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Coupled mass and angular momentum loss of massive main sequence stars

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Abstract. We investigate the interaction of mass loss and rotation during core hydrogen burning in massive stars. We compute their main sequence evolution assuming rigid rotation, and carry angular momentum as a passive quantity in the stellar interior but incorporate its effect on the stellar mass loss rate. We consider the example of a $60 M_\odot$ star assuming various initial rotation rates.

We show that rotation may substantially enhance the total main sequence mass loss of massive stars. Furthermore, we argue that the surface layers of rotating massive main sequence stars may reach the limit of hydrostatic stability ("Ω-limit") by achieving a considerable fraction of their Eddington luminosity. We show that this process is not catastrophic for the star, but rather that the coupling of mass and angular momentum loss limits the mass loss rate $\dot{M}_\Omega$ of main sequence stars at the Ω-limit. $\dot{M}_\Omega$ is determined through the angular momentum loss imposed by the Ω-limit rather than by atomic physics. For our $60 M_\odot$ sequences, it is $\dot{M}_\Omega \simeq 10^{-5} M_\odot\,\mathrm{yr}^{-1}$.

We find a convergence of the rotational velocities of main sequence stars of a given initial mass at the Ω-limit, but a strong dependance of their mass at core hydrogen exhaustion from the *initial* rotation rate. Since then also the post-main sequence evolution depends on the initial amount of angular momentum, we argue that this is a third independent initial parameter for the evolution of massive stars, as important as initial mass and metallicity.

We briefly discuss observable consequences of the coupling of mass and angular momentum loss, e.g. a significant decline of the projected rotational velocity $v\ \sin i$ towards the cool end of the main sequence, a period of strongly enhanced and aspherical mass loss, disks or rings in the equatorial plane of the star reminiscent of B[e]-stars, and highly bipolar circumstellar structures.

Key words: stars: rotation – stars: interiors – stars: early-type – stars: evolution – stars: mass loss

1. Introduction

Although we have a solid knowledge of the evolution of the deep interior of massive stars since a long time (Weaver et al. 1978, Kippenhahn & Weigert, 1990) it has become more and more clear during the last decades that our understanding of the evolution of their observable features is still rather incomplete. Among others, open problems concern the effective temperature evolution of moderately massive ($M_{\rm ZAMS} \simeq 10...30 M_\odot$) post main sequence stars (Langer & Maeder 1995) and the evolutionary connections between O stars, Luminous Blue Variables (LBVs) and Wolf-Rayet (WR) stars for higher masses (Schaller et al. 1992, Langer et al. 1994, Stothers & Chin 1996, Pasquali et al. 1997). Two major physical difficulties in the theoretical models of the observable evolutionary stages of massive stars have been identified and made responsible for the persisting lack of reliable models: mass loss and internal mixing processes (Meynet et al. 1994, Langer 1994, Deng et al. 1996). E.g., it has been found that the mass loss of massive main sequence stars should be roughly twice as high as what appears to be observed in order to understand many features of massive post main sequence stars (Meynet et al. 1994, Langer et al. 1994). Additionally, there is growing evidence that stellar rotation may considerably affect the evolution of massive stars (Maeder 1987, Langer 1991a, Fliegner et al. 1996, Maeder & Meynet 1996, Meynet & Maeder 1997, Langer 1997, Langer et al. 1997ab).

Clearly, massive main sequence stars are rapid rotators (cf. Fukuda 1982, Howarth et al. 1997). Massive stars are also very luminous, which brings about three major problems concerning their theoretical modeling. The first two, which have already been mentioned above, are mass loss — which is driven by their enormous photon fluxes (cf. Puls et al. 1996) — and internal mixing, which is much easier in massive stars due to the large contribution of the radiation pressure — which is independent of the mean molecular weight — to the total pressure in the star (e.g., Maeder 1987). In the present paper, we shall be concerned with a third problem brought along by the high luminosity, i.e. the proximity of the stellar surface to hydrodynamic instability, i.e. to the Eddington-limit. We want to show in the following that in rotating luminous stars the mass loss may be considerably enhanced over the (usually considered) nonrotating case, and that there is a maximum mass loss rate these stars may achieve which is actually controlled by the accompaniing angular momentum loss rate and not by the radiation force.

In the next section, we will spell out and discuss the physical methods and assumptions used for our investigation. Sect. 3 contains the results of exemplary stellar evolution calculations for the case of a $60 M_\odot$ stars. In Sect. 4, we shall discuss problems, consequences and further applications of the concept of coupled mass and angular momentum loss. Conclusions are given in Sect. 5.

2. Methods and assumptions

The results presented below have been obtained with a hydrodynamic stellar evolution code (cf. Langer et al. 1988, Langer 1991b), employing the OPAL opacities of Iglesias et al. (1992). We have used an outer boundary condition which takes the optical depth of the stellar wind selfconsistently into account within a grey approximation (Langer et al. 1994, Heger & Langer 1996).

