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ASTRONOMY AND ASTROPHYSICS

The determination of the parameters of fragmenting meteoroids

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Abstract. Taking into account the quasi-continuous fragmentation of meteor particles, the formulae describing the variation of both light and ionization along the atmospheric trajectory of meteoroids, are derived. Two methods of data processing are presented enabling the determination of the parameters of quasi-continuous fragmentation, R_0, and R_1, characterizing the parent body and the fragmentation products, as well as the bulk density of the parent meteoroid from the light and/or ionization curve. The use of these methods is shown by means of two examples of processing of model curves.

Key words: meteors, meteoroids

1. Introduction

The fragmentation of meteoroids during their flight in the atmosphere has been observed many times both visually and photographically. The amount of data evidencing fragmentation led Levin (1963) to the conclusion that, if fragmentation were not taken into consideration in processing the observations, erroneous results would result. According to Levin (1961), the following basic forms of fragmentation were recognized (see also Lebedinets 1980, Bronshten 1983):

1) disintegration of meteoroid into several pieces;
2) simultaneous detachment of much small debris from the parent meteor body (PMB), being classified as a flare;
3) progressive fragmentation during which also daughter particles fragment, and so on;
4) quasi-continuous fragmentation (QCF) during which a large number of small further nonfragmenting particles detaches from PMB.

Observations show that the QCF should be of greatest interest. First, this kind of fragmentation is most probable and frequently observed in practice. Second, there exist many facts (e. g. McCrosky 1958, Babadzhanov & Kramer 1968, Hawkes & Jones 1975, Lebedinets 1980, Bronshten 1983, Millman 1983) pointing out the QCF as the mechanism of disintegration of PMB. Third, the kinds of ablation of PMB such as pure evaporation or flare can be considered a special case of QCF. Fourth, QCF can explain the observed shapes of the radar-underdense Fresnel diffraction characteristics (e. g. Novikov & Pecina 1990). The basic physical mechanisms causing QCF are well known and were listed by Lebedinets (1980, 1987). The first numerical values of fragmentation energies were published by Lebedinets (1980) and Novikov et al. (1984 b). Their more precise values corresponding to various kinds of meteor matter were published by Lebedinets (1986), and by Babadzhanov (1993) – see his Table 2.

The basic theory of light curves of meteoroids ablating via QCF has been put forward by Simonenko (1973). Further development has been carried out by Lebedinets (1980), Kalenichenko (1980) and Novikov et al. (1984 a). Novikov & Konovalova (1995) utilized the theory of QCF in processing light curves without going into details of the derivation of basic formulae, and without treating the ionization.

Novikov et al. (1996 a, 1996 b) have used the theory of QCF of PMB to infer the bulk densities of meteoroids and to describe quantitatively the evolution and the structure of meteor coma observed photographically by the method of instantaneous exposure. However, the theory of QCF was not described by the authors. Here we derive the formulae describing both the light and ionization curves of meteors ablating by QCF of PMB; we also rectify some mistakes present in the old version of the theory of Novikov et al. (1984 a), and in Novikov & Konovalova (1996), and present the above formulae in compact form being more convenient for meteor physicists. To demonstrate the capability of the new approach, we solve the model tasks concerning determination of some parameters of PMB.

2. Theory

Novikov et al. (1984 a) have made the following assumptions when constructing the theory of QCF:

1) the only mechanism of ablation of PMB is QCF;

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2) the only mechanism of ablation of fragments is evaporation. This implies that the contribution to light and ionization curves comes from atoms and molecules of meteoroid matter which evaporated directly from the surface of fragments and not of PMB (this contribution is proportional to Q_f/Q, where Q_f represents the specific energy of QCF and Q the same quantity related to evaporation);
3) initial masses of all fragments are the same and equal to m_0;
4) deceleration of both the PMB and fragments is negligibly small;
5) the basic formulae of the physical theory of meteors (e. g. Bronshten 1983) are valid for both PMB and fragments.

We shall accept these assumptions, too. Contrary to Novikov et al. (1984 a), where the formula for fast QCF is erroneous, we shall derive below the correct one (Eq. (16)).

