies as σ^{-4} (where $\sigma = \sigma_0 - m\Omega$ is the local frequency). Zahn et al. consider only the damping in the critical layer where this local frequency goes to zero. There the waves are completely damped, and they deposit the totality of their momentum. At the same time, the frequency of their $-m$ counterparts increases, diminishing their damping. Thus they will be able to travel all the way to the core and back, to be re-absorbed by the convective zone[1]. On the other hand, Kumar & Quataert retained those waves for which the differential damping between the +m and $-m$ waves remains small, and they use a linearized form of that difference to study the deposition of momentum. The two approaches are thus complementary, and it is planned to combine both effects.

However in this first numerical study, we keep only the transport of angular momentum by the waves which are completely damped in their critical layer. This contribution will be added to that due to meridional circulation and shear turbulence.

2. Input physics

The meridional circulation is treated using the formalism described by Zahn (1992); it takes into account the radial differential rotation, with the assumption that the rotation rate Ω is nearly constant on isobars. The circulation velocity then depends on the third derivative of Ω, and the transport of angular momentum is a true advective process. The shear stresses are calculated with a turbulent viscosity which includes the weaken-

[1] If the differential rotation is strong, the local frequency can reach the Brunt-Väisälä frequency $\sigma = N$ and the wave will be reflected there.

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ing effect of thermal damping (as first described by Townsend, 1958). The details concerning that viscosity are not important here since the circulation, being an advective process, dominates the transport of momentum almost everywhere. The transport associated with gravity waves is assumed to follow the description of ZTM, which is relevant for strong differential rotation.

The equation for the evolution of angular momentum is then

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left[r^2\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right] - \frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathscr{L}_J(r) \quad (1)$$

where we used standard notations for the radius r and the density ρ and where ν_v is the vertical (turbulent) viscosity, U the amplitude of the vertical component of the circulation speed and $\mathscr{L}_J$ is the luminosity of angular momentum summed over the whole wave spectrum.

The upper boundary condition is also very important. It states that the angular momentum of the convection zone varies such as to balance the fluxes which enter and leave it:

$$\frac{\partial}{\partial t}\left[\Omega \int_{r_{\rm cz}}^{R} r^4 \rho \,\mathrm{d}r\right] = -\frac{1}{5} r^4 \rho \Omega U + \frac{1}{4\pi}\left[\mathscr{L}_{J_{\rm cz}} - \mathscr{L}_{J_{\rm core}}\right] + \mathscr{F}_\Omega. \quad (2)$$

Here $\mathscr{F}_\Omega$ accounts for the magnetic torque imposed on the convection zone by the solar wind and 'cz' refers to the base of that zone. For simplicity, we approximate the loss of angular momentum by a cubic law in Ω ($\mathscr{F}_\Omega = W\Omega^3$), including saturation to a linear law in Ω when $\Omega > 10^{-5}$ rad s^{-1}. The evolution of the surface velocity can then be compared to the square root law found by Skumanich (1972) or to the modern observations of rotational periods (see e.g. Bouvier et al. 1997).

An expression for $\mathscr{L}_J$ is given in ZTM. When differential rotation is small, the wave luminosity is proportional to $\Delta\Omega = \Omega - \Omega_{\rm cz}$. However, as $\Delta\Omega$ rises, some of the frequencies will disappear from the wave spectrum as even those waves with $\ell = 1$ will have reached their critical layer. This translates into an upper limit on $\Delta\Omega$ for which the absorption is linear

$$\left(\frac{\Delta\Omega}{\omega_c}\right)^3 < \left(\frac{I}{{\omega_c}^4}\right), \quad (3)$$

where ω_c characterizes the frequency of the largest convective eddy. $I(r) = \int_r^{r_c} K N N_T^2 r^{-3} \mathrm{d}r$ represents the damping by radiative processes (K is the thermal conductivity). There is also a lower limit for $\Delta\Omega$ (that is, a finite amount of differential rotation is required to get some extraction) corresponding to the fact that the maximal horizontal wave number is not infinite:

$$\frac{\omega_c}{N_c}\left(\frac{I}{{\omega_c}^4}\right) < \left(\frac{\Delta\Omega}{\omega_c}\right)^3, \quad (4)$$

where N_c is the buoyancy frequency at the base of the convective zone (assuming some penetration).