We have computed the proximity of the star to the Eddington-limit, i.e. the Eddington factor $\Gamma = L/L_{\rm edd} = \kappa L/(4\pi c G M)$ in the following way. It has been shown in Langer (1997) that the occurrence of convection and of density inversions makes the concept of the Eddington limit as a stability limit invalid in the stellar interior, i.e. the Eddington factor Γ has to be evaluated only at the stellar surface. Since the term "surface" is not unambiguously defined in this context, we considered Γ in layers with an optical depth of $\tau < 100$, where in fact neither a significant convective energy flux nor density inversions have been found in the investigated models. Thus, to estimate the distance to the Eddington limit, we used the maximum value of Γ occurring in the subsonic layers with $\tau < 100$. Furthermore, we used the OPAL opacity coefficient to compute the Eddington factor. Mass and luminosity are practically constant for $\tau < 100$ and equal to the total stellar mass and luminosity.

The choice of a maximum optical depth for the evaluation of Γ introduces some arbitrariness into our method. However, as we shall point out repeatedly, in the present state we can only draw qualitative conclusions. It turns out that the Eddington limit is only important in the equatorial surface layers of stars rotating close to their critical rotation rate (see below). The stratification of these layers can not yet be reliably computed by any existing stellar structure code, and it is ultimately our ignorance of the thermodynamical properties of these layers which introduce the major quantitative uncertainties into our results, not the choice of the maximum optical depth.

We have carried angular momentum in the stellar models only as a passive quantity, i.e. we ignore the centrifugal force in the stellar interior as well as the effect of mixing of chemical species due to rotationally induced instabilities (cf. Fliegner et al. 1996, Meynet & Maeder 1997). However, we do consider the centrifugal force at the stellar surface to evaluate the distance of the star from the Ω-limit (Langer 1997), i.e. from critical rotation, with

$$\Omega = v_{\rm rot}/v_{\rm crit}, \tag{1}$$

and

$$v_{\rm crit}^2 = GM(1-\Gamma)/R. \tag{2}$$

This is not unreasonable since for hot massive stars close to the Ω-limit, only the outer skin ($M_r/M \gtrsim 0.99999$) rotates close to critical while in the bulk of the star Ω is not close to one at all, due to the strong peak of the opacity in the outer layers.

Furthermore, we assume our models to be always rigidly rotating. Stellar models including differential rotation (e.g. Fliegner et al. 1996) show that this is a good approximation on the main sequence, since the time scale for angular momentum transport is of the order of the thermal time scale and also shorter than the time scale of rotationally induced chemical mixing (Chaboyer & Zahn 1992, Zahn 1992, Talon & Zahn 1997). According to Zahn (1994), the expected amount of differential rotation in a massive main sequence star is roughly $\Delta\omega/\omega \simeq \omega^2 R^3/(GM)$, with ω being the mean angular velocity and $\Delta\omega$ its difference between stellar core and surface. In the models presented below, this estimate gives $\Delta\omega/\omega \lesssim 0.01$. The approximation of rigid rotation may become invalid when the mass loss time scale becomes shorter than that of angular momentum transport, but this is not the case in our models.

To compute the mass loss rate for our stellar models, we have applied the empirical rate found by Lamers & Leitherer (1993) with the metallicity dependence obtained by Leitherer & Langer (1991). However, we have applied the correction factor derived by Bjorkman & Cassinelli (1993) as a fit to the results of Friend & Abbott (1986) to take the effect of rotation on the mass loss rate of hot star winds into account, i.e.

$$\dot M/\dot M(v_{\rm rot}=0) = \left(\frac{1}{1-\Omega}\right)^{\xi}, \tag{3}$$

with $\xi = 0.43$. Note that this increases the mass loss rate by less than one third for $\Omega \lesssim 0.5$, and to obtain an enhancement by a factor of two a value of $\Omega \simeq 0.8$ is required. Although the validity of Eq. (3) has been questioned by Owocki et al. (1996), its important consequence used below — i.e. the increase of the mass loss rate for increased Ω — may still remain valid.

Angular momentum loss is only considered through the effect of mass loss, i.e. the lost mass carries away its specific angular momentum. The effective coupling between mass and angular momentum loss will be described in Sect. 3 and Fig. 2 below. Effects of magnetic fields are not considered.

We thus have treated rotation in our models in the simplest possible way. This has the advantage that our results do not depend on the various physical effects which rotation introduces in the stellar interior but instead we concentrate on and single out those processes which are relevant for the coupling of mass and angular momentum loss, and the effect they have on the main sequence evolution of massive stars.