The most general theory of light and ionization taking into consideration the QCF has to consider also deceleration both of PMB and fragmentation products. Elements of such a theory have been put forward by McCrosky (1958) and a more detailed theory has been published by Babadzhanov et al. (1987). But there exist many observational results demonstrating that both luminosity and ionization can also be explained without taking into account the deceleration. These are

1) the observed heights of beginning, maximum light, and of end of meteor events as published by Hawkins & Southworth (1958);
2) the observed dependence of the height of maximum luminosity on the velocity of meteoroids as evidenced by the data of Jacchia et al. (1967);
3) the observed shift of the maximum light towards the beginning of the atmospheric trajectory of meteors as inferred by Hawkins & Southworth (1958), Jacchia et al. (1967), and Babadzhanov & Novikov (1987).

Therefore, we will neglect the deceleration when deriving formulae for light intensity, I, as well as for electron line density, α, characterizing the ionization state. Under these conditions the following formulae are valid (e. g. Bronshten 1983)

$$I = -\frac{\tau_v v^2}{2}\frac{dM_e}{dt}, \quad \alpha = -\frac{\beta_v}{\mu v}\frac{dM_e}{dt}, \tag{1}$$

where dM_e/dt stands for the rate of evaporation of meteoroid mass, τ_v represents the luminous efficiency, β_v the ionization efficiency, (both considered as a function of velocity, v). The mass of evaporated meteoric particle is designated as μ. We have adopted also the assumption of the dependence of atmospheric density on the height within the meteor zone, h, in the form $\rho = 3 \times 10^{-9} \exp[-(h-90)/6]$ where h is expressed in km (e. g. Poole & Nicholson 1975) (scale height of 6 km).

Now we will derive the formula for dM_e/dt needed for the evaluation of I and α. Let

$$J_e(t,t') = -\frac{\tau_v v^2}{2}\frac{dm_e(t,t')}{dt} \tag{2}$$

represent the luminosity produced at time t by an individual fragment detached from PMB at the instant t'. The function $m_e(t,t')$ is the mass of the fragment. Then all fragments which detached at time $t' = t - \tau$ and later (where τ stands for the lifetime of individual fragment counted from t' until its full evaporation), will contribute to ablation of PMB and, consequently, also to light and ionization at any instant t. The number of fragments detached from PMB per unit time can be expressed as

$$N(t') = -\frac{1}{m_0}\frac{dM}{dt'}, \tag{3}$$

where $M(t')$ denotes the mass of PMB at time t'. Taking into consideration the lifetime of fragments and of PMB, and summing up the contributions of all fragments we get the following formula for the meteor luminosity

$$I = -\Theta(T_e - t)\int_{(t-\tau)\Theta(t-\tau_b)}^{t} J_e(t,t')N(t')\Theta(\tau_o - t')dt', \tag{4}$$

with $\Theta(x)$ representing Heaviside's unit step function, τ_o denoting the lifetime of PMB to complete fragmentation, τ_b being the lifetime of the fragment having been released at the instant of beginning of fragmentation, $T_e = \tau_o + \tau_e$, where τ_e is the lifetime of a fragment which was detached at the end of fragmentation until its full evaporation. T_e can be considered the time of termination of the whole meteor event. Let

$$\alpha_e(t,t') = -\frac{\beta}{mv}\frac{dm_e(t,t')}{dt} \tag{5}$$

represent the electron line density produced by one fragment. Summing up contributions of all fragments we derive the following ionization equation

$$\alpha = -\Theta(T_e - t)\int_{(t-\tau)\Theta(t-\tau_b)}^{t} \alpha_e(t,t')N(t')\Theta(\tau_o - t')dt'. \tag{6}$$

By substituting for $J_e(t,t')$ from (2) and for $N(t')$ from (3) into (4) and for $\alpha_e(t,t')$ from (5) into (6), and comparing the formulae obtained in this way with (1) we can see that the expression for dM_e/dt attains the form

$$\frac{dM_e}{dt} = -\frac{\Theta(T_e - t)}{m_0}\int_{(t-\tau)\Theta(t-\tau_b)}^{t} \frac{dM}{dt'}\frac{dm_e}{dt}\ \Theta(\tau_o - t')dt', \tag{7}$$

common to both light and ionization. When changing in (7) the integration variable t' into ρ' we get

$$\frac{dM_e}{dt} = -\rho\frac{\Theta(\rho_e - \rho)}{m_0}\int_{\rho_l}^{\rho} \frac{dM}{d\rho'}\frac{dm_e}{d\rho}\Theta(b - \rho')\,d\rho', \tag{8}$$