When $\Delta\Omega$ becomes large, an important part of the wave spectrum has been damped before reaching the considered level and thus, cannot extract momentum. The extraction then varies as $\ln(\Delta\Omega)$ (see ZTM for the exact form). This somewhat augments the timescale calculated in ZTM.

In the present calculations, we reduced the magnitude of the wave luminosity with respect to the evaluation we made in ZTM by one order of magnitude. This may seem arbitrary, but we feel that the prescription we derived there, which was inspired by García López & Spruit (1991), probably overestimates the efficiency of wave generation. More work needs to be done to reduce the uncertainties associated with the crude treatments that have been used so far, and the validation of any theory of wave excitation will rely ultimately on the observational constraints provided by the chemical anomalies and surface rotation rates of low mass stars.

Equation (1) admits a stationary solution, with just enough differential rotation for the wave transport to balance the advection by the meridional circulation. A slight departure from this profile is sufficient to provide the flux extracted by the solar wind.

Let us stress that we considered here only the differential rotation in depth. It remains to be seen how the gravity waves act to homogenize the rotation in latitude below the convective zone. That would have to be compared with the horizontal turbulent diffusivity Spiegel & Zahn (1992) had to invoke to stop the spread of the differentially rotating region.

3. Numerical results

We solved the fourth order differential system describing the evolution of the rotation profile in the radiate zone using an implicit scheme, the internal structure being given by our stellar evolution code (a modified version of the Geneva code). The convective zones are assumed to rotate as solid bodies[2]. Here, we did not calculate the diffusion of chemicals associated with circulation and shear turbulence. More complete results will be presented in a forthcoming paper. Furthermore, we used a perfect gas law for the equation of state since the non ideal effects will not influence the evolution of the rotation profile.

Let us first examine the role played be each transport mechanism in the evolution of the rotation profile. If only circulation and shear turbulence are considered, the resulting Ω profile will have a very large slope, as the surface loses momentum through the wind much more rapidly than it can be transported by those two mechanisms (see Matias & Zahn 1997). The asymptotic regime described by Zahn (1992) in the case of a moderate wind is thus never achieved.

The solution for the wave transport (in the linear regime, when neglecting all variations except those of Ω) is

$$\Omega(r,t) = F\left(r + \int V_{\rm w}\,\mathrm{d}t\right), \quad (5)$$

[2] Actually, since we solve the problem only in radius, this corresponds to the observed absence of differential rotation in radius in the solar convection zone.

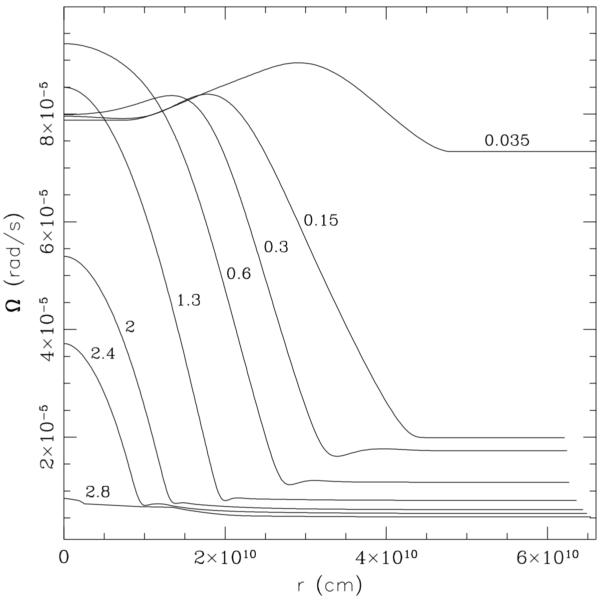

Fig. 1. Evolution of the solar rotation profile with time. The ages of the corresponding models are indicated in Gyrs. One sees clearly the advance of the front resulting from the action of gravity waves.