3. Evolutionary calculations

We have computed evolutionary models for stars with an initial mass of $60 M_\odot$ and a metallicity of $Z = 0.01$. The computations

Table 1. Computed $60M_{\odot}$ sequences. Given are: sequence number, initial equatorial rotation velocity $v_{\rm rot,i}$, initial ratio of rotation rate to critical rotation rate $\Omega_{\rm i}$, core hydrogen burning life time $\tau_{\rm H}$, final mass $M_{\rm f}$, average mass loss rate $\bar{\dot{M}}$, minimum rotational velocity $v_{\rm rot,min}$, and final values of luminosity log $L_{\rm f}/L_{\odot}$, effective temperature $T_{\rm eff,f}$ and radius $R_{\rm f}$. "Final" here means core hydrogen exhaustion.

Seq.#	$v_{\rm rot,i}$ [km s^{-1}]	$\Omega_{\rm i}$	$\tau_{\rm H}$ [10^6 yr]	$M_{\rm f}$ [$M_{\odot}$]	$\bar{\dot{M}}$ [$M_{\odot}$ yr^{-1}]	$v_{\rm rot,min}$ [km s^{-1}]	log $L_{\rm f}/L_{\odot}$	$T_{\rm eff,f}$ [1000 K]	$R_{\rm f}$ [$R_{\odot}$]
1	(200)	(0.46)	3.37	55.72	1.27E-6	173	5.92	30.7	32.6
2	100	0.23	3.37	53.23	2.01E-6	54.7	5.90	28.8	36.7
3	200	0.46	3.43	46.04	4.07E-6	41.5	5.87	25.0	46.5
4	300	0.68	3.49	43.27	4.80E-6	44.7	5.85	24.9	45.5
5	400	0.91	3.57	41.02	5.32E-6	51.5	5.82	27.3	36.6

are started on the zero age main sequence (ZAMS) and terminated shortly after core hydrogen exhaustion. We have computed four sequences with initial equatorial rotation velocities of $v_{\rm rot,i}$ = 100, 200, 300, and 400 km s^{-1} (sequence # 2 to 5, cf. Table 1). For comparison, we have also computed one sequence (#1) with an initial rotational velocity of 200 km s^{-1} without applying the mass loss enhancement due to rotation (cf. Sect. 2). All sequences start from the same initial model which is characterized by a surface temperature of $T_{\rm eff}$ = 50 000 K, a radius of $9.9R_{\odot}$, and a luminosity of log $L/L_{\odot}$ = 5.69.

Fig. 1 shows the time evolution of the critical rotational velocity (Eq. 2) and the actual rotational velocity for sequence #3. The critical rotational velocity has a pronounced minimum at roughly $t = 2.6\,10^6$ yr, which corresponds to a stellar effective temperature of $T_{\rm eff} \simeq 36\,500$ K, around which the iron opacity peak has its maximum effect. We see in Fig. 1 that, as $v_{\rm rot}$ approaches $v_{\rm crit}$ (i.e. $\Omega \rightarrow 1$) the rotation rate of the star declines such that the Ω-limit $\Omega = 1$ is never exceeded. This behavior is explained by the coupling of angular momentum and mass loss, which works as follows.

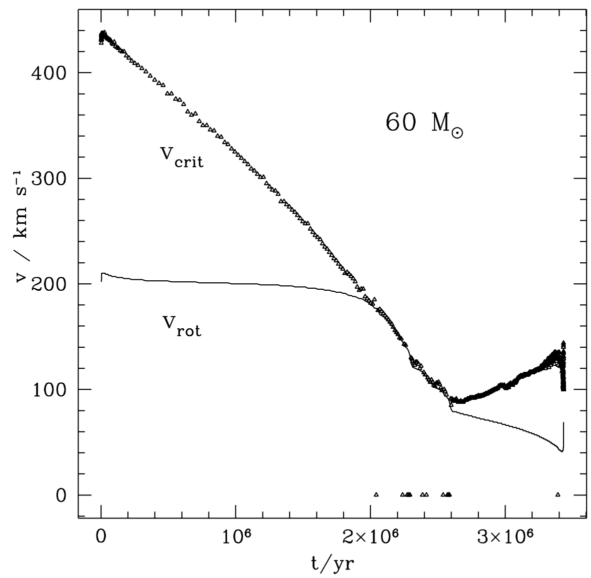

Fig. 1. Equatorial rotational velocity $v_{\rm rot}$ and the critical rotational velocity $v_{\rm crit}$ (cf. Eq. 2) as function of time during the main sequence evolution of the $60M_{\odot}$ sequence #3 (cf. Table 1).