where $\rho_l = (\rho - R_1)\Theta(\rho - a)$; $\rho_e = \rho_b + R_0 + R_1$ stands for the atmospheric density at the height of termination of the whole meteor event, h_e, $a = \rho_b + R_1$ represents atmospheric density at the height h_a of disappearance of fragments which were detached at the height of beginning of QCF, h_b with ρ_b being the corresponding atmospheric density, $b = \rho_b + R_0$ represents the atmospheric density at the height h_t of termination of fragmentation of PMB, ρ' designates the atmospheric density at any height, h'. The quantities R_0 and R_1 will be specified below.

The equations for dM/dt' and dm_e/dt have been derived by Lebedinets (1980). They read

$$\frac{dM}{dt'} = -\frac{\Lambda A M^{2/3} \rho' v^3}{2Q_f \delta_0^{2/3}}, \quad \frac{dm_e}{dt} = -\frac{\Lambda' A' m_e^{2/3} \rho v^3}{2(Q - Q_f)\delta^{2/3}}, \tag{9}$$

where Λ, A, δ_0 stand for the heat transfer coefficient, the shape-density coefficient and the bulk density of PMB respectively, and Λ', A', δ are analogous quantities valid for fragments, Q denotes the energy of evaporation, while Q_f is the energy of fragmentation. Using ρ as the new independent variable converts (9) into the alternative form:

$$\frac{dM}{d\rho'} = -\frac{\Lambda A H v^2 M^{2/3}}{2Q_f \delta_0^{2/3} \cos z_R}, \frac{dm_e}{d\rho} = -\frac{\Lambda' A' H v^2 m_e^{2/3}}{2(Q - Q_f)\delta^{2/3} \cos z_R}, \tag{10}$$

where H is the constant scale height used in the dependence $\rho(t) = \rho_b \exp(t/t_H)$ with ρ_b representing atmospheric density at the height of beginning of fragmentation, and $t_H = H/v \cos z_R$, z_R being zenith distance of the radiant.

In general $\Lambda = \Lambda(\rho')$ (e. g. Novikov et al. 1993). This functional dependence was inferred for large fireballs. However, we deal here with fainter meteors whose Λ can be kept constant. With our assumption of constant meteoroid velocity, v, the solution of (10) reads

$$M(\rho') = M_0 \left[1 - \frac{(\rho' - \rho_b)}{R_0}\right]^3,$$
$$m_e(\rho, \rho') = m_0 \left[1 - \frac{(\rho - \rho')}{R_1}\right]^3, \tag{11}$$

where we have introduced the auxiliary quantities

$$R_0 = 6Q_f (M_0 \delta_0^2)^{1/3} \cos z_R / \Lambda A H v^2,$$
$$R_1 = 6(Q - Q_f)(m_0 \delta^2)^{1/3} \cos z_R / \Lambda' A' H v^2. \tag{12}$$

It is necessary to distinguish two kinds of QCF (see e. g. Novikov et al. 1984 a, b). The first one, fast, labelled by the subscript, f, for which $b_f < a_f$ and the PMB is completely fragmented before the fragments released at the height of beginning of QCF can evaporate. Inserting (11) into (8) we get

$$\frac{dM_f}{dt} = -\frac{9M_0 v \cos z_R}{H R_{0f} R_{1f}} \rho \left\{ \left[\int_{\rho_b}^{\rho} F_f(\rho, \rho')\, d\rho'\right] \Theta(\rho - \rho_b) \times \Theta(b_f - \rho) + \left[\int_{\rho_b}^{b_f} F_f(\rho, \rho')\, d\rho'\right] \Theta(\rho - b_f)\Theta(a_f - \rho) + \left[\int_{\rho - R_{1f}}^{b_f} F_f(\rho, \rho')\, d\rho'\right] \Theta(\rho - a_f)\Theta(\rho_e - \rho) \right\}, \tag{13}$$

where now

$$F_f(\rho, \rho') = \left[1 - (\rho' - \rho_e)/R_{0f}\right]^2 \left[1 - (\rho - \rho')/R_{1f}\right]^2 \tag{14}$$

and

$$R_{0f} = 6Q_{ff}(M_0 \delta_{0f}^2)^{1/3} \cos z_R / \Lambda A H v^2,$$
$$R_{1f} = 6(Q - Q_{ff})(m_{0f} \delta_f^2)^{1/3} \cos z_R / \Lambda' A' H v^2. \tag{15}$$