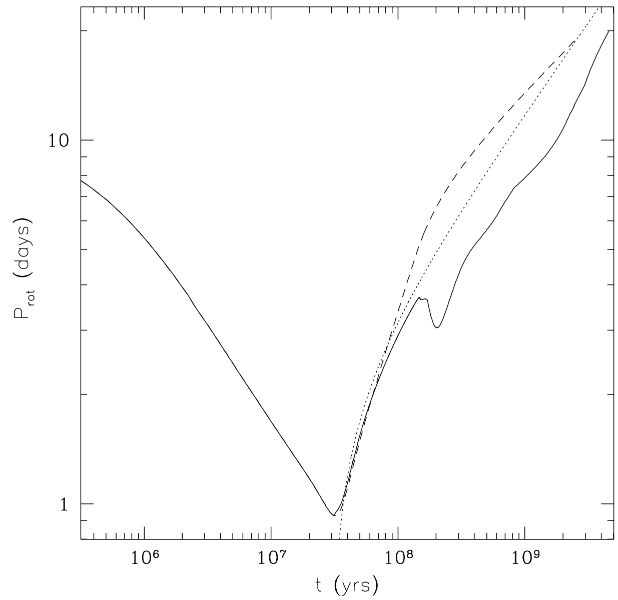

Fig. 2. Evolution of the rotation period. The solid line represents the actual calculation, the dashed line, the evolution of a sun in solid body rotation with the same initial angular momentum and the dotted line, Skumanich's law for the same rotation period at the beginning of the main sequence.

where

$$V_{\rm w} = \frac{1}{2}\frac{\rho_c v_c^3}{\rho r^2}\frac{1}{N_c \omega_c \ell_c}\left(\frac{{\omega_c}^4}{I}\right). \quad (6)$$

We thus expect to see the rotation profile move inwards, with a velocity which is inversely proportional to the damping integral, leaving behind a region rotating at the velocity of the convective zone. However, since we know that the linear regime is valid only as long as differential rotation is not too strong, we should expect the rotation profile to be modified as the "front" moves inwards.

When considering all three mechanisms together, we may recognize the characteristics of both solutions. This is illustrated in Fig. 1, which is the result of an evolution for a solar mass star, starting as a fully convective object on the Hayashi line. The advance of the "synchronized" portion is manifest, whereas the steep Ω profile below is governed by the circulation and shear turbulence in regions where the internal waves cannot penetrate. One verifies that the circulation, being of advective nature, is far more efficient in the transport of angular momentum than the shear turbulence. Note however that, due to the erosion by anisotropic turbulence, the circulation is far less effective in the transport of chemicals, as was shown by Chaboyer & Zahn (1992).

Fig. 2 illustrates the evolution of the rotation period. During the pre-main sequence evolution, the convective zone is quite deep and gravity waves are efficient enough to maintain a state of quasi-solid body rotation. The overall contraction leads to an acceleration of the surface. Reaching the main sequence phase, the wave production efficiency diminishes as the convection zone regresses and the stellar surface decelerates now more rapidly than the interior under the action of the magnetic torque. The retrograde waves are then not able to travel deeply into the radiative zone. At about 2×10^8 yrs, the wave front begins to penetrate into the radiative zone, from where it extracts angular momentum faster than the wind can carry it away. This leads to an increase of the surface velocity, at an age which depends on the magnitude of the total wave luminosity as well as on the initial angular momentum contained in the model. Unfortunately, no observations seem to exist that could confirm the existence of such a feature.

Let us stress once more that the wave luminosity is still very unsure. The existence of a rapidly rotating core in the Sun could help us estimate that value. However, the efficiency of the wave production will be in competition with the initial angular momentum of the model (that is, the core can be rotating rapidly either due to the poor wave transport or to the very high initial angular momentum of the model), and other observational constraints will be required to sort out all these effects.

4. Discussion

These calculations show that gravity waves may play an important role in the dynamics of radiative zones. They can clearly explain the flat solar rotation profile without invoking the action of a magnetic field.

Work is in progress in order to compare the predictions of our model with the rotation periods of solar-type stars and their lithium abundance. The results will be used to calibrate our prescription for the wave transport, and to suggest how the present, admittedly crude treatment should be improved.

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