For illustration, let us divide the process of mass and angular momentum loss into three discrete steps (cf. Fig. 2). First, consider a rigidly rotating star, rotating with an angular velocity ω_0. Since the specific angular momentum is $j = \omega r^2$, the layers with the largest specific angular momentum are those closest to the surface, where, initially, we have $j = j_0 = \omega_0 R_0^2$. In step one, let us take off a small amount of mass ΔM from the stellar surface ($a) \rightarrow b$) in Fig. 2). Before the star can react to this, its new surface specific angular momentum j_1 is smaller than j_0, since j is monotonically decreasing inwards for the case of rigid rotation. In this state, however, it is still $\omega_1 = \omega_0$. In the next step ($b) \rightarrow c$)), we allow the star to mechanically readjust, i.e. in particular its layers close to the surface will expand. In fact, if ΔM is not too large, the new stellar radius will be almost the same as the old one. Due to local angular momentum conservation, the expanding layers will spin down, i.e. the angular velocity in the outer layers will decrease. However, deep in the core we still have $\omega = \omega_0$. Finally, in the last step ($c) \rightarrow d$)) we allow angular momentum transport to reestablish rigid rotation, which will slow down the rotation of the core. It will again increase the surface angular velocity somewhat, but since the layers with the highest specific angular momentum have been lost it will be smaller than ω_0. In total, we have again a rigidly rotating star, but with a smaller mass *and* a smaller angular velocity. Note that this mechanism of coupled mass and angular momentum loss works in the same way for differentially rotating stars, as long as the time scale for the readjustment of the rotation profile is shorter than the mass loss time scale.

When the $60M_{\odot}$ sequence in Fig. 1 approaches $\Omega = 1$, the mass loss rate is increased according to Eq. 3. As a consequence, according to the coupling described above, the spin-down rate of the star increases as well. In this situation, i.e. at the Ω-limit, an equilibrium value $\Omega_{\rm eq} < 1$ will establish. It is defined such that if Ω is smaller than this value, mass and angular momentum loss are too small to prevent the star's continuous evolution towards the Ω-limit. If $\Omega > \Omega_{\rm eq}$, the corresponding angular momentum loss is so large that the star evolves away from the Ω-limit, i.e. Ω becomes smaller. In the case of the $60M_{\odot}$ sequence displayed in Fig. 1, it is $\Omega_{\rm eq} \simeq 0.991$.

Fig. 3 displays the mass loss rate of all models of sequence #3 as a function of time, in comparison to sequence #1.

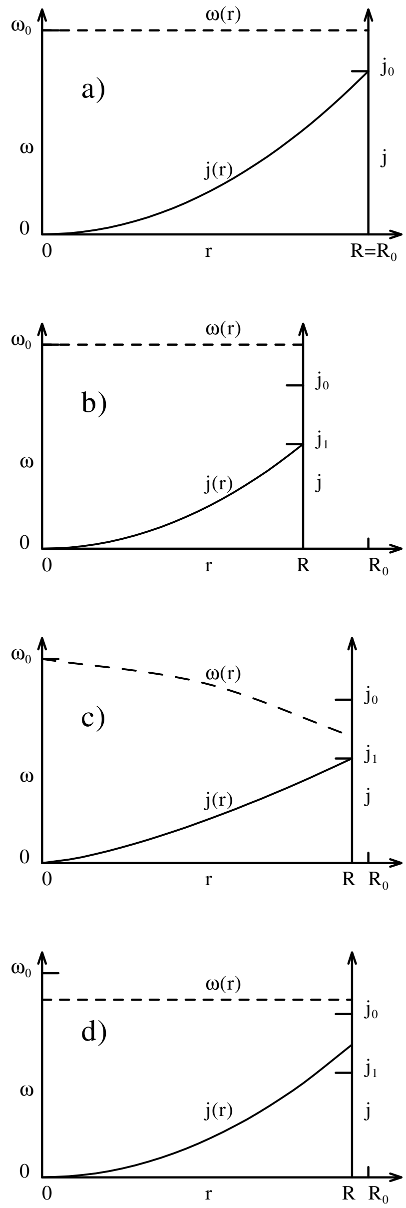

Fig. 2a–d. Schematic diagram of coupled mass and angular momentum loss in a rigidly rotating star, broken up into three discrete processes: mass loss without readjustment (**a**→**b**); reexpansion of the star (**b**→**c**); and reestablishment of rigid rotation (**c**→**d**). ω is the angular velocity, $j = \omega r^2$ the specific angular momentum, and R the stellar radius. Subscript 0 refers to the initial state. See text for more details.