Q_{ff} is the energy of fragmentation valid for this fast type. The first term in (13) corresponds to the fact that within the interval $\rho_b \le \rho \le b_f$ the PMB disintegrates due to fragmentation into fragments which further evaporate. But no fragment evaporates completely. The second term describes the situation when the PMB has already completely fragmented within the span $b_f \le \rho \le a_f$ and the fragments can be found in various states of evaporation, although no fragment completely disappears. And finally, the third term describes the circumstance that within the interval $a_f \le \rho \le \rho_e$ the evaporation of fragments still occurs but some fragments have already evaporated completely. On substituting (14) into (13) and carrying out the integration we arrive at

$$\frac{dM_f}{dt} = \frac{9M_0 v \cos z_R}{H(R_{0f} R_{1f})^3} \rho \left\{ \Theta(\rho - \rho_e)\Theta(b_f - \rho) \left[\frac{(\rho_e - \rho)^2}{3} \times \left[R_{0f}^3 - (b_f - \rho)^3\right] - \frac{\rho_e - \rho}{2}\left[R_{0f}^4 - (b_f - \rho)^4\right] + \frac{1}{5}\left[R_{0f}^5 - (b_f - \rho)^5\right]\right] + \Theta(\rho - b_f)\Theta(a_f - \rho) R_{0f}^3 \left[\frac{(\rho_e - \rho)^2}{3} - R_{0f} \times \frac{\rho_e - \rho}{2} + \frac{R_{0f}^2}{5}\right] + \Theta(\rho - a_f)\Theta(\rho_e - \rho)\frac{(\rho_e - \rho)^5}{30} \right\}. \tag{16}$$

The second type of the QCF, the slow one, labelled by the subscript s, for which $b_s > a_s$, concerns those fragments that have detached at the height of the beginning of fragmentation and disappeared due to evaporation before the PMB completely fragmented. Carrying out the same procedure as in the case of fast fragmentation we get

$$\frac{dM_s}{dt} = -\frac{9M_0 v \cos z_R}{H R_{0s} R_{1s}} \rho \left\{ \left[\int_{\rho_b}^{\rho} F_s(\rho, \rho')\, d\rho'\right] \Theta(\rho - \rho_b) \times \Theta(a_s - \rho) + \left[\int_{\rho - R_{1s}}^{\rho} F_s(\rho, \rho')\, d\rho'\right] \Theta(\rho - a_s)\Theta(b_s - \rho) + \left[\int_{\rho - R_{1s}}^{b_s} F_s(\rho, \rho')\, d\rho'\right] \Theta(\rho - b_s)\Theta(\rho_e - \rho) \right\}, \tag{17}$$

where now

$$F_s(\rho, \rho') = \left[1 - (\rho' - \rho_e)/R_{0s}\right]^2 \left[1 - (\rho - \rho')/R_{1s}\right]^2 \tag{18}$$

and

$$R_{0s} = 6Q_{fs}(M_0 \delta_{0s}^2)^{1/3} \cos z_R / \Lambda A H v^2,$$
$$R_{1s} = 6(Q - Q_{fs})(m_{0s} \delta_s^2)^{1/3} \cos z_R / \Lambda' A' H v^2. \tag{19}$$

Here Q_{fs} stands for the energy of fragmentation for the slow type. The first term in (17) describes the situation when within the interval $\rho_b \leq \rho \leq a_s$ the creation of fragments and their evaporation takes place. The second term shows that fragments still form due to fragmentation, and that they still evaporate. And finally, the third term corresponds to the situation when PMB has already fragmented completely within $b_s \leq \rho \leq \rho_e$, the evaporation of fragments still continues and some portion of them has already completely evaporated. On substituting (18) into (17) and performing the integration we obtain