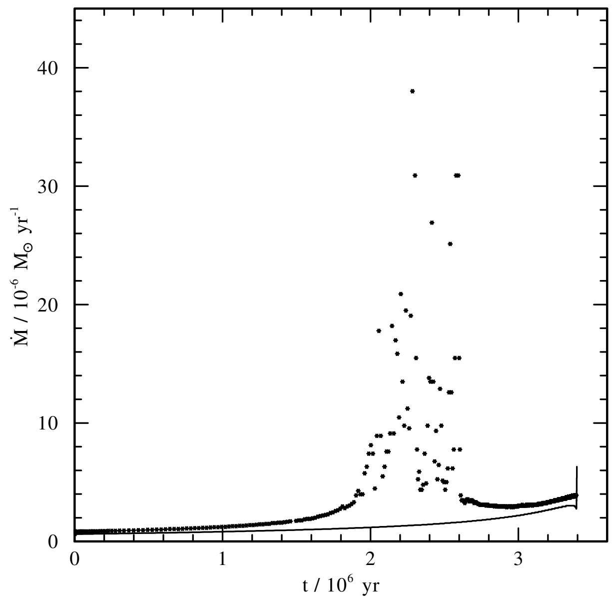

Fig. 3. Mass loss rate of the models of the $60 M_\odot$ sequence #3 (cf. Table 1; dots) compared to the mass loss rate as function of time for the $60 M_\odot$ without rotational mass loss enhancement (sequence #1, continuous line).

The big scatter during the period at the Ω-limit is purely numerical and results from the character of Eq. 3, i.e. it is $\dot{M} \rightarrow \infty$ for $\Omega \rightarrow 1$. In fact Fig. 4, which presents the time evolution of the stellar mass for all sequences, shows that the average mass loss rate during this phase is a well defined quantity. The average mass loss rate at the Ω-limit, which is represented by the slope of the curves in Fig. 4, is about $10^{-5} M_\odot\ \mathrm{yr}^{-1}$.

Note that for $t \gtrsim 2.6\ 10^6$ yr, the star evolves off the Ω-limit since the critical rotation rate increases again as a consequence of its further decreasing surface temperature during core hydrogen burning. At core hydrogen exhaustion, it has reached $T_{\rm eff} \simeq 25\,000$ K (cf. Table 1).

Figs. 4 and 5, which display stellar mass and rotation rate as function of time for all computes sequences, show that the stars which rotate rapidly initially behave qualitatively similar to sequence #3. As soon as they reach the Ω-limit they increase their mass loss rate to values around $10^{-5} M_\odot\ \mathrm{yr}^{-1}$ so that they spin down at the Ω-limit. Since the critical rotation velocity evolves almost equally for all considered sequences, this means that the stars leave the Ω-limit at almost the same time and with almost the same rotation rate. However, due to the fact that they spend different amounts of time at the Ω-limit (cf. Fig. 6), their masses at core hydrogen exhaustion are different, i.e. lower for higher initial rotation velocities (cf. Table 1 and Fig. 4).

It is important to realize that the equilibrium mass loss rate at the Ω-limit $\dot{M}_\Omega$ is determined by the condition of a sufficient angular momentum loss to not exceed the Ω-limit. Therefore, for a given critical rotational velocity, the mass loss rate of a rigidly rotating star at the Ω-limit is determined by its density distribution $\rho(r)$ and by its expansion rate due to the internal evolution. Within these assumptions, the mass loss rate does not depend on microscopic properties of the matter as e.g. in the case of the normal hot star winds.

The sequence with $v_{\rm rot,i}$ = 100 km s^{-1} does not quite reach the Ω-limit (cf. Fig. 6). However, it comes sufficiently close to critical rotation that its mass loss rate is also considerably increased compared to the "non-rotating" model.

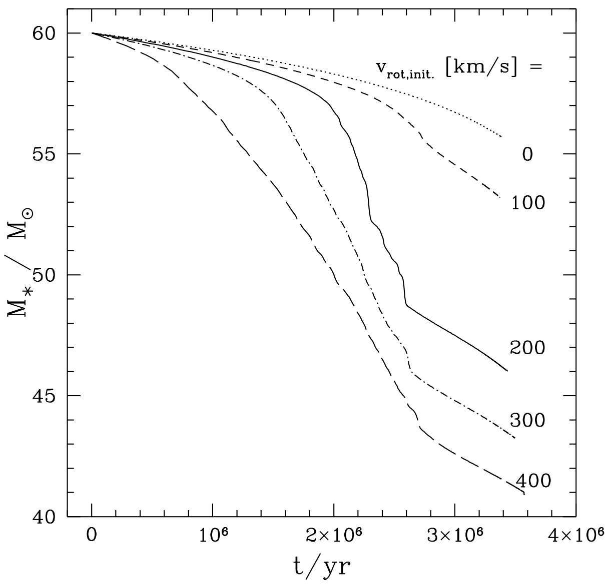

Fig. 4. Time dependence of the total stellar mass of the computed $60 M_\odot$ sequences with different initial rotation rates $v_{\rm rot,i}$ (sequences #2 to 5 in Table 1). For comparison, the time dependence of the mass of sequence #1 is also displayed and marked $v_{\rm rot,i}$ = 0.

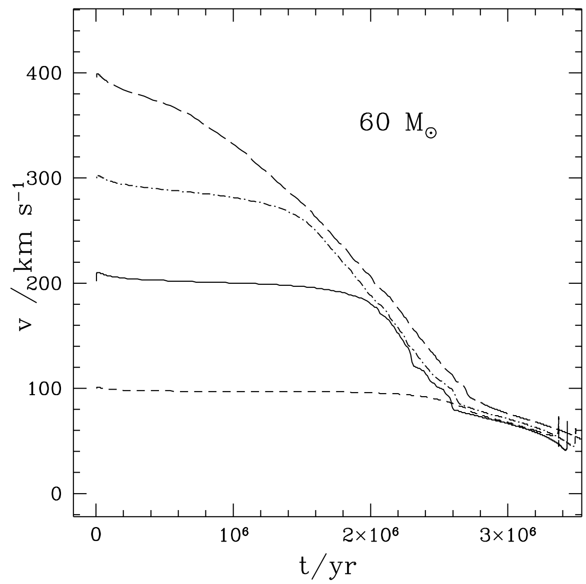

Fig. 5. Time dependence of the equatorial rotational velocity of the computed $60 M_\odot$ sequences #2 to #5 (cf. Table 1) with different initial rotation rates $v_{\rm rot,i}$ (see at t = 0). Since the time dependence of their critical rotational velocity is very similar, their tracks converge at the Ω-limit (cf. Fig. 1).

4. Discussion

In Sect. 3, we have discussed the evolution of stars at the Ω-limit for the case of a $60 M_\odot$ main sequence star at Z = 0.01. We have to emphasize again that this can only be a qualitative example. The general properties of the model, e.g. the existence of an equilibrium mass loss rate, its determination through the angular momentum loss, or the convergence of the rotational velocities at the Ω-limit are expected to be independent of the quantitative details. However, we are at present unable to state for which initial mass, metallicity and initial rotation rate massive main sequence stars do actually reach the Ω-limit. All we can say is that we have made reasonable assumptions to compute Ω for our $60 M_\odot$ star and thus it is plausible that stars of this mass may reach the Ω-limit.

The reason for this uncertainty is that, as mentioned above, it is impossible with the existing methods to compute the stratification of the surface layers of a mass losing star close to critical rotation (cf. Dupree 1995). Since Ω depends on the opacity coefficient κ through the Eddington factor Γ, and κ is a very sensitive function of temperature and density, the quantities $v_{\rm crit}$ and Ω must remain rather uncertain. E.g., one may think of the graph of $v_{\rm crit}$ in Fig. 1 to be shiftable up- or downwards.

One might try to limit the uncertainty through observations. Fig. 5 makes a rather clear prediction, i.e. the rotation velocities of all tracks converge to rather slow values (40...80 $\rm km\,s^{-1}$). Thus, sufficiently luminous (log $L/L_\odot \gtrsim$ 5.5; cf. Table 1) and cool ($T_{\rm eff} \lesssim 45\,000 \ldots 30\,000$ K) stars should be found to rotate with velocities in that range. This seems in fact to be at least not inconsistent with observations. Fukuda (1982) finds a remarkable decline in the rotation rate of supergiants (luminosity class I) at the transition from early to late spectral type O (cf. also Nieuwenhuijzen & de Jager 1988, Penny 1996, Howarth et al. 1997). Note that such a decline is not predicted by sequence #1 where the rotational mass loss enhancement is not considered. Due to the standard mass loss, this sequence does also lose some angular momentum. It evolves, from initially 200 $\rm km\,s^{-1}$ to a minimum value during core hydrogen burning of 173 $\rm km\,s^{-1}$, which means it hardly spins down at all. It is noteworthy that the radius increase of our stars on the main sequence by a factor of 3...4 (cf. Table 1) does not by itself lead to a spin-down of the surface layers since it is counterbalanced by an increasing density of the stellar core, i.e. the central density is increased by more than a factor of two during core hydrogen burning. However, although the number and quality observed rotation rates of hot luminous stars has increased significantly during the last decade, a solid conclusion appears difficult at present, also due to the fact that the interpretation of the spectral line widths is still a matter of debate (cf. Howarth et al. 1997).