$$\frac{dM_s}{dt} = \frac{9M_0 v \cos z_R}{H(R_{0s}R_{1s})^3}\rho\left\{\Theta(\rho-\rho_b)\Theta(a_s-\rho)\left[\frac{(\rho_e-\rho)^2}{3}\times\right.\right.$$
$$\times\left[R_{1s}^3-(a_s-\rho)^3\right]-\frac{\rho_e-\rho}{2}\left[R_{1s}^4-(a_s-\rho)^4\right]+\frac{1}{5}\left[R_{1s}^5-\right.$$
$$\left.\left.-(a_s-\rho)^5\right]\right]+\Theta(\rho-a_s)\Theta(b_s-\rho)R_{1s}^3\left[\frac{(\rho_e-\rho)^2}{3}-R_{1s}\times\right.$$
$$\left.\left.\times\frac{\rho_e-\rho}{2}+\frac{R_{1s}^2}{5}\right]+\Theta(\rho-b_s)\Theta(\rho_e-\rho)\frac{(\rho_e-\rho)^5}{30}\right\}. \quad (20)$$

It can easily be seen that the exchange of the former form of QCF for the latter one can simply be performed by substituting $R_{0f} \rightarrow R_{1s}$ and $R_{1f} \rightarrow R_{0s}$. Two pairs of parameters $R_0\{R_{0f};R_{0s}\}$ and $R_1\{R_{1s};R_{1f}\}$ can be inferred from observations of meteors. The observed light curve can result from either kind of fragmentation, the fast or the slow one which corresponds to two different types of meteoroid material. These kinds differ by bulk densities of PMB and fragments, by their initial masses, and also by their energies of fragmentation, Q_f. Thus, the possible ambiguity in the determination of physical parameters of PMB and fragments can be overcome by considering these differences.

We have now reached the part of our goals mentioned in the introduction concerning the derivation of the basic formulae of our approach. In order to illustrate the capability of the method to obtain the correct values of the basic parameters of PMB and fragments and to show how to apply it in practice, we shall use it to solve some simulated cases. We will construct a theoretical light curve and then will subject it to our approach in order to recover its original input parameters. Since we will arrive at the conclusion that the same basic function applies to both light and ionization cases, we will confine ourselves only to the processing of the light curve. Processing of the ionization curve would be quit analogous. This was carried out by Novikov & Zhdanov (1997).

3. Application of the technique

To demonstrate the application of the derived formulae, we shall show the determination of the parameters of QCF, R_0, and R_1, characterizing PMB and debris of its disintegration, from the observed light and/or ionization curve. We will confine ourselves to the case of the variability of the light curve along the meteoroid atmospheric trajectory, i.e. we will consider the light intensity to be a function of the atmospheric density, ρ, even though we will derive also the formula for the electron line density. It is sufficient to analyze only one kind of QCF, e. g. the fast one. Observations can yield M_0, z_R, v, and the light curve $I(\rho)$. Defining the auxiliary function

$$J_f(\rho) = I_f(\rho)/\left(\frac{9}{2}\tau_v\frac{M_0 v^3\cos z_R}{H}\rho\right), \quad (21)$$

we arrive with the help of (1) and (16) to its form useful for further computations

$$J_f(\rho) = \frac{1}{(R_{0f}R_{1f})^3}\left\{\left[\frac{(\rho_e-\rho)^2}{3}\left[R_{0f}-(b_f-\rho)^3\right]-\right.\right.$$
$$\left.-\frac{(\rho_e-\rho)}{2}\left[R_{of}^4-(b_f-\rho)^4\right]+\frac{1}{5}\left[R_{of}^5-(b_f-\rho)^5\right]\right]\times$$
$$\times\Theta(\rho-\rho_b)\Theta(b_f-\rho)+R_{0f}^3\left[\frac{(\rho_e-\rho)^2}{3}-\frac{R_{0f}}{2}(\rho_e-\rho)+\right.$$
$$\left.+\frac{R_{0f}^2}{5}\right]\Theta(\rho-b_f)\Theta(a_f-\rho)+\frac{(\rho_e-\rho)^5}{30}\Theta(\rho_e-\rho)\times$$
$$\left.\times\Theta(\rho-a_f)\right\}. \quad (22)$$

Radar observations yield the ionization curve $\alpha(\rho)$. Defining in analogy with the above case the normalized electron line density

$$\alpha_N(\rho) = \alpha(\rho)/\left(\frac{9\beta_v M_0\cos z_R}{\mu H}\rho\right) \quad (23)$$

and inserting into (1) from (16) we can see that $\alpha_N(\rho)$ corresponds to $J_f(\rho)$ defined by (22). Thus, we have justified the decision to confine ourselves only to the case of the light curve to show how to use our approach in practice. When employing the formulae of numerical differentiation, we can determine the second derivative $d^2J_f/d\rho^2$. Constructing within the interval $\rho_b \leq \rho < \rho_e$ the dependence $d^2J_f/d\rho^2$ as a function of ρ we can establish the interval within which $d^2J_f/d\rho^2 \simeq const$. It can easily be seen that J_f is maximum on the first part of the meteor trajectory inside the interval $\rho_b \leq \rho_m \leq b_f$. This implies that the first interval corresponds to the rising part as well as to some portion of the declining part of the J_f curve. On the other hand, the second interval, $b_f \leq \rho \leq a_f$, and the third one, $a_f \leq \rho \leq \rho_e$, correspond only to the declining part of the J_f curve. It can be proved from (22) that the following constraint holds true within the second interval

$$\frac{d^2J_f}{d\rho^2} = \frac{2}{3R_{1f}^3} = 2a_f = \text{const}. \quad (24)$$

Thus, knowing τ_v and determining $d^2J_f/d\rho^2$, we are able to determine R_{1f}. We would like to point out here that when constructing $d^2J_f/d\rho^2$ as a function of ρ, we can also determine b_f and a_f as bounds of the interval within which

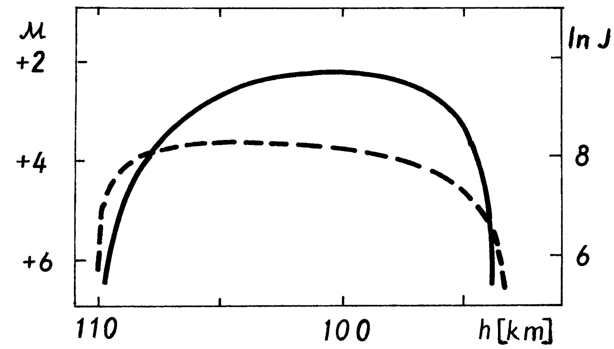

Fig. 1. Theoretical light curves. The left y-axis gives the intensity of light in stellar magnitudes, $\mathscr{M}$, while the right one gives ln J. The solid line corresponds to $\mathscr{M}$ and the dashed to ln J. The x-axis gives the height above the ground expressed in km. This case corresponds to the model No. 1.

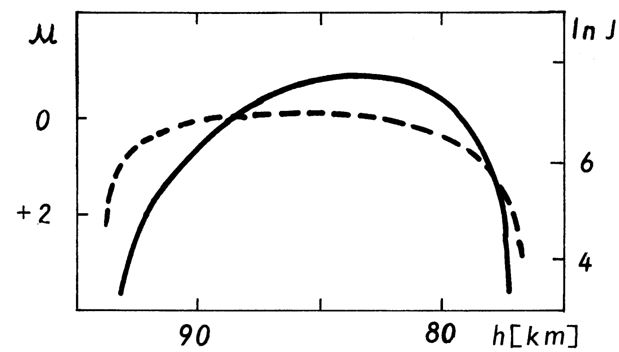

Fig. 2. The same as in Fig. 1 but for model No. 2.

$d^2J_f/d\rho^2 = const$. [Indeed, $d^2J_f/d\rho^2$ as a function of ρ possesses a region where it is largely constant.] The limits of this interval correspond to ρ_b and ρ_e, respectively. Hence, we find $\rho_b = a_f - R_{1f}$ from the formula for a_f, and consequently, also the height of beginning, h_b. On the other hand, we find $R_{0f} = b_f - \rho_b = b_f - a_f + R_{1f}$ from the formula for b_f. Determining ρ_b, R_{1f}, R_{0f}, we find $\rho_e = \rho_b + R_{0f} + R_{1f}$, and finally, also the height of the end of the whole meteor event, h_e.

When the light curve $I(\rho)$ is known from observations, we can construct $J_f(\rho)$ from that data. When using the above described method, we can determine not only the parameters R_{0f} and R_{1f} but also ρ_b and ρ_e and consequently, the theoretical heights h_b and h_e. From the parameters of fast QCF, we can easily find the parameters of slow QCF, R_{0s} and R_{1s}. The kind of QCF that occurs in each particular case depends on the type of meteoroid matter causing the meteor event.