We have neglected rotationally induced chemical mixing, for which some evidence exists (cf. Fliegner et al. 1996) and which may change the emerging picture as follows. The chemical mixing is stronger for larger initial rotation rates, and consequently rapid rotators would develop to larger luminosities than slow ones (Langer 1992, Fliegner et al. 1996) and would thus have smaller critical rotation velocities. Since the minimum of the critical rotation velocity determines the final rotational velocity (cf. Fig. 1), the surprising result is that stars which rotate faster initially end their core hydrogen burning rotating slower. However, all stars would evolve to rotational velocities lower

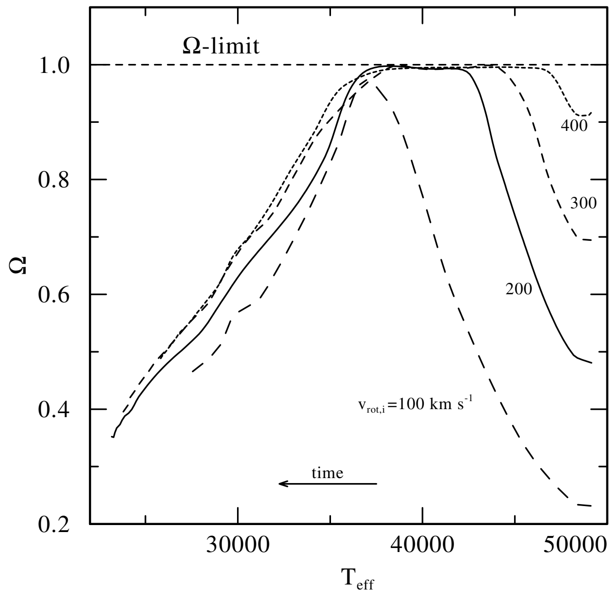

Fig. 6. Ω as function of the effective temperature during core hydrogen burning, for sequences #2 ... #5.

than the minimum critical rotational velocity obtained for the non-rotating case (cf. Figs. 1 and 5), unless the chemical mixing enhances the envelope helium content so much that they evolve towards hotter effective temperatures already on the main sequence (Maeder 1987, Langer 1992).

It is further worth noting that the $60 M_\odot$ tracks derived above are similar to those obtained by Langer et al. (1994) with completely different assumptions. In that paper, the mass loss rate in the effective temperature regime around $T_{\rm eff} \simeq 36\,000$ K has been assumed to be enhanced due to theoretically predicted strange mode pulsations. In fact, both, these pulsations and the Ω-limit depend on the iron opacity peak which has its maximum effect in the considered effective temperature range. Langer et al. (1994) have concluded that only the additional main sequence mass loss allows agreement between post-main sequence models and observations, e.g. concerning LBV masses and abundances or WR luminosities. Meynet et al. (1994) have found further evidence for strong main sequence mass loss. Our models involving Eq. (3) and the Ω-limit provide an additional source of enhancement of the main sequence mass loss, i.e. angular momentum.

It is not clear what happens to the matter lost by an almost critically rotating star. The Ω-limit itself guarantees only that the material is easily lifted off the stellar surface. It will depend then on the evolution of temperature and density of that matter whether the stellar radiation field will be able to push it to infinity. In case the opacity in the uplifted matter drops it can not be pushed further by the radiation, but it can also not fall back to the stellar surface since it has too much angular momentum. If there is a stationary state at all, the matter may remain at a finite distance to the star. At the Ω-limit, most of the material leaving the star will do so at latitudes close to the equator, and since the escape velocity there will be close to zero the material will leave the star with a small velocity. Consequently, the star may develop a disk or ring in the equatorial plane. At the poles, the star does not feel the centrifugal force and thus the effective gravity will be high there. Consequently, the surface regions around the poles may emit a fast radiation driven wind like normal main sequence stars in that mass range.

It has been pointed out to the author by P. Conti that the emerging picture is reminiscent of the so called B[e] stars. Those are hot stars which show the signature of a fast wind — which is typical for hot stars — but also narrow emission lines with widths of only tens of $\mathrm{km\,s^{-1}}$, and strong evidence for circumstellar dust. Zickgraf (1989) finds that a two-component model of a hot star with a fast polar wind and a slow equatorial wind which forms a dense disk can explain these features best.

In fact, our models allow an order of magnitude estimate of the equator-to-pole ratio of mass loss rate and outflow velocity at the Ω-limit. As the total mass loss rate is enhanced by a factor of the order of 10 over the normal radiation driven wind mass loss rate (cf. Figs. 3 and 4) a factor of that order may be expected for the mass flux density ratio between equator and pole. For the velocity at the poles we have to expect wind velocities found in normal hot stars (say $\sim 1000\ \mathrm{km\,s^{-1}}$), while at the equator the escape velocity is reduced by about 99% (cf. Sect. 3), i.e. we may expect some $10\ \mathrm{km\,s^{-1}}$. These numbers appear to be in the range which is found in B[e] stars (Zickgraf 1989).