We will illustrate the capability of the method developed here for the determination of the parameters of PMB and fragments in the following example. Figs. 1 and 2 show the theoretical light curve $I(h)$ by solid lines corresponding to the following two models. The left y-axis gives the magnitude, $\mathscr{M}$, while the right y-axis represents ln J as evaluated from (21). The x-axis represents the height expressed in km. Figs. 3 and 4 show the curve $d^2J/d\rho^2$ as a function of ρ along the meteor trajectory valid for model 1 and 2, respectively. The x-axis definition is the

Table 1. Results of data processing using the first method.

	Model No. 1		Model No. 2	
parameter	input	output	input	output
h_b [km]	110	113.0	93.6	96.9
$R_{0f} \times 10^{10}$ [g/cm^3]	2.21	2.52	73.0	82.2
h_{bf} [km]	103.28	103.5	83.4	83.3
$R_{1f} \times 10^9$ [g/cm^3]	1.56	1.55	23.4	23.2
h_{af} [km]	93.86	94.20	77.3	77.5
h_e [km]	92.33	92.75	75.44	75.83
$m_{of} \times 10^4$ [g]	5.2	3.1	500	320
δ_{0f} [g]	2.0	2.2	5.0	5.8

Table 2. Results of data processing using the second method.

	Model No. 1		Model No. 2	
parameter	input	output	input	output
h_b [km]	110.0	112.8	93.6	96.1
$R_{0f} \times 10^{10}$ [g/cm^3]	2.21	2.49	73.0	80.0
$R_{1f} \times 10^9$ [g/cm^3]	1.56	1.61	23.4	23.1
h_e [km]	92.81	92.70	75.44	75.35
$m_0 \times 10^4$ [g]	5.2	280.0	500	330

same as in the previous case. In constructing this picture, the parameters of PMB and fragments belonging to model task 1 were chosen as follows (for the numerical value of Q_{ff}, see Lebedinets 1986): M_0 = 3.06 × 10^{-2} g, v = 60 km/s, cos z_R = 0.6, δ_{0f} = 2 g/cm^3, δ_f = 2.5 g/cm^3, Q_{ff} = 4 × 10^9 erg/g, h_b = 110 km, m_{0f} = 5.2 × 10^{-4} g, Q = 8 × 10^{10} erg/g, $\Lambda = \Lambda^{'} = 1$, A = 1.5, $A^{'}$ = 1.21, H = 6 km. With these parameters, the light curve reaches a maximum within the interval $\rho_b \leq \rho_m \leq b_f$. The results of processing the light curve corresponding to model 1 are listed in Table 1.

Fig. 2 is analogous to Fig. 1, but with the following model parameters: M_0 = 1 g, v = 40 km/s, cos z_R = 0.6, δ_{0f} = 5 g/cm^3, δ = 5 g/cm^3, Q_{ff} = 10^{10} erg/g, h_b = 93.6 km, m_{af} = 0.05 g, while the values of Q, H, Λ, $\Lambda^{'}$, A, and $A^{'}$ are the same as above. For the numerical value of Q_{ff}, see again Lebedinets (1986). These data produced the maximum of the light curve within the interval $b_f \leq \rho_m \leq a_f$. The results of processing the light data of model 2 are listed in Table 1, as well. In solving the models, we adopted Öpik's b - model (1955) of the luminous efficiency, τ_v. It can be seen from Table 1 that the output results concerning R_{0f}, R_{1f}, h_b, h_e, and m_{0f}, coincide relatively well with the input values. The same holds also true with respect to δ_{0f}. However, we can also see that the method does not yield a sufficiently correct estimate of m_0.

Unfortunately, the results of numerical differentiation are very sensitive to imperfections in input data and consequently this procedure has to be applied with care. As a consequence, we will put forward another method of light curve analysis. The principle of this method consists in the following. Eq. (24) implies that

$$J_f = J = a\rho^2 + b\rho + c. \tag{25}$$

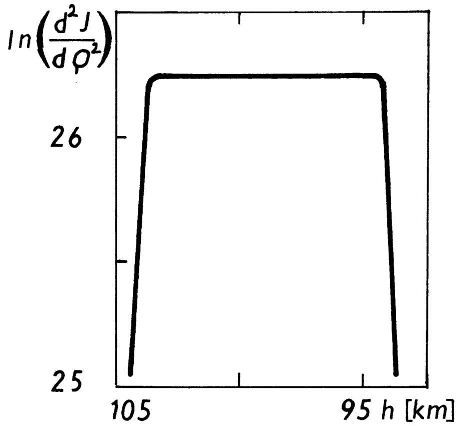

Fig. 3. The natural logarithm, $\ln(d^2J/d\rho^2)$ vs. height, h, expressed in km. Model No. 1 case.

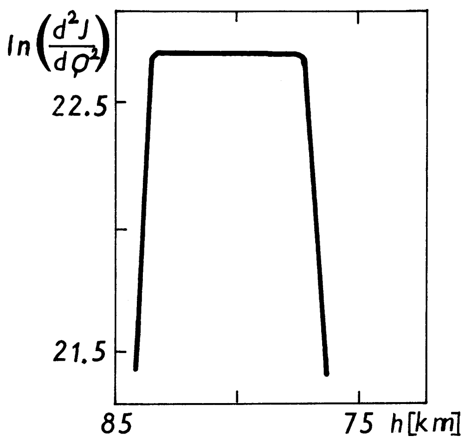

Fig. 4. The same as Fig. 3 but for model No. 2.

But on the other hand, according to Eq. (22) we can write for the second interval of light curve

$$J_f = \frac{1}{R_{1f}^3}\left[\frac{\rho^2}{3} + \left(\frac{R_{0f}}{2} - \frac{2}{3}\rho_e\right)\rho + \left(\frac{\rho_e^2}{3} + \frac{R_{0f}^2}{5} - \frac{R_{0f}\rho_e}{2}\right)\right]. \quad (26)$$

Comparison of Eq. (25) with Eq. (26) yields

$$a = \frac{1}{3R_{1f}}, \; b = \frac{1}{R_{1f}}\left(\frac{R_{0f}}{2} - \frac{2}{3}\rho_e\right),$$

$$c = \frac{1}{R_{1f}^3}\left(\frac{\rho_e^2}{3} + \frac{R_{0f}^2}{5} - \frac{R_{0f}\rho_e}{2}\right). \quad (27)$$

We need to know values of light intensity in at least three points in order to be able to construct the following set of linear equations for finding the coefficients a, b, c

$$J_i = a\rho_i^2 + b\rho_i + c, \quad i = 1, 2, 3. \quad (28)$$

Thus it can easily be shown that, having defined the following auxiliary quantities

$$A = (J_1 - J_2)/(\rho_1 - \rho_2), \quad B = (J_2 - J_3)/(\rho_2 - \rho_3),$$

the solution of Eq. (28) reads

$$a = \frac{A - B}{\rho_1 - \rho_3}, \; b = A - a(\rho_1 + \rho_2), \; c = J_3 - a\rho_3^2 + b\rho_3. \quad (29)$$

Now we can construct, according to Eq. (29), the dependence of a, b, c on the height, h, from the whole curve $J_f(\rho)$. We will accept only those values of a, b, c for which the subsequent values differ from the previous ones by less than the chosen accuracy, i. e. $a_{i+1} - a_i \rightarrow 0$ must hold true within the prescribed interval. We can compute R_{0f}, R_{1f}, ρ_e, and consequently, h_e, the theoretical height of disappearance of the whole meteor event, from (27). Then we find b_f from the relation $b_f = \rho_e - R_{1f}$. Since $\rho_b = \rho_e - (R_{0f} + R_{1f})$ holds true, we find also the theoretical height of the beginning of the QCF of the meteoroid, h_b. The results of the solution of the example processed by our second procedure are listed in Table 2. They indicate that the output parameters R_{0f}, R_{1f}, h_b, and h_e coincide again relatively well with the input values. As for the parameter m_0, we again see that even our second method is not capable of determining it correctly. We conclude that this method is not sufficiently sensitive to it.

The bulk densities of meteoroids resulting from the application of the above methods to a larger amount of observational data were published by Novikov et al. (1996 a). Application of the method to radar observations has been performed by Novikov & Zhdanov (1997).

4. Conclusions

We have developed two methods enabling the determination of the parameters of QCF of PMB and its fragments from the data processing of observed light and/or ionization curves produced by the corresponding meteor . We have applied them to process two model light curves, in order to assess the capability of the methods. We arrived also at the conclusion that these methods cannot yield sufficiently correct fragment masses.

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