Finally, we want to mention that the applicability of the concept of the Ω-limit is not restricted to massive main sequence stars. On the basis of 2-dimensional hydrodynamic simulations of the evolution of circumstellar matter it has been argued by García-Segura (1997) and Langer et al. (1997a) that the highly bipolar shape of LBV nebulae and the giant LBV outbursts which produce them are due to massive post-main sequence stars reaching the Ω-limit. As the internal rate of expansion during the hydrogen-shell burning evolution is at least two orders of magnitude smaller compared to core hydrogen burning, the mass loss rate at the Ω-limit in this case must be at least two orders of magnitude larger than those considered in Sect. 3. As the resulting mass loss time scale may then be smaller than the time scale of angular momentum transport, we can not treat this situation with the methods presented in Sect. 2.

Although in the case of LBVs again the proximity to the Eddington limit is thought of driving the stars to critical rotation (cf. Langer et al. 1997a), the Ω-limit is not restricted to situations close to the Eddington-limit. In fact, whichever instability at a spherically symmetric stellar surface is envisaged in the sense that the surface becomes force-free when the instability limit is reached, even small amounts of rotation will break the spherical symmetry, and instead of the envisaged instability to work all over the stellar surface critical rotation is achieved at the stellar equator even before the instability limit is reached. Two examples of this, which are discussed in the literature and where the Ω-limit may apply are an instability induced by turbulent convective pressure in hot luminous stars (Nieuwenhuijzen & de Jager, 1995) — which the authors also connect to LBVs and their outbursts —, and the recombination instability at the tip of the Asymptotic Giant Branch (Paczyński & Ziółkowski 1968, Han et al. 1994, Wagenhuber & Weiss 1994), which may be related to the "superwind" and planetary nebulae formation (Iben & Renzini 1983). For the latter, the Ω-limit has been suggested as explanation for the formation of highly bipolar planetary nebulae by García-Segura et al. (1997b).

5. Conclusions

In the previous sections we have shown that rotation may affect the evolution of massive main sequence stars in two ways. It can enhance the mass loss rate of the star over that in the non-rotating case (Friend & Abbott 1986) without affecting the angular momentum evolution very much. An example of this case is sequence #2, which has an initial rotational velocity of the order of its minimum critical rotational velocity during core hydrogen burning ($\sim 100\ \mathrm{km\,s^{-1}}$). However, we showed that it may occur that the initial rotation rate of a massive star is larger than this. Then the star will reach the Ω-limit of critical rotation. In this stage, the mass loss rate may be much larger than if no rotation were present. It is determined by the requirement of angular momentum loss, which is directly coupled to the mass loss (cf. Fig. 2).

Due to this circumstance, the resulting mass loss rate at the Ω-limit $\dot{M}_\Omega$ can be quantified rather reliably to $\dot{M}_\Omega \simeq 10^{-5} M_\odot$ for a $60 M_\odot$ star, while other interesting quantities, e.g. the function $f(M_{ZAMS}, \Omega_i, Z)$ which separates stars reaching the Ω-limit from those which do not in the relevant parameter space, must remain very uncertain at present (cf. Sect. 4). I.e., we can not be sure how many stars of a given population will reach their Ω-limit during core hydrogen burning, but we can deduce what happens to those which do.

Since it is also unclear which fraction of the mass lost at the Ω-limit can actually be pushed to infinity by the radiation force, there are several alternatives for the appearance of stars at the Ω-limit. If all the matter is blown away, the star may not look peculiar at first glance. It is important to note that it needs not even to be a rapid rotator (cf. Fig. 1). However, it's mass loss should be strongly enhanced and distinctly non-spherical. The latter might be observable through line profile shapes (cf. Petrenz & Puls 1996, Howarth et al. 1997). In this context, the anomalously high mass loss of some massive O stars in the LMC found by de Koter et al. (1997) may be interesting. The asphericity of the mass loss at the Ω-limit together with its spherical symmetry before and after that phase may result in bipolar circumstellar structures, perhaps like those found around the O6.5 star HD 148937 (Leitherer & Chavarria 1987). As mentioned in Sect. 4, a similarity to B[e] stars may be present if not all the matter is pushed to infinity.

The coupling of mass and angular momentum loss also opens the possibility to deduce average mass loss rates of main sequence stars from the dependence of the mean projected rotational velocity from the effective temperature. However, the observational data, although not inconsistent with the $60 M_\odot$ models presented above, allows no firm conclusions at present.

Finally, we want to emphasize the extreme importance of the initial rotation rate for the post main sequence evolution of massive stars. Even for stars which do not reach the Ω-limit, the mass loss enhancement may be appreciable (cf. Fig. 4, sequences #1 and #2). It is drastic for stars which do reach the Ω-limit, and initially faster rotating stars stay there longer than initially slower rotators. This makes the stellar mass at core hydrogen exhaustion a sensitive function of the initial rotation rate (see Table 1 and Fig. 4), which is of key importance for the further evolution of the stars.

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